
Chap. 13

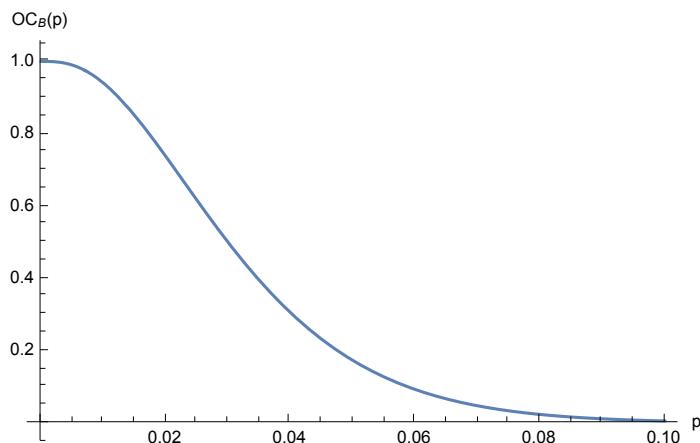
A few exercises from Chap. 13

Exercise 13.2

```
n = 89;
c = 2;
OCB[p_] = CDF[BinomialDistribution[n, p], c];

TableForm[Table[{p, OCB[p]}, {p, 0.01, 0.05, 0.01}],
  TableHeadings -> {None, {"p", "OCB(p)"}}]
Plot[OCB[p], {p, 0, .1}, AxesLabel -> {"p", "OCB(p)"}]
(* OC curve type B*)
```

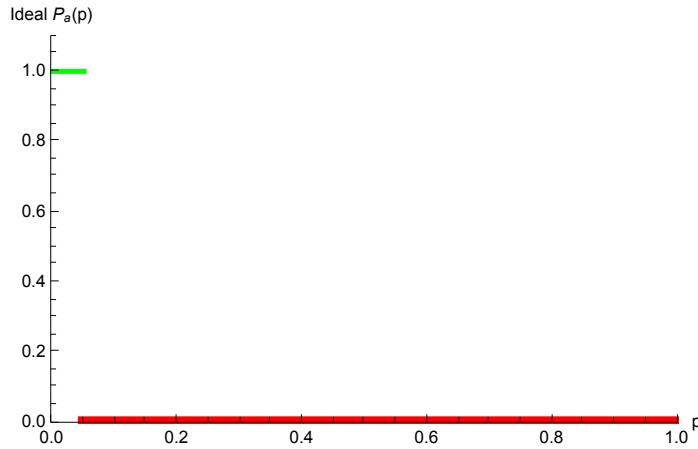
p	OC _B (p)
0.01	0.93969
0.02	0.736578
0.03	0.498483
0.04	0.304158
0.05	0.172077



```

p1 = 0.05;
idealOC[p_] = If[p ≤ p1, 1, 0];
G1 = Plot[idealOC[p], {p, 0, p1}, PlotStyle →
    {RGBColor[0, 1, 0], Thickness[0.008]}, AxesLabel → {"p", "Ideal Pa(p)"},
    PlotRange → {{0, 1}, {0, 1.1}}, DisplayFunction → Identity];
G2 = Plot[idealOC[p], {p, p1, 1}, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.02]},
    PlotStyle → Thickness[0.008],
    PlotRange → {{0, 1}, {0, 1.1}}, DisplayFunction → Identity];
Show[G1, G2, DisplayFunction → $DisplayFunction]

```



Exercise 13.3

```

p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.1; (* LTPD *)
β = 0.10; (* consumer's risk *)

Q[c_, x_] = Quantile[ChiSquareDistribution[2 × (c + 1)], x];
r[c_] =  $\frac{N[Q[c, 1 - \beta], 5]}{N[Q[c, \alpha], 5]}$ ;
i = 0;
While[r[i] >  $\frac{p_2}{p_1}$ , Print["Do not use acceptance number c=", i,
    " because r(c)=", r[i], ">  $\frac{p_2}{p_1} =$ ",  $\frac{p_2}{p_1}$ ];
    i++];
Print["Use acceptance number c=", i, " because r(c)=", r[i], " $\leq \frac{p_2}{p_1} =$ ",  $\frac{p_2}{p_1}$ ]

n[c_] = Ceiling[ $\frac{Q[i, 1 - \beta]}{2 \times p_2}$ ];
Print["Use the sample size n=", n[i]]

Do not use acceptance number c=0 because r(c)=44.8906> $\frac{p_2}{p_1}=10.$ 
Do not use acceptance number c=1 because r(c)=10.9458> $\frac{p_2}{p_1}=10.$ 
Use acceptance number c=2 because r(c)=6.50896 $\leq \frac{p_2}{p_1}=10.$ 
Use the sample size n=54

```

```

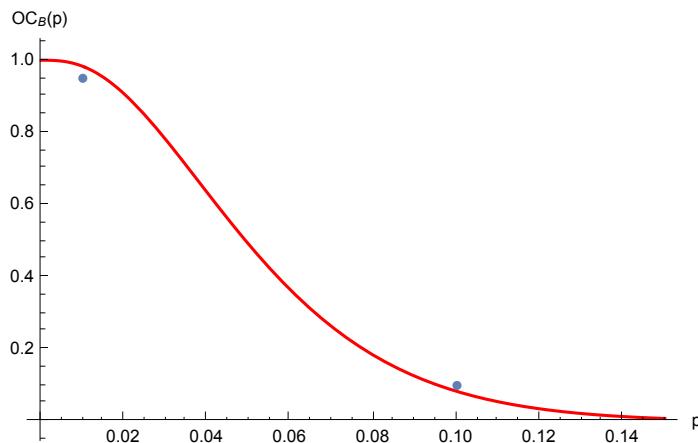
OCB[p_] = CDF[BinomialDistribution[n[i], p], i];

TableForm[Table[{p, OCB[p]}, {p, 0.005, 0.15, 0.005}],
  TableHeadings -> {None, {"p", "OCB(p)"}}

G1 = Plot[OCB[p], {p, 0, .15}, AxesLabel -> {"p", "OCB(p)"},
  PlotStyle -> RGBColor[1, 0, 0], DisplayFunction -> Identity];
riskpoints = {{p1, 1 - α}, {p2, β}};
G2 = ListPlot[riskpoints, PlotStyle -> PointSize[0.014],
  PlotStyle -> RGBColor[0, 1, 0], DisplayFunction -> Identity];
Show[G1, G2, DisplayFunction -> $DisplayFunction]
(* OC curve type B obtain via Wetherill and Brown's method;
producer's risk point (left) and consumer's risk point (right). *)

