iabilidade e Controlo de Qualidade LMAC

:ame de 2a. Época / 2o. Teste (1o. Teste) 2o. Semestre – 2002/03

ıração: 3 horas / 1 hora e 45 minutos 08/07/03 – 17 horas – V0.08

1. *Foram registados 5 tempos de reparação (em horas) que se crê serem provenientes do modelo* lognormal(μ, σ^2), *estando associados aos seguintes tempos entre conclusões sucessivas de reparações: 0.50, 0.41, 0.02, 0.65, 0.98.*

   (a) *Construa um papel de probabilidade que permita averiguar a adequação do modelo acima referido. Exemplifique a sua utilização com alguns cálculos.* ([1

   - **R.v.**
     T = repair time
   - **Conjectured model**
     $\{\text{LogNormal}(\mu, \sigma^2),\ \mu \in I\!R, \sigma^2 \in I\!R^+\}$
   - **Complete data**
     The times between consecutive conclusions of repairs are: $(t_{(1)}, t_{(2)} - t_{(1)}, t_{(3)} - t_{(2)}, t_{(4)} - t_{(3)}, t_{(5)} - t_{(4)}) = (0.50, 0.41, 0.02, 0.65, 0.98)$. Thus, the ordered repair times are: $(t_{(1)}, t_{(2)}, t_{(3)}, t_{(4)}, t_{(5)}) = (0.50, 0.50 + 0.41 = 0.91, 0.91 + 0.02 = 0.93, 0.93 + 0.65 = 1.58, 1.58 + 0.98 = 2.56)$.
   - **Probability paper**
     For any absolutely continuous model,
     $$F_T(T_{(i)}) \sim \text{Beta}(i, n - i + 1).$$
     Therefore, by considering as an estimate of
     $$p_i = F_T(t_{(i)}) = \Phi\left\{\frac{\ln\left[t_{(i)}\right] - \mu}{\sigma}\right\}$$
     the expected value
     $$\hat{p}_i = E[F_T(T_{(i)})] = E[\text{Beta}(i, n - i + 1)] = \frac{i}{n+1},$$
     we are supposed to confront directly (or indirectly) $\hat{p}_i$ and $F_T(t_{(i)})$, that is,
     $$\frac{i}{n+1} \quad \rightarrow \quad \Phi\left\{\frac{\ln\left[t_{(i)}\right] - \mu}{\sigma}\right\}$$
     $$\Phi^{-1}\left(\frac{i}{n+1}\right) \quad \rightarrow \quad \frac{\ln\left[t_{(i)}\right] - \mu}{\sigma}$$
     $$\mu + \sigma \times \Phi^{-1}\left(\frac{i}{n+1}\right) \quad \rightarrow \quad \ln\left[t_{(i)}\right].$$
     Thus, the probability paper for the lognormal model is a graph with the following points:
     $$\left(\Phi^{-1}\left(\frac{i}{n+1}\right), \ln\left[t_{(i)}\right]\right),\ i = 1, \ldots, n.$$
   - **Example (with two points)**

i	$\Phi^{-1}\left(\frac{i}{n+1}\right)$	$\ln\left[t_{(i)}\right]$
1	$\Phi^{-1}\left(\frac{1}{5+1}\right) \simeq \Phi^{-1}(0.167) \stackrel{table}{=} -0.9661$	$\ln(0.5) \simeq -0.693147$
2	$\Phi^{-1}\left(\frac{2}{5+1}\right) \simeq \Phi^{-1}(0.333) \stackrel{table}{=} -0.4316$	$\ln(0.91) \simeq -0.094311$

   (b) *Obtenha as abcissas e ordenadas dos dois primeiros pontos do correspondente gráfico TTT. Que aspecto esperaria para este gráfico?* ([1

   - **Total time on test up to time** $t_{(i)}$
     $$\tau\left(t_{(i)}\right) = \sum_{j=1}^{i}(n - j + 1)\left[t_{(j)} - t_{(j-1)}\right] = \tau\left(t_{(i-1)}\right) + (n - i + 1)\left[t_{(i)} - t_{(i-1)}\right]$$
   - **Abcissae of the TTT plot**
     $\frac{i}{n}$, $i = 0, 1, \ldots, n$
   - **Ordinates of the TTT plot**
     $\frac{\tau(t_{(i)})}{\tau(t_{(n)})}$, $i = 1, \ldots, n$; equal to 0, for $i = 0$.
   - **Two points of the TTT plot**

i	$\tau\left(t_{(i)}\right) = \tau\left(t_{(i-1)}\right) + (n - i + 1)\left[t_{(i)} - t_{(i-1)}\right]$	$\frac{i}{n}$	$\frac{\tau(t_{(i)})}{\tau(t_{(n)})}$
0	0	0	0
1	$0 + (5 - 1 + 1) \times 0.50 = 2.5$	$\frac{1}{5} = 0.2$	$\frac{2.5}{6.48} \simeq 0.3858$
2	$2.5 + (5 - 2 + 1) \times 0.41 = 4.14$	$\frac{2}{5} = 0.4$	$\frac{4.14}{6.48} \simeq 0.6389$
3	$4.14 + (5 - 3 + 1) \times 0.02 = 4.2$	...	...
4	$4.2 + (5 - 4 + 1) \times 0.65 = 5.5$	...	...
5	$5.5 + (5 - 5 + 1) \times 0.98 = 6.48$	...	...

   - **Aspect of the TTT plot**
     According to Remark 5.5, the TTT plot should be:
     - a line in case $T_i \sim \text{Exponential}(\lambda)$, $i = 1, \ldots, n$;
     - concave if $T_i \in IHR$, $i = 1, \ldots, n$;
     - convex if $T_i \in DHR$, $i = 1, \ldots, n$.

     Moreover, according to page 97 of the lecture notes, the log-normal distributions has a hazard rate function that increases and then decreases.
     By combining these results the TTT plot should be initially concave — following the initial increasing behavior of $\lambda_T(t)$ —, and then convex — on the account of the decreasing behavior of $\lambda_T(t)$ that follows.

2. *Com o objectivo de estudar o tempo (em minutos) até à replicação de certa estirpe de vírus foram observadas 10 caixas de Petri, tendo-se dado por terminada a recolha de informação aquando do registo da 8a. replicação. A amostra ordenada daí resultante é* $(0.123, 0.388, 0.938, 1.242, 1.626, 1.961, 2.957, 3.841)$ *com* $\sum_{i=1}^{8} t_{(i)} = 13.076$.

