A districting approach for a home hospitalization service network under uncertain demand

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Abstract

The benefits of home hospitalization are well established in the literature and justify its growing demand. Along with its rapid expansion, this model of care is also subject to higher uncertainty and variability compared to other home-based services. Given the logistical specificities of a home hospitalization service network, operational research tools that support decision-making become imperative, notably regarding the location and allocation of resources.

This paper addresses a districting problem motivated by a real-world case study of a home hospitalization service network in the district of Lisbon, Portugal. The problem is formulated as a multi-objective mixed integer linear programming model by considering criteria of complete and exclusive assignment, compatibility, distance and capacity limitations, compactness, and workload balance. The objective function combines the last three criteria using the lexicographic method, seeking to minimize the deviation from the average workload, the distance between basic units, and the use of under-capacity.

Computational results were presented for several generated instances representing increasing demand scenarios. The model was further explored for three situations: the gradual opening of potential new home hospitalization units, the districting plan when all units in the network are open, and the allocation of multiple teams within a single unit. Sensitivity analyses were performed on the main parameters to obtain managerial insights, concluding that it is possible to obtain both workload-balanced and compact districts with no expressive trade-off between the two objectives.

Keywords: Home Hospitalization, Districting Problem, Multi-objective Optimization, Mixed-Integer Linear Programming, Operational Research

1. Introduction

In order to address the growing growing global need for hospital beds and rising healthcare costs, technological and scientific innovation is becoming more and more crucial. Catalysed by the COVID-19 pandemic and aligned with today's demographic and economic trends, home hospitalization (HH) presents itself as an effective alternative to traditional hospital inpatient care, providing acute health care in a patient's home. It can replace hospital care entirely or reduce hospital length of stay through early discharge. The clinical results observed so far are very favorable, with an overall reduction in hospital days and a decrease in the risk of hospital readmission. In addition, HH has increased patient and family satisfaction while reducing hospitalization costs [1].

Home hospitalization is included in the spectrum of home care. Despite sharing several attributes with other home care services, such as the need to travel between patients' homes and the multidisciplinarity of the teams, the specific features of home hospitalization bring new logistical challenges. Some of these challenges are the need for greater proximity to the hospital, the possibility of sharing staff with other hospital services, and the increased uncertainty experienced due to the acute illness profile of patients and consequent shorter lengths of stay .

In Portugal, the Ministry of Health developed a strategy aimed at expanding the delivery of HH services in the National Health System, reaching 28 units at the end of 2020. In the same year, home hospitalization also became a reality in the portuguese private health sector with the opening of Provider X's HH unit. Currently, this unit is located in Hospital 1 and covers the subregion of Greater Lisbon. More specifically, the unit serves a 30-kilometer radius, a distance that allows the HH teams to assist patients in about 30 minutes in case their conditions deteriorate. In addition to Hospital 1, the region has three other Provider X hospitals for which home hospitalization service expansion is planned in the short term.

The environment of extreme uncertainty, variability, and ongoing change in which health services operate makes detailed information for decision-making strongly desirable [2]. Analysis of the geographic organization and distribution of health service capacity, from the local hospital level to services provided throughout a region or country as a whole, is often required. In this set, *districting* is a strategic-tactical planning decision that involves clustering a set of demand points, i.e., a group of patients aggregated according to their location, into districts that satisfy relevant criteria. Adopting a districting approach in a Home Hospitalization setting leads to increased reactivity and efficiency of caregivers. It also facilitates human resource management, improving the quality of care and increasing patient and provider satisfaction [3]. Despite the clear benefits of mathematical decision support tools such as districting, these issues have seldom been considered in the home hospitalization literature.

This paper aims to create a mathematical model capable of ensuring the optimal allocation of patients by a set of HH units, taking into account some criteria that will be discussed in the following chapters. As a case study, the model will be applied to candidate Provider X HH units in the Greater Lisbon region.

2. Literature Review

Districting aims at dividing a large geographical region into sub-areas, referred to as districts, for organizational or administrative purposes. It is the process of grouping small geographic areas, called basic units, into larger geographic clusters (districts) to optimize certain criteria and subject to some constraints [4, 5] and then assigning each cluster to a set of resources [6]. According to [6], districting encompasses three distinct strategic OR problems: partitioning, assignment, and classification.

As home hospitalization is framed within home care, it is assumed that the districting models for these two types of care are similar overall. Thus, the remainder of this section will look at districting solutions for healthcare and home care contexts.