```

p	OCB(p)
0.005	0.997437
0.01	0.98302
0.015	0.952447
0.02	0.906268
0.025	0.84742
0.03	0.779716
0.035	0.706983
0.04	0.632605
0.045	0.559329
0.05	0.489225
0.055	0.423727
0.06	0.363724
0.065	0.309664
0.07	0.26165
0.075	0.219535
0.08	0.182999
0.085	0.151615
0.09	0.124894
0.095	0.102326
0.1	0.0834077
0.105	0.0676558
0.11	0.0546237
0.115	0.0439056
0.12	0.0351398
0.125	0.0280082
0.13	0.022235
0.135	0.0175838
0.14	0.0138534
0.145	0.0108746
0.15	0.00850593



```

bdist[n_, pt_] := BinomialDistribution[n, pt];
pdist[n_, pt_] := PoissonDistribution[n * pt];
hdist[n_, pt_, ng_] := HypergeometricDistribution[n, Round[pt * ng], ng];

bin[x_, n_, pt_, ng_] := PDF[bdist[n, pt], x];
poi[x_, n_, pt_, ng_] := PDF[pdist[n, pt], x];
hip[x_, n_, pt_, ng_] := PDF[hdist[n, pt, ng], x];

Pa[p_, {n_, c_, ng_}, f_] :=  $\sum_{d=0}^c f[d, n, p, ng];$ 

Paatr[p_, {{a_, b_}, {e_, d_}}, ng_, f_] :=
  Pa[p, {planoamosatrib[{{a, b}, {e, d}}, ng, f][[1]],
    planoamosatrib[{{a, b}, {e, d}}, ng, f][[2]], ng}, f]

planoamosatrib[{{a_, b_}, {e_, d_}}, ng_, f_] :=
Module[{n, c},
  j = 0;
  t = 0;
  While[t == 0,
    i = 2;
    While[i <= ng && Pa[a, {i, j, ng}, f] >= 1 - b,
      i = i + 1]
    If[Pa[e, {i - 1, j, ng}, f] <= d, t = 1, t = 0];
    j = j + 1];
  While[
    Pa[a, {i - 1, j - 1, ng}, f] >= 1 - b && Pa[e, {i - 1, j - 1, ng}, f] <= d, i = i - 1];
  {n = i, c = j - 1}]

ntot = 800; (* lot size *)
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.1; (* LTPD *)
β = 0.10; (* consumer's risk *)

Print["Use the sample size and acceptance number, (n,c)=", 
  planoamosatrib[{{p1, α}, {p2, β}}, ntot, hip]]
(* By using the exact distribution, both n and c are smaller than the one obtained via the Poisson approximation *)

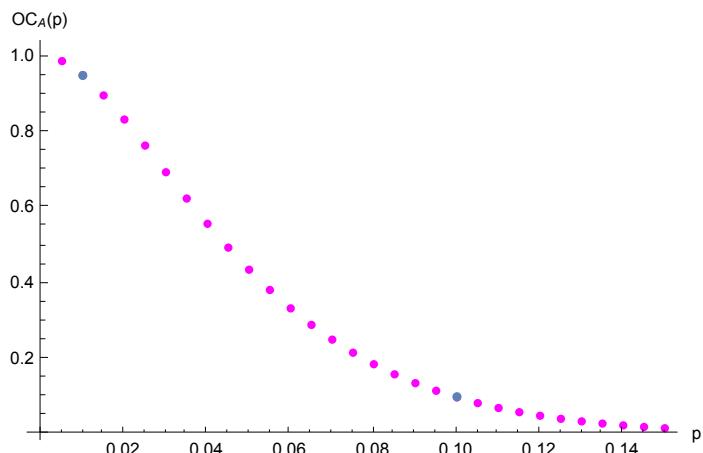
listpOCA =
  Table[{p, N[CDF[HypergeometricDistribution[37, Round[ntot × p], ntot], 1], 5]}, 
    {p, 0.005, 0.15, 0.005}];
TableForm[listpOCA, TableHeadings → {None, {"p", "OCA(p)"}}

G3 = ListPlot[listpOCA, AxesLabel → {"p", "OCA(p)"}, 
  PlotStyle → RGBColor[1, 0, 1], DisplayFunction → Identity];
riskpoints = {{p1, 1 - α}, {p2, β}};
G2 = ListPlot[riskpoints,
  PlotStyle → PointSize[0.014], DisplayFunction → Identity];
Show[G3, G2, DisplayFunction → $DisplayFunction]
(* OC curve type B, producer's risk point (left) and consumer's risk point (right). *)

Use the sample size and acceptance number, (n,c)={37, 1}

```

p	OC _A (p)
0.005	0.98822
0.01	0.95111
0.015	0.89737
0.02	0.83359
0.025	0.76465
0.03	0.69412
0.035	0.62454
0.04	0.55766
0.045	0.49462
0.05	0.43612
0.055	0.38249
0.06	0.33384
0.065	0.29010
0.07	0.25106
0.075	0.21646
0.08	0.18597
0.085	0.15924
0.09	0.13593
0.095	0.11568
0.1	0.098171
0.105	0.083084
0.11	0.070132
0.115	0.059048
0.12	0.049594
0.125	0.041554
0.13	0.034737
0.135	0.028972
0.14	0.024111
0.145	0.020021
0.15	0.016589



```

Print["Use the sample size and acceptance number, (n,c)=", 
planoamosatrib[{{p1, \[Alpha]}, {p2, \[Beta]}}, ntot, bin]
(* Both n and c are smaller than the one obtained via the Poisson approximation *)

OCB[p_] = CDF[BinomialDistribution[52, p], 2];

TableForm[Table[{p, OCB[p]}, {p, 0.005, 0.15, 0.005}],
TableHeadings \[Rule] {None, {"p", "OCB(p)"}}

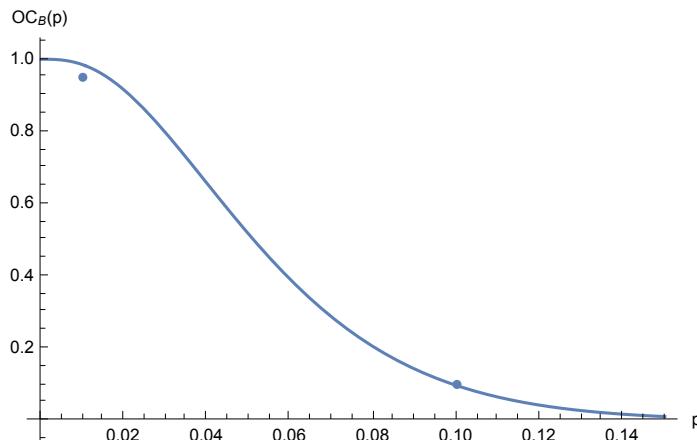
G4 = Plot[OCB[p], {p, 0, .15}, AxesLabel \[Rule] {"p", "OCB(p)"}, 
DisplayFunction \[Rule] Identity];

riskpoints = {{p1, 1 - \[Alpha]}, {p2, \[Beta]}};
G2 = ListPlot[riskpoints,
PlotStyle \[Rule] PointSize[0.014], DisplayFunction \[Rule] Identity];
Show[G4, G2, DisplayFunction \[Rule] \$DisplayFunction]
(* OC curve type B, producer's risk point (left) and consumer's risk point (right). *)

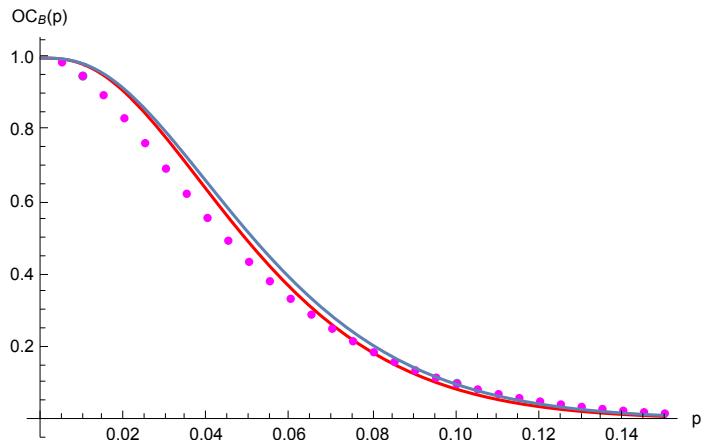
```