   (a) *Após ter identificado o teste de vida e considerado as hipóteses de trabalho que entender mais convenientes, calcule um intervalo de confiança a 95% para a duração esperada do tempo até à replicação do referido vírus.* (2

   - **Life test**
     Since the end of the test was determined by the $r = 8^{th}$ time to a replication of the virus and nothing in this exercise suggests that the $n = 10$ Petri dishes were replaced during the life test, we are dealing with a

- ∘ Type II/item censored testing without replacement.

- **Censored data**
$n = 10$
$r = 8$
$(t_{(1)}, \ldots, t_{(r)}) = (0.123, 0.388, 0.938, 1.242, 1.626, 1.961, 2.957, 3.841)$

- **Cumulative total time in test**
According to Definition 5.17, the cumulative total time in test is given by:
$$\begin{aligned} \tilde{t} &= \sum_{i=1}^{r} t_{(i)} + (n-r) \times t_{(r)} \\ &= 13.076 + (10-8) \times 3.841 \\ &= 20.758 \end{aligned}$$

- **Distribution assumption**
T_i = time until replication in the i^{th} Petri dish
$T_i \stackrel{i.i.d.}{\sim} T \sim \text{Exponential}(\lambda)$, $i = 1, \ldots, n$

- **Unknown parameters**
λ
$E(T) = \frac{1}{\lambda}$

- **Confidence interval for λ**
According to Table 5.16 of the lecture notes,
$$\begin{aligned} CI_{(1-\alpha)\times 100\%}(\lambda) &= [\lambda_l; \lambda_U] \\ &= \left[\frac{F_{\chi^2_{(2r)}}(\alpha/2)}{2 \times \tilde{t}}; \frac{F_{\chi^2_{(2r)}}(1-\alpha/2)}{2 \times \tilde{t}}\right] \\ CI_{95\%}(\lambda) &= \left[\frac{F_{\chi^2_{(16)}}(0.025)}{2 \times 20.758}; \frac{F_{\chi^2_{(16)}}(0.975)}{2 \times 20.758}\right] \\ &= \left[\frac{6.908}{2 \times 20.758}; \frac{28.85}{2 \times 20.758}\right] \\ &\simeq [0.166394; 0.694913] \end{aligned}$$

- **Confidence interval for $E(T)$**
Since $E(T) = \frac{1}{\lambda}$ is a decreasing function of $\lambda > 0$, we can state that
$$\begin{aligned} CI_{95\%}\left(\frac{1}{\lambda}\right) &= \left[\frac{1}{\lambda_U}; \frac{1}{\lambda_L}\right] \\ &= [1.439029; 6.009844]. \end{aligned}$$

(b) *Obtenha a redução na duração esperada do teste por se ter decidido concluí-lo aquando do registo da 8a. replicação. Comente o resultado.* (0

- **Reduction of the expected value of the test duration**
It is equal to
$$\begin{aligned} \frac{E\left[T_{(r:n)}\right]}{E\left[T_{(r:r)}\right]} &= \frac{\sum_{i=1}^{r} \frac{1}{n-i+1}}{\sum_{i=1}^{r} \frac{1}{r-i+1}} \\ &\stackrel{r=8, n=10}{\simeq} \frac{1.428968}{2.717857} \\ &\simeq 0.525770, \end{aligned}$$
according to (5.25).

- **Comment**
The decision of ending the test after the 8^{th} replication in those 10 Petri dishes reduces — by practically 50% — the duration of a test involving complete data associated not to 10 but to 8 Petri dishes. So it does pay off to censure the data in this case!

3. *Considere uma pequena central eléctrica que só será capaz de fornecer energia eléctrica a uma pequena povoação se possuir 2 dos 3 geradores a funcionar. Admita que o tempo (em meses) até falha dos geradores são v.a.s i.i.d. com função de sobrevivência comum $R(t) = e^{-t^{1.5}}, t \geq 0$.*

(a) *Caracterize o tempo até falha da central eléctrica quanto ao envelhecimento estocástico? Justifique e comente o resultado.* (2

- **System**
$2 - \text{out} - \text{of} - 3$

- **Individual durations of the components**
Since the common reliability function is equal to $\bar{F}(t) = e^{-t^{1.5}}$, $t \geq 0$, we can add that:
$T_i \stackrel{i.i.d.}{\sim} \text{Weibull}(\delta = 1, \alpha = 1.5)$, $i = 1, 2, 3$
(see result (4.22)).

- **Stochastic ageing of T_i**
The table in page 100 provides three possible stochastic ageing behaviours for Weibull distributions according to the value of the shape parameter α. Since in this case $\alpha = 1.5 > 1$, we get
$T_i \stackrel{i.i.d.}{\sim} IHR$, $i = 1, 2, 3$.

- **Duration of the $2 - \text{out} - \text{of} - 3$ system**
$T \stackrel{(2.6)}{=} T_{(n-k+1)} \stackrel{n=3, k=2}{=} T_{(2)}$

- **Stochastic ageing of T**
$T_i \stackrel{i.i.d.}{\sim} IHR, \; i = 1, 2, 3 \quad \stackrel{(3.15)}{\Rightarrow} \quad T_{(i)} \sim IHR, \; i = 1, 2, 3.$
Thus, $T = T_{(2)} \sim IHR$.

(b) *Obtenha a expressão exacta do tempo esperado até falha da central eléctrica, bem como a de um par de limites inferior e superior o mais estritos possível para esse mesmo valor esperado.* (2

- **Reliability function of T**
$$\begin{aligned} R_T(t) &\stackrel{(2.8)}{=} F_{Binomial(3, F(t))(3-2)} \\ &= \sum_{m=0}^{1} \binom{3}{m} \left(1 - e^{-t^{1.5}}\right)^m \left[1 - \left(1 - e^{-t^{1.5}}\right)\right]^{3-m} \\ &= e^{-3t^{1.5}} + 3\left(1 - e^{-t^{1.5}}\right) e^{-2t^{1.5}} \\ &= 3e^{-2t^{1.5}} - 2e^{-3t^{1.5}} \end{aligned}$$

