Chap. 10

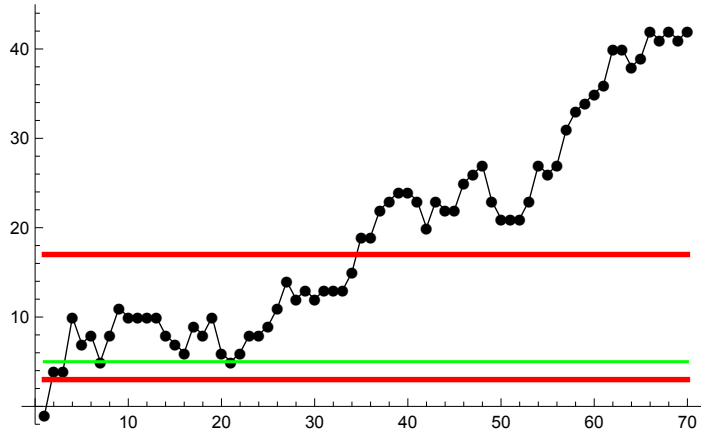
SOME HINTS for a few exercises from Chap. 10

Exercise 10.8

```
data = {4, 10, 5, 11, 2, 6, 2, 8, 8, 4, 5, 5, 5, 3, 4, 4, 8, 4, 7, 1,
   4, 6, 7, 5, 6, 7, 8, 3, 6, 4, 6, 5, 5, 7, 9, 5, 8, 6, 6, 5,
   4, 2, 8, 4, 5, 8, 6, 6, 1, 3, 5, 5, 7, 9, 4, 6, 9, 7, 6, 6,
   6, 9, 5, 3, 6, 8, 4, 6, 4, 6};

n = 100;
p0 = 0.05;
k = n × p0;
LCL = 3;
CL = n × p0;
UCL = 17;

z[0] = 0;
Do[z[i] = z[i - 1] + data[[i]] - k, {i, 1, Length[data]}]
cusum = Table[z[i], {i, 1, Length[data]}];

list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];

TableForm[Transpose[{data, list1, cusum, list3}],
 TableHeadings → {Automatic, {"yN", "LCL", "zN", "UCL"}}]

G1 = ListPlot[cusum, Joined → True, PlotStyle → {Black, Thickness[0.002]},
   PlotMarkers → Automatic, PlotRange → {-2, 45}, DisplayFunction → Identity];
G2 = ListPlot[list1, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.008]},
   Joined → True, DisplayFunction → Identity];
G3 = ListPlot[list2, PlotStyle → RGBColor[0, 1, 0],
   Joined → True, DisplayFunction → Identity];
G4 = ListPlot[list3, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.008]},
   Joined → True, DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[cusum] × Mean[
    Table[If[cusum[[i]] < LCL || cusum[[i]] > UCL, 1, 0], {i, 1, Length[cusum]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

	y_N	LCL	z_N	UCL
1	4	3	-1.	17
2	10	3	4.	17
3	5	3	4.	17
4	11	3	10.	17
5	2	3	7.	17
6	6	3	8.	17
7	2	3	5.	17
8	8	3	8.	17
9	8	3	11.	17
10	4	3	10.	17
11	5	3	10.	17

12	5	3	10.	17
13	5	3	10.	17
14	3	3	8.	17
15	4	3	7.	17
16	4	3	6.	17
17	8	3	9.	17
18	4	3	8.	17
19	7	3	10.	17
20	1	3	6.	17
21	4	3	5.	17
22	6	3	6.	17
23	7	3	8.	17
24	5	3	8.	17
25	6	3	9.	17
26	7	3	11.	17
27	8	3	14.	17
28	3	3	12.	17
29	6	3	13.	17
30	4	3	12.	17
31	6	3	13.	17
32	5	3	13.	17
33	5	3	13.	17
34	7	3	15.	17
35	9	3	19.	17
36	5	3	19.	17
37	8	3	22.	17
38	6	3	23.	17
39	6	3	24.	17
40	5	3	24.	17
41	4	3	23.	17
42	2	3	20.	17
43	8	3	23.	17
44	4	3	22.	17
45	5	3	22.	17
46	8	3	25.	17
47	6	3	26.	17
48	6	3	27.	17
49	1	3	23.	17
50	3	3	21.	17
51	5	3	21.	17
52	5	3	21.	17
53	7	3	23.	17
54	9	3	27.	17
55	4	3	26.	17
56	6	3	27.	17
57	9	3	31.	17
58	7	3	33.	17
59	6	3	34.	17
60	6	3	35.	17
61	6	3	36.	17
62	9	3	40.	17
63	5	3	40.	17
64	3	3	38.	17
65	6	3	39.	17
66	8	3	42.	17
67	4	3	41.	17
68	6	3	42.	17
69	4	3	41.	17
70	6	3	42.	17



There is (are) 37 observation(s) beyond the control limits.

Exercise 10.10

```
n = 10;
p = 0.5;
χ[z_] = PDF[BinomialDistribution[n, p], z];
k = 6;
LCL = 2;
UCL = 10;

(* The (infinite) probability transition matrix of the original Markov
   chain has entries P=[p_ij]_{i,j=...,LCL,...,UCL,...}= [p_{Bin(n,p)}(k+j-i)]_{i,j=...,LCL,...,UCL,...}*)

(* Q block of the probability transition matrix of the absorbing Markov chain *)
Q = Table[χ[k + j - i],
    {i, LCL, UCL}, {j, LCL, UCL}];
MatrixForm[Round[N[Q, 5] × 10 000] * 0.0001]
```

$$\begin{pmatrix} 0.2051 & 0.1172 & 0.0439 & 0.0098 & 0.001 & 0. & 0. & 0. & 0. \\ 0.2461 & 0.2051 & 0.1172 & 0.0439 & 0.0098 & 0.001 & 0. & 0. & 0. \\ 0.2051 & 0.2461 & 0.2051 & 0.1172 & 0.0439 & 0.0098 & 0.001 & 0. & 0. \\ 0.1172 & 0.2051 & 0.2461 & 0.2051 & 0.1172 & 0.0439 & 0.0098 & 0.001 & 0. \\ 0.0439 & 0.1172 & 0.2051 & 0.2461 & 0.2051 & 0.1172 & 0.0439 & 0.0098 & 0.001 \\ 0.0098 & 0.0439 & 0.1172 & 0.2051 & 0.2461 & 0.2051 & 0.1172 & 0.0439 & 0.0098 \\ 0.001 & 0.0098 & 0.0439 & 0.1172 & 0.2051 & 0.2461 & 0.2051 & 0.1172 & 0.0439 \\ 0. & 0.001 & 0.0098 & 0.0439 & 0.1172 & 0.2051 & 0.2461 & 0.2051 & 0.1172 \\ 0. & 0. & 0.001 & 0.0098 & 0.0439 & 0.1172 & 0.2051 & 0.2461 & 0.2051 \end{pmatrix}$$

Exercise 10.11

```
n = 80;
λ = 80 × 0.04;
dist = PoissonDistribution[λ];
χ[z_] = PDF[dist, z];
β[z_] = CDF[dist, z];
k = 2;
LCL = 0;
UCL = 2;

Q = Table[If[j == 0, β[k - i], χ[k + j - i]],
    {i, LCL, UCL}, {j, LCL, UCL}]; (* Check Equation (10.8). *)
MatrixForm[Round[N[Q, 5] × 10 000] * 0.0001]
```

$$\begin{pmatrix} 0.3799 & 0.2226 & 0.1781 \\ 0.1712 & 0.2087 & 0.2226 \\ 0.0408 & 0.1304 & 0.2087 \end{pmatrix}$$

Example 10.14 — CUSUM chart

Parameters

```
n = 100;
p0 = 0.02;
x = 6;
k = 3;
```

$$p_1 = \text{FindRoot}\left[\frac{n\ \text{Log}\left[\frac{1-p_0}{1-z}\right]}{\text{Log}\left[\frac{(1-p_0)\ z}{(1-z)\ p_0}\right]} == k,\ \{z,\ 1.1\ p_0,\ 0,\ 1\}\right];$$

```
p1 = {z} /. Dispatch[p1];
p1 = p1[[1]]
```

```
0.0427685
```

```
theta = {0, 0.001, 0.0025, 0.005, 0.0075, 0.01, p1 - p0, 0.03};
alpha = {0.05, 0.25, 0.5, 0.75, 0.9, 0.95};

χ[z_, δ_] = PDF[BinomialDistribution[n, δ], z];
β[z_, δ_] = CDF[BinomialDistribution[n, δ], z];
Q[δ_] = Table[If[j == 0, β[k - i, δ], χ[k + j - i, δ]],
   {i, 0, x}, {j, 0, x}];
```

Factorial moments — $E[RL^{(s)}]=E[RL\times(RL-1)\times\ldots\times(RL-s+1)]$

```
ID = IdentityMatrix[x + 1];

FM[i_, δ_, s_] := Apply[Plus, Factorial[s] × Table[If[j == i, 1, 0], {j, 0, x}] .
     MatrixPower[N[Q[δ], 10], s - 1].MatrixPower[Inverse[ID - N[Q[δ], 10]], s]];
```

4 Noncentral moments (in terms of the factorial moments) — $E[RL^s]$, s=1,...,4