Use the sample size and acceptance number, (n,c)={52, 2}

p	OCB(p)
0.005	0.997699
0.01	0.984647
0.015	0.9567
0.02	0.914066
0.025	0.859182
0.03	0.795393
0.035	0.726159
0.04	0.654624
0.045	0.583414
0.05	0.51457
0.055	0.449569
0.06	0.389391
0.065	0.334593
0.07	0.285401
0.075	0.241786
0.08	0.203539
0.085	0.170325
0.09	0.141737
0.095	0.117325
0.1	0.0966333
0.105	0.0792127
0.11	0.0646382
0.115	0.0525163
0.12	0.0424896
0.125	0.0342391
0.13	0.0274835
0.135	0.0219777
0.14	0.0175108
0.145	0.0139021
0.15	0.0109987



```
Show[G1, G2, G3, G4, DisplayFunction -> $DisplayFunction]
```



Exercise 13.4

```

ntot = 800; (* lot size *)
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.1; (* LTPD *)
β = 0.10; (* consumer's risk *)

(* Single sampling plans (n,c) according to: *)
(* (54,2), Binomial Distribution approximation + Wetherhill and Brown method RED *)
(* (52,2), Binomial Distribution approximation GREEN*)
(* (37,1), exact HyperGeometric Distribution BLUE *)
(* (80,2), Norm ANSI/ASQC Z1.4-1981 MAGENTA *)

OCWetherill[p_] = CDF[BinomialDistribution[54, p], 2];
OCB[p_] = CDF[BinomialDistribution[52, p], 2];
listpOCAWetherill =
Table[{p, N[CDF[HypergeometricDistribution[54, Round[ntot × p], ntot], 2], 5]}, {p, 0.005, 0.15, 0.005}]
listpOCABinomial = Table[
{p, N[CDF[HypergeometricDistribution[52, Round[ntot × p], ntot], 2], 5]}, {p, 0.005, 0.15, 0.005}]
listpOCA = Table[{p, N[CDF[HypergeometricDistribution[37, Round[ntot × p], ntot], 1], 5]}, {p, 0.005, 0.15, 0.005}]
listpOCAnorm = Table[{p, N[CDF[HypergeometricDistribution[80, Round[ntot × p], ntot], 1], 2]}, {p, 0.005, 0.15, 0.005}]

riskpoints = {{p1, 1 - α}, {p2, β}};

G1 = Plot[OCWetherill[p], {p, 0, .15}, PlotStyle → {RGBColor[0, 1, 0], Thickness[0.002]}, DisplayFunction → Identity];
G2 = Plot[OCB[p], {p, 0, .15}, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.002]}, AxesLabel → {"p", "OCB(p)"}, DisplayFunction → Identity];
G3 = ListPlot[listpOCA, PlotStyle → {RGBColor[0, 0, 1], Thickness[0.01]}, DisplayFunction → Identity];
G4 = ListPlot[listpOCAnorm, PlotStyle → {RGBColor[1, 0, 1], Thickness[0.01]}, DisplayFunction → Identity];
G5 = ListPlot[riskpoints, PlotStyle → PointSize[0.014], DisplayFunction → Identity];

Show[G1, G2, G3, G4, G5, DisplayFunction → $DisplayFunction]
(* OC curve associated to the Norm far from riskpoints *)
{{0.005, 0.99889}, {0.01, 0.98721}, {0.015, 0.95858}, {0.02, 0.91292}, {0.025, 0.85322}, {0.03, 0.78366}, {0.035, 0.70851}, {0.04, 0.63153}, {0.045, 0.55577}, {0.05, 0.48350}, {0.055, 0.41626}, {0.06, 0.35499}, {0.065, 0.30013}, {0.07, 0.25172}, {0.075, 0.20957}, {0.08, 0.17328}, {0.085, 0.14235}, {0.09, 0.11625}, {0.095, 0.094391}, {0.1, 0.076233}, {0.105, 0.061253}, {0.11, 0.048977}, {0.115, 0.038978}, {0.12, 0.030882}, {0.125, 0.024361}, {0.13, 0.019137}, {0.135, 0.014973}, {0.14, 0.011668}, {0.145, 0.0090579}, {0.15, 0.0070053}}