- **Expected value of T — exact expression**
$$\begin{aligned} E(T) &\stackrel{T \geq 0, (2.10)}{=} \int_0^{+\infty} R_T(t)\, dt \\ &= \int_0^{+\infty} \left[3e^{-2t^{1.5}} - 2e^{-3t^{1.5}}\right] dt \end{aligned}$$

$$
\begin{aligned}
&= 3\int_0^{+\infty} R_{Weibull(\delta=2^{-\frac{1}{1.5}},\alpha=1.5)}(t)\,dt \\
&\quad -2\int_0^{+\infty} R_{Weibull(\delta=3^{-\frac{1}{1.5}},\alpha=1.5)}(t)\,dt \\
&= 3\times E\left[\text{Weibull}(\delta=2^{-\frac{1}{1.5}},\alpha=1.5)\right] \\
&\quad -2\times E\left[\text{Weibull}(\delta=3^{-\frac{1}{1.5}},\alpha=1.5)\right] \\
&= 3\times 2^{-\frac{1}{1.5}}\times\Gamma\left(\frac{1}{1.5}+1\right)-2\times 3^{-\frac{1}{1.5}}\times\Gamma\left(\frac{1}{1.5}+1\right) \\
[&\overset{Mathematica}{=} 0.838092]
\end{aligned}
$$

- **Preliminaries**

  We are dealing with a coherent system characterized as follows:
  - $T_i \overset{i.i.d.}{\sim} IHR,\ i=1,2,3 \overset{Prop.\,3.36}{\Rightarrow} T_i \overset{i.i.d.}{\sim} IHRA,\ i=1,2,3;$
  - $\mu_i = E(T_i) = \mu^* = E[\text{Weibull}(\alpha=1.5,\delta=1)] = \Gamma\left(\frac{1}{1.5}+1\right);$
  - the minimal path sets are

    $$
    \begin{aligned}
    \mathcal{P}_1 &= \{1,2\} \\
    \mathcal{P}_2 &= \{1,3\} \\
    \mathcal{P}_3 &= \{2,3\} \\
    p &= 3 \text{ minimal path sets;}
    \end{aligned}
    $$
  - the minimal cut sets are

    $$
    \begin{aligned}
    \mathcal{K}_1 &= \{1,2\} \\
    \mathcal{K}_2 &= \{1,3\} \\
    \mathcal{K}_3 &= \{2,3\} \\
    q &= 3 \text{ minimal cut sets.}
    \end{aligned}
    $$

  Now, we can apply Theorem 3.69, and obtain a lower bound and an upper bound for $E(T)$.

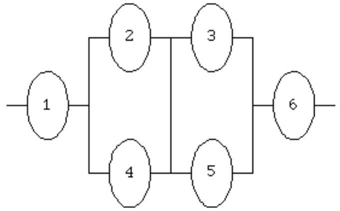
- **Lower bound for** $E(T)$

$$
\begin{aligned}
\mu &= E(T) \\
&\geq \max_{j=1,\dots,p}\left\{\left(\sum_{i\in\mathcal{P}_j}\mu_i^{-1}\right)^{-1}\right\} \\
&\overset{\mu_i=\mu^*}{=} \max_{j=1,\dots,p}\left\{\left(\frac{\#\mathcal{P}_j}{\mu^*}\right)^{-1}\right\} \\
&= \frac{\mu^*}{\min_{j=1,\dots,p}\{\#\mathcal{P}_j\}} \\
&= \frac{\mu^*}{2} \\
&= \frac{1}{2}\times\Gamma\left(\frac{1}{1.5}+1\right) \\
[&\overset{Mathematica}{\simeq} \frac{0.902745}{2} \\
&\simeq 0.451373].
\end{aligned}
$$

- **Upper bound for** $E(T)$

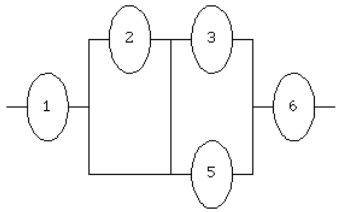
$$
\begin{aligned}
\mu &= E(T) \\
&\leq \min_{j=1,\dots,q}\int_0^{+\infty}\left[1-\prod_{i\in\mathcal{K}_j}\left(1-e^{-t/\mu_i}\right)\right]dt \\
&\overset{\mu_i=\mu^*}{=} \min_{j=1,\dots,q}\int_0^{+\infty}\left[1-\left(1-e^{-t/\mu^*}\right)^{\#\mathcal{K}_j}\right]dt \\
&= \int_0^{+\infty}\left[1-\left(1-e^{-t/\mu^*}\right)^{\min_{j=1,\dots,q}\#\mathcal{K}_j}\right]dt \\
&= \int_0^{+\infty}\left[1-\left(1-e^{-t/\mu^*}\right)^2\right]dt \\
&= \int_0^{+\infty}\left(2e^{-t/\mu^*}-e^{-2t/\mu^*}\right)dt \\
&= 2\mu^*-\frac{\mu^*}{2} \\
&= \frac{3\mu^*}{2} \\
[&\overset{Mathematica}{\simeq} \frac{3\times 0.902745}{2} \\
&\simeq 1.354118].
\end{aligned}
$$

4. *Uma rede de tratamento de águas residuais possui o figurino abaixo onde i denota a estação de tratamento i $(i=1,\dots 6)$.*



   (a) *Determine a função de estrutura por decomposição fulcral em torno da estação de tratamento 4.* (2

   - **Structure function by pivotal decomposition around component 4**
     - **Sub-system associated to** $(1_4, \underline{X})$



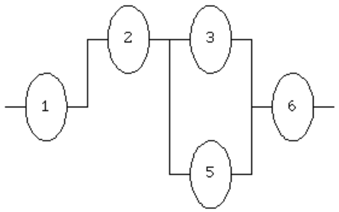
Minimal path sets

$$
\begin{aligned}
\mathcal{P}_1 &= \{1,3,6\} \\
\mathcal{P}_2 &= \{1,5,6\} \\
p^* &= 2 \text{ minimal path sets}
\end{aligned}
$$

Structure function

$$\begin{aligned}\phi(1_4,\underline{X}) &\overset{(1.13)}{=} 1-\prod_{j=1}^{p^*}\left(1-\prod_{i\in\mathcal{P}_j}X_i\right)\\ &= 1-(1-X_1X_3X_6)(1-X_1X_5X_6).\end{aligned}$$

– **Sub-system associated to** $(0_4,\underline{X})$



Minimal path sets

$$\begin{aligned}\mathcal{P}_1 &= \{1,2,3,6\}\\ \mathcal{P}_2 &= \{1,2,5,6\}\\ p^* &= 2 \text{ minimal path sets}\end{aligned}$$

Structure function

$$\begin{aligned}\phi(0_4,\underline{X}) &\overset{(1.13)}{=} 1-\prod_{j=1}^{p^*}\left(1-\prod_{i\in\mathcal{P}_j}X_i\right)\\ &= 1-(1-X_1X_2X_3X_6)(1-X_1X_2X_5X_6).\end{aligned}$$

