DM DEPARTAMENTO DE MATEMÁTICA TÉCNICO LISBOA

Reliability and Quality Control

MMA / LMAC

TEST 2
2nd. Semester — **2017/18**
2018/06/09 — **8AM**

- Duration: **1h30m**
- **Please justify your answers.**
- This test has **two pages** and **four questions**. The total of points is **20.0**.

1. (a) Elaborate on the following statement: *concerns about quality can be traced back to the Babylonian Empire and the Phoenician civilization.* **(1.0)**

- **Comment**
  [Concerns about quality can be traced back to the Babylonian Empire (Gitlow et al., 1989, pp. 8–9).] This is a curious fact that escapes most consumers nowadays.
  Indeed, the *Code of Hammurabi* [— a Babylonian law code, dating back to about 1772BC, named after the sixth Babylonian king, who enacted it —] consisted of 282 laws dealing with matters of contracts, terms of transactions or addressing household and family relationships such as inheritance, divorce, paternity and sexual behavior. For instance, in *Law 229* we can read: *If a builder builds a house for a man and does not make its construction firm, and the house which he has built collapse and cause the death of the owner of the house, that builder shall be put to death.*
  This *eye for an eye* approach to quality was also adopted by Phoenicians inspectors, who eliminated any repeated violations of quality standards by chopping off the hand of the maker of the defective product (Gitlow et al., 1989, p. 9).

(b) Discuss (briefly) the importance of the detection of decreases in the fraction defective (p) in an industrial process. **(0.5)**

- **Discussion**
  A decrease in fraction defective (p) represents process improvement — to be noted and possibly incorporated. It is also possible that a downward shift in p is caused by errors in the inspection of units, rather than by an actual change in the process parameter; for example, it can actually mean that a new inspector may not have been trained properly to inspect the industrial process output.

2. Printed circuit boards used in several different avionics devices are tested for defects. Defect data from the last 10 days of production are shown below.

Part number	1	2	3	4	5	6	7	8	9	10
Number of defects	16	8	28	8	8	53	25	10	15	16

(a) Set up a $c-$chart with 3–sigma limits and target expected number of defects (per part) equal to $\lambda_0 = 10$. **(1.0)**
Could the production process be deemed in statistical control?

- **Control statistic of the $c-$chart and its distribution**
  Y_N = number of defects in the N^{th} part, $N \in \mathbb{N}$
  $Y_N \sim \text{Poisson}(\lambda)$

- **3-sigma control limits**
  [The control statistic takes values in $\mathbb{N}_0$, thus the control limits are given by the following ceiling and floor functions of the target expected number of defects, λ_0:]

$$\begin{aligned} LCL &= \left\lceil \max\{0, \lambda_0 - 3\times\sqrt{\lambda_0}\}\right\rceil \\ &= \lceil \max\{0, 0.513167\}\rceil \\ &= 1 \end{aligned}$$

$$\begin{aligned} UCL &= \left\lfloor \lambda_0 + 3\times\sqrt{\lambda_0}\right\rfloor \\ &= \lfloor 19.486833 \rfloor \\ &= 19. \end{aligned}$$

- **Examining the process for statistical control**
  Since $y_3, y_6, y_7 \notin [LCL, UCL] = [1, 19]$, we deem the production process out-of-control.

(b) Verify that ARL of the chart described in (a) is approximately equal to: **(2.5)**

- 0.0035^{-1}, in the absence of assignable causes;
- 0.0006^{-1}, when the expected number of defects (per part) shifts from its target value to 8.

Comment on these two ARL values.