```

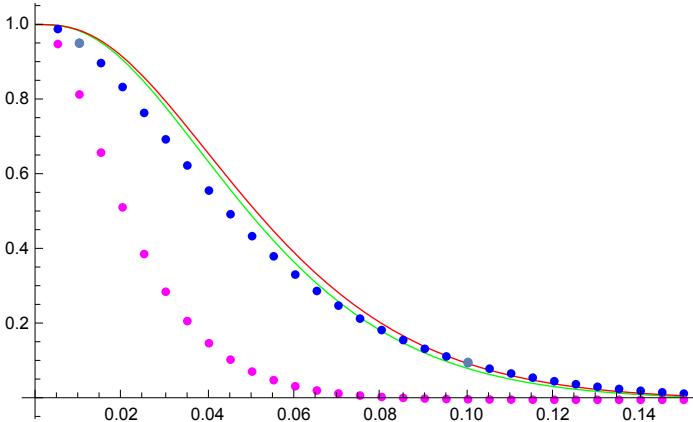
```

{{0.005, 0.99901}, {0.01, 0.98849}, {0.015, 0.96245},
{0.02, 0.92047}, {0.025, 0.86499}, {0.03, 0.79965}, {0.035, 0.72827},
{0.04, 0.65434}, {0.045, 0.58077}, {0.05, 0.50980}, {0.055, 0.44303},
{0.06, 0.38149}, {0.065, 0.32575}, {0.07, 0.27600}, {0.075, 0.23219},
{0.08, 0.19402}, {0.085, 0.16112}, {0.09, 0.13302}, {0.095, 0.10921},
{0.1, 0.089198}, {0.105, 0.072488}, {0.11, 0.058629}, {0.115, 0.047204},
{0.12, 0.037839}, {0.125, 0.030205}, {0.13, 0.024012}, {0.135, 0.019014},
{0.14, 0.014999}, {0.145, 0.011787}, {0.15, 0.0092291}},

{{0.005, 0.98822}, {0.01, 0.95111}, {0.015, 0.89737},
{0.02, 0.83359}, {0.025, 0.76465}, {0.03, 0.69412}, {0.035, 0.62454},
{0.04, 0.55766}, {0.045, 0.49462}, {0.05, 0.43612}, {0.055, 0.38249},
{0.06, 0.33384}, {0.065, 0.29010}, {0.07, 0.25106}, {0.075, 0.21646},
{0.08, 0.18597}, {0.085, 0.15924}, {0.09, 0.13593}, {0.095, 0.11568},
{0.1, 0.098171}, {0.105, 0.083084}, {0.11, 0.070132}, {0.115, 0.059048},
{0.12, 0.049594}, {0.125, 0.041554}, {0.13, 0.034737},
{0.135, 0.028972}, {0.14, 0.024111}, {0.145, 0.020021}, {0.15, 0.016589}},

{{0.005, 0.95}, {0.01, 0.81}, {0.015, 0.66}, {0.02, 0.51}, {0.025, 0.39},
{0.03, 0.29}, {0.035, 0.21}, {0.04, 0.15}, {0.045, 0.11}, {0.05, 0.075},
{0.055, 0.052}, {0.06, 0.036}, {0.065, 0.025}, {0.07, 0.017},
{0.075, 0.011}, {0.08, 0.0077}, {0.085, 0.0052}, {0.09, 0.0034},
{0.095, 0.0023}, {0.1, 0.0015}, {0.105, 0.00098}, {0.11, 0.00064},
{0.115, 0.00042}, {0.12, 0.00027}, {0.125, 0.00017}, {0.13, 0.00011},
{0.135, 0.000071}, {0.14, 0.000045}, {0.145, 0.000029}, {0.15, 0.000018}}

```



Exercise 13.6

```

ntot = 800; (* lot size *)

AOQ[n_, c_, p_] =  $\frac{1}{n_{tot}}(n_{tot} - n) \times p \times \text{CDF}[\text{BinomialDistribution}[n, p], c];$ 
(* Average Outgoing Quality (AOQ) or percentage of defective due to rectifying inspection in a
single sampling plan and using the binomial approximation to the acceptance probability *)

Plot[{AOQ[80, 2, p], AOQ[37, 1, p], p}, {p, 0.001, 0.15},
PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
AxesLabel -> {"p", "AOQ(p)"}]
(* AOQ of single sampling plan using the Norm RED *)
(* AOQ of single sampling plan obtained by solving (13.5) with the exact Hypergeometric distribution GREEN *)

FindMaximum[AQO[80, 2, p], {p, 0.001, 1}]
FindMaximum[AQO[37, 1, p], {p, 0.001, 1}]

Plot[{ $\left(1 - \frac{\text{AOQ}[80, 2, p]}{p}\right) \times 100, \left(1 - \frac{\text{AOQ}[37, 1, p]}{p}\right) \times 100\right),$ 
{p, 0.001, 0.15}, AxesLabel -> {"p", "AOQ rel. reduction"}, 
PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}]
(* Associated relative reduction of the percentage of defective *)

```

AOQ(p)

{0.0154001, {p → 0.0280931}}

{0.0214757, {p → 0.0427029}}

AOQ rel. reduction

Exercise 13.7

```

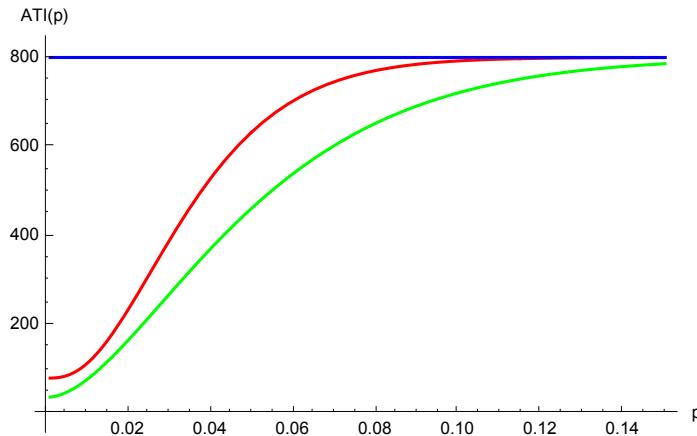
ntot = 800; (* lot size *)

ATI[n_, c_, p_] = ntot + (n - ntot) × CDF[BinomialDistribution[n, p], c];
(* Average Total Inspection (ATI) or percentage of defective due to rectifying inspection in
   a single sampling plan and using the binomial approximation to the acceptance probability *)
ATITable = Table[{p, ATI[80, 2, p], ATI[37, 1, p]}, {p, 0, 0.15, 0.01}];
TableForm[ATITable,
  TableHeadings → {None, {"p", "ATI(n=80,c=2)", "ATI(n=37,c=1)"}}

Plot[{ATI[80, 2, p], ATI[37, 1, p], ntot}, {p, 0.001, 0.15},
  PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
  AxesLabel → {"p", "ATI(p)"}]
(* ATI of single sampling plan using the Norm RED *)
(* ATI of single sampling plan obtained by solving (13.5) with the exact Hypergeometric distribution GREEN *)

```

p	ATI (n=80,c=2)	ATI (n=37,c=1)
0.	80	37
0.01	113.518	77.3459
0.02	235.218	165.85
0.03	390.951	269.885
0.04	530.153	371.768
0.05	633.953	462.92
0.06	703.199	540.095
0.07	745.976	603.006
0.08	770.928	652.864
0.09	784.838	691.512
0.1	792.308	720.93
0.11	796.193	742.977
0.12	798.158	759.28
0.13	799.128	771.188
0.14	799.595	779.793
0.15	799.816	785.946



Exercise | 3.8

```

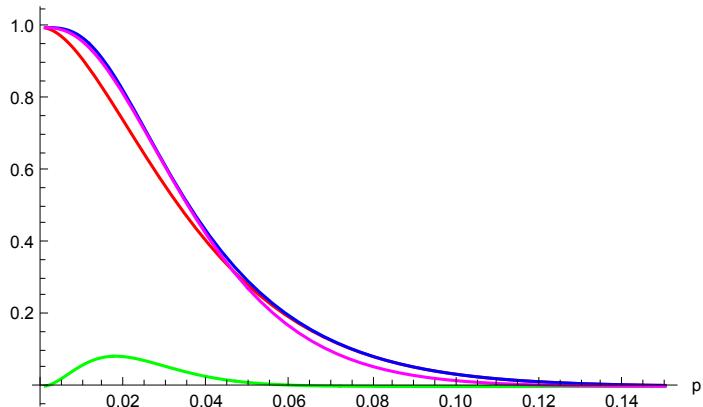
n1 = 50; (* Collect a first sample of size n1 *)
c1 = 1; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 100; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject otherwise *)

pI[p_] = CDF[BinomialDistribution[n1, p], c1]; (* RED *)
pII[p_] = Sum[PDF[BinomialDistribution[n1, p], k] *
  CDF[BinomialDistribution[n2, p], c2 - k]; (* GREEN *)
pa[p_] = pI[p] + pII[p]; (* BLUE *)

OCB[p_] = CDF[BinomialDistribution[75, p], 2]; (* MAGENTA *)

Plot[{pI[p], pII[p], pa[p], OCB[p]}, {p, 0.001, 0.15},
  PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0],
    RGBColor[0, 0, 1], RGBColor[1, 0, 1]}, AxesLabel -> {"p", ""}]
Null