- **Structure function of the original system**

$$\begin{aligned}\phi(\underline{X}) &\overset{(1.7)}{=} X_4\times\phi(1_4,\underline{X})+(1-X_4)\times\phi(0_4,\underline{X})\\ &= X_4\times[1-(1-X_1X_3X_6)(1-X_1X_5X_6)]\\ &\quad +(1-X_4)\times[1-(1-X_1X_2X_3X_6)(1-X_1X_2X_5X_6)].\end{aligned}$$

(b) *Após ter definido o tempo até falha da rede de tratamento de águas residuais, determine um par de limites inferior e superior para a função de fiabilidade da rede para o período de um mês, caso as estações sejam associadas (positivamente) e possuam tempos até falha (em meses) com distribuição comum* Weibull$(\delta=1,\alpha=0.25)$. (1

- **Individual durations**
  $T_i\sim$ Weibull$(\delta=1,\alpha=0.25)$
  $R_i(t)=R(t)=e^{-t^{0.25}}$, $t\geq 0$.

- **Minimal path sets**

$$\begin{aligned}\mathcal{P}_1 &= \{1,2,3,6\}\\ \mathcal{P}_2 &= \{1,2,5,6\}\\ \mathcal{P}_3 &= \{1,4,5,6\}\\ \mathcal{P}_4 &= \{1,3,4,6\}\\ p^* &= 4 \text{ minimal path sets}\end{aligned}$$

- **Minimal cut sets**

$$\begin{aligned}\mathcal{K}_1 &= \{1\}\\ \mathcal{K}_2 &= \{2,4\}\\ \mathcal{K}_3 &= \{3,5\}\\ \mathcal{K}_4 &= \{6\}\\ q &= 4 \text{ minimal cut sets}\end{aligned}$$

- **System duration**
  $T=\max\{\min\{T_1,T_2,T_3,T_6\},\ \min\{T_1,T_2,T_5,T_6\},\ \min\{T_1,T_4,T_5,T_6\},\ \min\{T_1,T_3,T_4,T_6\}\}$
  or
  $T=\min\{T_1,\ \max\{T_2,T_4\},\ \min\{T_3,T_5\},\ T_6\}$

- **Lower and upper bounds for the reliability function** $R_T(t)$
  Since the 6 components form a coherent system and operate in a positively associated fashion, we can apply Theorem 2.22, namely result (2.18), and get the following lower and upper bounds:

$$\begin{aligned}R_T(t) &\overset{(1.42)}{\geq} \max_{j=1,\dots,p^*}\prod_{i\in\mathcal{P}_j}R_i(t)\\ &\overset{R_i(t)=R(t)}{=} \max_{j=1,\dots,p^*} p^{\#\mathcal{P}_j}\\ &\overset{\#\mathcal{P}_j=4,\forall j}{=} [R(t)]^4\\ &= \left(e^{-t^{0.25}}\right)^4\\ &\overset{t=1}{=} e^{-4}\\ &= 0.018316;\\ R_T(t) &\overset{(1.42)}{\leq} \min_{j=1,\dots,q}\left[1-\prod_{i\in\mathcal{K}_j}[1-R_i(t)]\right]\\ &\overset{R_i(t)=R(t)}{=} \min_{j=1,\dots,q}\left\{1-[1-R(t)]^{\#\mathcal{K}_j}\right\}\\ &= 1-[1-R(t)]^{\min_{j=1,\dots,q}\#\mathcal{K}_j}\\ &= 1-[1-R(t)]\\ &= R(t)\\ &= e^{-t^{0.25}}\\ &\overset{t=1}{=} e^{-1}\\ &\simeq 0.367879.\end{aligned}$$

5. *Os dados seguintes dizem respeito ao número de defeitos em 22 cabos de* $1000\,m$ *de fibra óptica: 1, 1, 3, 7, 8, 10, 5, 13, 0, 11, 24, 6, 9, 12, 14, 8, 3, 6, 7, 4, 8, 11.*

   (a) *Permitiriam estes dados concluir que o processo de produção está sob controlo? Defina os limites de controlo da carta que entender mais razoável para o controlo da produção futura.* (0

   - **Control statistic / quality characteristic**
     $Y_N=$ number of defects in the N^{th} optical fiber cable with 1000m

- **Distributions**
  $Y_N \sim \text{Poisson}(\lambda_0)$, IN CONTROL
  $Y_N \sim \text{Poisson}(\lambda = \lambda_0 + \delta)$, OUT OF CONTROL, where δ represents the magnitude of a shift in λ

- **Obtaining the target value of λ**
  Taking into account the data $\underline{y} = (y_1, \dots, y_{22}) = (1, \dots, 11)$, associated to unitarian samples ($n = 1$), and the fact that the underlying model is Poisson, we obtain the following maximum likelihood estimate for the target value of λ:
  $$\begin{aligned} \hat\lambda_0 &= \bar y \\ &= \frac{1 + \ldots + 11}{22} \\ &= \frac{171}{22} \\ &\simeq 7.773. \end{aligned}$$
  Please note that we shall use $\hat\lambda_0$ to estimate/obtain the target and the lower and upper control limits of chart described below.

- **(Estimate of the upper) control limit of the standard u chart with $3\,sigma$ limits**
  $$\begin{aligned} LCL &= \max\left\{0, \hat\lambda_0 - 3 \times \sqrt{\hat\lambda_0}\right\} \\ &= \max\left\{0, 7.773 - 3\sqrt{7.773}\right\} \\ &\simeq \max\{0, -0.591156\} \\ &= 0 \quad \text{(we ended up with an upper one-sided chart)} \\ UCL &= \hat\lambda_0 + 3 \times \sqrt{\hat\lambda_0} \\ &= 7.773 + 3\sqrt{7.773} \\ &\simeq 16.136611 \end{aligned}$$

- **State of the process and reestimation the target and control limits**
  Please note that $y_{11} = 24 > UCL$, thus, the process does not seem to be in control and we need to omit this observation and reestimate λ_0, etc.:
  $$\begin{aligned} \hat\lambda_0 &= \frac{171 - 24}{22 - 1} \\ &= 7 \\ LCL &= \max\left\{0, \hat\lambda_0 - 3 \times \sqrt{\hat\lambda_0}\right\} \\ &= \max\left\{0, 7 - 3\sqrt{7}\right\} \\ &\simeq \max\{0, -0.937254\} \\ &= 0 \\ UCL &= \hat\lambda_0 + 3 \times \sqrt{\hat\lambda_0} \\ &= 7 + 3\sqrt{7} \\ &\simeq 14.937245. \end{aligned}$$
  Now, all the observations are below UCL and we are going to assume that the estimate of UCL is the true value of the upper control limit of this u chart.