- **Probability of triggering a signal**
  When $\lambda = \lambda_0 + \delta$ ($\delta \in (-\lambda_0, +\infty)$), the $c-$chart triggers a signal with probability

$$\begin{aligned} \xi(\theta) &= P(Y_N \notin [LCL, UCL] \mid \lambda = \lambda_0 + \delta) \\ &= 1 - P(LCL \le Y_N \le UCL \mid \lambda = \lambda_0 + \delta) \\ &= 1 - \left[F_{Poisson(\lambda_0+\delta)}(UCL) - F_{Poisson(\lambda_0+\delta)}(LCL-1)\right]. \end{aligned}$$

  Probability of a false alarm

$$\begin{aligned} \xi(0) &= 1 - \left[F_{Poisson(10+0)}(19) - F_{Poisson(10+0)}(1-1)\right] \\ &\overset{tables}{\simeq} 1 - (0.9965 - 0.0000) \\ &= 0.0035 \end{aligned}$$

  Probability of a valid signal when $\lambda = 10 + \delta = 8$

$$\begin{aligned} \xi(-2) &= 1 - \left[F_{Poisson(10-2)}(19) - F_{Poisson(10-1)}(1-1)\right] \\ &\overset{tables}{\simeq} 1 - (0.9997 - 0.0003) \\ &= 0.0006 \end{aligned}$$

- **Requested values of ARL**
  We are dealing with a Shewhart chart thus the number of samples collected until this alternative chart triggers a signal given δ, $RL(\delta)$, is such that $RL(\delta) \sim \text{Geometric}(\xi(\delta))$ and $ARL(\delta) = [\xi(\delta)]^{-1}$. Consequently, the requested in-control and out-of-control ARL are

$$\begin{aligned} ARL(0) &\simeq 0.0035^{-1} \\ ARL(-2) &\simeq 0.0006^{-1}. \end{aligned}$$

- **Comment**
  Since

$$ARL(0) \simeq 0.0035^{-1} \simeq 285.714 < 1666.67 \simeq 0.0006^{-1} \simeq ARL(-2),$$

  this $c-$chart will take longer (in average) to detect a particular decrease in λ (from the target value 10 to 8) than to trigger a false alarm — a very undesirable property.
  [Using *Mathematica* instead of the tables, we would have obtained $ARL(0) \simeq 0.00349974^{-1} \simeq 285.735$ and $ARL(-2) \simeq 0.000588402^{-1} \simeq 1699.518$.]

(c) Let $1 - \xi(\delta) = F_{Poisson(\lambda_0+\delta)}(UCL) - F_{Poisson(\lambda_0+\delta)}(LCL-1)$, where LCL and UCL are control limits of the $c-$chart. **(1.5)**
Prove that the root of $\frac{d[1-\xi(\delta)]}{d\delta} = 0$ is given by $\delta = \left[\frac{UCL!}{(LCL-1)!}\right]^{\frac{1}{UCL-LCL+1}} - \lambda_0$, when $LCL > 0$.
Note: This root equals $argmax_{\delta' \in (-\lambda_0, +\infty)} ARL(\delta') \simeq 7.93 - 10$, when $LCL = 1$ and $UCL = 19$.

Hint: Recall that $F_{Poisson(\lambda_0+\delta)}(n) = 1 - F_{Gamma(n+1,1)}(\lambda_0+\delta)$, for $n = 0,1,2,\ldots$

- **Proof**

  For $n = 0,1,2,\ldots$, we have $F_{Poisson(\lambda_0+\delta)}(n) = 1 - F_{Gamma(n+1,1)}(\lambda_0+\delta)$.

  When $LCL > 0$,

$$\begin{aligned} 1-\xi(\delta) &= F_{Poisson(\lambda_0+\delta)}(UCL) - F_{Poisson(\lambda_0+\delta)}(LCL-1) \\ &= [1-F_{Gamma(UCL+1,1)}(\lambda_0+\delta)] - [1-F_{Gamma(LCL-1+1,1)}(\lambda_0+\delta)] \\ &= F_{Gamma(LCL,1)}(\lambda_0+\delta) - F_{Gamma(UCL+1,1)}(\lambda_0+\delta) \\ \frac{d\,[1-\xi(\delta)]}{d\delta} &= f_{Gamma(LCL,1)}(\lambda_0+\delta) - f_{Gamma(UCL+1,1)}(\lambda_0+\delta). \end{aligned}$$

