Department of Mathematics, IST — Probability and Statistics Unit

Reliability and Quality Control

2nd. Test 1st. Semester — **2016/17**

Duration: **1h30m** **2017/01/12** — **8AM**, Room **P8**

- Please justify your answers.
- This test has **two pages** and **four questions**. The total of points is **20.0**.

1. Consider the following multiple choice questions. Select and justify the best possible answer.

(a) A hospital is interested in control charting the mortality rate for a surgical procedure. If the number of surgeries varies each month, the most adequate control chart to use is the: **(0.5)**

(A) $c-$chart; (B) $np-$chart; (C) $p-$chart.

- **Best possible answer**
  C. Since each patient either dies or does not die, the data is Bernoulli/binomial and the $c-$chart cannot be used.
  Moreover, the sample size (i.e., the number of surgical patients per month) varies, thus the $np-$chart cannot be used and we should resort instead to a $p-$chart.

(b) A sales process has a target fraction of orders paid with credit card equal to $p_0 = 0.14$. A sample of 88 orders is obtained with 25 orders paid using credit card. If we use a $p-$chart (with 3-sigma control limits), then this sample leads us to deem the fraction of orders paid with credit card: **(1.0)**

(A) in-control; (B) out-of-control.

- **Best possible answer**
  B. The observed value of the control statistic is $t = 25/88 \simeq 0.284$. Since it is above the center line $p_0 = 0.14$, we only need to calculate the

$$UCL = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}} = 0.14 + 3\sqrt{\frac{0.14(1-0.14)}{88}} \simeq 0.251,$$

  and conclude that $t > UCL$, thus this sample lead us to deem the fraction of orders paid with credit card out-of-control.

2. A single batch of fruit-flavored syrup is produced each day and checked for defects during quality control evaluations.[1] The following data was obtained over ten production days:

Batch	1	2	3	4	5	6	7	8	9	10
Number of defects	11	17	23	14	35	6	14	18	9	14

(a) Set up a $c-$chart with 3–sigma limits and in-control expected number of defects (per batch) equal to $\lambda_0 = 16$. Does the production process appear to be in statistical control? **(1.0)**

- **Control statistic of the $c-$chart and its distribution**
  $Y_N =$ number of defects in the N^{th} batch, $N \in \mathbb{N}$
  $Y_N \sim \text{Poisson}(\lambda)$
- **3-sigma control limits**
  The control statistic only takes values in $\mathbb{N}_0$, therefore the control limits are given by the following ceiling and floor functions of the target expected number of defects (per batch), λ_0:

[1] The defects commonly found are foreign matter, temperature above 49^oF, poor color, pH over 5.3, weak flavor, incorrect concentration or viscosity, excessive bacteria, yeast, or moulds.

$$\begin{aligned} LCL &= \left\lceil \max\{0, \lambda_0 - 3\times\sqrt{\lambda_0}\}\right\rceil \\ &= \left\lceil \max\left\{0, 16 - 3\times\sqrt{16}\right\}\right\rceil \\ &= 4 \\ UCL &= \left\lfloor \lambda_0 + 3\times\sqrt{\lambda_0}\right\rfloor \\ &= \left\lfloor 16 + 3\times\sqrt{16}\right\rfloor \\ &= 28. \end{aligned}$$

- **Checking whether the process is in statistical control**
  Since $y_5 = 35 \notin [LCL, UCL] = [4, 28]$, we deem the production process out-of-control.

(b) Verify that the chart described in (a) triggers a signal with probability approximately equal to: **(2.0)**

- 0.0023, in the absence of assignable causes;
- 0.0011, when the expected number of defects (per batch) shifts from its target value to 15.

Comment on these two probabilities.

- **Probability of triggering a signal**
  When $\lambda = \lambda_0 + \theta$ ($\theta \in (-\lambda_0, +\infty)$), the $c-$chart triggers a signal with probability

$$\begin{aligned} \xi(\theta) &= P(Y_N \notin [LCL, UCL] \mid \lambda = \lambda_0 + \theta) \\ &= 1 - P(LCL \le Y_N \le UCL \mid \lambda = \lambda_0 + \theta) \\ &= 1 - \left[F_{Poisson(\lambda_0+\theta)}(UCL) - F_{Poisson(\lambda_0+\theta)}(LCL-1)\right]. \end{aligned}$$