```



Exercise | 3.9

```

n1 = 50; (* Collect a first sample of size n1 *)
c1 = 2; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 100; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 6; (* Accept the lot if D1+D2≤c2, reject otherwise *)

ASN[p_] = n1 + n2 × (CDF[BinomialDistribution[n1, p], c2] -
CDF[BinomialDistribution[n1, p], c1]); (* RED *)
(* Average Sample Number *)

Plot[{ASN[p], 79}, {p, 0.001, 0.15},
PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}, AxesLabel -> {"p", "ASN(p)"}]

ASN(p)

```

Exercise | 3.11

```

n1 = 60; (* Collect a first sample of size n1 *)
c1 = 2; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 120; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject AS SOON AS D1+D2>c2, *)

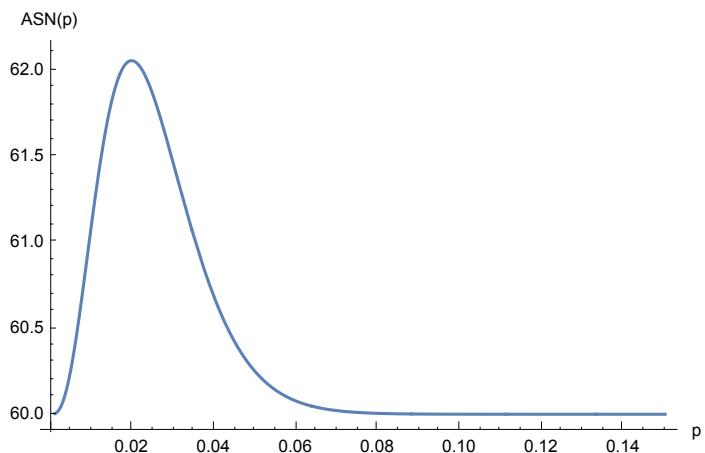
ASN[p_] = n1 + ∑_{j=c1+1}^{c2} PDF[BinomialDistribution[n1, p], j] ×
( n2 × CDF[BinomialDistribution[n2, p], c2 - j] +
(C2 - j + 1) / p × PDF[BinomialDistribution[n2 + 1, p], c2 - j + 2] );
(* Average Sample Number - double sampling plan with curtailment *)

TableForm[Table[{p, ASN[p]}, {p, 0.005, 0.15, 0.005}],
TableHeadings -> {None, {"p", "ASN(p)"}}

Plot[ASN[p], {p, 0.001, 0.15}, AxesLabel -> {"p", "ASN(p)"}]

```

p	ASN(p)
0.005	60.2756
0.01	61.1169
0.015	61.8346
0.02	62.0549
0.025	61.8529
0.03	61.4504
0.035	61.0265
0.04	60.6734
0.045	60.4161
0.05	60.2449
0.055	60.1384
0.06	60.0756
0.065	60.04
0.07	60.0206
0.075	60.0104
0.08	60.0051
0.085	60.0025
0.09	60.0012
0.095	60.0005
0.1	60.0003
0.105	60.0001
0.11	60.0001
0.115	60.
0.12	60.
0.125	60.
0.13	60.
0.135	60.
0.14	60.
0.145	60.
0.15	60.

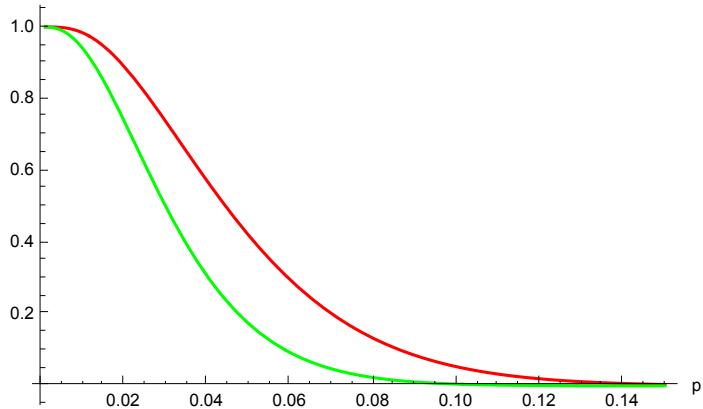


```

pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] = Sum[PDF[BinomialDistribution[n1, p], k] *
  CDF[BinomialDistribution[n2, p], c2 - k];
pa[p_] = pI[p] + pII[p]; (* RED *)
OCB[p_] = CDF[BinomialDistribution[89, p], 2]; (* GREEN *)

Plot[{pa[p], OCB[p]}, {p, 0.001, 0.15},
  PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}, AxesLabel -> {"p", ""}]

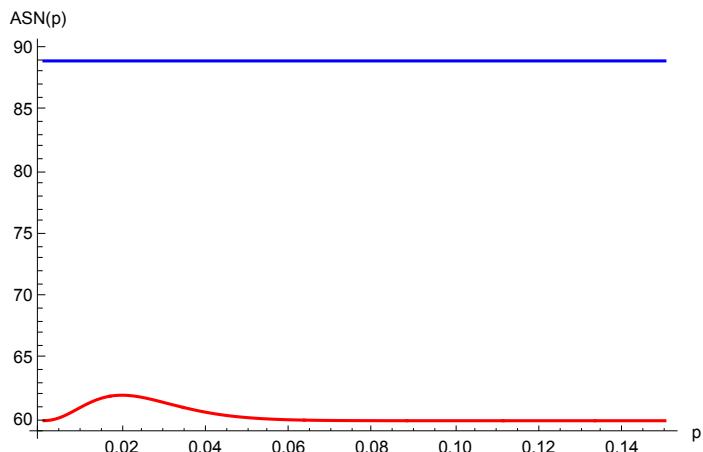
```



```

Plot[{ASN[p], 89}, {p, 0.001, 0.15},
  PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 0, 1]}, AxesLabel -> {"p", "ASN(p)"}]

```



Exercise 13.12

```

p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.06; (* LTPD *)
β = 0.10; (* consumer's risk *)

n1 = 60; (* Collect a first sample of size n1 *)
c1 = 1; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 2 × n1; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject otherwise *)

pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] = Sum[PDF[BinomialDistribution[n1, p], k] *
  CDF[BinomialDistribution[n2, p], c2 - k];
pa[p_] = pI[p] + pII[p];

Plot[pa[p], {p, 0.001, 0.15}, AxesLabel → {"p", "Pa(p)"}]
(* Primary OC curve of type B of a double sampling
plan (without rectifying inspection or curtailment) *)