  By equating this derivative to zero and checking the formulae for the p.d.f. of the gamma distribution, we get

$$\begin{aligned} \frac{(\lambda_0+\delta)^{LCL-1}\,e^{-(\lambda_0+\delta)}}{(LCL-1)!} &= \frac{(\lambda_0+\delta)^{UCL}\,e^{-(\lambda_0+\delta)}}{UCL!} \\ (\lambda_0+\delta)^{UCL-LCL+1} &= \frac{UCL!}{(LCL-1)!} \\ \delta &= \left[\frac{UCL!}{(LCL-1)!}\right]^{\frac{1}{UCL-LCL+1}} - \lambda_0. \end{aligned}$$

(d) Since the quality engineer in charge of the quality control evaluations anticipated both downward and upward shifts, she decided to adopt an ARL-unbiased $c-$chart with control limits $LCL^\star = 2$ and $UCL^\star = 21$ and randomization probabilities $\gamma_{LCL^\star} = 0.6876$ and $\gamma_{UCL^\star} = 0.8328$. **(2.0)**

How should she use the ARL-unbiased $c-$chart?

Obtain the corresponding in-control ARL.

- **How to use the ARL-unbiased $c-$chart**

  According to the slides (2018-04-04-Slides-CandS2 charts.pdf, p. 12), using a ARL-unbiased $c-$chart means to trigger a signal with:
  - probability one if the sample number of defects is below $LCL^\star$ or above $UCL^\star$;
  - probabilities $\gamma_{LCL^\star}$ and $\gamma_{UCL^\star}$ if the sample number of defects is equal to $LCL^\star$ and $UCL^\star$, respectively.

  [Once more according to the slides (2018-04-04-Slides-CandS2 charts.pdf, p. 14), the randomization of the emission of the signal can be done in practice by incorporating the generation of a pseudo-random number from a Bernoulli distribution with parameter $\gamma_{LCL^\star}$ (resp. $\gamma_{UCL^\star}$) in the software used to monitor the data fed from the production line, whenever the observed number of defects is equal to $LCL^\star$ (resp. $UCL^\star$).]

- **Probability of a false alarm**

  Judging by the description above and the slides (2018-04-04-Slides-CandS2 charts.pdf, p. 14), when $\lambda = \lambda_0 + 0$, the ARL-unbiased $c-$chart triggers a signal with probability

$$\begin{aligned} \xi^\star(0) &= 1 \times P(Y_N \notin [LCL^\star, UCL^\star] \mid \lambda = \lambda_0+0) \\ &\quad + \gamma_{LCL^\star} \times P(Y_N = LCL^\star \mid \lambda = \lambda_0+0) \\ &\quad + \gamma_{UCL^\star} \times P(Y_N = UCL^\star \mid \lambda = \lambda_0+0) \\ &= 1 - \left[F_{Poisson(\lambda_0+0)}(UCL^\star) - F_{Poisson(\lambda_0+0)}(LCL^\star-1)\right] \\ &\quad + \gamma_{LCL^\star} \times \left[F_{Poisson(\lambda_0+0)}(LCL^\star) - F_{Poisson(\lambda_0+0)}(LCL^\star-1)\right] \\ &\quad + \gamma_{UCL^\star} \times \left[F_{Poisson(\lambda_0+0)}(UCL^\star) - F_{Poisson(\lambda_0+0)}(UCL^\star-1)\right] \\ &= 1 - \left[F_{Poisson(10+0)}(21) - F_{Poisson(10+0)}(2-1)\right] \\ &\quad + 0.6876 \times \left[F_{Poisson(10+0)}(2) - F_{Poisson(10+0)}(2-1)\right] \\ &\quad + 0.8328 \times \left[F_{Poisson(10+0)}(21) - F_{Poisson(16+0)}(21-1)\right] \end{aligned}$$

$$\begin{aligned} \xi^\star(0) &\overset{tables}{=} 1-(0.9993-0.0005) + 0.6876 \times (0.0028-0.0005) + 0.8328 \times (0.9993-0.9984) \\ &\simeq 0.0035. \end{aligned}$$