(b) *Suponha que as novas unidades de inspecção são cabos de* $2000\,m$ *de fibra óptica. Determine a dimensão amostral mínima e os limites de controlo de modo a lidarmos com uma carta padrão com limites 3-*sigma *que assinale um aumento de 3 unidades no número esperado de defeitos por cabo de* $1000\,m$ *com probabilidade superior ou igual a 0.10?* (1

- **New control statistic / quality characteristic**
  Y_N^* = number of defects in the N^{th} sample of n optical fiber cables, each with 2000m

- **Distributions**
  $Y_N^* \sim \text{Poisson}(2n \times \lambda_0)$, IN CONTROL
  $Y_N \sim \text{Poisson}(2n \times \lambda = 2n \times (\lambda_0 + \delta))$, OUT OF CONTROL, where δ represents the magnitude of a shift in λ

- **(Estimates of the) control limits**
  $$\begin{aligned} LCL^* &= \max\left\{0, 2n\hat\lambda_0 - 3 \times \sqrt{2n\hat\lambda_0}\right\} \\ &= \max\left\{0, 14n - 3\sqrt{14n}\right\} \\ UCL^* &= 2n\hat\lambda_0 + 3 \times \sqrt{2n\hat\lambda_0} \\ &= 14n + 3\sqrt{14n} \end{aligned}$$

- **Run length**
  We are dealing with a Shewhart chart, thus, the number of samples collected until the chart triggers a signal given δ, $RL(\delta)$, has the following distribution:
  $$RL(\delta) \sim \text{Geometric}\,(\xi(\delta))\,.$$

- **(Estimate of the) probability of triggering a signal**
  $$\begin{aligned} \xi(\delta) &= P(Y_N^* \not\in [LCL^*, UCL^*] \mid \delta) \\ &= 1 - \left[F_{Poisson(2n\times(\hat\lambda_0+\delta))}(UCL^*) - F_{Poisson(2n\times(\hat\lambda_0+\delta))}((LCL^*)^-)\right] \\ &\overset{\hat\lambda_0=7,\,\delta=3}{=} 1 - \left[F_{Poisson(20n)}(14n + 3\sqrt{14n}) - F_{Poisson(20n)}((14n - 3\sqrt{14n})^-)\right]. \end{aligned}$$

- **Obtaining n**
  The minimum value of n such that $\xi(\delta) \geq 10$ is one because in this case we get
  $$\begin{aligned} \xi(\delta) &= 1 - \left[F_{Poisson(20)}(25) - F_{Poisson(20)}(2)\right] \\ &\overset{table}{=} 1 - (0.8878 - 0.0000) \\ &= 0.1122. \end{aligned}$$

6. *Um aparelho de ar condicionado é classificado de "inimigo do ambiente" (IA) caso a quantidade de clorofluorcarbonetos (CFCs) que liberta por hora não pertença ao intervalo* 30.00 ± 0.06 *(ppm).*

   (a) *Admita que é recolhida uma amostra de dimensão 2 cuja média* ($\bar x = \frac{x_{(1)}+x_{(2)}}{2}$) *e amplitude* ($r = x_{(2)} - x_{(1)}$) *são iguais a 30.05 e 0.03, respectivamente.* (0
   Prove que a esta amostra está associado um aparelho de ar condicionado IA.

   - **R.v.**
     X = quantity of CFC released in an hour by an air conditioned
   - **Indicator r.v.**
     $$Y = \begin{cases} 1, & \text{if } X \not\in [30 \pm 0.06] = [29.94, 30.06] \text{(NON eco-friendly air conditioned)} \\ 0, & \text{otherwise} \end{cases}$$

(b) **Data**

$(x_{(1)}, x_{(2)})$
$\bar{x} = 30.05$ (sample mean)
$r = 0.03$(sample range)

(c) **Obtaining $x_{(1)}$ and $x_{(2)}$**

$$(x_{(1)}, x_{(2)}) \ : \ \begin{cases} \frac{x_{(1)}+x_{(2)}}{2} = \bar{x} \\ x_{(2)} - x_{(1)} = r \end{cases}$$

$$\begin{cases} x_{(1)} = \bar{x} - \frac{r}{2} \\ x_{(2)} = \bar{x} + \frac{r}{2} \end{cases}$$

$$\begin{cases} x_{(1)} = 30.05 - \frac{0.03}{2} = 30.035 \in [29.94,\ 30.06] \text{ (eco-friendly air conditioned)} \\ x_{(2)} = 30.05 + \frac{0.03}{2} = 30.065 \notin [29.94,\ 30.06] \text{ (NON eco-friendly air cond.)} \end{cases}$$

(d) *Suponha que a quantidade de CFCs libertados por hora é, sob controlo, uma v.a. com distribuição normal$(30, 0.025^2)$.* (2
Qual o valor nominal (i.e., sob controlo) do número esperado de aparelhos IA em amostras de dimensão $n = 25$ e o valor exacto da probabilidade de ser emitido um falso alarme por uma carta padrão com limites 3-sigma somente após a recolha de 100 amostras?

- **In-control distribution of X**
  $X \sim \text{Normal}(\mu = \mu_0 = 30, \sigma^2 = \sigma_0^2 = 0.0025^2)$

- **Nominal value of the probability of selecting a NON eco-friendly air conditioned**

$$\begin{aligned} p_0 &= P(X \notin [29.94,\ 30.06] | \mu = \mu_0 = 30, \sigma^2 = \sigma_0^2 = 0.0025^2) \\ &= 1 - \left[\Phi\left(\frac{30.06-30}{0.025}\right) - \Phi\left(\frac{29.94-30}{0.025}\right)\right] \\ &= 1 - [\Phi(2.4) - \Phi(-2.4)] \\ &= 2 \times [1 - \Phi(2.4)] \\ &\overset{table}{=} 2 \times (1 - 0.9918) \\ &= 0.0164 \end{aligned}$$

- **Requested number**
  The expected number of NON eco-friendly appliances in a sample of size $n = 25$ is equal to:

$$\begin{aligned} np_0 &= 25 \times 0.0164 \\ &= 0.41 \end{aligned}$$

- **Control statistic / quality characteristic**
  $Z_N =$ number of NON eco-friendly appliances in the N^{th} sample of size n

- **Distributions**
  $Z_N \sim \text{Binomial}(n, p_0)$, IN CONTROL
  $Y_N \sim \text{Binomial}(n, p = p_0 + \delta)$, OUT OF CONTROL, where δ ($\delta \in (-p_0, 1 - p_0)$) represents the magnitude of a shift in p