```

```

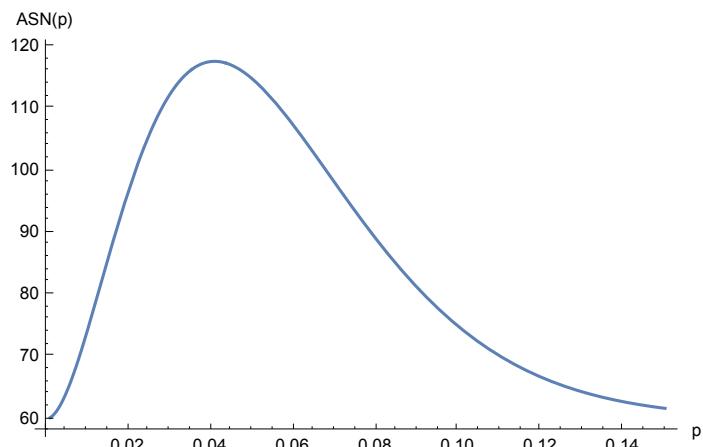
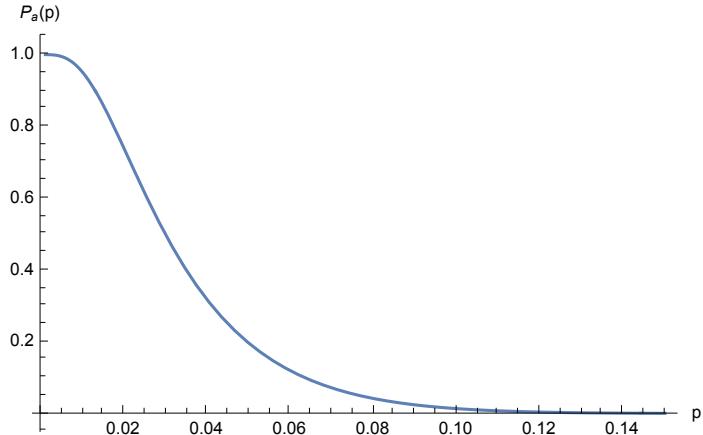
ASN[p_] = n1 + n2 × (CDF[BinomialDistribution[n1, p], c2] -
  CDF[BinomialDistribution[n1, p], c1]); (* RED *)
(* Average Sample Number *)

```

```

Plot[ASN[p], {p, 0.001, 0.15}, AxesLabel → {"p", "ASN(p)"}]

```



Exercise 13.13

```

ntot = 800; (* lot size *)

p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.06; (* LTPD *)
β = 0.10; (* consumer's risk *)

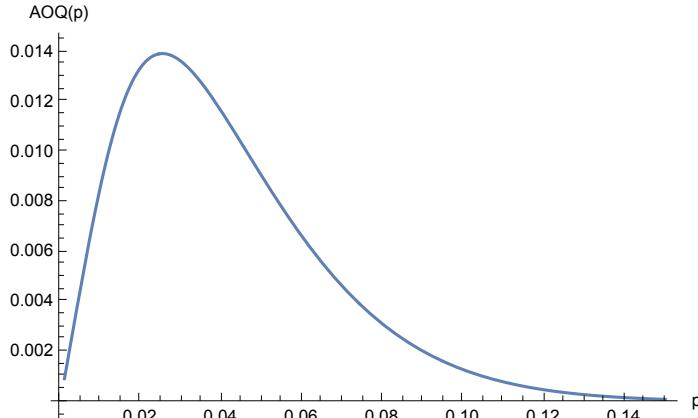
n1 = 60; (* Collect a first sample of size n1 *)
c1 = 1; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 2 × n1; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 3; (* Accept the lot if D1+D2≤c2, reject otherwise *)

pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] = Sum[PDF[BinomialDistribution[n1, p], k] *
  CDF[BinomialDistribution[n2, p], c2 - k];
pa[p_] = pI[p] + pII[p];

AOQ[p_] = 1/ntot p × ((ntot - n1) × pI[p] + (ntot - n1 - n2) × pII[p]);
Plot[AOQ[p], {p, 0.001, 0.15}, AxesLabel → {"p", "AOQ(p)"}]
(* AOQ of a double sampling (WITH rectifying inspection and NO curtailment) *)

```

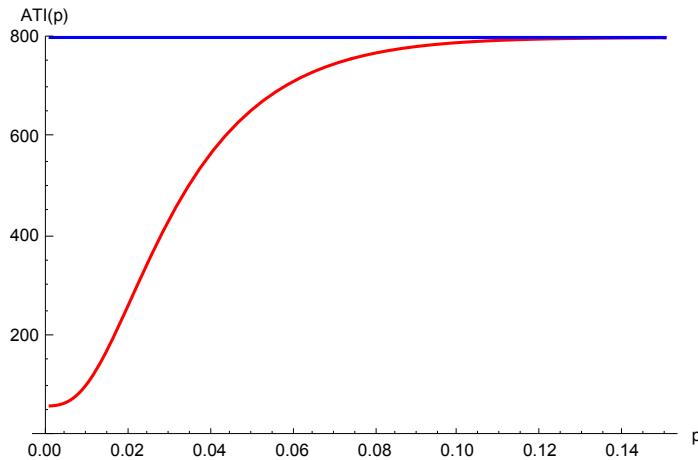
```
FindMaximum[AOQ[p], {p, 0.001, 1}]
```



```

ATI[p_] = n1*pI[p] + (n1 + n2)*pII[p] + ntot*(1 - pa[p]);
Plot[{ATI[p], 800}, {p, 0.001, 0.15}, PlotStyle -> {RGBColor[1, 0, 0],
RGBColor[0, 0, 1]}, AxesLabel -> {"p", "ATI(p)"}, PlotRange -> {0, ntot}]
(* ATI of a double sampling plan(WITH rectifying inspection and NO curtailment) *)