2.1. Modeling approaches

There is a consensus in the field to model districting as a mixed-integer programming (MIP) problem. MIP models deal with problems where some decision variables are constrained to integer values at the optimal solution. Capturing the discrete nature of some decisions greatly expands the scope of practical optimization problems that can be defined and solved. When the models do not have quadratic characteristics, they are called Mixed Integer Linear Programming (MILP) problems. There are also non-linear formulations in the literature, although far less frequent [7, 8, 9].

Depending on the specific application, location-allocation and set partitioning are two common formulations for districting problems [10]. A location-allocation model takes a fixed set of district centers and assigns each basic unit to exactly one district center. The objective is to minimize the total cost of assigning those units to district centers whilst being subject to certain constraints. This formulation is beneficial in situations where the district center acts as a depot, being the starting and finishing point for all routes within the district [10]. In the Set Partitioning formulation, a set of potentially feasible districts are heuristically generated and then selected to optimize the overall balance of the district plan [4]. The objective function minimizes the total cost of all selected districts while ensuring that each unit is assigned to a single district and that a chosen number of districts is generated [11]. This formulation enables the modeler to design and evaluate the cost of complex district restrictions within an auxiliary problem, outside of the core optimization problem. This confers an advantage to the Set Partitioning formulation since almost any criterion can be applied to the generation of candidate districts [4].

2.2. Solution approaches

Ideally, it would be possible to solve districting models using exact methods that guarantee to find an optimal solution. Exact methods are typically solved using solvers such as CPLEX [3, 12], Gurobi [13], or Xpress [11]. However, most of the districting problems are NP-hard. Thus, large-scale instances are intractable by exact optimization algorithms [4]. For that reason, many papers in this field developed heuristics and metaheuristics. These approaches have the flexibility to include almost any practical criterion and can handle complex constraints. Consider the case of [13], where the authors started by solving the model using the Gurobi Optimizer, which proved feasible for their problem. However, they noted that the solver could be time-consuming and suggested a greedy heuristic method. Other authors implemented well-known metaheuristics, specifically Tabu Search [7, 8] and Genetic Algorithms [14, 15, 9, 5].

2.3. Uncertainty in districting models

Mathematical models represent a simplified version of a real system and should be able to explain previous observations, integrate current data, and anticipate the system's response to planned stresses [16]. A deterministic model is one in which state variables are determined solely by model parameters and sets of previous states. The home-care deterministic districting problem has a comprehensive literature base [7, 8, 17, 3, 11, 13, 9]. These models do not account for uncertainty, neglecting the effect of unpredictable variables in the solution [16].

No stochastic work could be identified for home-care districting. Within healthcare districting, [12] presents an approach that, although not stochastic, uses a multi-period model that allows for better planning adjustment according to demand and supply changes and potentially improves the efficiency and quality of the health service. The authors' districting model was applied to the primary care scheme of Istanbul, Turkey.

[18] handles the allocation of patients and the facility location, employing a stochastic approach to LA through a geographic simulation model using Delphi programming. The incorporation of stochasticity in their model allows for a better fit of variable factors such as patient flows or transportation time. It also makes the model more generic and easier to use by different users. The model was used in two case studies at the district and regional levels in Eastern England. There is no objective function in this work, and an optimal solution is not necessarily found. However, it encourages discussions between stakeholders and allows for the rapid configuration of new scenarios. [5] is the only paper in this literature review that estimates demand, doing so through hedonic models. They use two methods to deal with uncertainty, allowing the decision-makers to adjust their results according to their attitude toward risk. They obtain higher values for the original objective when the worst-case scenario is not considered and lower values when protection against it is increased. This work was applied to residential areas in Iran with positive results compared to the existing districting decisions.

Table 1: Key findings from the literature review on the districting problem.