- **Probability of a false alarm**

$$\begin{aligned} \xi(0) &= 1 - \left[F_{Poisson(16+0)}(28) - F_{Poisson(16+0)}(4-1)\right] \\ &\overset{tables}{=} 1 - (0.9978 - 0.0001) \\ &= 0.0023 \end{aligned}$$

- **Probability of a valid signal when $\lambda = 16 + \theta = 15$**

$$\begin{aligned} \xi(-1) &= 1 - \left[F_{Poisson(16-1)}(28) - F_{Poisson(16-1)}(4-1)\right] \\ &\overset{tables}{=} 1 - (0.9991 - 0.0002) \\ &= 0.0011 \end{aligned}$$

- **Comment**
  Since

$$\xi(-1) = 0.0011 < 0.0023 = \xi(0),$$

  this $c-$chart triggers false alarms more frequently than valid signals when the expected number of defects decrease from its target value $\lambda_0 = 16$ to 15 — a very undesirable property. [Using *Mathematica* instead of the tables, we would have obtained $\xi(0) = 0.00228171$ and $\xi(-1) = 0.00107210$.]

(c) Since the statistician in charge of the quality control evaluations anticipated both downward and upward shifts, she decided to adopt an alternative $c-$chart. This chart has control limits $LCL^\star = 5$ and $UCL^\star = 30$ and triggers a signal with: **(2.5)**

- probability one if the sample number of defects is below $LCL^\star$ or above $UCL^\star$;
- probabilities $\gamma_{LCL^\star} = 0.9258$ and $\gamma_{UCL^\star} = 0.7483$ if the sample number of defects is equal to $LCL^\star$ and $UCL^\star$, respectively.

Compare the in-control ARL with the out-of-control ARL when the expected number of defects (per batch) decreases from its target value $\lambda_0 = 16$ to 15. Comment.

- **Probability of triggering a signal**
  Judging by the description above, when $\lambda = \lambda_0 + \theta$ ($\theta \in (-\lambda_0, +\infty)$), this alternative $c-$chart triggers a signal with probability

$$\begin{aligned}\xi^\star(\theta) &= 1\times P(Y_N \notin [LCL^\star, UCL^\star] \mid \lambda=\lambda_0+\theta)\\ &\quad +\gamma_{LCL^\star}\times P(Y_N = LCL^\star \mid \lambda=\lambda_0+\theta)\\ &\quad +\gamma_{UCL^\star}\times P(Y_N = UCL^\star \mid \lambda=\lambda_0+\theta)\\ &= 1-\left[F_{Poisson(\lambda_0+\theta)}(UCL^\star)-F_{Poisson(\lambda_0+\theta)}(LCL^\star-1)\right]\\ &\quad +\gamma_{LCL^\star}\times\left[F_{Poisson(\lambda_0+\theta)}(LCL^\star)-F_{Poisson(\lambda_0+\theta)}(LCL^\star-1)\right]\\ &\quad +\gamma_{UCL^\star}\times\left[F_{Poisson(\lambda_0+\theta)}(UCL^\star)-F_{Poisson(\lambda_0+\theta)}(UCL^\star-1)\right].\end{aligned}$$

- **Probability of a false alarm**

$$\begin{aligned}\xi^\star(0) &= 1-\left[F_{Poisson(16+0)}(30)-F_{Poisson(16+0)}(5-1)\right]\\ &\quad +0.9258\times\left[F_{Poisson(16+0)}(5)-F_{Poisson(16+0)}(5-1)\right]\\ &\quad +0.7483\times\left[F_{Poisson(16+0)}(30)-F_{Poisson(16+0)}(30-1)\right]\\ &\overset{tables}{=} 1-(0.9994-0.0004)+0.9258\times(0.0014-0.0004)+0.7483\times(0.9994-0.9989)\\ &\simeq 0.0023\end{aligned}$$

- **Probability of a valid signal when $\lambda = 16+\theta = 15$**

$$\begin{aligned}\xi^\star(-1) &= 1-\left[F_{Poisson(16-1)}(30)-F_{Poisson(16-1)}(5-1)\right]\\ &\quad +0.9258\times\left[F_{Poisson(16-1)}(5)-F_{Poisson(16-1)}(5-1)\right]\\ &\quad +0.7483\times\left[F_{Poisson(16-1)}(30)-F_{Poisson(16-1)}(30-1)\right]\\ &\overset{tables}{=} 1-(0.9998-0.0009)+0.9258\times(0.0028-0.0009)+0.7483\times(0.9998-0.9996)\\ &\simeq 0.0030\end{aligned}$$