```



```

p1 = 0.01; (* AOL *)
α = 0.05; (* producer's risk *)
p2 = 0.06; (* LTPD *)
β = 0.10; (* consumer's risk *)

Q[c_, x_] = Quantile[ChiSquareDistribution[2*(c + 1)], x];
r[c_] =  $\frac{N[Q[c, 1-\beta], 5]}{N[Q[c, \alpha], 5]}$ ;
i = 0;
While[r[i] >  $\frac{p_2}{p_1}$ , Print["Do not use acceptance number c=",
i, " because r(c)=" , r[i], ">  $\frac{p_2}{p_1} =$ ",  $\frac{p_2}{p_1}$ ];
i++]
Print["Use acceptance number c=", i, " because r(c)=" , r[i], " $\leq \frac{p_2}{p_1} =$ ",  $\frac{p_2}{p_1}$ ]

n[c_] = Ceiling[ $\frac{Q[i, 1-\beta]}{2 \times p_2}$ ];
Print["Use the sample size n=", n[i]]

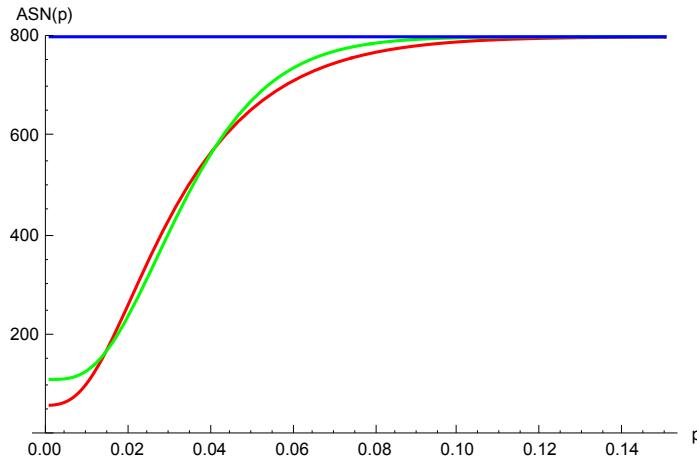
ATISingle[p_] = ntot + (n[i] - ntot) * CDF[BinomialDistribution[n[i], p], i];
(* ATI of a single sampling plan(WITH rectifying inspection) *)

Plot[{ATI[p], ATISingle[p], 800}, {p, 0.001, 0.15},
PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]},
AxesLabel -> {"p", "ASN(p)"}, PlotRange -> {0, ntot}]

```

Do not use acceptance number $c=0$ because $r(c)=44.8906 > \frac{p_2}{p_1} = 6$.
 Do not use acceptance number $c=1$ because $r(c)=10.9458 > \frac{p_2}{p_1} = 6$.
 Do not use acceptance number $c=2$ because $r(c)=6.50896 > \frac{p_2}{p_1} = 6$.
 Use acceptance number $c=3$ because $r(c)=4.88962 \leq \frac{p_2}{p_1} = 6$.

Use the sample size $n=112$



Exercise 13.14

```

p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];

nσ = Ceiling[(Ω[1 - α] - Ω[β])^2 / (Ω[p2] - Ω[p1])];
kσ = (Ω[p2] × Ω[1 - α] - Ω[p1] × Ω[β]) / (Ω[β] - Ω[1 - α]);
PVar[n_, p_] = Φ[√n × (-kσ - Ω[p])];

i = nσ;
While[PVar[i, p1] < 1 - α || PVar[i, p2] > β,
  Print["Do not use sample size nσ=", i, " because Pa[p1]=", PVar[i, p1], "<", 1 - α, " or Pa[p2]=", PVar[i, p2], ">", β];
  i++]

Print["Use sample size nσ=", i, " and acceptance constant kσ=", kσ,
  " because Pa[p1]=", PVar[i, p1], "≥", 1 - α, " and Pa[p2]=", PVar[i, p2], "≤", β]

```

Use sample size $n_\sigma=12$ and acceptance constant $k_\sigma=1.84827$ because $Pa[p_1]=0.951149 \geq 0.95$ and $Pa[p_2]=0.0984707 \leq 0.1$

```

bdist[n_, pt_] := BinomialDistribution[n, pt];
pdist[n_, pt_] := PoissonDistribution[n * pt];
hdist[n_, pt_, ng_] := HypergeometricDistribution[n, Round[pt * ng], ng];

bin[x_, n_, pt_, ng_] := PDF[bdist[n, pt], x];
poi[x_, n_, pt_, ng_] := PDF[pdist[n, pt], x];
hip[x_, n_, pt_, ng_] := PDF[hdist[n, pt, ng], x];

Pa[p_, {n_, c_, ng_}, f_] :=  $\sum_{d=0}^c f[d, n, p, ng];$ 

Paatr[p_, {{a_, b_}, {e_, d_}}, ng_, f_] :=
  Pa[p, {planoamosatrib[{{a, b}, {e, d}}, ng, f][[1]],
    planoamosatrib[{{a, b}, {e, d}}, ng, f][[2]], ng}, f]

planoamosatrib[{{a_, b_}, {e_, d_}}, ng_, f_] :=
Module[{n, c},
  j = 0;
  t = 0;
  While[t == 0,
    i = 2;
    While[i <= ng && Pa[a, {i, j, ng}, f] >= 1 - b,
      i = i + 1];
    If[Pa[e, {i - 1, j, ng}, f] <= d, t = 1, t = 0];
    j = j + 1];
  While[
    Pa[a, {i - 1, j - 1, ng}, f] >= 1 - b && Pa[e, {i - 1, j - 1, ng}, f] <= d, i = i - 1];
  {n = i, c = j - 1}]

ntot = 500; (* lot size *)
p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

Print["Use the sample size and acceptance number, (n,c)=", 
  planoamosatrib[{{p1, α}, {p2, β}}, ntot, hip]]
Use the sample size and acceptance number, (n,c)={72, 2}

```

```

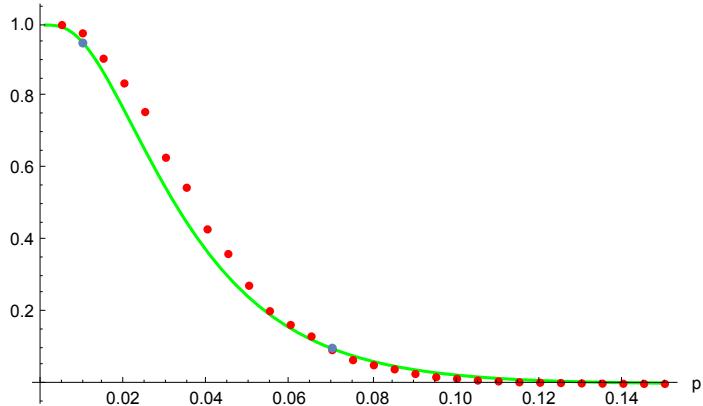
G1 = Plot[PVar[12, p], {p, 0.001, 0.15}, PlotStyle -> RGBColor[0, 1, 0],
  AxesLabel -> {"p", ""}, DisplayFunction -> Identity];
(* GREEN *)

listpOCA =
  Table[{p, N[CDF[HypergeometricDistribution[72, Round[ntot × p], ntot], 2], 5]}, 
    {p, 0.005, 0.15, 0.005}]];
TableForm[listpOCA, TableHeadings -> {None, {"p", "OCA(p)"}}];
G2 =
  ListPlot[listpOCA, PlotStyle -> RGBColor[1, 0, 0], DisplayFunction -> Identity];
(* RED *)

riskpoints = {{p1, 1 - α}, {p2, β}};
G3 = ListPlot[riskpoints,
  PlotStyle -> PointSize[0.014], DisplayFunction -> Identity];

Show[G1, G2, G3, DisplayFunction -> $DisplayFunction]
(* OC curve type B, producer's risk point (left) and consumer's risk point (right). *)

