Capítulo 8

3 a) eq.(6.9): $L = \frac{\rho \ell}{A} = \frac{10^3 \times 50}{0{,}01} = 5 \times 10^6$ (SI)

b) $\underbrace{\rho g \bar{h}}_{\text{pressão no fundo do tanque}} = \bar{P_r} \Rightarrow \bar{h} = \frac{8\times10^4}{10^3\times 9{,}8} = 8{,}16\,m$

c) $Q_r = 0{,}3\times10^{-4} N\sqrt{P_r} \xrightarrow{\text{condições nominais}} 0{,}01 = 0{,}3\times10^{-4}\bar{N}\sqrt{8\times10^4} \Leftrightarrow$

$\Leftrightarrow \bar{N} = \frac{0{,}01}{0{,}3\times10^{-4}\sqrt{8\times10^4}} = 1{,}18$

valor nominal de N

$$Q_r - \bar{Q_r} = \left.\frac{\partial Q_r}{\partial N}\right|_{\bar{N},\bar{P_r}} \times (N-\bar{N}) + \left.\frac{\partial Q_r}{\partial P_r}\right|_{\bar{N},\bar{P_r}} \times (P_r-\bar{P_r})$$

desvios face aos valores nominais — derivadas parciais nas condições nominais

$$Q_r = 0{,}01 + 0{,}3\times10^{-4}\sqrt{8\times10^4}\,(N-1{,}18) + 0{,}3\times10^{-4}\times1{,}18\,\frac{1}{2\sqrt{8\times10^4}}\,(P_r-8\times10^4) =$$

$$= 0{,}01 + 8{,}5\times10^{-3}(N-1{,}18) + 6{,}25\times10^{-8}(P_r - 8\times10^4)$$

(pois $\frac{\partial Q_r}{\partial N} = 0{,}3\times10^{-4}\sqrt{P_r}$ e $\frac{\partial Q_r}{\partial N} = 0{,}3\times10^{-4} N \times \frac{1}{2} \times \frac{1}{\sqrt{P_r}}$)

d) $P_r = \rho g h$ logo, em condições nominais, $\bar{P_r} = \rho g \bar{h}$ e subtraindo vem

$$P_r - \bar{P_r} = \rho g (h-\bar{h})$$

isto é 0 porque as condições nominais são constantes

$Q - Q_r = A\dot{h}$ logo, em condições nominais $\bar{Q} - \bar{Q_r} = A\dot{\bar{h}}$ e subtraindo vem

conservação da massa, a.k.a. a água para algum lado tem de ir

$$Q-\bar{Q} - Q_r + \bar{Q_r} = A(\dot{h} - \dot{\bar{h}}) = A\frac{d}{dt}(h-\bar{h})$$

eq.(6.8)
eq.(6.9)
derivada

$\xrightarrow{\text{ou então (6.12)+15'}}$ $P - P_r = L\dot{Q}$ em condições nominais $\bar{P} - \bar{P_r} = L\dot{\bar{Q}}$ e subtraindo

$$P-\bar{P} - P_r + \bar{P_r} = L(\dot{Q} - \dot{\bar{Q}}) = L\frac{d}{dt}(Q-\bar{Q})$$

e) Temos 4 equações; aplicando $\mathcal{L}$ vem

$$\begin{cases} \Delta P_v = \rho g \Delta h \\ \Delta Q - \Delta Q_v = A s \Delta h \\ \Delta P - \Delta P_v = L s \Delta Q \\ \Delta Q_v = \underbrace{\alpha}_{8{,}5\times10^{-3}} \Delta N + \underbrace{\beta}_{6{,}25\times10^{-8}} \Delta P_v \end{cases} \qquad \begin{cases} \Delta h = \frac{1}{\rho g}\Delta P_v \\ \Delta Q = \Delta Q_v + \frac{As}{\rho g}\Delta P_v \\ \Delta P = \Delta P_v + Ls\Delta Q_v + \frac{LAs^2}{\rho g}\Delta P_v = \Delta Q_v \times Ls + \Delta P_v \times \left(1 + \frac{LAs^2}{\rho g}\right) \\ \Delta Q_v = \alpha \Delta N + \beta \Delta P_v \end{cases}$$