Publication	Application	Uncertainty	Model	Approach	Solution	Case Study	Location	Stakeholders
(Blais et al., 2003) [7]	Home Care	D	NLP	MH	TS	√	Canada	√
(Harper et al., 2005) [18]	Healthcare	S	-	-	GS	√	England	√
(Lahrichi et al., 2006) [8]	Home Care	D	NLP	MH	TS	√	Canada	
(Sahin et al., 2010) [17]	Home Care	D	MIP	E	-		-	
(Benzarti et al., 2013) [3]	Home Care	D	MIP	E	CPLEX		-	
(Datta et al., 2013) [14]	Healthcare	D	MIP	MH	GA	√	England	
(Steiner et al., 2015) [15]	Healthcare	D	MIP	MH	GA	√	Brazil	
(Gutiérrez-Gutierrez and Vidal, 2015) [11]	Home Care	D	MIP	E	Xpress-IVE	√	Colombia	√
(Lin et al., 2017) [13]	Home Care	D	MIP	E, H	Gurobi, Greedy	√	China	
(Yanik et al., 2019) [12]	Healthcare	D	MIP	E	CPLEX	√	Istambul	
(Lin et al., 2020) [9]	Home Care	D	MINLP	MH	GA	√	China	√
(Darmian et al., 2021) [5]	Healthcare	S	MIP	MH	GA	√	Iran	√

2.4. Districting Criteria

The vast majority of the reviewed papers consider multi-objective optimization. This section describes the most used criteria in the literature, distinguishing whether they are formulated as a constraint or part of the objective function.

Accessibility: Also referred to as mobility, this criterion measures the ease of moving from one location to another within a district. It is necessary since caregivers use public transportation in most home-care case studies. This criterion was incorporated as a constraint in [7, 8, 17, 3, 13, 12].

Balance: Balance describes the desire for districts of similar size concerning some performance measure. The most common balance criterion concerns workload. Usually expressed in hours per year, the workload corresponds to the sum of the service time, or "care" workload, and of the average travel time between the district center and the demand points [7, 17]. Workload balance is considered essential in district design, hence being mentioned in most districting literature [7, 8, 17, 3, 11, 12, 9, 5]. Other types of balance can be looked for, such as population size [15, 14] or even population characteristics such as age [14].

Capacity limitation: This criterion seeks to respect the capacity limitations of each district, ensuring that supply meets demand. Rarely mentioned in the literature, it was used as a constraint in [13, 12].

Compactness: A geographically compact district is somewhat round-shaped, undistorted, and without holes [4, 5]. Different approaches can be used to measure compactness since its definition strongly depends on the geometric representation of basic units. It can be ensured by minimizing travel distances [14, 3, 15, 12] or travel times [17, 11, 13, 9, 5], thus improving a provider's efficiency. Compactness is a consensual criterion, formulated as an objective in [17, 14, 11, 15, 13, 9] and as a constraint in [14, 12, 9]. In the case of [3], compactness is integrated into the model first as a hard constraint and then as subject to minimization.

Complete and Exclusive Assignment: Also referred to as the indivisibility of basic units or integrity, this criterion states that each basic unit must be assigned to one and only one district, allowing the establishment of long-term relationships with patients and avoiding overlapping caregiver responsibilities. This is one of the fundamental constraints of a districting problem, only not included in the formulation of [12], that allowed the gradual assignment of basic units to one or more districts.

Contiguity: A contiguity or connectivity criterion guarantees that one can travel between any two points of a certain district without going through any other district. It is a desirable property not only for administrative reasons but also because it facilitates the reduction of travel distances. It is possible to ensure contiguity through both geometrical [7, 8, 11] and graph-based measure constraints [14, 15, 5]. Only in [5], this criterion is part of the objective function.

District number: The number of districts to create can be predefined [14, 3, 15, 11], limited to a specific range [5], or minimized as in [13, 9]. In the last two papers, an upper bound is also established.

Respect for administrative boundaries: The districts designed must conform to the administrative boundaries, either municipalities or civil parishes. This simplifies the organization of health care delivery procedures and indirectly assures district contiguity. It is considered a constraint in [7, 8, 17, 3, 11] and an objective in [14].

2.5. Contributions

To date and the best of the author's knowledge, no article addresses the problem of districting considering the specific characteristics of home hospitalization. Most papers reviewed used real healthcare case studies, but less than half of the authors explicitly refer to the inclusion of stakeholders in the decision-making process. There is also a shortage of models that encompass the uncertainty inherent to the healthcare setting. This dissertation will tackle these three shortcomings.

Next section provides the description of the general problem and the mathematical programming model.

3. Mathematical Model

The following chapter proposes a generic deterministic multi-objective mixed-integer linear programming (MILP) formulation to address the districting problem for a home hospitalization unit network. Subsection 3.1 introduces the formal problem description along with the underlying assumptions. The mathematical formulation can be consulted in Subsection 3.2 .