- **In-control ARL**

  We are still dealing with a Shewhart chart, thus the number of samples collected until this alternative chart triggers a signal given δ, $RL^\star(\delta)$, is such that $RL^\star(\delta) \sim \text{Geometric}(\xi^\star(\delta))$ and

$$ARL^\star(\delta) = [\xi^\star(\delta)]^{-1}.$$

  Consequently, the in-control ARL is

$$\begin{aligned} ARL^\star(0) &\simeq 0.0035^{-1} \\ &\simeq 285.714. \end{aligned}$$

3. The quality-improvement team of a company uses the *flow time to process a check request* as the quality characteristic for control chart analysis. This characteristic is assumed to have a normal distribution with target mean and standard deviation equal to $\mu_0 = 30$ (days) and $\sigma_0 = 2.7$ (days).

(a) Admit that n completed check requests are randomly selected each day and the sample mean of the flow time is plotted on an UPPER one-sided $\bar{X}-$chart with $UCL_\mu = \mu_0 + \Phi^{-1}(1-1/500) \times \frac{\sigma_0}{\sqrt{n}}$. **(2.5)**

What should be the minimum sample size, n^*, if we require that the out-of-control ARL of this chart does not exceed 1.25 (days), in the presence of an increase from the target mean to 35 (days) AND a 10% decrease in the target standard deviation?

- **Quality characteristic**

  X = flow time to process a check request

  $X \sim \text{Normal}(\mu, \sigma^2)$, where μ and σ^2 represent the process mean and variance, respectively.

- **Control statistic**

  $\bar{X}_N$ = mean of the N^{th} random sample of size n

- **Relevant distribution**

  $\bar{X}_N \sim \text{Normal}\left(\mu = \mu_0 + \delta \times \frac{\sigma_0}{\sqrt{n}}, \frac{\sigma^2}{n} = \frac{(\theta\sigma_0)^2}{n}\right)$, where $\delta = \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}} \in \mathbb{R}_0^+$ (resp. $\theta = \frac{\sigma}{\sigma_0} \in \mathbb{R}^+$) represents the magnitude of a shift in μ (resp. in σ).

- **Upper control limit of the individual chart for μ**

  $UCL_\mu = \mu_0 + \gamma_\mu \times \frac{\sigma_0}{\sqrt{n}}$, where $\gamma_\mu = \Phi^{-1}(1-1/500) \simeq 2.8782$

- **Probability of triggering a signal**

  Taking into account the distribution of the control statistic, this UPPER one-sided $\bar{X}-$chart triggers a signal with probability

$$\begin{aligned} \xi_\mu(\delta,\theta) &= P\left(\bar{X}_N \notin [LCL_\mu, UCL_\mu] \mid \delta, \theta\right) \\ &= \ldots \\ &= 1 - \Phi\left(\frac{\gamma_\mu - \delta}{\theta}\right), \delta \in \mathbb{R}, \theta \in \mathbb{R}_0^+. \end{aligned}$$

- **Run length**

  We are dealing with a Shewhart chart thus the number of samples collected until the chart triggers a signal given δ and θ, $RL_\mu(\delta,\theta)$, is such that:

$$\begin{aligned} RL_\mu(\delta,\theta) &\sim \text{Geometric}(\xi_\mu(\delta,\theta)); \\ ARL_\mu(\delta,\theta) &= \frac{1}{\xi_\mu(\delta,\theta)}. \end{aligned}$$