```
NM[i_, δ_, 1] := FM[i, δ, 1];
NM[i_, δ_, 2] := FM[i, δ, 2] + FM[i, δ, 1];
NM[i_, δ_, 3] := FM[i, δ, 3] + 3 × FM[i, δ, 2] + FM[i, δ, 1];
NM[i_, δ_, 4] := FM[i, δ, 4] + 6 × FM[i, δ, 3] + 7 × FM[i, δ, 2] + FM[i, δ, 1];
```

3 Central moments (in terms of the factorial moments) — $E(\{RL-E(RL)\}^s)$, s=1,...,4

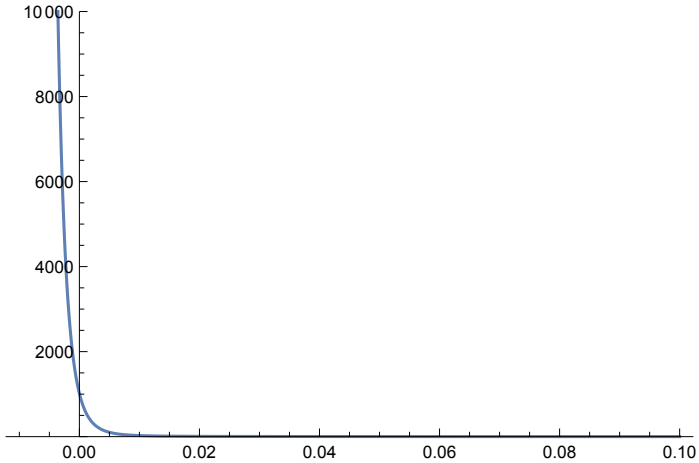
```
CM[i_, δ_, 2] := NM[i, δ, 2] - (NM[i, δ, 1])^2;
CM[i_, δ_, 3] := NM[i, δ, 3] - 3 × NM[i, δ, 2] × NM[i, δ, 1] + 2 × (NM[i, δ, 1])^3;
CM[i_, δ_, 4] := NM[i, δ, 4] - 4 × NM[i, δ, 3] × NM[i, δ, 1] +
  6 × NM[i, δ, 2] × (NM[i, δ, 1])^2 - 3 × (NM[i, δ, 1])^4;
```

ARL (Average Run Length)

```
ARL[i_, δ_] := NM[i, δ, 1];

ARL[0, p0 + 0.]
```

```
1015.71
```

```
Plot[ARL[0, p0 + δ], {δ, -.01, .1}, PlotRange → {0, 10000}]
```



SDRL (Standard Deviation of the Run Length)

```
SDRL[i_, δ_] := √CM[i, δ, 2] ;
```

CVRL (Coefficient of Variation of the Run Length)

```
CVRL[i_, δ_] := SDRL[i, δ] / ARL[i, δ];
```

CSRL (Coefficient of Skewness of the Run Length) — Assimetria

```
CSRL[i_, δ_] := CM[i, δ, 3] / (SDRL[i, δ])^3;
```

CKRL (Coefficient of Kurtosis of the Run Length) — Achatamento

```
CKRL[i_, δ_] := CM[i, δ, 4] / (SDRL[i, δ])^4 - 3;
```

All RL related measures — 0% HS

```
TableForm[Transpose[Table[{theta[[i]],
    ARL[0, p0 + theta[[i]]],
    SDRL[0, p0 + theta[[i]]],
    CVRL[0, p0 + theta[[i]]],
    CSRL[0, p0 + theta[[i]]],
    CKRL[0, p0 + theta[[i]]]}, {i, 1, Dimensions[theta][[1]]}]],
 TableAlignments -> {Right}]
```

0	0.001	0.0025	0.005	0.0075	0.01	0.0227685
1015.71	591.724	284.121	102.081	46.2269	25.4584	5.93204
1012.18	588.012	280.175	97.8955	42.0217	21.4193	3.32168
0.996523	0.993726	0.986113	0.959002	0.909031	0.841345	0.559956
1.99999	1.99996	1.99978	1.9979	1.98855	1.96056	1.52326
5.99995	5.99983	5.99911	5.99149	5.95264	5.83267	3.81437

$\alpha\times 100\%$ Percentage points of the Run Length — 0% HS

```
S[i_, m_, δ_] :=
    Apply[Plus,
   Table[If[j == i, 1, 0], {j, 0, x}] . MatrixPower[ N[Q[δ], 10], Floor[m]]];

MatrixForm[Transpose[Table[
        If[j == 1, m = 0];
        While[
    If[S[0, m, p0 + theta[[i]]] <= 1 - alpha[[j]], 1, 0] == 0, m++];
     m, {i, 1, Dimensions[theta][[1]]}, {j, 1, Dimensions[alpha][[1]]}]]]
```

$$\begin{pmatrix} 55 & 34 & 18 & 9 & 6 & 4 & 2 & 2 \\ 295 & 173 & 85 & 32 & 16 & 10 & 4 & 3 \\ 705 & 411 & 198 & 72 & 33 & 19 & 5 & 4 \\ 1407 & 819 & 392 & 140 & 63 & 34 & 7 & 5 \\ 2334 & 1358 & 649 & 230 & 101 & 53 & 10 & 7 \\ 3036 & 1765 & 843 & 297 & 130 & 68 & 12 & 8 \end{pmatrix}$$

Example 10.14 — np-chart

Parameters

```
n = 100;
p0 = 0.02;
k = 3;
UCL = 7;
```

$$p_1 = \text{FindRoot}\left[\frac{n\ \text{Log}\left[\frac{1-p_0}{1-z}\right]}{\text{Log}\left[\frac{(1-p_0)\ z}{(1-z)\ p_0}\right]} == k,\ \{z,\ 1.1\ p_0,\ 0,\ 1\}\right];$$

```
p1 = {z} /. Dispatch[p1];
p1 = p1[[1]]

theta = {0., 0.001, 0.0025, 0.005, 0.0075, 0.01, 0.02, p1 - p0, 0.03};
```

0.0427685

Parameter of RL

```
β[dim_, δ_] = 1 - CDF[BinomialDistribution[dim, δ], UCL];
```

ARL (Average Run Length)

$$\text{ARL}[\text{dim_},\ \delta_] := \frac{1}{N[\beta[\text{dim},\ \delta],\ 10]};$$

SDRL (Standard Deviation of the Run Length)

$$\text{SDRL}[\text{dim_},\ \delta_] := \frac{\sqrt{1 - N[\beta[\text{dim},\ \delta],\ 10]}}{N[\beta[\text{dim},\ \delta],\ 10]};$$

CVRL (Coefficient of Variation of the Run Length)

$$\text{CVRL}[\text{dim_},\ \delta_] := \sqrt{1 - N[\beta[\text{dim},\ \delta],\ 10]};$$

CSRL (Coefficient of Skewness of the Run Length) — Assimetria

```
CSRL[dim_, δ_] := (2 - N[β[dim, δ], 10]) / Sqrt[1 - N[β[dim, δ], 10]];
```

CKRL (Coefficient of Kurtosis of the Run Length) — Achatamento

```
CKRL[dim_, δ_] := 4 + 1/(1 - N[β[dim, δ], 10]) + 1 - N[β[dim, δ], 10];
```

All RL related measures

```
TableForm[Transpose[Table[{theta[[i]],
    ARL[n, p0 + theta[[i]]],
    SDRL[n, p0 + theta[[i]]],
    CVRL[n, p0 + theta[[i]]],
    CSRL[n, p0 + theta[[i]]],
    CKRL[n, p0 + theta[[i]]]}, {i, 1, Dimensions[theta][[1]]}]],
 TableAlignments -> {Right}]
```

0.	0.001	0.0025	0.005	0.0075	0.01	0.02	0.0227685	0.03
1073.03	787.737	512.346	270.112	154.275	94.1282	21.0473	15.3692	7.81491
1072.53	787.237	511.846	269.611	153.774	93.6269	20.5412	14.8608	7.2978
0.999534	0.999365	0.999024	0.998147	0.996754	0.994674	0.975955	0.96692	0.933831
2.	2.	2.	2.	2.00001	2.00003	2.00059	2.00113	2.00469
6.	6.	6.	6.00001	6.00004	6.00011	6.00237	6.00453	6.01878

Survival Function

```
S[m_, dim_, δ_] := Power[1 - N[β[dim, δ], 10], If[m < 0, 0, Floor[m]]];
```

$\alpha\times$100% Percentage points of the Run Length

```
alpha = {0.05, 0.25, 0.5, 0.75, 0.9, 0.95};

MatrixForm[Transpose[Table[
        If[j == 1, m = 0];
        While[
    If[S[m, n, p0 + theta[[i]]] <= 1 - alpha[[j]], 1, 0] == 0, m++];
     m, {i, 1, Dimensions[theta][[1]]}, {j, 1, Dimensions[alpha][[1]]}]]]
```

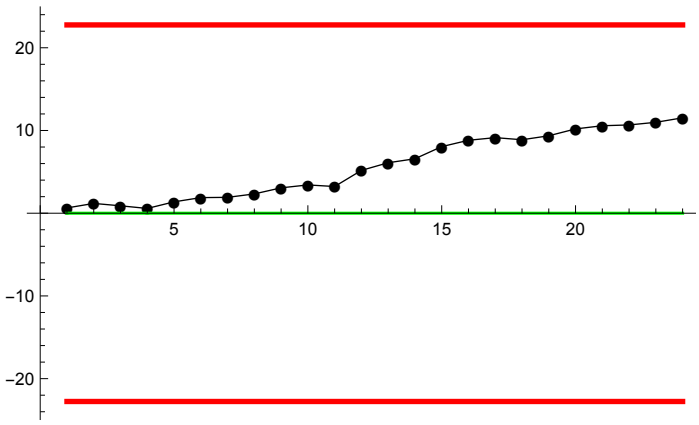
$$\begin{pmatrix} 56 & 41 & 27 & 14 & 8 & 5 & 2 & 1 & 1 \\ 309 & 227 & 148 & 78 & 45 & 27 & 6 & 5 & 3 \\ 744 & 546 & 355 & 187 & 107 & 65 & 15 & 11 & 6 \\ 1487 & 1092 & 710 & 374 & 214 & 130 & 29 & 21 & 11 \\ 2470 & 1813 & 1179 & 621 & 355 & 216 & 48 & 35 & 17 \\ 3214 & 2359 & 1534 & 808 & 461 & 281 & 62 & 45 & 22 \end{pmatrix}$$

Exercise 10.17(a)