3.1. Problem Statement

Given a set of $d \in D$ demand points and a set of $u \in U$ home hospitalization units or supply points, the districting problem consists of grouping the patients' locations into good districts according to relevant criteria. The goal is to minimize the travel distances and travel times within districts, thus making districts as compact as possible while assuring workload balance between the region's HH units. The districting plan must also guarantee that care is provided according to the different capacities of the HH units. Thus, minimizing the periods in which the units cannot satisfy demand is also part of the objective function. The time horizon of the districting plan is an input of the model; for this dissertation, one year will be considered.

Each demand point represents the aggregate demand of a particular civil parish or parish cluster. It is assumed that these demand points have been defined *a priori*. Demand points are described by a certain number of patients, about whom the month they were admitted to HH care and the number of days of hospitalization is known. The number of daily visits per month needed to treat the patients of each demand point (given by h_d^m) is an input of the problem. To obtain this number, the hospitalization days of each month's patients are summed. Assume that a daily visit represents the complete daily care provided to a patient and may, in practice, entail two trips to the patient's home. HH units are characterized by their capacity, meaning the number of patients each unit can treat daily. Of the supply points, only those that are open, denoted by $u \in U^{open}$, will be considered in the districting decision. The number of districts to design is predetermined and equal to the number of HH open facilities. Each demand point is assigned to precisely one district.

The decisions to be made include the allocation of demand points to HH units, given by the decision variable x_{du}, the maximum workload deviation, the longest distance between two demand points, the longest distance between a demand point and its assigned HH unit, and the maximum under-capacity on which each HH unit would operate on, denoted by Δ, Ω, Θ, and Ψ, respectively. This formulation also quantifies each supply point's monthly and annual workload (wl_u^m and wl_u), that is, what percentage of its capacity is deployed in those periods. In addition, the model identifies units in under-capacity and in which months that occurs; the auxiliary variable $UnderCap_u^m$ represents the monthly value of under-capacity for each HH unit.

3.1.1 Assumptions

Without losing generality, it is assumed that:

A.1 The districting is done once for a relatively long time. The period of a common calendar year is considered, totaling 365 days.

A.2 The HH structure can treat all patients, and all the demand points are covered, meaning that all the patients admitted to the HH must be assigned to a district.

A.3 Each open HH unit serves exactly one district.

A.4 The number of patients admitted to the HH structure is known in advance and does not change while considering the districting problem.

A.5 As there are no unallocated demand points and supply is limited, under-capacity may occur in specific units in certain months.

A.6 Each demand point has a predefined monthly number of required daily visits, given by the sum of the hospitalization days of the aggregate demand.

A.7 Patients admitted on any day of a particular month are accounted as entering on the first day of that month.

A.8 The coordinates of each demand point correspond to the midpoint location of each parish.

A.9 Given that the road networks are dense in urban locations, geodesic distances can be utilized to approximate road distances and journey durations.

A.10 All patients are homogeneous in terms of care requirements and service demand.

A.11 HH units and their teams are homogeneous regarding skills, contracts, and workload capacity.

3.2. Mathematical Formulation

The following section introduces the model's notation and presents the objective functions and constraints.

Sets and Subsets

$d \in D$ set of demand points
$u \in U$ set of home hospitalization units
$m \in M$ set of months in the planning horizon
$(d, d') \in E$ set of demand points pairs $(d, d') \in E$ where $(d, d') \in E$ if and only if $e_{dd'} = 1$
$u \in U^{open}$ subset of potential new home hospitalization units

Parameters and weights

$\delta_{dd'}$: Distance between demand points d and d'
δ_{du} : Distance between demand points d and home hospitalization unit u
δ_{Dmax} : Maximum distance allowed between 2 demand points $d, d' \in D$ assigned to the same district $u \in U$
δ_{Umax} : Maximum distance allowed between a demand point $d \in D$ and its assigned home hospitalization unit $u \in U$
τ : Time frame considered in the districting decision, expressed in days
γ_m: Number of days of month $m \in M$
h_d^m : Number of hospitalization days of demand point $d \in D$ in month $m \in M$
Cap_u: Daily capacity of each HH unit $u \in U$, expressed in number of patients
$MonthSupply_u^m$: Capacity of each HH unit $u \in U$ for month $m \in M$, $MonthSupply_u^m = Cap_u * \gamma_m$
$Supply_u$: Yearly capacity of each HH unit $u \in U$, $Supply_u = Cap_u * \tau$
$e_{dd'} = \{0, 1\}$: Compatibility index, 1 if demand points d and d' are compatible; 0 otherwise
$open_u = \{0, 1\}$: Potential open home hospitalization units, 1 if $u \in U^{open}$; 0 otherwise