- **Obtaining the minimum requested sample size**

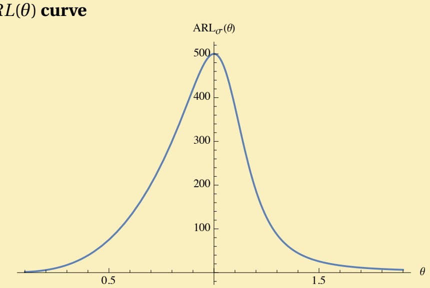
  Since the samples are drawn every day, $\mu_0 = 30$, $\sigma_0 = 2.7$, $\mu = 35$, $\delta = \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}$ and $\theta = \sigma/\sigma_0 = 0.9$, the sample size should satisfy

$$
\begin{aligned}
n \quad : \quad & ARL_\mu\left(\frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}, \theta\right) \le 1.25 \\
& \frac{1}{1-\Phi\left(\frac{\gamma_\mu - \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}}{\theta}\right)} \le 1.25 \\
& \frac{\gamma_\mu - \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}}{\theta} \le \Phi^{-1}\left(1-\frac{1}{1.25}\right) \\
& n \ge \left\{\frac{\sigma_0}{|\mu-\mu_0|} \times \left[\gamma_\mu - \theta \times \Phi^{-1}(0.2)\right]\right\}^2 \\
& n \ge \left[\frac{2.7}{|35-30|} \times [2.8782 - 0.9 \times (-0.8416)]\right]^2 \\
& n \ge 3.85433.
\end{aligned}
$$

Hence, the minimum requested sample size is $n^* = 4$.

(b) A member of the quality-improvement team suggested the use of an ARL-unbiased S^2–chart with in-control ARL equal to 500 (days) and control limits given by $LCL_\sigma = \frac{\sigma_0^2}{n-1} \times 0.0349$ and $UCL_\sigma = \frac{\sigma_0^2}{n-1} \times 18.9214$, where $n = 4$. **(0.5)**
Sketch and comment the ARL profile of this chart.

- **(Sketch of the)** $ARL(\theta)$ **curve**



- **Comment**
The ARL curve achieves a maximum at $\theta = 1$ (in-control situation), thus this chart takes longer in average to trigger a false alarm than to detect any increase or decrease in the process standard deviation.

(c) What sort of shifts in the process mean and variance is the quality-improvement team able to detect (in a timely fashion) by using each of the individual charts described in (a) and (b)? **(0.5)**

- **Shifts likely to be detected by each individual chart**
We are dealing with:
  - an UPPER one-sided $\bar{X}$–chart meant to detect solely increases in the process mean (in a timely fashion);
  - an ARL-unbiased S^2–chart able to detect both increases and decreases in the process variance (in an expedient manner).

(d) Find the median of the IN-CONTROL run length of the joint scheme whose constituent charts are described above. **(1.5)**

- **Another control statistic and relevant distribution**
S_N^2 = variance of the N^{th} random sample of size n

$\frac{(n-1)S_N^2}{(\theta\sigma_0)^2} \sim \chi^2_{(n-1)}$

- **Probability of false alarm**
The individual $\bar{X}$ and S^2 charts are set in such way that their in-control ARL are equal to

$$
\begin{aligned}
ARL_\mu(0,1) &= \frac{1}{\xi_\mu(0,1)} = 500 \\
ARL_\sigma(1) &= \frac{1}{\xi_\sigma(1)} = 500.
\end{aligned}
$$

Moreover, following Exercise 10.38 of the lecture notes, the probability of a false alarm being triggered by the joint scheme is given by:

$$
\begin{aligned}
\xi_{\mu,\sigma}(0,1) &= \xi_\mu(0,1) + \xi_\sigma(1) - \xi_\mu(0,1) \times \xi_\sigma(1) \\
&= \frac{1}{500} + \frac{1}{500} - \frac{1}{500} \times \frac{1}{500} \\
&= 0.0039960.
\end{aligned}
$$

- **In-control RL of the joint scheme**
The in-control RL of this Shewhart-type joint scheme is such that

$$
\begin{aligned}
RL_{\mu,\sigma}(0,1) &\stackrel{st}{=} \min\{RL_\mu(0,1), RL_\sigma(1)\} \\
&\sim \text{Geometric}\left(\xi_{\mu,\sigma}(0,1)\right).
\end{aligned}
$$