```
data = {34.5, 34.2, 31.6, 31.5, 35.0, 34.1, 32.6, 33.8, 34.8, 33.6, 31.9, 38.6,
   35.4, 34.0, 37.1, 34.9, 33.5, 31.7, 34.0, 35.1, 33.7, 32.8, 33.5, 34.2};
Length[data]

n = 5;
μ0 = 32.5;
σ0 = √49 ;
k = μ0;
h = 22.7610;
(* The in-control ARL of this standard CUSUM chart for μ is approximately 500. Its control statistic is a
   deviation between the sample mean and the target value (measured in in-control standard deviations of x̄). *)
LCL = -h;
CL = 0;
UCL = h;

z[0] = 0;
Do[z[i] = z[i - 1] + (data[[i]] - k)/(σ0/√n), {i, 1, Length[data]}]
cusum = Table[z[i], {i, 1, Length[data]}];

list1 = Table[LCL, {i, 1, Length[data]}];
list2 = Table[CL, {i, 1, Length[data]}];
list3 = Table[UCL, {i, 1, Length[data]}];

TableForm[Transpose[{data, list1, cusum, list3}],
 TableHeadings → {Automatic, {"yN", "LCL", "zN", "UCL"}}]

G1 = ListPlot[cusum, Joined → True, PlotStyle → {Black, Thickness[0.002]},
   PlotMarkers → Automatic, PlotRange → {-25, 25}, DisplayFunction → Identity];
G2 = ListPlot[list1, Joined → True, PlotStyle →
    {RGBColor[1, 0, 0], Thickness[0.008]}, DisplayFunction → Identity];
G3 = ListPlot[list2, Joined → True, PlotStyle → RGBColor[0, 1, 0],
   DisplayFunction → Identity];
G4 = ListPlot[list3, Joined → True, PlotStyle →
    {RGBColor[1, 0, 0], Thickness[0.008]}, DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[cusum] × Mean[
    Table[If[cusum[[i]] < LCL || cusum[[i]] > UCL, 1, 0], {i, 1, Length[cusum]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

24

	y_N	LCL	z_N	UCL
1	34.5	-22.761	0.638877	22.761
2	34.2	-22.761	1.18192	22.761
3	31.6	-22.761	0.894427	22.761
4	31.5	-22.761	0.574989	22.761
5	35.	-22.761	1.37358	22.761
6	34.1	-22.761	1.88469	22.761
7	32.6	-22.761	1.91663	22.761
8	33.8	-22.761	2.3319	22.761
9	34.8	-22.761	3.06661	22.761
10	33.6	-22.761	3.41799	22.761
11	31.9	-22.761	3.22633	22.761
12	38.6	-22.761	5.1749	22.761
13	35.4	-22.761	6.10127	22.761
14	34.	-22.761	6.58043	22.761
15	37.1	-22.761	8.04984	22.761
16	34.9	-22.761	8.8165	22.761
17	33.5	-22.761	9.13593	22.761
18	31.7	-22.761	8.88038	22.761
19	34.	-22.761	9.35954	22.761
20	35.1	-22.761	10.1901	22.761
21	33.7	-22.761	10.5734	22.761
22	32.8	-22.761	10.6692	22.761
23	33.5	-22.761	10.9887	22.761
24	34.2	-22.761	11.5317	22.761



There are no observations beyond the control limits.

Exercise 10.17(b)

Sample size and distribution function of the standard normal and the chi-square with (n-1) degrees of freedom

```
n = 5;
Φ[z_] = CDF[NormalDistribution[0, 1], z];
χ[z_] = CDF[ChiSquareDistribution[n - 1], z];
```

Definition of the sub-stochastic matrix $Q_\mu(\delta, \theta)$ of the chart CUSUM-μ

```
hμ = 22.7610;
kμ = 0.0;
xμ = 20;

Aμ[δ_, θ_] = Table[
  Φ[1/θ × ((2 hμ ((j + 1) - (i + 1/2)))/(2 xμ + 1) - δ)], {i, -xμ, xμ}, {j, -xμ - 1, xμ}];

Tμ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, -xμ, xμ + 1}, {j, -xμ, xμ}];

Qμ[δ_, θ_] = Aμ[δ, θ].Tμ;
```

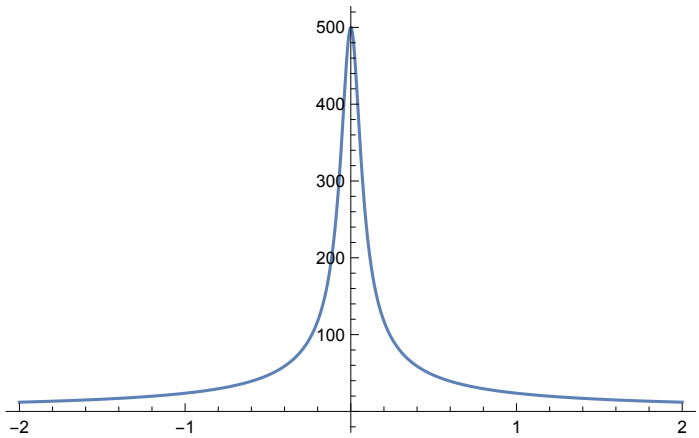
Definition of $ARL^{i}_{C-\mu}(\delta, \theta)$

```
IDμ = IdentityMatrix[2 xμ + 1];
Mμ[δ_, θ_] := Inverse[IDμ - Qμ[δ, θ]];
umμ = Table[1, {i, -xμ, xμ}];
bμ[i_] = Table[If[j == i, 1, 0], {j, -xμ, xμ}];
ARLμ[i_, δ_, θ_] := bμ[i] . Mμ[δ, θ] . umμ;
```

In-control $ARL^{0}_{C-\mu}(\delta, \theta)$ and plot of $ARL^{0}_{C-\mu}(\delta, 1)$

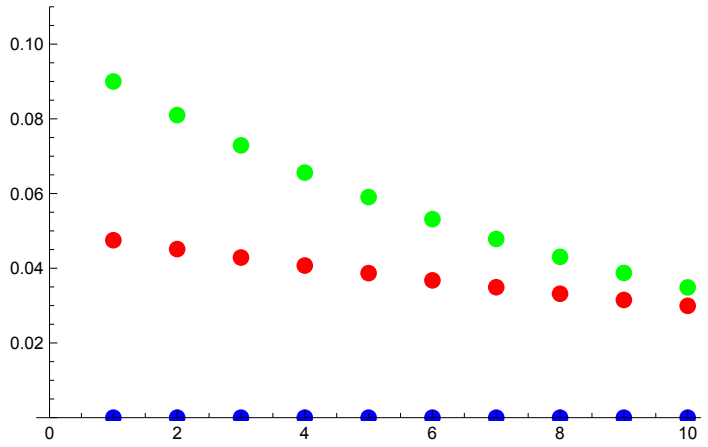
```
ARLμ[0, 0, 1]
Plot[ARLμ[0, δ, 1], {δ, -2, 2}, PlotRange → Full, AxesOrigin → {0, 0}]
```

500.001

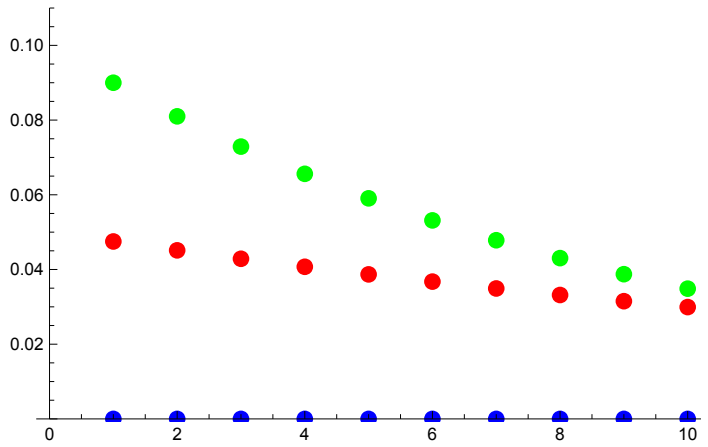


Exercise 10.21

```
λ = {0.05, 0.1, 1.};
G1 = ListPlot[Table[λ[[1]] (1 - λ[[1]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[1, 0, 0], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
G2 = ListPlot[Table[λ[[2]] (1 - λ[[2]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[0, 1, 0], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
G3 = ListPlot[Table[λ[[3]] (1 - λ[[3]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[0, 0, 1], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
Show[G1, G2, G3, DisplayFunction → $DisplayFunction]
```



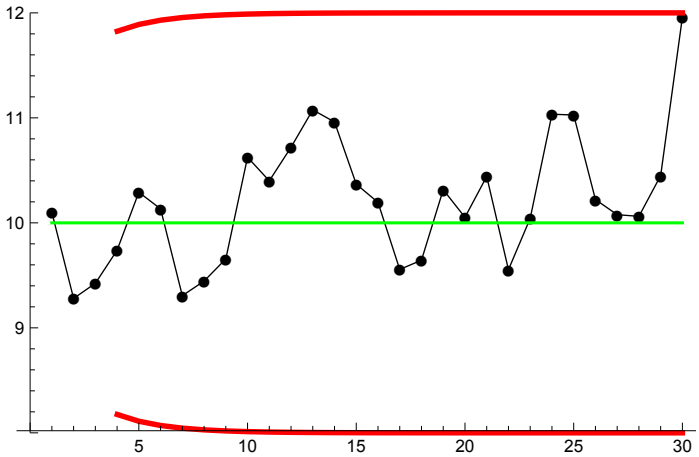

```
λ = {0.05, 0.1, 1.};
G1 = ListPlot[Table[λ[[1]] × (1 - λ[[1]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[1, 0, 0], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
G2 = ListPlot[Table[λ[[2]] × (1 - λ[[2]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[0, 1, 0], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
G3 = ListPlot[Table[λ[[3]] × (1 - λ[[3]])^j, {j, 1, 10}],
    PlotStyle → {RGBColor[0, 0, 1], PointSize[0.025]},
    PlotRange → {0, .11}, DisplayFunction → Identity];
Show[G1, G2, G3, DisplayFunction → $DisplayFunction]
```