Decision Variables

$x_{du} = \{0, 1\}$: 1 if demand point d is assigned to district u; 0 otherwise
Δ : The maximum deviation, expressed as a percentage, between the care workload associated to each HH unit $u \in U$ and the average care workload among all districts
Ω : The maximum distance between two demand points $d, d' \in D$ assigned to the same unit $u \in U$
Θ : The maximum distance between a demand point $d \in D$ and it's assigned HH unit $u \in U$
Ψ : The maximum undercapacity on which a HH unit $u \in U$ would operate

wl_u : Percentage of utilized care workload out of the total annual supply of district u
wl_u^m : Percentage of utilized care workload out of the total supply of district u in month m
$\overline{wl}$: Average workload among all districts
$MonthDemand_u^m$: The number of required daily visits attributed to district $u \in U$ during month $m \in M$
$Demand_u$: The total number of required daily visits attributed to district $u \in U$
$UnderCap_u^m$: The number of extra daily visits that unit $u \in U$ would have to operate for there to be no under-capacity in month $m \in M$

Mathematical Model

$$\min \Delta \tag{1}$$

$$\Delta \geq wl_u - \overline{wl}, \forall u = 1, ..., U \tag{2}$$

$$\Delta \geq \overline{wl} - wl_u, \forall u = 1, ..., U \tag{3}$$

$$Demand_u = \sum_{d=1}^{D} \sum_{m=1}^{M} x_{du} * h_d^m \tag{4}$$

$$wl_u = \frac{Demand_u}{Supply_u} \tag{5}$$

$$\overline{wl} = \sum_{u=1}^{U} \frac{wl_u}{U} \tag{6}$$

$$\min \Theta \tag{7}$$

$$\Theta \geq \delta_{du} * x_{du}, \forall d = 1, ..., D, u = 1, ..., U \tag{8}$$

$$\min \Omega \tag{9}$$

$$\Omega \geq \delta_{dd'}(x_{du} + x_{d'u} - 1), , \forall d, d' \in D, u \in U \tag{10}$$

$$\min \Psi \tag{11}$$

$$Supply_u + \Psi \geq Demand_u \tag{12}$$

$$MonthSupply_u^m + \Psi \geq MonthDemand_u^m, \forall u = 1, ..., U \tag{13}$$

$$MonthDemand_u^m = \sum_{d=0}^{D} x_{du} * h_d^m \tag{14}$$

Subject to:

$$x_{du} \leq open_u, \forall d = 1, ..., D, \forall u = 1, ..., U \tag{15}$$

$$\sum_{u=1}^{U} x_{du} = 1, \forall d = 1, ..., D \tag{16}$$

$$x_{du} \in \{0, 1\}, \forall d = 1, ..., D, u = 1, ..., U \tag{17}$$

$$x_{d'u} \geq x_{du}, \forall (d, d') \in E, \forall u \in U \tag{18}$$

$$\delta_{du} x_{du} \leq \delta_{Umax}, \forall d \in D, u \in U \tag{19}$$

$$\delta_{dd'}(x_{du} + x_{d'u} - 1) \leq \delta_{Dmax}, \forall d, d' \in D, u \in U \tag{20}$$

The model considers four objective functions. The workload balance objective is written in equations 1, 2, and 3 that minimize the relative deviation of the district workload from the mean district workload. Equations 7 and 8 minimize the distance between a given HH unit and its' assigned demand points, while equations 9 and 10 minimize the maximum distance between two demand points assigned to the same HH unit. The fourth objective ensures that the allocation is carried out with minimum values of under-capacity and is defined in 11, 12, and 13 where the monthly demand is given by 14.

Constraint 15 defines if supply points are considered open or not. Constraint 17 defines the binary and integer decision variables while 16 assures the complete and exclusive assignment of demand points. Making use of the set $(d, d') \in E$ containing all pairs of demand points considered compatible, the constraint 18 ensures that if the demand point d is assigned to the district $u \in U$, then the point d' will also be. Finally, equations 19 and 20 regard distance limitations.

