Formulário de Complementos de Probabilidades e Estatística

Distribuição	Suporte	$P(X=x)$	$E[X]$	$Var[X]$	$M_X(t)$
Uniforme$(\{1,\ldots,n\})$	$\{1,\ldots,n\}$	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$	$\frac{e^t(1-e^{tn})}{n(1-e^t)}$
Binomial(n,p)	$\{0,\ldots,n\}$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$[pe^t+(1-p)]^n$
HiperG(N,M,n)	$\{\max\{0,n-N+M\},\ldots,\min\{n,M\}\}$	$\frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$	$n\frac{M}{N}$	$n\frac{M}{N}\frac{N-M}{N}\frac{N-n}{N-1}$	sem interesse
BinomialN(r,p)	$\{r,r+1,\ldots\}$	$\binom{x-1}{r-1}p^r(1-p)^{x-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$
BinomialN$^*(r,p)$	$\{0,1,\ldots\}$	$\binom{y+r-1}{r-1}p^r(1-p)^y$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$
Poisson(λ)	$\mathbb{N}_0$	$e^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ	$e^{\lambda(e^t-1)}$

Distribuição	Suporte	$f_X(x)$	$E[X]$	$Var[X]$	$M_X(t)$ ou $E[X^k]$
Uniforme(α,β)	$[\alpha,\beta]$	$\frac{1}{\beta-\alpha}$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	$\frac{e^{\beta t}-e^{\alpha t}}{(\beta-\alpha)t},\ t\neq 0$
Gama(α,β)	$\mathbb{R}_0^+$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha,\ t<\beta$
$\chi^2_{(n)}\equiv$ Gama$(\frac{n}{2},\frac{1}{2})$					
Exp$(\lambda)\equiv$ Gama$(1,\lambda)$					
$t_{(n)}$	$\mathbb{R}$	$\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$	$0,\ n>1$	$\frac{n}{n-2},\ n>2$	$E[X^k]=\frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(n-\frac{k}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{n}{2}\right)}n^{\frac{k}{2}},\ k$ par
Beta(α,β)	$[0,1]$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	$1+\sum_{k=1}^{+\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^k}{k!}$
Normal(μ,σ^2)	$\mathbb{R}$	$\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2	$e^{\mu t+\frac{(t\sigma)^2}{2}}$
LogNormal(μ,σ^2)	$\mathbb{R}_0^+$	$\frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{(\log x-\mu)^2}{2\sigma^2}}$	$e^{\mu+\frac{\sigma^2}{2}}$	$\left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}$	$E[X^k]=e^{k\mu+\frac{k^2\sigma^2}{2}}$
Cauchy(μ,σ)	$\mathbb{R}$	$\frac{1}{\pi\sigma}\frac{1}{1+\left(\frac{x-\mu}{\sigma}\right)^2}$	não existe	não existe	não existem
Weibull(α,β)	$\mathbb{R}_0^+$	$\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}e^{-\left(\frac{x}{\alpha}\right)^\beta}$	$\alpha\Gamma\left(1+\frac{1}{\beta}\right)$	$\alpha^2\left(\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma\left(1+\frac{1}{\beta}\right)^2\right)$	$E[X^k]=\alpha^k\Gamma\left(1+\frac{k}{\beta}\right)$
F$_{(n_1,n_2)}$	$\mathbb{R}_0^+$	$\frac{\Gamma\left(\frac{n_1+n_2}{2}\right)}{\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)}\left(\frac{n_1}{n_2}\right)^{\frac{n_1}{2}}\frac{x^{\frac{n_1-2}{2}}}{\left(1+\frac{n_1}{n_2}x\right)^{\frac{n_1+n_2}{2}}}$	$\frac{n_2}{n_2-2},\ n_2>2$	$\frac{2n_2^2(n_1+n_2-2)}{n_1(n_2-2)^2(n_2-4)},\ n_2>4$	$E[X^k]=\frac{\Gamma\left(\frac{n_1+2k}{2}\right)\Gamma\left(\frac{n_2-2k}{2}\right)}{\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)}\left(\frac{n_1}{n_2}\right)^k,\ k<\frac{n_2}{2}$

Cap. 1

$F_{BinomialN(r,p)}(x) = 1 - F_{Binomial(x,p)}(r-1)$

$\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1} e^{-x} dx,\ \alpha > 0;\quad \Gamma(n) = (n-1)!,\ n \in \mathbb{N};\quad \Gamma(\alpha+1) = \alpha\Gamma(\alpha),\ \alpha > 0;\quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$

$F_{Erlang(n,\beta)}(x) = 1 - F_{Poisson(\beta x)}(n-1)$

$F_{Gama(\alpha,\beta)}(x) = F_{\chi^2_{(2\alpha)}}(2\beta x)$

$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$

$F_{Beta(\alpha,\beta)}(x) = 1 - F_{Binomial(\alpha+\beta-1,x)}(\alpha-1)$

$E(X) = \int_0^{+\infty} [1 - F_X(x)]\, dx$, para $X \geq 0$

$E(X^k) = \int_0^{+\infty} kx^{k-1}[1 - F_X(x)]\, dx$, para $X \geq 0$

$SC(X) = \frac{E\{[X-E(X)]^3\}}{[SD(X)]^3}$

$KC(X) = \frac{E\{[X-E(X)]^4\}}{[SD(X)]^4} - 3$

$E[X(X-1)\cdots(X-k+1)] = \left.\frac{d^k P_X(z)}{dz^k}\right|_{z=1},\ k = 1, 2, \ldots$