Exercise 10.25

```
μ0 = 10.0;
w0 = μ0;
λ = 0.2;
s = 2;(*target standard deviation of the sample mean*)
(*σ0=2;*)
γ = 3;
(*n=Length[data];*)

data = {10.5, 6, 10, 11, 12.5, 9.5, 6, 10, 10.5, 14.5, 9.5, 12,
  12.5, 10.5, 8, 9.5, 7, 10, 13, 9, 12, 6, 12, 15, 11, 7, 9.5, 10, 12, 18};
W = Function[N, (1 - λ)^N × w0 + Sum[λ × (1 - λ)^j × data[[N - j]], {j, 0, N-1}]]; (* Check Equation (10.19). *)
ewma = Table[W[i], {i, 1, Length[data]}];

LCLexact = Table[μ0 - γ × s × Sqrt[λ × (1 - (1 - λ)^(2 N)) / (2 - λ)], {N, 1, Length[data]}];
CL = Table[μ0, {N, 1, Length[data]}];
UCLexact = Table[μ0 + γ × s × Sqrt[λ × (1 - (1 - λ)^(2 N)) / (2 - λ)], {N, 1, Length[data]}];

TableForm[Transpose[{data, LCLexact, ewma, UCLexact}],
 TableHeadings → {Automatic, {"x̄N", "LCLN", "wN", "UCLN"}}]

G1 = ListPlot[ewma, Joined → True, PlotStyle → {Black, Thickness[0.002]},
   PlotMarkers → Automatic, PlotRange → {8, 12}, DisplayFunction → Identity];
G2 = ListPlot[LCLexact, Joined → True, PlotStyle →
    {RGBColor[1, 0, 0], Thickness[0.008]}, DisplayFunction → Identity];
G3 = ListPlot[CL, Joined → True, PlotStyle → RGBColor[0, 1, 0],
   DisplayFunction → Identity];
G4 = ListPlot[UCLexact, Joined → True, PlotStyle →
    {RGBColor[1, 0, 0], Thickness[0.008]}, DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[ewma] ×
   Mean[Table[If[ewma[[i]] < LCLexact[[i]] || ewma[[i]] > UCLexact[[i]], 1, 0],
     {i, 1, Length[ewma]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

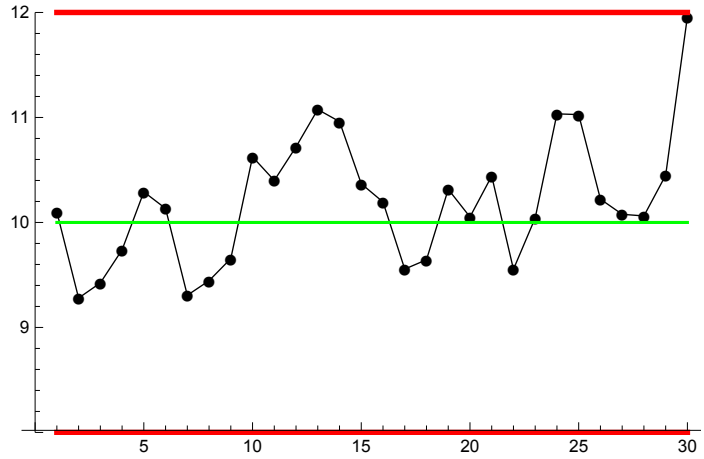
	$\overline{x}_N$	LCL_N	w_N	UCL_N
1	10.5	8.8	10.1	11.2
2	6	8.46325	9.28	11.5367
3	10	8.28203	9.424	11.718
4	11	8.17547	9.7392	11.8245
5	12.5	8.11042	10.2914	11.8896
6	9.5	8.06994	10.1331	11.9301
7	6	8.04447	9.30647	11.9555
8	10	8.02835	9.44518	11.9717
9	10.5	8.0181	9.65614	11.9819
10	14.5	8.01156	10.6249	11.9884
11	9.5	8.00739	10.3999	11.9926
12	12	8.00473	10.7199	11.9953
13	12.5	8.00302	11.076	11.997
14	10.5	8.00194	10.9608	11.9981
15	8	8.00124	10.3686	11.9988
16	9.5	8.00079	10.1949	11.9992
17	7	8.00051	9.55591	11.9995
18	10	8.00032	9.64473	11.9997
19	13	8.00021	10.3158	11.9998
20	9	8.00013	10.0526	11.9999
21	12	8.00009	10.4421	11.9999
22	6	8.00005	9.55368	11.9999
23	12	8.00003	10.0429	12.
24	15	8.00002	11.0344	12.
25	11	8.00001	11.0275	12.
26	7	8.00001	10.222	12.
27	9.5	8.00001	10.0776	12.
28	10	8.	10.0621	12.
29	12	8.	10.4497	12.
30	18	8.	11.9597	12.



There are no observations beyond the control limits.

```
LCLasympt = Table[μ0 - γ × s × Sqrt[λ/(2 - λ)], {N, 1, Length[data]}];
CL = Table[μ0, {N, 1, Length[data]}];
UCLasympt = Table[μ0 + γ × s × Sqrt[λ/(2 - λ)], {N, 1, Length[data]}];

TableForm[Transpose[{data, LCLasympt, ewma, UCLasympt}],
 TableHeadings → {Automatic, {"x̄N", "LCLa", "wN", "UCLa"}}]

G1 = ListPlot[ewma, Joined → True, PlotStyle → {Black, Thickness[0.002]},
   PlotMarkers → Automatic, PlotRange → {8, 12}, DisplayFunction → Identity];
G2 = ListPlot[LCLasympt, Joined → True, PlotStyle →
    {RGBColor[1, 0, 0], Thickness[0.008]}, DisplayFunction → Identity];
G3 = ListPlot[CL, Joined → True, PlotStyle → RGBColor[0, 1, 0],
   DisplayFunction → Identity];
G4 = ListPlot[UCLasympt, Joined → True, PlotStyle →
    {RGBColor[1, 0, 0], Thickness[0.008]}, DisplayFunction → Identity];
Show[G1, G2, G3, G4, DisplayFunction → $DisplayFunction]

numberobsbeyondcontrollimits = Length[ewma] ×
   Mean[Table[If[ewma[[i]] < LCLasympt[[i]] || ewma[[i]] > UCLasympt[[i]], 1, 0],
     {i, 1, Length[ewma]}]];
If[numberobsbeyondcontrollimits > 0, Print["There is (are) ",
  numberobsbeyondcontrollimits, " observation(s) beyond the control limits."],
 Print["There are no observations beyond the control limits."]]
```

	$\bar{x}_N$	LCL_a	w_N	UCL_a
1	10.5	8.	10.1	12.
2	6	8.	9.28	12.
3	10	8.	9.424	12.
4	11	8.	9.7392	12.
5	12.5	8.	10.2914	12.
6	9.5	8.	10.1331	12.
7	6	8.	9.30647	12.
8	10	8.	9.44518	12.
9	10.5	8.	9.65614	12.
10	14.5	8.	10.6249	12.
11	9.5	8.	10.3999	12.
12	12	8.	10.7199	12.
13	12.5	8.	11.076	12.
14	10.5	8.	10.9608	12.
15	8	8.	10.3686	12.
16	9.5	8.	10.1949	12.
17	7	8.	9.55591	12.
18	10	8.	9.64473	12.
19	13	8.	10.3158	12.
20	9	8.	10.0526	12.
21	12	8.	10.4421	12.
22	6	8.	9.55368	12.
23	12	8.	10.0429	12.
24	15	8.	11.0344	12.
25	11	8.	11.0275	12.
26	7	8.	10.222	12.
27	9.5	8.	10.0776	12.
28	10	8.	10.0621	12.
29	12	8.	10.4497	12.
30	18	8.	11.9597	12.



There are no observations beyond the control limits.

Exercise 10.35

Upper one-sided Shewhart chart

```
Φ[z_] = CDF[NormalDistribution[0, 1], z];
n = 5;
pfaμ = 1 / 500.0;
γS-μ = Quantile[NormalDistribution[0, 1], 1 - pfaμ];
ARLS-μ[δ_, θ_] = 1 / (1 - Φ[(γS-μ - δ) / θ]);
ARLS-μ[0, 1]
```

500.

Upper one-sided EWMA chart (0% Head Start)

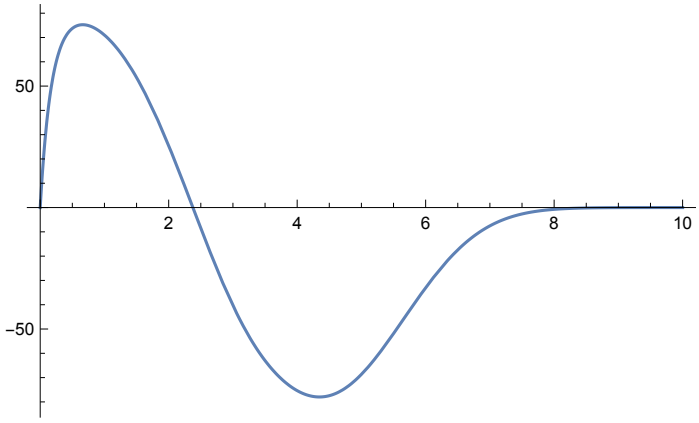
```
γE-μ = 2.8116;
λμ = 0.134;
xμ = 40;
Φ[z_] = CDF[NormalDistribution[0, 1], z];
n = 5;