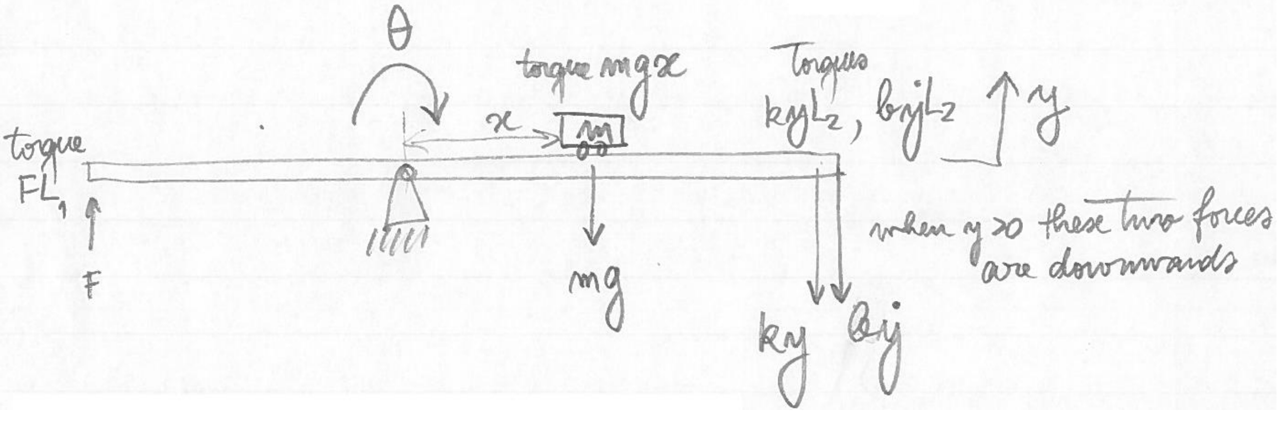
$$\Delta P = \alpha L s \Delta N + \beta L s \Delta P_v + \Delta P_v\left(1 + \frac{LA}{\rho g}s^2\right) =$$

$$= \alpha L s \Delta N + \left(1 + \beta L s + \frac{LA}{\rho g}s^2\right)\Delta P_v$$

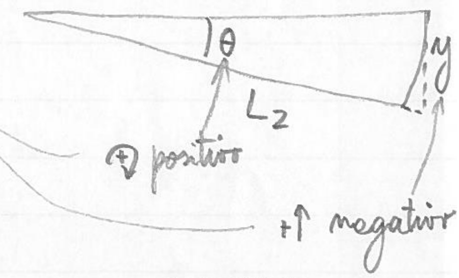
P/ achar a F.T. fazemos $\Delta P = 0$ e vem $\dfrac{\Delta P_v}{\Delta N} = \dfrac{-\alpha L s}{\frac{LA}{\rho g}s^2 + \beta L s + 1} =$

$$= \frac{-8{,}5\times10^{-3}\times 5\times10^{6} s}{\frac{50\times0{,}01}{9{,}8\times10^{3}} s^2 + 6{,}25\times10^{-8}\times5\times10^{6} s + 1}$$

5 a)



$$I\ddot{\theta} = FL_1 + mgx + kyL_2 + b\dot{y}L_2$$

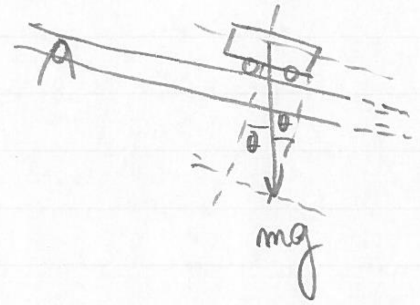


se $\theta \simeq 0$ então $\sin\theta \simeq \theta$ logo

$$\theta \simeq \frac{-y}{L_2} \iff y = -\theta L_2$$

$$I\ddot{\theta} = FL_1 + mgx - k\theta L_2^2 - b\dot{\theta}L_2^2$$

equação 1



peso: mg

componente perpendicular: $mg\cos\theta \simeq mg$

" tangencial: $mg\sin\theta \simeq mg\theta$

$$m\ddot{x} = mg\theta \iff \ddot{x} = g\theta \quad \text{(equação 2)}$$

b)
$$\begin{cases} I s^2 \Theta = F L_1 + mg X - k L_2^2 \Theta - b L_2^2 s \Theta \\ X s^2 = g \Theta \end{cases}$$

$$\begin{cases} \left(I s^2 + k L_2^2 + b L_2^2 s\right) \dfrac{s^2}{g} X = F L_1 + mg X \\ \Theta = \dfrac{s^2}{g} X \end{cases}$$

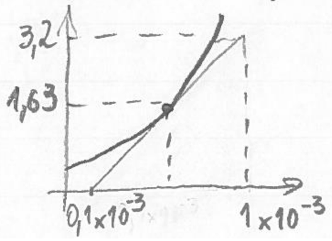
$$\left(I s^4 + b L_2^2 s^3 + k L_2^2 s^2\right) X - mg^2 X = F L_1 g$$

$$\frac{X}{F} = \frac{L_1 g}{I s^4 + b L_2^2 s^3 + k L_2^2 s^2 - m g^2}$$

6 a) Em equilíbrio, o balanço de momentos é

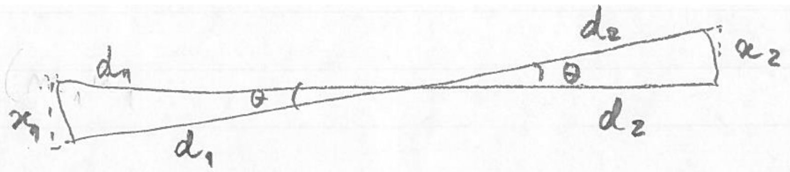
$$0 = \overline{F_m}\underbrace{d_1}_{0,6} + \underbrace{m_1 g d_1}_{1,5\times 9,8\times 0,6} - \underbrace{m_2 g d_2}_{2\times 9,8\times 0,4} \Rightarrow \overline{F_m} = \frac{9,8}{0,6}\left(2\times 0,4 - 1,5\times 0,6\right) = 1,63$$