- **Median in-control RL**
We have

$$
\begin{aligned}
F^{-1}_{RL_\mu(0,1)}(p) &\stackrel{Table\ 9.2}{=} \inf\{m \in \mathbb{N} : F_{RL_\mu(0,1)}(m) \ge p\} \\
&= 1 - \left[1-\xi_{\mu,\sigma}(0,1)\right]^m \ge p \\
&= m \times \ln\left[1-\xi_{\mu,\sigma}(0,1)\right] \le \ln(1-p) \\
&\stackrel{\ln[1-\xi_{\mu,\sigma}(0,1)]<0}{=} m \ge \frac{\ln(1-p)}{\ln\left[1-\xi_{\mu,\sigma}(0,1)\right]} \\
&\stackrel{p=0.5,\,etc}{=} m \ge \frac{\ln(1-0.5)}{\ln(1-0.0039960)} \\
&= m \ge 173.113,
\end{aligned}
$$

thus, $F^{-1}_{RL_\mu(0,1)}(0.5) = 174$.

4. Admit a single sampling plan by VARIABLES with KNOWN STANDARD DEVIATION is going to be adopted by a quality engineer to screen lots of thermostatic mixing valves.

(a) Set such a plan with risk points $(p_1, 1-\alpha) = (1\%, 1-0.01)$ and $(p_2, \beta) = (5\%, 0.05)$. **(2.5)**
Confirm that if she takes $n_\sigma = 34$ and $k_\sigma = 1.927141$ then $P_a(p_1) \ge 1-\alpha$ and $P_a(p_2) \le \beta$.

- **Single sampling plan by variables with** KNOWN STANDARD DEVIATION
n_σ (sample size)
k_σ (acceptance constant)
σ (known standard deviation)
U (upper specification limit)

- **Producer's and consumer's risk points**
$(p_1, 1-\alpha) = (1\%, 0.99)$
$(p_2, \beta) = (5\%, 0.05)$.

- **Obtaining** n_σ **and** k_σ
According to (13.32),

$$
(n_\sigma, k_\sigma) \quad : \quad \begin{cases} n_\sigma = \left[\frac{\Phi^{-1}(1-\alpha)-\Phi^{-1}(\beta)}{\Phi^{-1}(p_2)-\Phi^{-1}(p_1)}\right]^2 \\ k_\sigma = \frac{\Phi^{-1}(p_2)\Phi^{-1}(1-\alpha)-\Phi^{-1}(p_1)\Phi^{-1}(\beta)}{\Phi^{-1}(\beta)-\Phi^{-1}(1-\alpha)}. \end{cases}
$$

$$(n_\sigma, k_\sigma) \quad : \quad \begin{cases} n_\sigma = \left[\frac{\Phi^{-1}(0.99)-\Phi^{-1}(0.05)}{\Phi^{-1}(0.05)-\Phi^{-1}(0.01)}\right]^2 \\ k_\sigma = \frac{\Phi^{-1}(0.05)\,\Phi^{-1}(0.99)-\Phi^{-1}(0.01)\,\Phi^{-1}(0.05)}{\Phi^{-1}(0.05)-\Phi^{-1}(0.99)}. \end{cases}$$

$$\begin{cases} n_\sigma \overset{table}{=} \left[\frac{2.3263-(-1.6449)}{(-1.6449)-(-2.3263)}\right]^2 \simeq 33.965598 \\ k_\sigma \overset{table}{=} \frac{(-1.6449)\times 2.3263-(-2.3263)\times(-1.6449)}{(-1.6449)-2.3263} \simeq 1.927141. \end{cases}$$