Aμ[δ_, θ_] = Table[
   If[j == -1, 0, Φ[1/θ ((γE-μ ((j + 1) - (1 - λμ) (i + 1/2)))/((xμ + 1) Sqrt[λμ (2 - λμ)]) - δ)]],
   {i, 0, xμ}, {j, -1, xμ}];

Tμ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, 0, xμ + 1}, {j, 0, xμ}];

Qμ[δ_, θ_] = Aμ[δ, θ].Tμ;

IDμ = IdentityMatrix[xμ + 1];
Mμ[δ_, θ_] := Inverse[IDμ - Qμ[δ, θ]];
umμ = Table[1, {i, 0, xμ}];
ARLE-μ[i_, δ_, θ_] := Table[If[j == i, 1, 0], {j, 0, xμ}] . Mμ[δ, θ] . umμ;
ARLE-μ[0, 0, 1]

shift =
  {0, 0.05, 0.10, 0.20, 0.30, 0.40, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 3.0};

TableForm[Table[{shift[[i]], ARLE-μ[0, shift[[i]], 1], ARLS-μ[shift[[i]], 1]},
  {i, 1, Length[shift]}], TableHeadings → {Automatic, {"δ", "ARLE-μ", "ARLS-μ"}}]
(* ARL values for the EWMA and Shewhart charts *)
```

500.047

	δ	$ARL_{E-\mu}$	$ARL_{S-\mu}$
1	0	500.047	500.
2	0.05	342.199	427.203
3	0.1	239.007	365.849
4	0.2	124.546	270.17
5	0.3	71.1556	201.354
6	0.4	44.4944	151.445
7	0.5	30.1927	114.948
8	0.6	21.9546	88.04
9	0.7	16.8793	68.0411
10	0.8	13.5571	53.0582
11	0.9	11.2646	41.7445
12	1.	9.61043	33.1351
13	1.5	5.52942	11.8939
14	2.	3.92261	5.26515
15	3.	2.55816	1.8232

EWMA vs. Shewhart chart (percentual reduction in the ARL when the Shewhart chartis replaced by a EWMA chart)

```
Plot[(1 - ARLE-μ[0, δ, 1] / ARLS-μ[δ, 1]) × 100, {δ, 0, 10}]
```



Exercise 10.37

Upper one-sided EWMA chart (0% Head Start)

```
n = 5;
sigma0 = 1;
λσ = 0.05;
LCLE-σ = 0;
UCLE-σ = 0.157079;
```

$$\gamma_{E-\sigma} = \frac{UCL_{E-\sigma} - Log\left[\left(sigma_0\right)^2\right]}{\sqrt{PolyGamma\left[1, \frac{n-1}{2}\right] \times \frac{\lambda_\sigma}{2-\lambda_\sigma}}};$$

```
χ[z_] = CDF[ChiSquareDistribution[n - 1], z];
xσ = 20;
Aσ[θ_] = Table[If[j == -1, 0,
    χ[(n - 1)/θ^2 ×
      Exp[(γE-σ × Sqrt[PolyGamma[1, (n - 1)/2]] ((j + 1) - (1 - λσ) (i + 1/2)))/((xσ + 1) Sqrt[λσ (2 - λσ)])]
    ]], {i, 0, xσ}, {j, -1, xσ}];

Tσ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, 0, xσ + 1}, {j, 0, xσ}];

Qσ[θ_] = Aσ[θ].Tσ;

IDσ = IdentityMatrix[xσ + 1];
Mσ[θ_] := Inverse[IDσ - Qσ[θ]];
umσ = Table[1, {i, 0, xσ}];
bσ[i_] = Table[If[j == i, 1, 0], {j, 0, xσ}];
ARLE-σ[i_, θ_] := bσ[i]. Mσ[θ] . umσ;

ARLE-σ[0, 1]
```

370.411

Upper one-sided Shewhart chart

```
UCLS-σ = 4.06285;
```

$$\gamma_{S-\sigma} = UCL_{S-\sigma} \times \frac{n-1}{1};$$

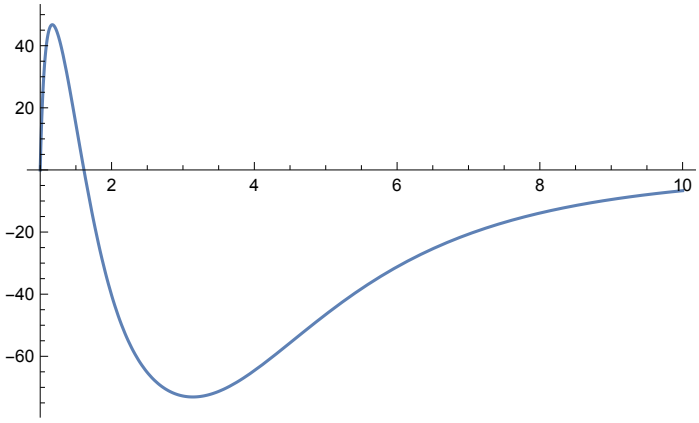
$$ARL_{S-\sigma}[\theta_] = \frac{1}{1 - \chi\left[\frac{\gamma_{S-\sigma}}{\theta^2}\right]};$$

```
ARLS-σ[1]
```

370.408

EWMA vs. Shewhart chart (percentual reduction in the ARL when the Shewhart chartis replaced by a EWMA chart)

```
Plot[(1 - ARL_{E-σ}[0, θ]/ARL_{S-σ}[θ]) × 100, {θ, 1, 10}, AxesOrigin → {1, 0}]
```



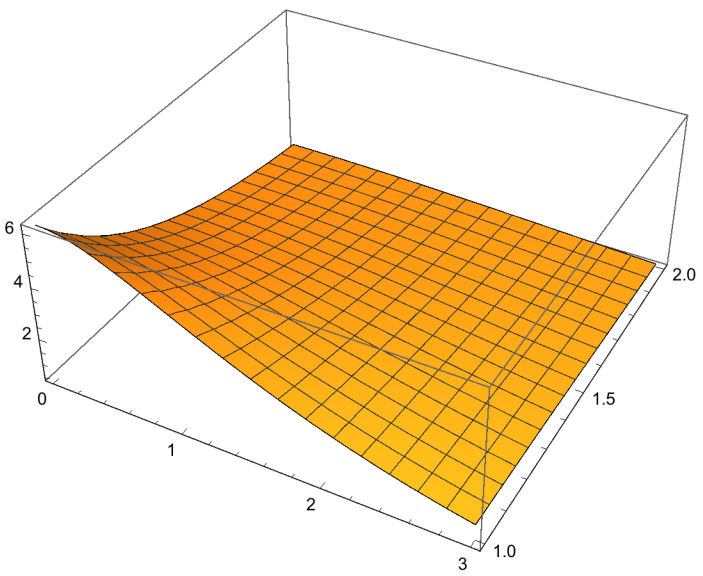
Exercise 10.38

```
n = 5;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfaμ = 1 / 1000.;
γμ = Quantile[NormalDistribution[0, 1], 1 - pfaμ/2];

pfaσ = 1/1000.;
γσ = Quantile[ChiSquareDistribution[n - 1], 1 - pfaσ];
ARLμ[δ_, θ_] = 1/(1 - (Φ[(γμ - δ)/θ] - Φ[(-γμ - δ)/θ]));
ARLσ[θ_] = 1/(1 - χ[γσ/θ^2]);
ARLμ,σ[δ_, θ_] = 1/(1 - (Φ[(γμ - δ)/θ] - Φ[(-γμ - δ)/θ]) × χ[γσ/θ^2]);
ARLμ[0, 1]
ARLσ[1]
ARLμ,σ[0, 1]
Plot3D[Log[ARLμ,σ[δ, θ]], {δ, 0, 3}, {θ, 1, 2}]
```

1000.

1000.

500.25



```
ARLμ,σ[δ_, θ_] = (ARLμ[δ, θ] × ARLσ[θ]) / (ARLμ[δ, θ] + ARLσ[θ] - 1);
(* Relating the ARL of the joint scheme and ssthe ones of the individual charts... *)

ARLμ[0, 1]
ARLσ[1]
ARLμ,σ[0, 1]
```

1000.

1000.

500.25

Exercise 10.39

```
n = 10;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

datamu = {1.04, 1.06, 1.09, 1.05, 1.07, 1.06, 1.05, 1.10, 1.09,
   1.05, 0.99, 1.06, 1.05, 1.07, 1.11, 1.04, 1.03, 1.05, 1.06, 1.04};
datasigma = {0.87, 0.85, 0.90, 0.85, 0.73, 0.80, 0.78, 0.83, 0.87, 0.860  .86,
   0.79, 0.82, 0.75, 0.76, 0.89, 0.91, 0.85, 0.83, 0.79, 0.85};

pfamu = 0.002;
γμ = Quantile[NormalDistribution[0, 1], 1 - pfamu/2];
pfasigma = 0.002;
γσ = Quantile[ChiSquareDistribution[n - 1], 1 - pfasigma];

μ0 = Mean[datamu];
σ0 = Sqrt[Mean[datasigma]];
LCLμ = μ0 - γμ × σ0/Sqrt[n]
UCLμ = μ0 + γμ × σ0/Sqrt[n]
LCLσ = 0
UCLσ = σ0^2/(n - 1) × γσ
```

0.171486

1.94451

0

2.38266