- **Requested values of ARL**
  We are still dealing with a Shewhart chart thus the number of samples collected until this alternative chart triggers a signal given θ, $RL^\star(\theta)$, $\sim$ Geometric($\xi^\star(\theta)$) and

$$ARL^\star(\theta) = \frac{1}{\xi^\star(\theta)}.$$

  Consequently, the requested in-control and out-of-control ARL are given by:

$$\begin{aligned}ARL^\star(0) &\simeq \frac{1}{0.0023}\\ &\simeq 434.783;\\ ARL^\star(-1) &\simeq \frac{1}{0.0030}\\ &\simeq 333.333.\end{aligned}$$

- **Comment**
  In this case, we have

$$ARL^\star(-1) \simeq 333.333 < 434.783 = ARL^\star(0).$$

  Unlike the $c-$chart with 3–sigma limits, this alternative $c-$chart is able to signal a decrease in λ (from the target value 16 to 15) sooner (in average) than to trigger a false alarm — a very desirable property.
  [In fact, we are dealing with an ARL-unbiased $c-$chart. In contrast with the $c-$chart with 3–sigma limits, the ARL profile of this alternative chart achieves a maximum in the in-control situation. Consequently, the ARL-unbiased $c-$chart requires, in average, less time to trigger a signal in the presence of ANY shift in λ than to trigger a false alarm.]

3. A quality engineer at a manufacturing plant decided to collect samples of size $n = 5$ every 30 minutes and to set up a joint scheme to monitor the mean and variance of the outside diameter of a motor shaft being machined in a CNC turning center.[2] This quality characteristic is assumed to have a normal distribution with nominal mean and variance equal to $\mu_0 = 2.125$ and $\sigma_0^2 = 0.001^2$, respectively.[3] Furthermore, the control limits of the individual:

- $\bar{X}-$chart are equal to $LCL_\mu = \mu_0 - \gamma_\mu \times \frac{\sigma_0}{\sqrt{n}}$ and $UCL_\mu = \mu_0 + \gamma_\mu \times \frac{\sigma_0}{\sqrt{n}}$, where $\gamma_\mu = \Phi^{-1}(1-0.001)$;
- S^2-chart are given by $LCL_\sigma = 0$ and $UCL_\sigma = \frac{\sigma_0^2}{n-1}\times\gamma_\sigma$, where $\gamma_\sigma = F^{-1}_{\chi^2_{(n-1)}}(1-0.005)$.

(a) What sort of shifts in the process mean and variance is the quality engineer able to detect (in a timely fashion) by using this joint scheme? (**0.5**)

- **Shifts likely to be detected**
  Given the control limits, we are dealing with:
  - a STANDARD $\bar{X}-$chart meant to detect both increases and decreases in the process mean;
  - an UPPER ONE-SIDED S^2-chart able to detect solely increases in the process variance in a timely fashion.

(b) Admit that: the process mean μ has increased and the variance is on-target; the magnitude of the upward shift in the process mean is equal to $\delta = \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}} = 0.05$. (**3.0**)
Find the associated ARL of the two individual charts, $ARL_\mu(\delta,1)$ and $ARL_\sigma(1)$, and also of the joint scheme, $ARL_{\mu,\sigma}(\delta,1)$.

- **Quality characteristic**
  X = outside diameter of a motor shaft
  $X \sim \text{Normal}(\mu,\sigma^2)$, where μ and σ^2 represent the process mean and variance, respectively.
- **Control statistics**
  $\bar{X}_N$ = mean of the N^{th} random sample of size n
  S_N^2 = variance of the N^{th} random sample of size n
- **Distributions**
  $\bar{X}_N \sim \text{Normal}\left(\mu = \mu_0 + \delta\times\frac{\sigma_0}{\sqrt{n}}, \frac{\sigma^2}{n} = \frac{(\theta\sigma_0)^2}{n}\right)$,
  where $\delta = \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}} \leq 0$ (resp. $\theta = \frac{\sigma}{\sigma_0} \geq 1$) represents the magnitude of a shift in μ (resp. an upward shift in σ).
  $\frac{(n-1)S_N^2}{(\theta\sigma_0)^2} \sim \chi^2_{(n-1)}$
- **Control limits of the individual charts**
  $LCL_\mu = \mu_0 - \gamma_\mu\times\frac{\sigma_0}{\sqrt{n}}$
  $UCL_\mu = \mu_0 + \gamma_\mu\times\frac{\sigma_0}{\sqrt{n}}$, where $\gamma_\mu = \Phi^{-1}(1-0.001)$
  $LCL_\sigma = 0$
  $UCL_\sigma = \frac{\sigma_0^2}{n-1}\times\gamma_\sigma$, where $\gamma_\sigma = F^{-1}_{\chi^2_{(n-1)}}(1-0.005)$
- **Probabilities of triggering a signal**
  Taking into account the distribution of the control statistics, the STANDARD $\bar{X}-$chart and the UPPER ONE-SIDED S^2-chart trigger a signal with probabilities:

$$\begin{aligned}\xi_\mu(\delta,\theta) &= P\left(\bar{X}_N \notin [LCL_\mu, UCL_\mu] \mid \delta,\theta\right)\\ &= \ldots\\ &= 1-\left[\Phi\left(\frac{\gamma_\mu-\delta}{\theta}\right)-\Phi\left(\frac{-\gamma_\mu-\delta}{\theta}\right)\right], \delta\in\mathbb{R}, \theta\geq 1;\end{aligned}$$

$$\begin{aligned}\xi_\sigma(\theta) &= P\left(S_N^2 \notin [LCL_\sigma, UCL_\sigma] \mid \theta\right)\\ &= \ldots\\ &= 1-F_{\chi^2_{(n-1)}}\left(\frac{\gamma_\sigma}{\theta^2}\right), \theta\geq 1.\end{aligned}$$

  According to Exercise 10.38, the joint scheme triggers a signal with probability

[2] Usually the term *turning* is reserved for the generation of external surfaces by this cutting action (https://en.wikipedia.org/wiki/Turning).
[3] The measurements are made in inches to the nearest ten-thousandth of an inch (e.g. 2.1248 and 2.1254 inches, etc.).

$$\xi_{\mu,\sigma}(\delta,\theta) = P\left(\bar{X}_N \notin [LCL_\mu, UCL_\mu] \text{ or } S_N^2 \notin [LCL_\sigma, UCL_\sigma] \mid \delta,\theta\right)$$
$$= \xi_\mu(\delta,\theta) + \xi_\sigma(\theta) - \xi_\mu(\delta,\theta) \times \xi_\sigma(\theta),\ \delta \in \mathbb{R}, \theta \geq 1.$$

[Recall that: the joint scheme signals as soon as at least one of its constituent charts triggers a signal; and $\bar{X}_N$ and S_N^2 are independent control statistics given δ and θ.]

- **Run lengths and ARL**
  We are dealing with two individual charts and a joint scheme that are in any case of the Shewhart-type. Consequently, the associated RL, $RL_\mu(\delta,\theta)$, $RL_\sigma(\theta)$ and $RL_{\mu,\sigma}(\delta,\theta)$, are geometrically distributed r.v. Moreover:

$$ARL_\mu(\delta,\theta) = \frac{1}{\xi_\mu(\delta,\theta)};$$
$$ARL_\sigma(\theta) = \frac{1}{\xi_\sigma(\theta)};$$
$$ARL_{\mu,\sigma}(\delta,\theta) = \frac{1}{\xi_{\mu,\sigma}(\delta,\theta)}.$$

- **Requested ARL**
  Since the shift in the process mean has magnitude $\delta = \frac{\mu-\mu_0}{\sigma_0/\sqrt{n}} = 0.05$ and the variance is on target, that is, $\theta = \sigma/\sigma_0 = 1$, we get the following probabilities of signal:

$$\xi_\mu(0.05,1) \overset{tables}{=} 1 - \left[\Phi\left(\frac{3.0902-0.05}{1}\right) - \Phi\left(\frac{-3.0902-0.05}{1}\right)\right]$$
$$\simeq 1 - \{\Phi(3.04) - (1-\Phi(-3.14))\}$$
$$\overset{tables}{=} 1 - [0.998817 - (1-0.999155)]$$
$$= 0.002028;$$
$$\xi_\sigma(1) = 1 - F_{\chi^2_{(n-1)}}\left(\frac{F^{-1}_{\chi^2_{(n-1)}}(1-0.005)}{1^2}\right)$$
$$\overset{tables}{=} 1-(1-0.005)$$
$$= 0.005;$$
$$\xi_{\mu,\sigma}(0.05,1) = 0.002028 + 0.005 - 0.002028 \times 0.005$$
$$= 0.00701786.$$