- **Control limits of the standard np chart**
  If we consider a standard np chart with 3 sigma limits, we get:

$$\begin{aligned} LCL &= \max\left\{0, np_0 - 3\sqrt{np_0(1-p_0)}\right\} \\ &= \max\left\{0, 25 \times 0.0164 - 3\sqrt{25 \times 0.0164 \times (1 - 0.0164)}\right\} \\ &= \max\{0, -1.495120\} \\ &= 0 \\ UCL &= np_0 + 3\sqrt{np_0(1-p_0)} \\ &= 25 \times 0.0164 + 3\sqrt{25 \times 0.0164 \times (1 - 0.0164)} \\ &\simeq 2.315120 \end{aligned}$$

- **Run length**
  We are dealing with a Shewhart chart, thus, the number of samples collected until the chart triggers a signal given δ, $RL(\delta)$, has the following distribution:
  $RL(\delta) \sim \text{Geometric}\,(\xi(\delta))$.

- **Probability of triggering a false alarm**

$$\begin{aligned} \xi(\delta) &= P(Y_N \notin [LCL, UCL] \mid \delta) \\ &\overset{Y_N \geq 0,\, LCL=0}{=} P\left(Y_N > UCL \mid \delta\right) \\ &= 1 - F_{Binomal(n, p=p_0+\delta)}(UCL) \\ &= 1 - F_{Binomial(25, 0.0164)}(2.315120) \\ &= 1 - F_{Binomial(25, 0.0164)}(2) \\ &= 1 - \sum_{i=0}^{2}\binom{25}{i}0.0164^i(1-0.0164)^{25-i} \\ &\simeq 0.007746 \end{aligned}$$

- **Requested probability**
  Since $RL(\delta) \sim \text{Geometric}\,(\xi(\delta))$ we get:

$$\begin{aligned} P[RL(\delta) > m] &= [1 - \xi(\delta)]^m \\ &= [1 - \xi(0)]^{100} \\ &\simeq (1 - 0.007746)^{100} \\ &\simeq 0.459509. \end{aligned}$$

7. *O diâmetro é uma característica importante de determinada fibra têxtil. Foram recolhidas 10 amostras com dimensão igual a $n = 5$ tendo-se resumido na tabela abaixo as variâncias corrigidas amostrais (s_N^2) e alguns valores observados da estatística sumária de carta EWMA unilateral superior (v_N) com os seguintes parâmetros: $\lambda_\sigma = 0.05, v_0 = \ln(\sigma_0^2) = \ln(0.83)$.*

N	1	2	3	4	5	6	7	8	9	10
s_N^2	0.87	0.85	0.90	0.85	0.73	0.80	0.78	0.83	0.87	0.86
v_N		-0.183	-0.179	-0.178	-0.185	-0.186	-0.186	-0.186	-0.184	

(a) *Preencha a tabela acima, obtenha os limites de controlo da carta* EWMA *unilateral superior na situação em que $\gamma_{EWMA} = 1.5$ e averigue se alguma das 10 observações foi responsável por um sinal por parte desta carta.* (0

- **Quality characteristic**
  $X =$ diameter of the textile fiber
  $X \sim \text{Normal}(\mu, \sigma^2)$

- **Nominal values of σ^2**
  $\sigma_0^2 = 0.83$
- **Estimator of σ^2**
  $S_N^2 =$ variance of the N^{th} random sample of size n, $N \in I\!N$
- **Control limits of the upper one-sided EWMA chart for σ^2**
  To obtain the control limits of this chart, recall that $\lambda_\sigma = 0.05$, $\gamma_{EWMA} = 1.5$, $n = 5$ and $\psi'\left(\frac{n-1}{2}\right) = \psi'(2) \overset{(10.31)}{=} \psi'(1) - \frac{1}{1^2} = 1.6449340668 - \frac{1}{1^2} = 0.6449340668$. Thus, according to Table 10.10, we get:

$$\begin{aligned} LCL_{EWMA} &= \ln\left(\sigma_0^2\right) \\ &= \ln(0.83^2) \\ &\simeq -0.186 \\ UCL_{EWMA} &= \ln\left(\sigma_0^2\right) + \gamma_\sigma \times \sqrt{\psi'\left(\frac{n-1}{2}\right) \times \frac{\lambda_\sigma}{2-\lambda_\sigma}} \\ &= \ln\left(0.1^2\right) + 1.5 \times \sqrt{0.6449340668 \times \frac{0.05}{2-0.05}} \\ &= 0.006563 \end{aligned}$$

- **On the observed values of the control statistic**
  According to Table 10.10, the control statistic is given by

$$V_N = \begin{cases} v_0 = \ln\left(\sigma_0^2\right), & N = 0 \\ \max\{\ln\left(\sigma_0^2\right), (1-\lambda_\sigma) \times V_{N-1} + \lambda_\sigma \times \ln\left(S_N^2\right)\}, & N \in I\!N. \end{cases}$$

  Thus, the first and the last observed values of the control statistic are:

$$\begin{aligned} v_1 &= \max\{-0.186, (1-0.05) \times (-0.186) + 0.05 \times \ln(0.87)\} \\ &\simeq -0.184; \\ v_{10} &= \max\{-0.186, (1-0.05) \times (-0.184) + 0.05 \times \ln(0.86)\} \\ &\simeq -0.182. \end{aligned}$$

  Since $v_1, \ldots, v_{10} \in [LCL_{EWMA}, UCL_{EWMA}]$ the process variance is deemed in control.

(b) *Caracterize a carta S^2 unilateral superior com $ARL_\sigma(1) = 1000$ e determine o 1o. quartil do número de amostras recolhidas até à emissão de sinal válido por parte desta carta aquando de alteração em σ com magnitude $\theta = 1.2$. Comente.* (1

Nota: Na impossibilidade de obter um valor exacto obtenha um intervalo de valores para este quartil e para as probabilidades que se seguirem.