```
probsignalμ,σ[δ_, θ_] = 1 - (Φ[(γμ - δ)/θ] - Φ[(-γμ - δ)/θ]) × χ[γσ/θ^2];
probsignalμ,σ[0, 1]
```

0.003996

```
probvalidsignal = probsignal_{μ,σ}[0.1, 1.1];
dist = GeometricDistribution[probvalidsignal] ;
(* geometric distribution for the number of trials before the first success,
where the probability of success in a trial is p *)
CDF[dist, 10 - 1]
```

0.145223

$$\mathbf{ARL}_{\mu}[\delta_, \theta_] = \frac{1}{1 - \left(\Phi\left[\frac{\gamma_u - \delta}{\theta}\right] - \Phi\left[\frac{-\gamma_u - \delta}{\theta}\right]\right)};$$

$$\mathbf{ARL}_{\sigma}[\theta_] = \frac{1}{1 - \chi\left[\frac{\gamma_a}{\theta^2}\right]};$$

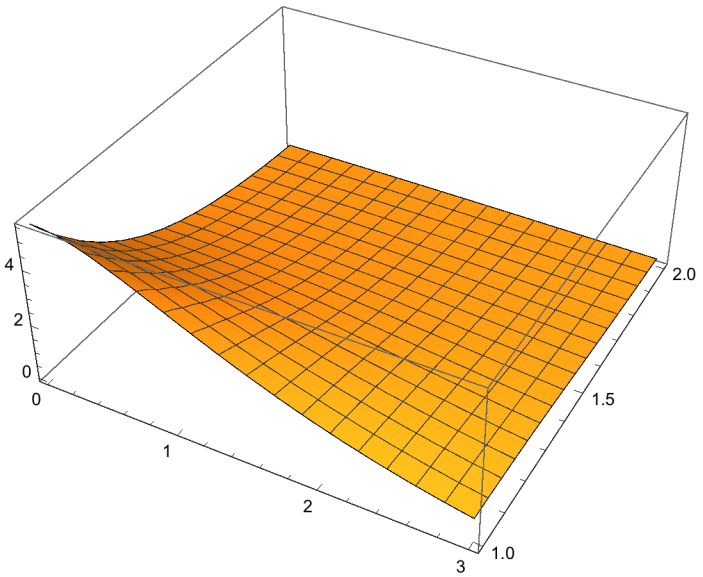
$$\mathbf{ARL}_{\mu,\sigma}[\delta_, \theta_] = \frac{\mathbf{ARL}_{\mu}[\delta, \theta] \times \mathbf{ARL}_{\sigma}[\theta]}{\mathbf{ARL}_{\mu}[\delta, \theta] + \mathbf{ARL}_{\sigma}[\theta] - 1};$$

```
ARL_μ[0, 1]
ARL_σ[1]
ARL_{μ,σ}[0, 1]
Plot3D[Log[ARL_{μ,σ}[δ, θ]], {δ, 0, 3}, {θ, 1, 2}]
```

500.

500.

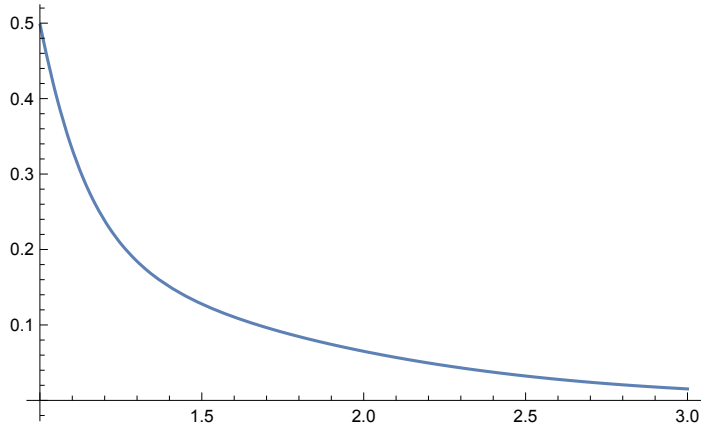
250.25



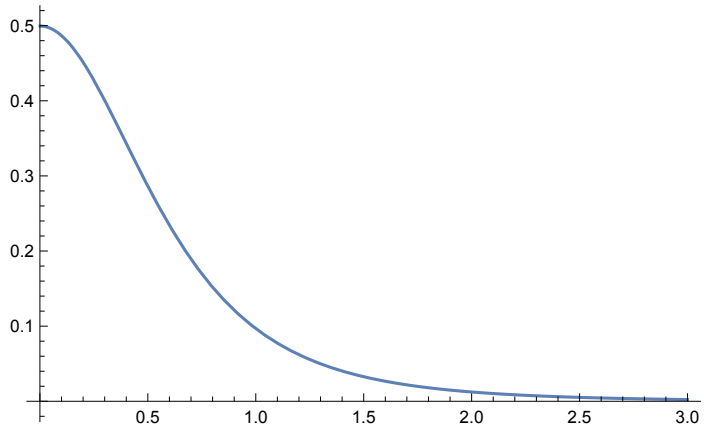
Exercise 10.44 (with n=9, $ARL_\mu(0,1)=ARL_\sigma(1)=500$)

```
n = 9;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfaμ = 1 / 500.0;
γμ = Quantile[NormalDistribution[0, 1], 1 - pfaμ/2];

pfaσ = 1 / 500.0;
γσ = Quantile[ChiSquareDistribution[n - 1], 1 - pfaσ];

PMSIII[θ_] = (1 - (Φ[γμ/θ] - Φ[-γμ/θ])) / (1/χ[γσ/θ^2] - (Φ[γμ/θ] - Φ[-γμ/θ]));
theta = {1.01, 1.03, 1.05, 1.10, 1.20,
   1.30, 1.40, 1.50, 1.60, 1.70, 1.80, 1.90, 2.00, 3.00};
TableForm[Table[{theta[[i]], PMSIII[theta[[i]]]}, {i, 1, Length[theta]}],
 TableHeadings → {Automatic, {"θ", "PMSIII"}}]
Plot[{PMSIII[θ]}, {θ, 1.001, 3}, AxesOrigin -> {1, 0}]


PMSIV[δ_] = (1 - χ[γσ]) / (1/(Φ[γμ-δ]-Φ[-γμ-δ]) - χ[γσ]);
delta =
  {0.05, 0.10, 0.20, 0.30, 0.40, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 3.0};
TableForm[Table[{delta[[i]], PMSIV[delta[[i]]]}, {i, 1, Length[delta]}],
 TableHeadings → {Automatic, {"δ", "PMSIV"}}]

Plot[{PMSIV[δ]}, {δ, 0.001, 3}, AxesOrigin -> {0, 0}]
```

	θ	PMS_{III}
1	1.01	0.478689
2	1.03	0.439872
3	1.05	0.404805
4	1.1	0.332453
5	1.2	0.238189
6	1.3	0.184291
7	1.4	0.150771
8	1.5	0.127747
9	1.6	0.110428
10	1.7	0.096442
11	1.8	0.0845854
12	1.9	0.0742359
13	2.	0.0650638
14	3.	0.0152266



	δ	PMS_{IV}
1	0.05	0.496258
2	0.1	0.48673
3	0.2	0.451344
4	0.3	0.400673
5	0.4	0.343289
6	0.5	0.286308
7	0.6	0.234262
8	0.7	0.189271
9	0.8	0.151773
10	0.9	0.121258
11	1.	0.0967965
12	1.5	0.0326784
13	2.	0.0123586
14	3.	0.00230455



Exercise 10.45

```
n = 5;
Φ[x_] = CDF[NormalDistribution[0, 1], x];
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfaμ = 1 / 500.0;
γμ = Quantile[NormalDistribution[0, 1], 1 - pfaμ];

pfaσ = 1 / 500.0;
γσ = Quantile[ChiSquareDistribution[n - 1], 1 - pfaσ];
```

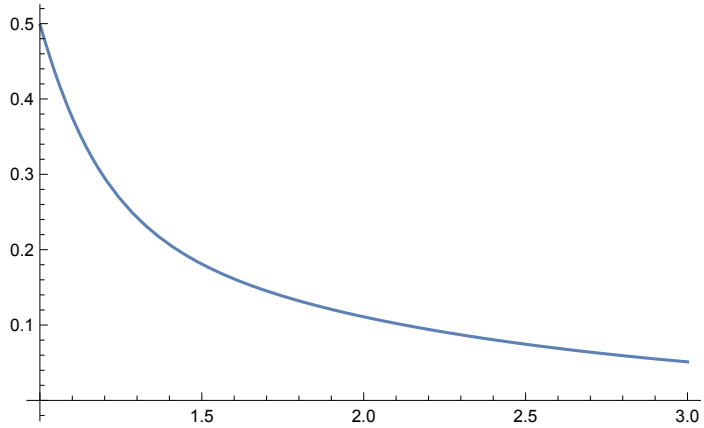
$$\text{PMS}_{\text{III}}[\theta_] = \frac{1 - \Phi\left[\frac{\gamma_\mu}{\theta}\right]}{\frac{1}{\chi\left[\frac{\gamma_\sigma}{\theta^2}\right]} - \Phi\left[\frac{\gamma_\mu}{\theta}\right]};$$

