Chapter 10

3 see next page

4 a) ω = 0.01 rad/s gain 33.9 dB phase -0.6°

ω = 0.1 rad/s gain 28 dB phase -5.7°

ω = 1 rad/s gain -3.1 dB phase -45°

4 b) ω = 0.1 rad/s gain -20 dB phase 84°

ω = 1 rad/s gain 0 dB phase 0°

ω = 10 rad/s gain -20 dB phase -84°

4 c) ω = 1 rad/s gain 17 dB phase -90°

ω = 10 rad/s gain -3 dB phase -90°

ω = 100 rad/s gain -23 dB phase -90°

5 1.2 m

7 a) $0 < K_1 < 30$

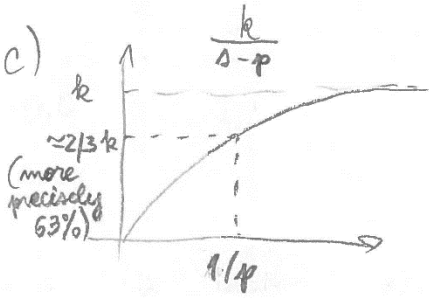
7 b) $K_2 > 0.5$

7 c) $K_1 > K_2/2\text{-}1$ and $K_2 > 0$

8 a) 10

8 b) 30

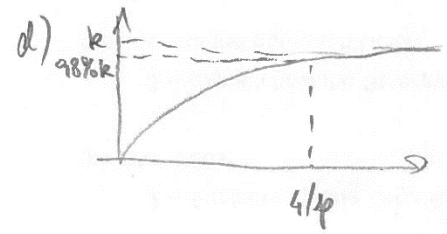
c)



pole: $s = -\frac{1}{10}$ (i.e. $p = \frac{1}{10}$) $20 = \frac{2}{3} \times 30$

About 10 s.

d)



About 40 s.

e) $y(t) = 30\mathcal{L}^{-1}\left[\frac{3}{s}\,\frac{10}{10s+1}\right] = 30\mathcal{L}^{-1}\left[\frac{\frac{1}{10}}{s\left(s+\frac{1}{10}\right)}\right] = 30\left(1 - e^{-\frac{1}{10}t}\right) = 30 - 30e^{-0,1t}$

$y(t) = 20 \Leftrightarrow 20 = 30 - 30e^{-0,1t} \Leftrightarrow e^{-0,1t} = \frac{1}{3} \Leftrightarrow -0,1t = \log_e 1/3 \Leftrightarrow t = -10\log_e 1/3 = 11,0$ s

$y(t) = 0,98 \times 30 \Leftrightarrow 0,98 \times 30 = 30 - 30e^{-0,1t} \Leftrightarrow e^{-0,1t} = 0,02 \Leftrightarrow -0,1t = \log_e 0,02 \Leftrightarrow t = -10\log_e 0,02 = 39,1$ s

8 f) the same

3 a) $u(t) = \sin(2t)$ at 2 rad/s

gain is 55 dB i.e. $10^{55/20} = 562$ in abs. value

phase in $-340° = -5{,}9$ rad

$y(t) = 562 \sin(2t - 5{,}9)$

$u(t) = \sin(2t + \frac{\pi}{2})$ $\quad y(t) = 562 \sin(2t - 4{,}3)$

$\frac{\pi}{2} \approx 1{,}6$ rad

$u(t) = \sin(1000t)$ at 1000 rad/s

gain is -60 dB i.e. $10^{-6/2} = 10^{-3} = 0{,}001$ in abs. value

phase is $-180° = -\pi$ rad

$y(t) = 10^{-3} \sin(10^3 t - \pi)$

$u(t) = 10 \sin(1000t)$ $\quad y(t) = 10^{-2} \sin(10^3 t - \pi)$

$\pi \approx 3{,}14$ rad

$u(t) = \frac{1}{3} \sin(0{,}1t - \frac{\pi}{4}) + \sin(2t + \frac{\pi}{2}) + 10 \sin(1000t)$

$\frac{\pi}{4} \approx 0{,}8$ rad $\quad \frac{\pi}{2} \approx 1{,}6$ rad

at 0,1 rad/s gain is 110 dB i.e. $10^{11/2} = 3{,}16 \times 10^5$

phase is $-315° = -5{,}5$ rad

$y(t) = 1{,}05 \sin(0{,}1t - 6{,}3) + 562 \sin(2t - 4{,}3) + 0{,}01 \sin(10^3 t - 3{,}1)$

b)

ω	2	10^3	10^{-1}
gain [dB]	-58	-32	-52
gain	$10^{-58/20} = 1{,}3 \times 10^{-3}$	$10^{-32/20} = 2{,}5 \times 10^{-2}$	$10^{-52/20} = 2{,}5 \times 10^{-3}$
phase [°]	135°	0°	175°
phase [rad]	2,4	0	3,1

$u(t) = \sin(2t)$ $\quad y(t) = 1{,}3 \times 10^{-3} \sin(2t + 2{,}4)$

$u(t) = \sin(2t + \frac{\pi}{2})$ $\quad y(t) = 1{,}3 \times 10^{-3} \sin(2t + 4{,}0)$

$u(t) = \sin(1000t)$ $\quad y(t) = 2{,}5 \times 10^{-2} \sin(10^3 t)$

$u(t) = 10 \sin(1000t)$ $\quad y(t) = 2{,}5 \times 10^{-1} \sin(10^3 t)$

$u(t) = \frac{1}{3} \sin(0{,}1t - \frac{\pi}{4}) + \sin(2t + \frac{\pi}{2}) + 10 \sin(1000t)$

$y(t) = 0{,}8 \times 10^{-3} \sin(0{,}1t + 2{,}3) + 1{,}3 \times 10^{-3} \sin(2t + 4{,}0) + 2{,}5 \times 10^{-1} \sin(10^3$