b) Por leitura no gráfico, o declive é $\dfrac{3,2}{0,9\times 10^{-3}} = 3,56\times 10^{3}$



logo $F_m = \underbrace{1,63}_{\overline{F_m}} + 3,56\times 10^{3}\, x_1$

c)



$$\theta \simeq \sin\theta = \frac{x_1}{d_1} \qquad\qquad \theta \simeq \sin\theta = \frac{x_2}{d_2}$$

$$x_1 = \theta d_1 \qquad\qquad x_2 = \theta d_2$$

Balanço de momentos:

$$\big(\underbrace{m_1 d_1^2 + m_2 d_2^2}_{\text{momento de inércia } J}\big)\ddot{\theta} = \tau - F_m d_1 - F_b d_2 + m_1 g d_1 - m_2 g d_2$$

$$J\ddot{\theta} = \tau - (\underbrace{F_m d_1} + 3{,}56\times10^3 x_1 d_1) - b\dot{x}_2 d_2 + \underbrace{m_1 g d_1 - m_2 g d_2}$$

estes termos cortam (ver alínea a)

$$J\ddot{\theta} = \tau - 3{,}56\times10^3 d_1^2\theta - b d_2^2\dot{\theta}$$

substituindo

$$(1{,}5\times0{,}6^2 + 2\times0{,}4^2)\ddot{\theta} + 3{,}56\times10^3\times0{,}6^2\theta + 20\times0{,}4^2\dot{\theta} = \tau$$

$$0{,}86\ddot{\theta} + 3{,}2\dot{\theta} + 1282\theta = \tau$$

$\mathcal{L}$

$$(0{,}86s^2 + 3{,}2s + 1282)\Theta = T \Rightarrow \frac{\Theta}{T} = \frac{1}{0{,}86s^2 + 3{,}2s + 1282}$$

Balanço de forças:

o fiel não sobe nem desce

$$0 = F - m_1 g - m_2 g + F_m - F_b - \underbrace{m_1\ddot{x}_1 + m_2\ddot{x}_2}_{\text{"forças" resultantes do movimento}}$$

$$F = \underbrace{m_1 g + m_2 g - 1{,}63}_{\text{constantes}} - 3{,}56\times10^3\underbrace{x_1}_{\theta d_1} + b\underbrace{\dot{x}_2}_{\dot{\theta} d_2} + \underbrace{m_1\ddot{x}_1}_{m_1\ddot{\theta}d_1} - \underbrace{m_2\ddot{x}_2}_{m_2\ddot{\theta}d_2}$$

$$\Delta F = \underbrace{(-2\times0{,}4 + 1{,}5\times0{,}6)}_{0{,}1}\ddot{\theta} + \underbrace{(20\times0{,}4)}_{8}\dot{\theta} - \underbrace{3{,}56\times10^3\times0{,}6}_{2136}\theta$$

$$\frac{\Delta F}{\Theta} = 0{,}1s^2 + 8s - 2136$$

Logo $\frac{\Delta F}{T} = \frac{\Delta F}{H} \times \frac{H}{T} = \frac{0{,}1s^2+8s-2136}{0{,}86s^2+3{,}2s+1282}$

7 a) Em condições nominais, $\bar{q}_c = \bar{q}_1 = 0{,}2$ logo

$q_1 = 0{,}15\, x_v \sqrt{\Delta p} \Rightarrow 0{,}2 = 0{,}15\, x_v \sqrt{\rho g (h_0 - h_1)} = 0{,}15\, x_v \sqrt{9{,}8\times 10^3 \times 2}$

$\Rightarrow x_v = \frac{0{,}2}{0{,}15\sqrt{9{,}8\times 2\times 10^3}} = 9{,}5\times 10^{-3}$

$$q_1 - \bar{q}_1 = \underbrace{\left.\frac{\partial q_1}{\partial x_v}\right|_{\bar{x}_v, \bar{\Delta p}}}_{\substack{0{,}15\sqrt{\bar{\Delta p}} \\ = \\ 0{,}15\sqrt{9{,}8\times10^3\times 2} \\ = \\ 21}} \times (x_v - \bar{x}_v) + \underbrace{\left.\frac{\partial q_1}{\partial \Delta p}\right|_{\bar{x}_v, \bar{\Delta p}}}_{\substack{0{,}15\,\bar{x}_v \frac{1}{2}\frac{1}{\sqrt{\bar{\Delta p}}} \\ = \\ 0{,}15\times 9{,}5\times10^{-3}\times\frac{1}{2}\times\frac{1}{\sqrt{9{,}8\times10^3\times 2}} \\ = \\ 5{,}09\times10^{-6}}} \times (\Delta p - \bar{\Delta p})$$

$$\underbrace{\Delta q_1}_{q_1 - 0{,}2} = 21\, \underbrace{\Delta x_v}_{x_v - 9{,}5\times10^{-3}} + 5{,}09\times 10^{-6}(\Delta p - 2)$$