  The corresponding ARL are:

$$ARL_\mu(0.05,1) \simeq \frac{1}{0.002028} \simeq 493.097$$
$$ARL_\sigma(1) = \frac{1}{0.005} = 200$$
$$ARL_{\mu,\sigma}(0.05,1) \simeq \frac{1}{0.00701786} \simeq 142.494.$$

(c) Obtain the value of the probability of a misleading signal of Type IV when $\delta = 0.05$. (**1.0**)

- **Probability of a misleading signal of Type IV**

$$PMS_{IV}(\delta) \overset{Table\,10.12}{=} \frac{1-F_{\chi^2_{(n-1)}}(\gamma_\sigma)}{\left[\Phi(\gamma_\mu-\delta)-\Phi(-\gamma_\mu-\delta)\right]^{-1} - F_{\chi^2_{(n-1)}}(\gamma_\sigma)}$$
$$= \frac{\xi_\sigma(1)}{\left[1-\xi_\mu(\delta,1)\right]^{-1} - [1-\xi_\sigma(1)]}$$

- **Requested PMS of Type IV**

$$PMS_{IV}(0.05) \overset{(b)}{=} \frac{0.005}{(1-0.002028)^{-1} - (1-0.005)} \simeq 0.711023.$$

(d) Prove that, for any $\delta \neq 0$: $\quad PMS_{IV}(\delta) > 0.5 \quad \Leftrightarrow \quad ARL_\mu(\delta,1) - 1 > ARL_\sigma(1)$. (**2.0**)
Comment the value of the PMS of Type IV you obtained in (b) in light of this equivalence.
Hint: Capitalize on the fact that $PMS_{IV}(\delta) = \frac{\xi_\sigma(1)[1-\xi_\mu(\delta,1)]}{\xi_\mu(\delta,1)+\xi_\sigma(1)-\xi_\mu(\delta,1)\xi_\sigma(1)}$.

- **Requested proof**
  Capitalizing on the hint and on the fact that the ARL function of a Shewhart-type chart is the reciprocal of the probability of a signal, we get:

$$PMS_{IV}(\delta) > 0.5 \quad \Leftrightarrow \quad \frac{\xi_\sigma(1)[1-\xi_\mu(\delta,1)]}{\xi_\mu(\delta,1)+\xi_\sigma(1)-\xi_\mu(\delta,1)\times\xi_\sigma(1)} > 0.5$$
$$2\xi_\sigma(1) - 2\xi_\sigma(1)\times\xi_\mu(\delta,1) > \xi_\mu(\delta,1) + \xi_\sigma(1) - \xi_\mu(\delta,1)\times\xi_\sigma(1)$$
$$\xi_\sigma(1) - \xi_\sigma(1)\times\xi_\mu(\delta,1) > \xi_\mu(\delta,1)$$
$$\frac{1}{\xi_\mu(\delta,1)} - 1 > \frac{1}{\xi_\sigma(1)}$$
$$ARL_\mu(\delta,1) - 1 > ARL_\sigma(1).$$

- **Comment**
  The probability of misidentifying a shift in μ with magnitude $\delta = 0.05$ by a shift in σ, $PMS_{IV}(\delta)$, we obtained in (b) is larger than 0.5 because we have indeed

$$ARL_\mu(\delta,1) - 1 = 492.097 > 142.494 = ARL_\sigma(1).$$

4. Admit a double-sampling plan (for attributes) was adopted used to screen lots of $N = 1000$ motor shafts. This acceptance plan comprises RECTIFYING INSPECTION and is characterized by $n_1 = 30$, $c_1 = 0$, $n_2 = 50$ and $c_2 = 2$.

(a) The quality engineer in charge collected a first sample of size $n_1 = 30$ and found one nonconforming motor shaft. (**2.0**)
How should she proceed according to the adopted acceptance plan?

- **How to proceed...**
  [Bear in mind that the double-sampling plan (for attributes):
  - comprises RECTIFYING INSPECTION;
  - is characterized by $N = 1000$, $n_1 = 30$, $c_1 = 0$, $n_2 = 50$, and $c_2 = 2$.

