α^{1}

1a) $\dfrac{Y(s)}{U(s)} = \dfrac{s}{s^2+12s+20}$ $\longleftarrow s=0$ one zero

$\longleftarrow s = \dfrac{-12 \pm \sqrt{144-80}}{2} = \dfrac{-12 \pm 8}{2} < \begin{matrix} -10 \\ -2 \end{matrix}$ two poles

order 2, strictly proper

$\Rightarrow Ys^2 + 12Ys + 20Y = Us \Rightarrow y''(t) + 12y'(t) + 20y(t) = u'(t)$

b) $\dfrac{Y(s)}{U(s)} = \dfrac{s+1}{s-5}$ $\begin{matrix} \longleftarrow s=-1 \text{ one zero} \\ \longleftarrow s=5 \text{ one pole} \end{matrix}$ order 1, proper

$\Downarrow$

$Ys - 5Y = Us + U \Rightarrow y'(t) - 5y(t) = u'(t) + u(t)$

c) $\dfrac{Y(s)}{U(s)} = \dfrac{s^2+2s+10}{s^3-5s^2+15{,}25s}$ $\longleftarrow s = \dfrac{-2 \pm \sqrt{4-40}}{2} = \dfrac{-2 \pm 6j}{2} = -1 \pm 3j$ two zeros

$\longleftarrow s = 0 \vee s = \dfrac{5 \pm \sqrt{25-61}}{2} = \dfrac{5 \pm 6j}{2} = 2{,}5 \pm 3j$ two poles

order 3, strictly proper

$\Rightarrow Ys^3 - 5Ys^2 + 15{,}25Ys = Us^2 + 2Us + 10U \Rightarrow$

$\Rightarrow y'''(t) - 5y''(t) + 15{,}25y(t) = u''(t) + 2u(t) + 10u(t)$

d) $\dfrac{Y(s)}{U(s)} = \dfrac{10}{(s+1)^2(s^2+5s+6)}$ $\longleftarrow$ no zeros

$\longleftarrow s = -1$ (double root) $\vee s = \dfrac{-5 \pm \sqrt{25-24}}{2} < \begin{matrix} -3 \\ -2 \end{matrix}$ four poles (three distinct poles, one is double)

$\Downarrow$

$Y(s^2+2s+1)(s^2+5s+6) = 10U \Leftrightarrow Y(s^4 + 5s^3 + 6s^2 + 2s^3 + 10s^2 + 12s +$

$+ s^2 + 5s + 6) = 10U \Leftrightarrow Y(s^4 + 7s^3 + 17s^2 + 17s + 6) = 10U \Rightarrow$

$y^{(4)}(t) + 7y'''(t) + 17y''(t) + 17y'(t) + 6y(t) = 10u(t)$

order 4 strictly proper

e) $\dfrac{Y(t)}{U(t)} = \dfrac{s^2+2}{s^2(s+3)(s+50)}$ $\longleftarrow s = \sqrt{-2} = \sqrt{2}j$ two zeros (one double zero)

$\longleftarrow s=0$ (double) $\vee s=-3 \vee s=-50$ four poles, three distinct

order 4 strictly proper

$Y(s^2+53s+150)s^2 = Us^2 + 2U \Rightarrow y^{(4)}(t) + 53y'''(t) + 150y''(t) = u''(t) + 2u(t)$

f) $$\frac{Y(s)}{U(s)} = \frac{s^4+6s^3+8{,}75s^2}{(s^2+4s+4)^2}$$

$\leftarrow s=0 \vee s=\frac{-6\pm\sqrt{36-35}}{2} < \begin{matrix} -3{,}5 \\ -2{,}5 \end{matrix}$ double