- **Control statistic of the upper one-sided S^2 chart**
  $S_N^2 =$ variance of the N^{th} random sample of size n, $N \in I\!N$
- **Distribution**
  $\frac{(n-1)S_N^2}{\sigma_0^2} \sim \chi^2_{(n-1)}$, IN CONTROL
  $\frac{(n-1)S_N^2}{(\theta\sigma_0)^2} \sim \chi^2_{(n-1)}$, OUT OF CONTROL, where θ represents the magnitude of the upward shift in the standard deviation σ.
- **Control limits of the upper one-sided S^2- chart**
  $LCL_\sigma = 0$
  $UCL_\sigma = \frac{\sigma_0^2}{n-1} \times \gamma_\sigma$

- **Probability of triggering a signal**

$$\begin{aligned} \xi_\sigma(\theta) &= P\left(\bar{S}_N^2 \notin [LCL_\sigma, UCL_\sigma] \mid \theta\right) \\ &= 1 - F_{\chi^2_{(n-1)}}\left[\frac{(n-1)\,UCL_\sigma}{\sigma^2}\right] \\ &= 1 - F_{\chi^2_{(n-1)}}\left(\frac{\gamma_\sigma}{\theta^2}\right), \ \theta \geq 1. \end{aligned}$$

- **Run length**
  We are dealing once again with a Shewhart chart, thus,

$$RL_\sigma(\theta) \sim \text{Geometric}\left(\xi_\sigma(\theta)\right).$$

- **Obtaining γ_σ**

$$\begin{aligned} \gamma_\sigma \ : \ \frac{1}{\xi_\sigma(1)} &= ARL_\sigma(1) \\ 1 - F_{\chi^2_{(n-1)}}(\gamma_\sigma) &= \frac{1}{ARL_\sigma(1)} \\ \gamma_\sigma &= F_{\chi^2_{(n-1)}}\left[1 - \frac{1}{ARL_\sigma(1)}\right] \\ \gamma_\sigma &= F_{\chi^2_{(5-1)}}\left(1 - \frac{1}{1000}\right) \\ \gamma_\sigma &= F_{\chi^2_{(5-1)}}(0.999) \\ \gamma_\sigma &\overset{table}{=} 18.47 \end{aligned}$$

- **Probability of triggering a valid signal when $\theta = 1.2$**

$$\begin{aligned} \xi_\sigma(\theta) &= 1 - F_{\chi^2_{(n-1)}}\left(\frac{\gamma_\sigma}{\theta^2}\right) \\ &= 1 - F_{\chi^2_{(5-1)}}\left(\frac{18.47}{1.2^2}\right) \\ &\simeq 1 - F_{\chi^2_{(4)}}(12.826) \\ &\in (0.010; 0.025) \end{aligned}$$

  because $F^{-1}_{\chi^2_{(4)}}(0.975) = 11.14 < 12.826 < 13.28 = F^{-1}_{\chi^2_{(4)}}(0.990)$.

- **Obs.**
  $\xi_\sigma(1.2) \overset{Mathematica}{=} 0.012158$
- **Request percentage point**

$$\begin{aligned} F^{-1}_{RL_\sigma(\theta)}(p) &\overset{Table\,9.2}{=} \inf\left\{m \in I\!N : F_{RL_\sigma(\theta)}(m) \geq p\right\} \\ &= \quad 1 - [1 - \xi_\sigma(\theta)]^m \geq p \\ &= \quad [1 - \xi_\sigma(\theta)]^m \leq 1 - p \\ &= \quad m \times \ln[1 - \xi_\sigma(\theta)] \leq \ln(1-p) \\ &\overset{\ln(1-p)<0}{=} \quad m \geq \frac{\ln(1-p)}{\ln[1-\xi_\sigma(\theta)]} \\ &\overset{p=0.25}{=} \quad m \geq \frac{\ln(1-0.5)}{\ln(1-(0.010; 0.025))} \\ &\in \quad (12; 29) \end{aligned}$$

- **Obs.**
  $F^{-1}_{RL_\sigma(1.2)}(0.25) \overset{Mathematica}{=} 24$

- **Comment**
The probability of a valid signal within the first $F^{-1}_{RL_\sigma(1.2)}(0.25)$ samples is of at least 25%, thus, suggesting a reasonably quick detection of such a shift.

8. *Considere um esquema conjunto para μ e σ^2 que faça uso de amostras de dimensão $n = 5$ e de cartas* Shewhart *padrão para μ e unilateral superior para σ^2 cujos limites de controlo são tais que:*

- *a probabilidade de emissão de sinal válido quando $(\delta, \theta) = (0, 2.4)$ pela carta para μ é igual a 0.2;*
- *o número esperado de amostras recolhidas até à emissão de falso alarme por parte da carta para σ^2 é de 1000 amostras.*

(a) *Calcule o número esperado de amostras recolhidas até que o esquema conjunto emita um falso alarme.* (1

- **Quality characteristic**
$X \sim \text{Normal}(\mu, \sigma^2)$
- **Control statistics**
$\bar{X}_N =$ mean of the N^{th} random sample of size n
$S^2_N =$ variance of the N^{th} random sample of size n, $N \in I\!N$
- **Distributions**
$\bar{X}_N \sim \text{Normal}\left(\mu = \mu_0, \frac{\sigma^2}{n} = \frac{\sigma_0^2}{n}\right)$, IN CONTROL.
$\bar{X}_N \sim \text{Normal}\left(\mu = \mu_0 + \delta \times \frac{\sigma_0}{\sqrt{n}}, \frac{\sigma^2}{n} = \frac{(\theta\sigma_0)^2}{n}\right)$, OUT OF CONTROL, where δ $(\delta \neq 0)$ represents the magnitude of the shift (a decrease or increase!) in μ and θ $(\theta > 1)$ represents a shift (an increase!) in the standard deviation σ
$\frac{(n-1)S^2_N}{\sigma_0^2} \sim \chi^2_{(n-1)}$, IN CONTROL
$\frac{(n-1)S^2_N}{(\theta\sigma_0)^2} \sim \chi^2_{(n-1)}$, OUT OF CONTROL
- **Control limits of the standard $\bar{X}-$ chart and the upper one-sided S^2- chart**
$LCL_\mu = \mu_0 - \gamma_\mu \frac{\sigma_0}{\sqrt{n}}$
$UCL_\mu = \mu_0 - \gamma_\mu \frac{\sigma_0}{\sqrt{n}}$
$LCL_\sigma = 0$
$UCL_\sigma = \frac{\sigma_0^2}{n-1} \times \gamma_\sigma$
- **Probability of triggering signals**
Taking into account the distribution of the control statistics, the individual chart for μ and σ trigger signals with probabilities equal to:

$$\begin{aligned}
\xi_\mu(\delta,\theta) &= P\left(\bar{X}_N \notin [LCL_\mu, UCL_\mu] \mid \delta, \theta\right) \\
&= 1 - \left[\Phi\left(\frac{UCL_\mu - \mu}{\frac{\sigma}{\sqrt{n}}}\right) - \Phi\left(\frac{LCL_\mu - \mu}{\frac{\sigma}{\sqrt{n}}}\right)\right] \\
&= 1 - \left[\Phi\left(\frac{\gamma_\mu - \delta}{\theta}\right) - \Phi\left(\frac{-\gamma_\mu - \delta}{\theta}\right)\right],\ \delta \in I\!R,\ \theta \geq 1; \\
\xi_\sigma(\theta) &= P\left(S^2_N \notin [LCL_\sigma, UCL_\sigma] \mid \theta\right) \\
&= 1 - F_{\chi^2_{(n-1)}}\left[\frac{(n-1)\,UCL_\sigma}{\sigma^2}\right] \\
&= 1 - F_{\chi^2_{(n-1)}}\left(\frac{\gamma_\sigma}{\theta^2}\right),\ \theta \geq 1.
\end{aligned}$$