```
theta = {1.01, 1.03, 1.05, 1.10, 1.20,
   1.30, 1.40, 1.50, 1.60, 1.70, 1.80, 1.90, 2.00, 3.00};
TableForm[Table[{theta[[i]], PMSIII[theta[[i]]]}, {i, 1, Length[theta]}],
 TableHeadings → {Automatic, {"θ", "PMSIII"}}]
Plot[{PMSIII[θ]}, {θ, 1.001, 3}, AxesOrigin -> {1, 0}]
```

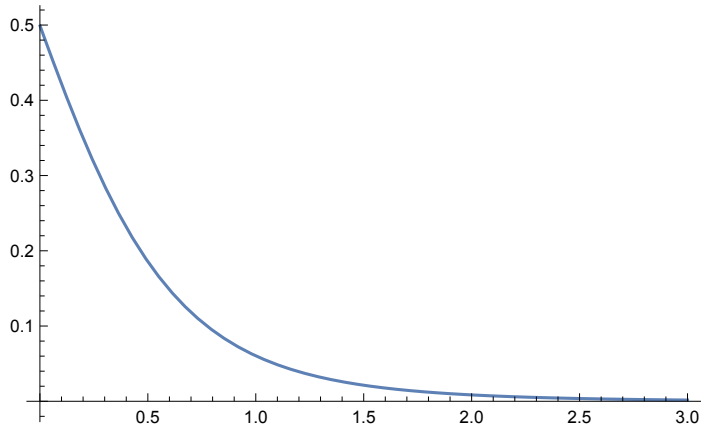
$$\text{PMS}_{\text{IV}}[\delta_] = \frac{1 - \chi[\gamma_\sigma]}{\frac{1}{\Phi[\gamma_\mu - \delta]} - \chi[\gamma_\sigma]};$$

```
delta =
  {0.05, 0.10, 0.20, 0.30, 0.40, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 3.0};
TableForm[Table[{delta[[i]], PMSIV[delta[[i]]]}, {i, 1, Length[delta]}],
 TableHeadings → {Automatic, {"δ", "PMSIV"}}]

Plot[{PMSIV[δ]}, {δ, 0.001, 3}, AxesOrigin -> {0, 0}]
```

	θ	PMS_{III}
1	1.01	0.484676
2	1.03	0.456701
3	1.05	0.430911
4	1.1	0.375334
5	1.2	0.295048
6	1.3	0.242637
7	1.4	0.206805
8	1.5	0.180893
9	1.6	0.161108
10	1.7	0.14527
11	1.8	0.132095
12	1.9	0.120806
13	2.	0.11092
14	3.	0.05117



	δ	PMS_{IV}
1	0.05	0.460162
2	0.1	0.421864
3	0.2	0.349949
4	0.3	0.286075
5	0.4	0.231295
6	0.5	0.185599
7	0.6	0.148269
8	0.7	0.11823
9	0.8	0.0942984
10	0.9	0.075349
11	1.	0.0603889
12	1.5	0.0213232
13	2.	0.00845816
14	3.	0.00164369



Sample size and distribution function of the standard normal and the chi-square with (n-1) degrees of freedom

```
n = 5;
Φ[z_] = CDF[NormalDistribution[0, 1], z];
χ[z_] = CDF[ChiSquareDistribution[n - 1], z];
```

Definition of the sub-stochastic matrix $Q_\mu(\delta, \theta)$ of the chart EWMA-μ

```
γμ = 2.8116;
λμ = 0.134;
xμ = 40;

Aμ[δ_, θ_] = Table[
  If[j == -1, 0, Φ[1/θ × ((γμ ((j + 1) - (1 - λμ) (i + 1/2)))/((xμ + 1) Sqrt[λμ (2 - λμ)]) - δ)]],
  {i, 0, xμ}, {j, -1, xμ}];

Tμ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, 0, xμ + 1}, {j, 0, xμ}];

Qμ[δ_, θ_] = Aμ[δ, θ].Tμ;
```

Definition of $\mathrm{ARL}^{i}_{E-\mu}(\delta, \theta)$

```
IDμ = IdentityMatrix[xμ + 1];
Mμ[δ_, θ_] := Inverse[IDμ - Qμ[δ, θ]];
umμ = Table[1, {i, 0, xμ}];
bμ[i_] = Table[If[j == i, 1, 0], {j, 0, xμ}];
ARLμ[i_, δ_, θ_] := bμ[i] . Mμ[δ, θ] . umμ;

ARLμ[0, 0, 1]
500.047
```

Definition of the sub-stochastic matrix $Q_{\sigma}(\theta)$ of the chart EWMA-σ

```
γσ = 1.2198;
λσ = 0.043;
xσ = 40;

Aσ[θ_] = Table[If[j == -1, 0,
    χ[n - 1/θ^2 ×
      Exp[(γσ × Sqrt[PolyGamma[1, (n - 1)/2]] ((j + 1) - (1 - λσ) (i + 1/2)))/((xσ + 1) Sqrt[λσ (2 - λσ)])]
    ]], {i, 0, xσ}, {j, -1, xσ}];

Tσ = Table[If[i == j, -1, If[i == j + 1, 1, 0]], {i, 0, xσ + 1}, {j, 0, xσ}];

Qσ[θ_] = Aσ[θ].Tσ;
```

Definition of $\mathrm{ARL}^{i}_{E-\sigma}(\theta)$

```
IDσ = IdentityMatrix[xσ + 1];
Mσ[θ_] := Inverse[IDσ - Qσ[θ]];
umσ = Table[1, {i, 0, xσ}];
bσ[i_] = Table[If[j == i, 1, 0], {j, 0, xσ}];
ARLE-σ[i_, θ_] := bσ[i]. Mσ[θ] . umσ;

ARLE-σ[0, 1]
500.027
```

Survival function of $\mathrm{RL}^{i}_{E-\mu}(\delta, \theta)$

$\overline{F}_{\mathrm{RL}^{j}_{E-\mu}\ (\delta,\theta)}(k)$

```
Sμ[i_, k_, δ_, θ_] := bμ[i] .MatrixPower[ Qμ[δ, θ], If[k ≥ 0, Floor[k], 0]] . umμ;
```

Survival function of $\mathrm{RL}^{j}_{E-\sigma}(\theta)$

$\overline{F}_{\mathrm{RL}^{j}_{E-\sigma}\ (\theta)}(k)$

```
Sσ[j_, k_, θ_] :=
  bσ[j] .MatrixPower[ Qσ[θ], If[k ≥ 0, Floor[k], 0]] . umσ;
```

"Probability of Misleading Signals" — EE^{+}

Type III — $\mathrm{PMS}_{\mathrm{EE}^{+}}(\mathrm{III};\ \theta)$
$\mathrm{HS}_{\mu} = 0\ \%$, $\mathrm{HS}_{\sigma} = 0\ \%$

```
theta = {1.01, 1.03, 1.05, 1.10, 1.20,
   1.30, 1.40, 1.50, 1.60, 1.70, 1.80, 1.90, 2.00, 3.00};

headμ = {0};
headσ = {0};
```

```
Do[Do[Do[{
δ = 0;
θ = theta[[l]];
i = head_μ[[m]];
j = head_σ[[p]];
cu_μ = Q_μ[δ, θ];
cu_σ = Q_σ[θ];
err = 0.000001;
erel = 2 err;
oldsum = 0;
pot1_μ = ID_μ;
pot2_μ = cu_μ ;
pot_σ = cu_σ;
oldparc_μ = b_μ[i] .(pot1_μ - pot2_μ) . um_μ;
oldparc_σ = b_σ[j] .pot_σ . um_σ;
oldparc = oldparc_μ * oldparc_σ;
nparc = 1;

While[erel > err, {soma = oldsum + oldparc,
      erel = Abs[(soma - oldsum) / soma],
  oldsum = soma,
         oldpot2_μ = pot2_μ,
         oldpot_σ = pot_σ,
         pot1_μ = pot2_μ,
     pot2_μ = cu_μ.oldpot2_μ,
         pot_σ = cu_σ.oldpot_σ,
         oldparc_μ = b_μ[i] .(pot1_μ - pot2_μ) . um_μ,
         oldparc_σ = b_σ[j] .pot_σ . um_σ,
         oldparc = oldparc_μ * oldparc_σ,
      nparc = nparc + 1,
         If[Mod[nparc - 1, 500] == 0, Print[
          "iter=", nparc - 1,
             " erro relat=", erel,
             " valor aprox=", oldsum,
           " δ=", δ, " θ=", θ]]}],

     PMS_III[i, j, θ] = oldsum;
      iter[i, j, θ] = nparc - 1;

      Print[
    " δ=", δ, " θ=", θ,
    " i=", i, " j=", j,
    " aproximate value=", oldsum,
    " iterations=", nparc - 1,
    " relative error=", erel]},