$\leftarrow s = \frac{-4\pm\sqrt{16-16}}{2} = -2$ quadruple

(P)

four zeros, three distinct zeros
" poles, one " pole

order 4, proper

$$Y(s^4+4s^3+4s^2+4s^3+16s^2+16s+4s^2+16s+16) = Us^4+6Us^3+8{,}75Us^2$$

$$\Rightarrow Y(s^4+8s^3+24s^2+32s+16) = Us^4+6Us^3+8{,}75Us^2 \Rightarrow$$

$$y^{(4)}(t)+8y'''(t)+24y''(t)+32y'(t)+16y(t) = u^{(4)}(t)+6u'''(t)+8{,}75u''(t)$$

2a) b) c) d)

$$y = Gu = G(d+Ke) = Gd+GKe = Gd+GK(r-C(m+H(n+y)))$$
$$= Gd+GKr-GKC(m+H(n+y)) =$$
$$= Gd+GKr-GKCm-GKCH(n+y) =$$
$$= Gd+GKr-GKCm-GKCHn-GKCHy \Rightarrow$$
$$\Rightarrow (1+GKCH)y = Gd+GKr-GKCm-GKCHn \Rightarrow$$
$$\Rightarrow y = \underbrace{\frac{G}{1+GKCH}}_{\frac{y}{d}} d + \underbrace{\frac{GK}{1+GKCH}}_{\frac{y}{r}} r + \underbrace{\frac{-GKC}{1+GKCH}}_{\frac{y}{m}} m + \underbrace{\frac{-GKCH}{1+GKCH}}_{\frac{y}{n}} n$$

e) f) g) h) $y = Gu \Rightarrow u = \frac{y}{G} \Rightarrow$

$$\Rightarrow u = \underbrace{\frac{1}{1+GKCH}}_{\frac{u}{d}} d + \underbrace{\frac{K}{1+GKCH}}_{\frac{u}{r}} r + \underbrace{\frac{-KC}{1+GKCH}}_{\frac{u}{m}} m + \underbrace{\frac{-KCH}{1+GKCH}}_{\frac{u}{n}} n$$

1j) k) l) $e = r - C(m + H(n+y)) = r - Cm - CH(n+y) =$

$= r - Cm - CHn - CHy = r - Cm - CHn - CHGu =$

$= r - Cm - CHn - CHG(d + Ke) =$

$= r - Cm - CHn - CHGd - CHGKe \Rightarrow$

$\Rightarrow (1 + CHGK)e = r - Cm - CHn - CHGd$

$$\Rightarrow e = \underbrace{\frac{1}{1+CHGK}}_{\frac{e}{r}} r + \underbrace{\frac{-C}{1+CHGK}}_{\frac{e}{m}} m + \underbrace{\frac{-CH}{1+CHGK}}_{\frac{e}{n}} n + \underbrace{\frac{-CHG}{1+CHGK}}_{\frac{e}{d}} d$$

3) $Y = D + GU = D + GG^{-1}E = D + GG^{-1}(R - (Y - \hat{Y})) =$

$E = R - (Y - \hat{Y}) = R - Y + \hat{Y}$ $\cancel{- Y + GG^{-1}\hat{Y} =}$

$\hat{Y} = G^{*}U = G^{*}G^{-1}E$

a) $G^{*} = G \Rightarrow \hat{Y} = GG^{-1}E \Rightarrow E = R - Y + GG^{-1}E =$

$= R - (D + \cancel{GG^{-1}E}) + \cancel{GG^{-1}E} = R - D$

b) $GG^{-1} = 1 \Rightarrow Y = D + E = D + R - D = R$

c) We will not assume $G^{*} = G$ neither $GG^{-1} = 1$:

$E = R - Y + G^{*}G^{-1}E \Rightarrow (1 - G^{*}G^{-1})E = R - Y \Rightarrow$

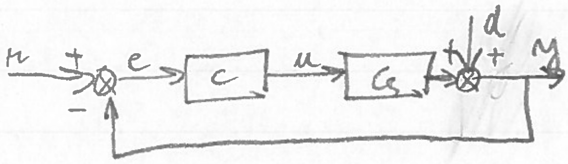
$$\Rightarrow E = \frac{1}{1 - G^{*}G^{-1}} R + \frac{-1}{1 - G^{*}G^{-1}} Y$$

$$\Rightarrow Y = D + \frac{GG^{-1}}{1 - G^{*}G^{-1}} R + \frac{-GG^{-1}}{1 - G^{*}G^{-1}} Y \Rightarrow$$

$$\left(1 + \frac{GG^{-1}}{1 - G^{*}G^{-1}}\right) Y = D + \frac{GG^{-1}}{1 - G^{*}G^{-1}} R \Rightarrow$$

$$\frac{1-G^*G^{-1}+GG^{-1}}{1-G^*G^{-1}}\,Y = D + \frac{GG^{-1}}{1-G^*G^{-1}}\,R \Rightarrow$$

$$Y = \frac{1-G^*G^{-1}}{1-G^*G^{-1}+GG^{-1}}\,D + \frac{GG^{-1}}{1-G^*G^{-1}+GG^{-1}}\,R$$



$$Y = D + GC(R-Y) = D + GCR - GCY \Rightarrow$$

$$\Rightarrow (1+GC)Y = D + GCR \Rightarrow$$

$$\Rightarrow Y = \frac{1}{1+GC}\,D + \frac{GC}{1+GC}\,R$$

$$C = \frac{G^{-1}}{1-G^{-1}G^*} \longrightarrow Y = \frac{1}{1+\frac{GG^{-1}}{1-G^{-1}G^*}}\,D + \frac{\frac{GG^{-1}}{1-G^{-1}G^*}}{1+\frac{GG^{-1}}{1-G^{-1}G^*}}\,R =$$

$$= \frac{1-G^*G^{-1}}{1-G^*G^{-1}+GG^{-1}}\,D + \frac{GG^{-1}}{1-G^*G^{-1}+GG^{-1}}\,R$$

which is the same as above.

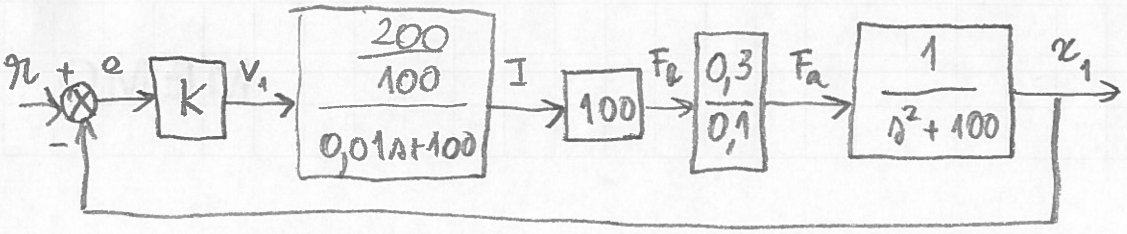
4 a) $Y_1 = G_1C_1(U_2 - H_1Y_1) \Rightarrow Y_1(1+G_1C_1H_1) = G_1C_1U_2 \Rightarrow \dfrac{Y_1}{U_2} = \dfrac{G_1C_1}{1+G_1C_1H_1}$

b) $Y_2 = G_2\frac{Y_1}{U_2}C_2(R-H_2Y_2) \Rightarrow Y_2\left(1+G_2\frac{Y_1}{U_2}C_2H_2\right) = G_2\frac{Y_1}{U_2}C_2R \Rightarrow \dfrac{Y_2}{R} = \dfrac{G_2\frac{Y_1}{U_2}C_2}{1+G_2\frac{Y_1}{U_2}C_2H_2} =$

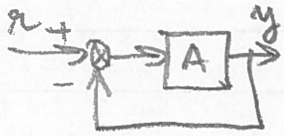
$$= \frac{\frac{G_2C_2G_1C_1}{1+G_1C_1H_1}}{1+\frac{G_2C_2G_1C_1H_2}{1+G_1C_1H_1}} = \frac{G_1C_1G_2C_2}{1+G_1C_1H_1+G_1C_1G_2C_2H_2}$$