The suggested solution approach uses the lexicographic method to obtain an exact solution. This approach considers the stakeholders' preferences, guarantees Pareto-efficient solutions, and does not require normalization of the objective functions.

4. Results and Discussion

The present section discusses the application of the proposed model to Provider X's case study. The developed model was applied to various instances that simulated different demand scenarios. Subsection 4.1 describes the test instances. Next, the computational results are analysed for Provider X's current HH unit and its three potential new supply points in 4.2. Finally, some managerial insights are presented in 4.3.

4.1. Growing demand scenarios and test instances

Three demand scenarios were created to study the possible combinations between the upcoming HH units. Later, a fourth scenario was created to analyze districting with all units open. For each scenario, two data frames were set: one for demand and one for supply. The supply data frame contains the assessed hospitals' list, location, daily capacity, and whether or not their HH unit is open. The last two parameters were varied for the different scenarios and discussed later in this section.

Both the number of patients per parish per month and the number of hospitalization days were randomly generated following a triangular distribution ($X \sim$ triangular(a, c, b)). The number of hospital days per patient followed $X \sim$ triangular$(1, 7, 70)$, attempting to represent the distribution observed in Provider X's patients. In turn, the number of patients per month per parish was calculated based on the number of inhabitants in each parish and an arbitrary monthly service utilization rate. A symmetric distribution was considered, given by $X \sim \text{triangular}(\frac{2p}{3}, p, \frac{4p}{3})$ where p is the product of a monthly utilization rate by the number of inhabitants in each parish. The same utilization rate was used throughout all the municipalities. This simplification is not representative of reality and was corrected in scenario 4, as discussed later.

The distance limit and planning horizon parameters were defined and did not differ between scenarios. The current Provider X hospitalization services' catchment area was studied for one year. Ideally, patients should not be more than 30 kilometers away from the hospital. However, as the feasible region should cover all demand points, the value of parameter δ_{Umax} was increased to 45 kilometers for this case study.

4.1.1 Scenario 1

Scenario 1 (S1) is based on the demand for HH services in the Hospital 1 unit in 2021 and the first quarter of 2022. In this scenario, the population residing in all parishes of the municipalities of Lisbon, Cascais, Sintra, Odivelas, Loures, Mafra, and Oeiras was considered. A service utilization rate of 0.014% generated an arbitrary number of patients per parish, making the distribution of patients proportional to the number of inhabitants in each parish in 2021, obtained from the 2021 census [1]. The rate was calculated to produce approximately the same number of patients treated by Provider X in 2021, totaling 147 annual patients. Regarding supply, each hospital can treat 12 patients simultaneously except Hospital 2, where the capacity is expected to be 6 since the decision-makers reported that the unit would always be smaller than the remainder.

4.1.2 Scenario 2

Scenario 2 (S2) assumes that demand will equal supply if the latter increases. Therefore, this scenario represents a demand proportional to that in Scenario 1, where the number of patients in S2 roughly matches the number of patients in S1 multiplied by the number of open HH units. This scenario's effective demand extends to the Amadora municipality, the only municipality in the region that has not yet obtained Provider X home hospitalization services. There are three subscenarios within S2 whose utilization rates are commensurate to the number of units opened and adjusted considering the coverage of an additional city (Amadora). By request of the stakeholders, the units' capacity was increased to 15 patients per day for all hospitals except Hospital 2, where the capacity is assumed to be 8.

4.1.3 Scenario 3

Scenario 3 (S3) forecasts a 3-year high-demand scenario. It considers the sub-scenarios described in S2 and Portugal's historical growth of Home Hospitalization services. A growth rate of 16.5% per year was considered. This value accounted for the growth experienced between August 2021 and the same month in 2022, as documented by [19]. When running this scenario, the same supply as in S2 was considered. In the discussion of the results, it was evaluated how much additional capacity was needed to meet the new demand figures.

4.1.4 Scenario 4

So far, the number of patients per parish has been based solely on the number of inhabitants and considering a common utilization rate for all parishes. However, both potential and effective demand reflect several other factors. To obtain a more accurate estimate of the number of patients, it would be necessary to comprehensively characterize the patient profiles for this type of service and quantify them for each parish. One could, for example, incorporate economic factors or break down the demand by age group, given that senior citizens are the most in need of HH services. Scenario 4 seeks to approximate the proportion of patients per parish felt in Provider X's reality observed up to now to obtain districting solutions that are of practical use for decision-makers. To this end, different rates were considered for each parish instead of considering a single utilization rate. Patients were scaled according to: $demand[Lisboa] = 2*demand[Cascais] = 4*demand[Sintra] = 8*demand[Oeiras] = 10*demand[Amadora, Loures, Mafra, Odivelas]$. These rates reflect a high-demand scenario, with a total of 773 patients per year, an intermediate number between S2.3 and S3.3.