```



Exercise 13.15

```

In[1]:= p1 = 0.01; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];

nσ = Ceiling[ (Ω[1 - α] - Ω[β])^2 ];
kσ = Ω[p2] × Ω[1 - α] - Ω[p1] × Ω[β];
Ω[β] - Ω[1 - α]
PVar[n_, p_] = Φ[ √n × (-kσ - Ω[p]) ];

i = nσ;
While[PVar[i, p1] < 1 - α || PVar[i, p2] > β,
Print["Do not use sample size nσ=", i, " because Pa[p1]=", PVar[i, p1], "<", 1 - α, " or Pa[p2]=", PVar[i, p2], ">", β];
i++]

Print["Use sample size nσ=", i, " and acceptance constant kσ=", kσ,
" because Pa[p1]=", PVar[i, p1], "≥", 1 - α, " and Pa[p2]=", PVar[i, p2], "≤", β]

Use sample size nσ=12 and acceptance constant kσ=
1.84827 because Pa[p1]=0.951149≥0.95 and Pa[p2]=0.0984707≤0.1

```

```

In[14]:= nσ = 12;
kσ = 1.848273;
u = 3 × nσ × (kσ2 - 2) + 8;
v = 3 × nσ2 × kσ2;
ns = Ceiling[nσ + (u + Sqrt[u2 + 24 v])/12];
ks = Sqrt[(3 × ns - 3)/(3 × ns - 4)] × kσ;
PVarDesc[n_, k_, p_] = Φ[(Ω[1 - p] - k × Sqrt[3 × n - 4]/Sqrt[3 × n - 3])/
                           Sqrt[(1 + 3 × n × k2)/(6 × n - 8)]];
(* PVarDesc[n_,k_,p_]=
  CDF[NoncentralStudentTDistribution[n-1,Sqrt[n]×Ω[p]],-Sqrt[n]×k]; *)
i = ns;
While[PVarDesc[i, Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, p1] < 1 - α || PVarDesc[i, Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, p2] > β,
      Print["Do not use sample size ns=", i, " and acceptance constant ks=",
            Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, " because Pa[p1]=", PVarDesc[i, Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, p1],
            "<", 1 - α, " or Pa[p2]=", PVarDesc[i, Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, p2], ">", β];
      i++];
Print["Use sample size ns=", i, " and acceptance constant ks=",
      Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, " because Pa[p1]=", PVarDesc[i, Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, p1],
      "≥", 1 - α, " and Pa[p2]=", PVarDesc[i, Sqrt[(3 × i - 3)/(3 × i - 4)] × kσ, p2], "≤", β]

```

Use sample size n_s=34 and acceptance constant k_s=
 1.85768 because P_a[p₁]=0.952257≥0.95 and P_a[p₂]=0.0969855≤0.1

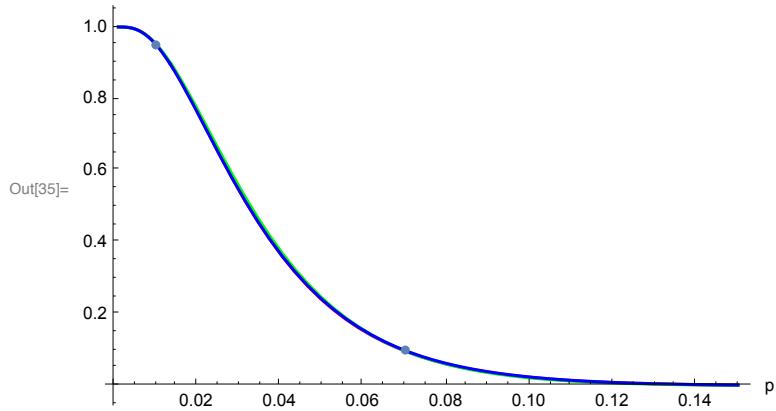
```
In[30]:= G1 = Plot[PVarDesc[34, 1.857679, p], {p, 0.001, 0.15}, PlotStyle ->
    RGBColor[1, 0, 0], AxesLabel -> {"p", ""}, DisplayFunction -> Identity];
(* RED *)

G2 =
Plot[CDF[NoncentralStudentTDistribution[34 - 1, Sqrt[34] * Ω[p]], -Sqrt[34] * 1.857679],
{p, 0.001, 0.15}, PlotStyle -> RGBColor[0, 1, 0], DisplayFunction -> Identity];
(* GREEN *)

G3 = Plot[Sqrt[12] * (-1.848273 - Ω[p]), {p, 0.001, 0.15},
PlotStyle -> RGBColor[0, 0, 1], DisplayFunction -> Identity];
(* BLUE *)

riskpoints = {{p1, 1 - α}, {p2, β}};
G4 = ListPlot[riskpoints,
PlotStyle -> PointSize[0.014], DisplayFunction -> Identity];

Show[G1, G2, G3, G4, DisplayFunction -> $DisplayFunction]
```



Exercise 13.17

```

In[36]:= p1 = 0.01; (* AOL *)
α = 0.05; (* producer's risk *)
p2 = 0.07; (* LTPD *)
β = 0.10; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];

PVarDesc[n_, k_, p_] = Φ[
$$\frac{\Omega[1-p] - k \times \sqrt{\frac{3xn-4}{3xn-3}}}{\sqrt{\frac{1 + \frac{3xn \times k^2}{6xn-8}}{n}}}]$$
];

G1 = Plot[PVarDesc[34, 1.857679, p], {p, 0.001, 0.15}, PlotStyle →
RGBColor[1, 0, 0], AxesLabel → {"p", ""}, DisplayFunction → Identity];
(* RED *)

G2 =
Plot[CDF[NoncentralStudentTDistribution[34 - 1,  $\sqrt{34} \times \Omega[p]$ ], - $\sqrt{34} \times 1.857679$ ],
{p, 0.001, 0.15}, PlotStyle → RGBColor[0, 1, 0], DisplayFunction → Identity];
(* GREEN *)

G3 = Plot[PVarDesc[25, 1.85, p], {p, 0.001, 0.15},
PlotStyle → RGBColor[0, 0, 1], DisplayFunction → Identity];
(* BLUE *)

G4 = Plot[CDF[NoncentralStudentTDistribution[25 - 1,  $\sqrt{25} \times \Omega[p]$ ], - $\sqrt{25} \times 1.85$ ],
{p, 0.001, 0.15}, PlotStyle → RGBColor[1, 0, 1], DisplayFunction → Identity];
(* MAGENTA *)

riskpoints = {{p1, 1 - α}, {p2, β}};
G5 = ListPlot[riskpoints,
PlotStyle → PointSize[0.014], DisplayFunction → Identity];

Show[G1, G3, G5, DisplayFunction → $DisplayFunction]

Show[G2, G4, G5, DisplayFunction → $DisplayFunction]

Show[G1, G2, G3, G4, G5, DisplayFunction → $DisplayFunction]

```