We should take $n_\sigma = \lceil 33.965598 \rceil = 34$ and $k_\sigma = 1.927141$. In fact,

$$\begin{aligned} P_a(p_1) &\overset{(13.34)}{=} \Phi\left(\sqrt{n_\sigma}\left[-k_\sigma - \Phi^{-1}(p_1)\right]\right) \\ &= \Phi\left(\sqrt{34}\left[-1.927141-(-2.3263)\right]\right) \\ &\simeq \Phi(2.33) \\ &\overset{table}{=} 0.9901 \\ &\geq 1-\alpha = 0.99 \\ P_a(p_2) &= \Phi\left(\sqrt{n_\sigma}\left[-k_\sigma - \Phi^{-1}(p_2)\right]\right) \\ &= \Phi\left(\sqrt{34}\left[-1.927141-(-1.6449)\right]\right) \\ &\simeq \Phi(-1.65) \\ &\overset{table}{=} 1-0.9505 \\ &= 0.0495 \\ &\leq \beta = 0.05. \end{aligned}$$

(b) Suppose incoming lots of size $N = 200$ contain 5% nonconforming items. **(2.5)**

Find the corresponding AOQ (average outgoing quality) and ATI (average total inspection) if the quality engineer considers a a single sampling plan with known standard deviation and $(n_\sigma, k_\sigma) = (34, 1.927141)$, and complements it with RECTIFYING INSPECTION.

Comment on the values of AOQ and ATI.

- **Lot size, etc.**
  $N = 200$
  $(n_\sigma, k_\sigma) = (34, 1.927141)$

- **Requested average outgoing quality (AOQ) / Comment**
  Inspired by (13.14), we can write

$$\begin{aligned} AOQ(p) &= \frac{p\,(N-n_\sigma)\,P_a(p)}{N} \\ AOQ(p_2) &\overset{(a)}{\simeq} \frac{0.05\times(200-34)\times 0.0495}{200} \\ &\simeq 0.002054. \end{aligned}$$

  Due to the rectifying inspection:
  - $AOQ(0.05)$ is smaller than $p = p_2 = 0.05$;
  - the relative reduction in the percentage defective is very substancial,

$$\begin{aligned} \left[1-\frac{AOQ(p)}{p}\right]\times 100\% &\simeq \left(1-\frac{0.002054}{0.05}\right)\times 100\% \\ &\simeq 95.892\%, \end{aligned}$$

    after all $p = p_2 = LTPD$.

- **Requested average total inspection (ATI) / Comment**
  Following (13.15), we get

$$\begin{aligned} ATI(p) &= n_\sigma\,P_a(p) + N\,[1-P_a(p)] \\ ATi(p_2) &\simeq 34\times 0.0495 + 200\times(1-0.0495) \\ &\simeq 191.783. \end{aligned}$$

Since $p = p_2 = LTPD$, the probability of lot acceptance is smaller than $\beta = 0.05$. Consequently, we have to reject the lot more than 95% of the time and thus inspect the remaining $N - n_\sigma = 200 - 34$ items. Unsurprisingly, the average number of items we have to inspected is very close to the lot size, $N = 200$.

(c) Admit now that the quality engineer: obtained $n_\sigma = 34$ measurements leading to the sample mean $\bar{x} = 40.88$; considered the known standard deviation $\sigma = 0.1$, the upper specification limit $U = 41$, and the acceptance constant $k_\sigma = 1.927141$. **(1.5)**

Make the necessary calculations to determine whether or not the quality engineer should accept the lot.

How should she proceed according to the acceptance plan described in (b) ?

- **Checking whether or not the lot should be accepted**
  The lot should be accepted iff

$$Q = \frac{U-\bar{x}}{\sigma} \geq k_\sigma,$$

  where Q, U, $\bar{x}$, σ, and k_σ represent the quality index, the upper specification limit, the mean of a sample with size n_σ, the known standard deviation, and the acceptance constant (respectively).
  For this sample, we have

$$\begin{aligned} q &= \frac{41-40.88}{0.1} \\ &= 1.2 \\ &\not\geq 1.927141, \end{aligned}$$

  therefore we should reject the lot.

- **How to proceed...**
  Since the lot has been reject and rectifying inspection has been adopted, the quality engineer has to:
  (i) inspect the remaining $N - n_\sigma = 166$ items;
  (ii) replace all the nonconforming items found in this particular lot.