{p, 1, Dimensions[head_σ][[1]]}], {l, 1, Dimensions[theta][[1]]}],
 {m, 1, Dimensions[head_μ][[1]]}]
```

iter=500 erro relat=0.000495864 valor aprox=0.412857 δ=0 θ=1.01

iter=1000 erro relat=0.0000410471 valor aprox=0.45152 δ=0 θ=1.01

iter=1500 erro relat=3.68745×10^{-6} valor aprox=0.45502 δ=0 θ=1.01

δ=0 θ=1.01 i=0 j=0 aproximate value=
0.455274 iterations=1772 relative error=9.97658×10^{-7}

iter=500 erro relat=0.000253896 valor aprox=0.363451 δ=0 θ=1.03

iter=1000 erro relat=8.53256×10^{-6} valor aprox=0.37667 δ=0 θ=1.03

δ=0 θ=1.03 i=0 j=0 aproximate value=
0.377092 iterations=1320 relative error=9.94111×10^{-7}

iter=500 erro relat=0.0000960605 valor aprox=0.310034 δ=0 θ=1.05

δ=0 θ=1.05 i=0 j=0 aproximate value=
0.313194 iterations=991 relative error=9.95982×10^{-7}

iter=500 erro relat=1.38164×10^{-6} valor aprox=0.206127 δ=0 θ=1.1

δ=0 θ=1.1 i=0 j=0 aproximate value=
0.206131 iterations=517 relative error=9.94621×10^{-7}

δ=0 θ=1.2 i=0 j=0 aproximate value=
0.114615 iterations=194 relative error=9.75375×10^{-7}

δ=0 θ=1.3 i=0 j=0 aproximate value=
0.0811304 iterations=101 relative error=9.14844×10^{-7}

δ=0 θ=1.4 i=0 j=0 aproximate value=
0.0656048 iterations=64 relative error=8.40646×10^{-7}

δ=0 θ=1.5 i=0 j=0 aproximate value=
0.057295 iterations=45 relative error=9.44502×10^{-7}

δ=0 θ=1.6 i=0 j=0 aproximate value=
0.0525313 iterations=35 relative error=7.51473×10^{-7}

δ=0 θ=1.7 i=0 j=0 aproximate value=
0.0497684 iterations=28 relative error=8.43009×10^{-7}

δ=0 θ=1.8 i=0 j=0 aproximate value=
0.0482488 iterations=24 relative error=5.77129×10^{-7}

δ=0 θ=1.9 i=0 j=0 aproximate value=
0.047556 iterations=20 relative error=9.46289×10^{-7}

δ=0 θ=2. i=0 j=0 aproximate value=
0.0474388 iterations=18 relative error=6.35696×10^{-7}

δ=0 θ=3. i=0 j=0 aproximate value=
0.0599576 iterations=9 relative error=2.37278×10^{-7}

```
MatrixForm[Table[{
   theta[[l]],
   (* head_μ[[i]],
   head_σ[[j]],
   iter[head_μ[[i]],head_σ[[j]],theta[[l]]], *)
   PMS_III[head_μ[[i]], head_σ[[j]], theta[[l]]]},
  {j, 1, Dimensions[head_σ][[1]]},
{i, 1, Dimensions[head_μ][[1]]},
  {l, 1, Dimensions[theta][[1]]}]]
```

$$\begin{pmatrix} \begin{pmatrix} 1.01 & 0.455274 \\ 1.03 & 0.377092 \\ 1.05 & 0.313194 \\ 1.1 & 0.206131 \\ 1.2 & 0.114615 \\ 1.3 & 0.0811304 \\ 1.4 & 0.0656048 \\ 1.5 & 0.057295 \\ 1.6 & 0.0525313 \\ 1.7 & 0.0497684 \\ 1.8 & 0.0482488 \\ 1.9 & 0.047556 \\ 2. & 0.0474388 \\ 3. & 0.0599576 \end{pmatrix} \end{pmatrix}$$

Type IV — $PMS_{EE^+}(IV; \delta)$
$HS_\mu = 0\,\%$, $HS_\sigma = 0\,\%$

```
delta = {0.05, 0.10, 0.20, 0.30, 0.40, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0, 3.0};

head_μ = {0};
head_σ = {0};
```

```
Do[Do[Do[{
δ = delta[[k]];
θ = 1;
i = head_μ[[m]];
j = head_σ[[p]];
cu_μ = Q_μ[δ, θ];
cu_σ = Q_σ[θ];
err = 0.000001;
erel = 2 err;
oldsum = 0;
pot1_σ = ID_σ;
pot2_σ = cu_σ ;
pot_μ = cu_μ;
oldparc_σ = b_σ[j] .(pot1_σ - pot2_σ) . um_σ;
oldparc_μ = b_μ[i] .pot_μ . um_μ;
oldparc = oldparc_μ * oldparc_σ;
nparc = 1;

While[erel > err, {soma = oldsum + oldparc,
     If[nparc > 1, erel = Abs[(soma - oldsum) / soma]],
  oldsum = soma,
        oldpot2_σ = pot2_σ,
        oldpot_μ = pot_μ,
        pot1_σ = pot2_σ,
     pot2_σ = cu_σ.oldpot2_σ,
        pot_μ = cu_μ.oldpot_μ,
        oldparc_σ = b_σ[j] .(pot1_σ - pot2_σ) . um_σ,
        oldparc_μ = b_μ[i] .pot_μ . um_μ,
        oldparc = oldparc_μ * oldparc_σ,
     nparc = nparc + 1,
        If[Mod[nparc - 1, 500] == 0, Print[
         "iter=", nparc - 1,
            " erro relat=", erel,
            " valor aprox=", oldsum,
         " δ=", δ, " θ=", θ]]}],

    PMS_Iv[i, j, δ] = oldsum;
     iter[i, j, δ] = nparc - 1;

     Print[
    " δ=", δ, " θ=", θ,
    " i=", i, " j=", j,
    " aproximate value=", oldsum,
    " iterations=", nparc - 1,
    " relative error=", erel]},

{p, 1, Dimensions[head_σ][[1]]}], {k, 1, Dimensions[delta][[1]]}],
 {m, 1, Dimensions[head_μ][[1]]}]
```

iter=500 erro relat=0.000462764 valor aprox=0.370002 δ=0.05 θ=1

iter=1000 erro relat=0.0000347943 valor aprox=0.401293 δ=0.05 θ=1

iter=1500 erro relat=2.81943×10^{-6} valor aprox=0.403845 δ=0.05 θ=1

δ=0.05 θ=1 i=0 j=0 aproximate value=
0.403991 iterations=1707 relative error=9.9845×10^{-7}

iter=500 erro relat=0.000291121 valor aprox=0.305314 δ=0.1 θ=1

iter=1000 erro relat=0.0000116966 valor aprox=0.318696 δ=0.1 θ=1

δ=0.1 θ=1 i=0 j=0 aproximate value=
0.319232 iterations=1388 relative error=9.96504×10^{-7}

iter=500 erro relat=0.0000583307 valor aprox=0.190622 δ=0.2 θ=1

δ=0.2 θ=1 i=0 j=0 aproximate value=
0.191651 iterations=885 relative error=9.95265×10^{-7}

iter=500 erro relat=2.89932×10^{-6} valor aprox=0.114198 δ=0.3 θ=1

δ=0.3 θ=1 i=0 j=0 aproximate value=
0.11421 iterations=561 relative error=9.85603×10^{-7}

δ=0.4 θ=1 i=0 j=0 aproximate value=
0.0697674 iterations=364 relative error=9.96163×10^{-7}

δ=0.5 θ=1 i=0 j=0 aproximate value=
0.0441521 iterations=246 relative error=9.92946×10^{-7}

δ=0.6 θ=1 i=0 j=0 aproximate value=
0.0288977 iterations=174 relative error=9.54561×10^{-7}

δ=0.7 θ=1 i=0 j=0 aproximate value=
0.0194324 iterations=128 relative error=9.25287×10^{-7}

δ=0.8 θ=1 i=0 j=0 aproximate value=
0.0133272 iterations=97 relative error=9.69104×10^{-7}

δ=0.9 θ=1 i=0 j=0 aproximate value=
0.00926217 iterations=76 relative error=9.91704×10^{-7}

δ=1. θ=1 i=0 j=0 aproximate value=
0.00649049 iterations=62 relative error=8.42018×10^{-7}

δ=1.5 θ=1 i=0 j=0 aproximate value=
0.00112638 iterations=28 relative error=8.45159×10^{-7}

δ=2. θ=1 i=0 j=0 aproximate value=
0.000184517 iterations=17 relative error=6.50647×10^{-7}

δ=3. θ=1 i=0 j=0 aproximate value=
3.64592×10^{-6} iterations=9 relative error=4.75845×10^{-7}

```
MatrixForm[Table[{
    delta[[l]],
    (* head_μ[[i]],
    head_σ[[j]],
    iter[head_μ[[i]],head_σ[[j]],delta[[l]]], *)
    PMS_IV[head_μ[[i]], head_σ[[j]], delta[[l]]]},
   {j, 1, Dimensions[head_σ][[1]]},
{i, 1, Dimensions[head_μ][[1]]},
   {l, 1, Dimensions[delta][[1]]}]]
```

$$\begin{pmatrix} \begin{pmatrix} 0.05 & 0.403991 \\ 0.1 & 0.319232 \\ 0.2 & 0.191651 \\ 0.3 & 0.11421 \\ 0.4 & 0.0697674 \\ 0.5 & 0.0441521 \\ 0.6 & 0.0288977 \\ 0.7 & 0.0194324 \\ 0.8 & 0.0133272 \\ 0.9 & 0.00926217 \\ 1. & 0.00649049 \\ 1.5 & 0.00112638 \\ 2. & 0.000184517 \\ 3. & 3.64592\times 10^{-6} \end{pmatrix} \end{pmatrix}$$

Exercise 10.47

```
n = 10;
χ[x_] = CDF[ChiSquareDistribution[n - 1], x];

pfa_σ = 1 / 100.0;
γ_σ = Quantile[ChiSquareDistribution[n - 1], 1 - pfa_σ];
1 - χ[γ_σ / .9^2]
```

0.00153939

```
pfa_σ = 1 / 100.0;
a = Quantile[ChiSquareDistribution[n - 1], pfa_σ / 2];
b = Quantile[ChiSquareDistribution[n - 1], 1 - pfa_σ / 2];
1 - (χ[b / .9^2] - χ[a / .9^2])
```

0.0115981

```
probmisleadingsignal_upper = 1 - χ[γσ/.9^2];
dist = GeometricDistribution[probmisleadingsignal_upper] ;
(* geometric distribution for the number of trials before the first success,
where the probability of success in a trial is p *)
CDF[dist, 100 - 1]
```

```
probmisleadingsignal_standard = 1 - (χ[b/.9^2] - χ[a/.9^2]);
dist = GeometricDistribution[probmisleadingsignal_standard] ;
(* geometric distribution for the number of trials before the first success,
where the probability of success in a trial is p *)
CDF[dist, 100 - 1]
```

```
0.142778

0.688574
```