  Moreover, recall that the quality engineer has already collected a first sample of size $n_1 = 30$ and found $d_1 = 1$ nonconforming motor shaft. As a consequence...]
  The quality engineer should proceed as follows, according to Figure 13.1 etc. of the lecture notes:
  (i) collect a second sample of $n_2 = 50$ motor shafts because $c_1 = 0 < d_1 = 1 \leq c_2 = 2$;
  (ii) count the number of nonconforming motor shafts, d_2, in this second sample;
  (iii) if the total number of nonconforming motor shafts verifies $d_1 + d_2 > c_2 = 2$, then the quality engineer should reject the lot, inspect all the remaining $N - n_1 - n_2 = 920$ motor shafts, and replace every nonconforming motor shaft in this lot, namely the one in the first sample;
  (iv) if $d_1 + d_2 \leq c_2 = 2$, then she should accept the lot and replace the $d_1 + d_2$ nonconforming motor shafts she found in the two samples.

(b) Calculate the probability of acceptance on the first sample, on the second sample and in total if the lots contain 2% of nonconforming motor shafts. (**3.0**)

- **Double sampling plan (for attributes)**
  $n_1 = 30$, $n_2 = 50$ (sample sizes)
  $c_1 = 0$, $c_2 = 2$ (acceptance numbers)

- **Auxiliary r.v. and their approximate distributions**
  D_i = number of defective units in the i^{th} sample, $i = 1,2$
  $D_i \stackrel{a}{\sim}$ Binomial(n_i, p), $i = 1,2$

- **Probability of accepting the lot in the first stage of the plan**

$$\begin{aligned} P_a^I(p) &\stackrel{(13.16)}{=} P(D_1 \le c_1) \\ &\simeq F_{Binomial(n_1,p)}(c_1) \\ &= F_{Binomial(30,p)}(0) \\ &= \binom{30}{0} \times p^0 \times (1-p)^{30-0} \\ &= (1-p)^{30} \\ &\stackrel{p=0.02}{=} 0.545484 \end{aligned}$$

- **Probability of accepting the lot in the second stage of the plan**

$$\begin{aligned} P_a^{II}(p) &\stackrel{(13.17)}{=} P(c_1 < D_1 \le c_2,\ D_1 + D_2 \le c_2) \\ &= \sum_{k=c_1+1}^{c_2} P(D_1 = k) \times P(D_2 \le c_2 - k) \\ &\simeq \sum_{k=c_1+1}^{c_2} P_{Binomial(n_1,p)}(k) \times F_{Binomial(n_2,p)}(c_2 - k) \\ &= \sum_{k=1}^{2} P_{Binomial(30,p)}(k) \times F_{Binomial(50,p)}(2-k) \\ &= \left[\binom{30}{1} \times p^1 \times (1-p)^{30-1}\right] \times \left[(1-p)^{50} + 50 \times p \times (1-p)^{50-1}\right] \\ &\quad + \left[\binom{30}{2} \times p^2 \times (1-p)^{30-2}\right] \times (1-p)^{50} \\ &\stackrel{p=0.02}{\simeq} 0.333970 \times 0.735771 + 0.098828 \times 0.364170 \\ &\simeq 0.281716 \end{aligned}$$

- **Probability of accepting the lot in the double sampling plan**

$$\begin{aligned} P_a(p) &\stackrel{(13.18)}{=} P_a^I(p) + P_a^{II}(p) \\ &\stackrel{p=0.02}{\simeq} 0.545484 + 0.281716 \\ &= 0.8272. \end{aligned}$$

(c) Find the corresponding AOQ (average outgoing quality) and ATI (average total inspection). (**1.5**)

- **Lot size**
  $N = 1000$

- **Average outgoing quality of the double-sampling plan with rectifying inspection**

$$\begin{aligned} AOQ(p) &\stackrel{(13.26)}{=} \frac{p[(N-n_1)\,P_a^I(p) + (N-n_1-n_2)\,P_a^{II}(p)]}{N} \\ &\stackrel{p=0.02,\,(b)}{=} \frac{0.02 \times [(1000-30) \times 0.545484 + (1000-30-50) \times 0.281716]}{1000} \\ &= 0.015766 \end{aligned}$$

- **Average total inspection (ATI) of the double-sampling plan with rectifying inspection**

$$\begin{aligned} ATI(p) &\stackrel{(13.27)}{=} n_1 P_a^I(p) + (n_1+n_2) P_a^{II}(p) + N[1 - P_a(p)] \\ &\stackrel{p=0.02,\,(b)}{=} 30 \times 0.545484 + (30+50) \times 0.281716 + 300 \times (1 - 0.8272) \\ &\simeq 211.702. \end{aligned}$$