$E(X^k) = \left.\frac{d^k M_X(t)}{dt^k}\right|_{t=0}$

$F_{Y=g(X)}(y) = P[X \in g^{-1}((-\infty, y])]$

$f_{Y=g(X)}(y) = f_X[g^{-1}(y)] \times \left|\frac{dg^{-1}(y)}{dy}\right|$

Cap. 2

$M_{\underline{X}}(\underline{t}) = E\left[\exp\left(\sum_{i=1}^n t_i X_i\right)\right]$

$E\left(\prod_{i=1}^n X_i^{k_i}\right) = \left.\frac{\partial^{\sum_{i=1}^n k_i} M_{\underline{X}}(\underline{t})}{\partial t_1^{k_1} \ldots \partial t_n^{k_n}}\right|_{\underline{t}=\underline{0}}$

$F_{\underline{Y}=\underline{g}(\underline{X})}(\underline{y}) = P\left[\underline{X} \in \underline{g}^{-1}\left(\prod_{i=1}^m (-\infty, y_i]\right)\right]$

$f_{\underline{Y}=\underline{g}(\underline{X})}(\underline{y}) = f_{\underline{X}}\left[\underline{g}^{-1}(\underline{y})\right] \times |J(\underline{y})|$

$J(\underline{y}) = \begin{vmatrix} \frac{\partial g_1^{-1}(y)}{\partial y_1} & \ldots & \frac{\partial g_1^{-1}(y)}{\partial y_n} \\ \vdots & \ldots & \vdots \\ \frac{\partial g_n^{-1}(y)}{\partial y_1} & \ldots & \frac{\partial g_n^{-1}(y)}{\partial y_n} \end{vmatrix}$

$P(N_1 = n_1, \ldots, N_d = n_d) = \frac{n!}{\prod_{i=1}^d n_i!} \times \prod_{i=1}^d p_i^{n_i}$

$\{(n_1, \ldots, n_d) \in \mathbb{N}_0^d : \sum_{i=1}^d n_i = n\}$

$M_{N_1,\ldots,N_{d-1}}(t_1, \ldots, t_{d-1}) = \left[\left(\sum_{i=1}^{d-1} p_i\, e^{t_i}\right) + p_d\right]^n$

$Cov(N_i, N_j) = -n\, p_i\, p_j,\ i \neq j$

$\underline{N}_{-j}|N_j = n_j \sim \text{Multinomial}_{d-2}(n - n_j, \underline{p}_{-j|j})$

$\underline{p}_{-j|j} = \left(\frac{p_1}{1-p_j}, \ldots, \frac{p_{j-1}}{1-p_j}, \frac{p_{j+1}}{1-p_j}, \ldots, \frac{p_{d-1}}{1-p_j}, \frac{p_d}{1-p_j}\right)$

$f_{X_1,X_2}(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right) + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right]\right\}$

$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)\times|\underline{\Sigma}|^{\frac{1}{2}}} \times \exp\left[-\frac{1}{2}(\underline{x} - \underline{\mu})^\top \underline{\Sigma}^{-1}(\underline{x} - \underline{\mu})\right]$

$\underline{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix},\ \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ \underline{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix},\ \underline{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$

$M_{X_1,X_2}(t_1, t_2) = \exp\left[(\mu_1 t_1 + \mu_2 t_2) + \frac{1}{2}\left(\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2\right)\right]$

$X_1|X_2 = x \sim \text{Normal}\left(\mu_1 + \frac{\rho\sigma_1}{\sigma_2}(x - \mu_2),\ \sigma_1^2(1 - \rho^2)\right)$

Cap. 3

$P[X_{(n-k+1)} > x] = 1 - F_{Binomial(n,1-F_X(x))}(k-1)$

$f_{X_{(1)},\ldots,X_{(n)}}(x_{(1)}, \ldots, x_{(n)}) = n! \times \prod_{i=1}^n f_X(x_{(i)})$

$F_{X_{(i)}}(x) = 1 - F_{Binomial(n,F_X(x))}(i-1)$

$f_{X_{(i)}}(x) = \frac{n!}{(i-1)!(n-i)!}[F_X(x)]^{i-1}[1 - F_X(x)]^{n-i} f_X(x)$

$f_{(X_{(i)},X_{(j)})}(x, y) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}[F_X(x)]^{i-1}[F_X(y) - F_X(x)]^{j-i-1}[1 - F_X(y)]^{n-j} f_X(x) f_X(y),\ x < y$

$X_n \xrightarrow{P} X \Leftrightarrow \forall\epsilon > 0,\ \lim_{n\to+\infty} P(|X_n - X| > \epsilon) = 0$

$P(|X - \mu| \geq c\,\sigma) \leq \frac{1}{c^2},\ c > 0$

$X_n \xrightarrow{d} X \Leftrightarrow \lim_{n\to+\infty} F_{X_n}(x) = F_X(x)$, em todos os pontos de continuidade de $F_X(x)$