ε'

5)



$$\frac{x_1}{e} = \frac{200\times 3\,K}{(0{,}01s+100)(s^2+100)} = \frac{600K}{0{,}01s^3+100s^2+s+10^4}$$

Since 

$y=A(r-y)\Rightarrow y(1+A)=Ar \Rightarrow \frac{y}{r}=\frac{A}{1+A}$

then

$$\frac{x_1}{r} = \frac{\dfrac{600K}{0{,}01s^3+100s^2+s+10^4}}{1+\dfrac{600K}{0{,}01s^3+100s^2+s+10^4}} = \frac{600k}{0{,}01s^3+100s^2+s+(10^4+600K)}$$

6.1a) $Y=B\big(A(R-CDY)+DY\big)=B(AR-ACDY+DY)\Rightarrow$

$$\Rightarrow Y+BACDY-BDY = BAR \Rightarrow \frac{Y}{R} = \frac{BA}{1-BD+BACD}$$

b) $$\frac{Y}{R} = \frac{\frac{10}{s+1}\,\frac{1}{s}}{1-\frac{10}{s+1}\,\frac{s+0{,}1}{s+2}+\frac{1}{s}\,\frac{10}{s+1}\,20\,\frac{s+0{,}1}{s+0{,}2}} =$$

$$= \frac{1}{\frac{s+1}{10}s - s\frac{s+0{,}1}{s+2}+\frac{20s+2}{s+0{,}2}} = \frac{1}{\frac{s^2+s}{10}-\frac{s^2+0{,}1s}{s+2}+\frac{20s+2}{s+0{,}2}} =$$

$$= \frac{1}{\dfrac{(s^2+s)(s+2)(s+0{,}2)-(s^2+0{,}1s)10(s+0{,}2)+(20s+2)10(s+2)}{10(s+2)(s+0{,}2)}} =$$

$$= \frac{10(s^2+2{,}2s+0{,}4)}{(s^2+s)(s^2+2{,}2s+0{,}4)-10(s^3+0{,}2s^2+0{,}1s^2+0{,}02s)+10(20s^2+40s+2s+4)} =$$

$$= \frac{10s^2+22s+4}{s^4+2{,}2s^3+0{,}4s^2+s^3+2{,}2s^2+0{,}4s-10s^3-3s^2-0{,}2s+200s^2+420s+40} =$$

$$= \frac{10s^2+22s+4}{s^4-6{,}8s^3+199{,}6s^2+420{,}2s+40}$$

6.2a) $Y = BU$

$$U = A(DBU + R - CU) = ADBU + AR - ACU = (ADB - AC)U + AR \Rightarrow$$

$$\Rightarrow U(1+AC-ADB) = AR \Rightarrow \frac{U}{R} = \frac{A}{1+AC-ADB} \Rightarrow \frac{Y}{R} = \frac{AB}{1+AC-ADB}$$

b) $$\frac{Y}{R} = \frac{\frac{1}{s}\,\frac{10}{s+1}}{1+\frac{1}{s}2-\frac{1}{s}\,\frac{s+0{,}1}{s+2}\,\frac{10}{s+1}} = \frac{1}{\frac{s+1}{10}s+2\frac{s+1}{10}-\frac{s+0{,}1}{s+2}} =$$

$$= \frac{1}{\frac{s^2+3s+2}{10}-\frac{s+0{,}1}{s+2}} = \frac{1}{\frac{(s^2+3s+2)(s+2)-(s+0{,}1)10}{10s+20}} =$$

$$= \frac{10s+20}{s^3+2s^2+3s^2+6s+2s+4-10s-1} = \frac{10s+20}{s^3+5s^2-2s+3}$$