Obtaining γ_μ and γ_σ

$$\begin{aligned}
\gamma_\mu \ :\ & \xi_\mu(0, 2.4) = 0.2 \\
& 1 - \left[\Phi\left(\frac{\gamma_\mu}{\theta}\right) - \Phi\left(\frac{-\gamma_\mu}{\theta}\right)\right] = 0.2 \\
& 2 - 2 \times \Phi\left(\frac{\gamma_\mu}{2.4}\right) = 0.2 \\
& \gamma_\mu = 2.4 \times \Phi^{-1}(0.90) \\
& \gamma_\mu \overset{table}{=} 2.4 \times 1.2816 \\
& \gamma_\mu = 3.07584 \\
\gamma_\sigma \ :\ & ARL_\sigma(1) = 1000 \\
& \gamma_\sigma \overset{(a)}{=} 18.47
\end{aligned}$$

- **Probability of a signal by the joint scheme for μ and σ**
The joint scheme triggers a signal if either of the individual charts triggers an alarm; moreover, the control statistics of the individual charts are independent given (δ, θ). As a consequence, the joint scheme for μ and σ triggers a false alarm with probability equal to:

$$\begin{aligned}
\xi_{\mu,\sigma}(\delta,\theta) &= P\left(\bar{X}_N \notin [LCL_\mu, UCL_\mu] \text{ or } S^2_N \notin [LCL_\sigma, UCL_\sigma] \mid \delta, \theta\right) \\
&= \xi_\mu(\delta,\theta) + \xi_\sigma(\theta) - \xi_\mu(\delta,\theta) \times \xi_\sigma(\theta) \\
&\overset{(\delta,\theta)=(0,1)}{\simeq} \{1 - [\Phi(\gamma_\mu) - \Phi(-\gamma_\mu)]\} + \left[1 - F_{\chi^2_{(n-1)}}(\gamma_\sigma)\right] \\
&\quad - \{1 - [\Phi(\gamma_\mu) - \Phi(-\gamma_\mu)]\} \times \left[1 - F_{\chi^2_{(n-1)}}(\gamma_\sigma)\right] \\
&\simeq 0.00207 + 0.001 - 0.00207 \times 0.001 \\
&\simeq 0.00306793.
\end{aligned}$$

- **In-control RL and ARL**
The in-control run length of this joint scheme for μ and σ, $RL_{\mu,\sigma}(0,1)$, has the following distribution and expected value:

$$\begin{aligned}
RL_{\mu,\sigma}(0,1) &\sim \text{Geometric}(\xi_{\mu,\sigma}(0,1)); \\
ARL_{\mu,\sigma}(0,1) &= \frac{1}{\xi_{\mu,\sigma}(0,1)} \\
&\simeq \frac{1}{0.00306793} \\
&\simeq 325.953 \text{ samples.}
\end{aligned}$$

(b) *Determine a probabilidade de ocorrência de sinal erróneo de Tipo III (IV) quando $\theta = 1.2$ ($\delta = 0.5$). Comente.* (1

- **Probability of a misleading signal of type III**

$$\begin{aligned}
PMS_{III}(\theta) &\overset{Table\ 10.12}{=} \frac{1 - [\Phi(\gamma_\mu/\theta) - \Phi(-\gamma_\mu/\theta)]}{[F_{\chi^2_{(n-1)}}(\gamma_\sigma/\theta^2)]^{-1} - [\Phi(\gamma_\mu/\theta) - \Phi(-\gamma_\mu/\theta)]} \\
&\overset{\theta=1.2}{=} \frac{1 - [\Phi(3.07584/1.2) - \Phi(-3.07584/1.2)]}{[F_{\chi^2_{(5-1)}}(18.47/1.2^2)]^{-1} - [\Phi(3.07584/1.2) - \Phi(-3.07584/1.2)]} \\
&\simeq \frac{2 \times [1 - \Phi(2.56)]}{[F_{\chi^2_4}(12.826)]^{-1} - [2 \times \Phi(2.56) - 1]} \\
&\in \left(\frac{2 \times (1 - 0.9948)}{0.975^{-1} - (2 \times 0.9948 - 1)}; \frac{2 \times (1 - 0.9948)}{0.990^{-1} - (2 \times 0.9948 - 1)}\right)
\end{aligned}$$

$$\simeq \quad (0.288560;\ 0.507292)$$

because $F^{-1}_{\chi^2_{(4)}}(0.975) = 11.14 < 12.826 < 13.28 = F^{-1}_{\chi^2_{(4)}}(0.990)$.

- **Probability of a misleading signal of type IV**

$$
\begin{aligned}
PMS_{IV}(\delta) &\overset{Table\ 10.12}{=} \frac{1 - F_{\chi^2_{(n-1)}}(\gamma_\sigma)}{[\Phi(\gamma_\mu - \delta) - \Phi(-\gamma_\mu - \delta)]^{-1} - F_{\chi^2_{(n-1)}}(\gamma_\sigma)} \\
&\overset{\delta=0.5}{=} \frac{1 - F_{\chi^2_{(5-1)}}(18.47)}{[\Phi(3.07584 - 0.5) - \Phi(-3.07584 - 0.5)]^{-1} - F_{\chi^2_{(5-1)}}(18.47)} \\
&\simeq \frac{1 - F_{\chi^2_{(4)}}(18.47)}{[\Phi(2.58) - \Phi(-3.58)]^{-1} - F_{\chi^2_{(4)}}(18.47)} \\
&\overset{table}{=} \frac{1 - 0.999}{[0.9951 - (1 - 0.999828)]^{-1} - 0.999} \\
&\simeq 0.130478.
\end{aligned}
$$

- **Comment**

  The PMS are from being small, in particular when the standard deviation has increased 20%.