Table 2: Characterization of the scenarios and instances used for the case study districting

Scenario	**S1**	**S2.1**	**S2.2**	**S2.3**
Utilization rate	0.014%	0.018%	0.027%	0.036%
Patients	147	273	513	677
Binary variables	276	300	88	88
Continuous variables	109	109	109	109
Constraints	39144	46136	4321	4323
Scenario	**S3.1**	**S3.2**	**S3.3**	**S4**
Utilization rate	0.028%	0.042%	0.056%	variable
Patients	486	803	1068	773
Binary variables	300	88	88	88
Continuous variables	109	109	109	109
Constraints	46136	4321	4323	4323

4.2. Computational Experiments Results

The model was implemented in a *Python* script using the library *docplex - IBM Decision Optimization CPLEX*. All tests were run on a Macbook Pro computer with an Apple M1 processor and 16 GB of RAM, running *macOS Monterey* (version 12.2.1). This section presents the results of the model implementation. Objectives were ranked according to their priority for the stakeholders in decreasing order of importance: Ψ, Δ, Θ, and Ω. Note that this order was maintained for all studies. An analysis of potential trade-offs when varying the order of preference was later conducted.

[1] Provisional data from Instituto Nacional de Estatística, url: $https://www.ine.pt/scripts/db_censos_2021.html$

4.2.1 Optimal launch order for the upcoming Provider X HH units

The possible combinations of 2 and 3 HH units within the four units under study were explored to sustain the best launch order decision. The review considered three scenarios (S1, S2, and S3) to ensure that decisions were appropriate to current and future demand. Figures 1 and 2 provide a comparison between the objective values for the different districting decisions. In the figures, the values for each objective were normalized to facilitate comparison.

Districting between two HH units

The first decision concerns the location for the second HH unit to be opened. Three pairs of units were analyzed: H1 and H3, H1 and H2, H1 and H4 [2]. It is assumed that H3 and H4 units are similar in everything except their location, having the same capacity and no preference on the part of the decision-makers to choose one unit over the other. The H2 unit, on the other hand, is a smaller unit that historically serves fewer people. Hence it is assumed that this unit has half the installed capacity of the other facilities. Furthermore, if the districting performance is equally satisfactory for all three cases, it would be preferable to open the Hospital 3 or Hospital 4 unit rather than Hospital 2.

Overall, when running the model for the three scenarios, the results for the three combinations are somewhat balanced. H2 would have some advantage in terms of location for the first two instances run, managing to be more central for patients from Loures, Mafra, and Sintra. There are, however, two reasons to discard the H1 + H2 pair. On the one hand, the instances' demand is based solely on population density in the municipalities covered by Provider X. While this is a fair measure to represent potential demand, it does not represent effective demand. The effective demand is impacted by several other drivers that were not considered when creating the test instances. When looking at the actual distribution of Provider X's patients in 2021 and 2022, the number of patients in Sintra's surroundings is less than that observed in, for example, Cascais. On the other hand, the fact that the H2 hospital is smaller than the others and is equipped to serve fewer people makes it a less evident alternative for high-demand scenarios.

When looking at the two other options, Hospital 3 or Hospital 4, it is more advantageous to open Hospital 3 first. Although the difference is not substantial, opening H3 allows the patient furthest from their HH unit to be 3 kilometers closer than if H4 had opened. The fact that the effective demand for Provider X in 2021/2022 in Cascais was proportionally higher than in the test instances reinforces the choice of opening the H3 unit first.

Districting between three HH units

The trios behave similarly concerning workload balance for the first two scenarios, not exceeding a 2.5% difference between them. The difference in compactness objectives is quite significant in the first scenario, where opening the Hospital 2 unit would be advantageous. However, this difference is attenuated in scenarios S2 and S3, and the Θ value differs in less than 2 kilometres.

[2] H3: Hospital Hospital 3, H4: Hospital Hospital 4, H1: Hospital Hospital 1, H2: Hospital Hospital 2

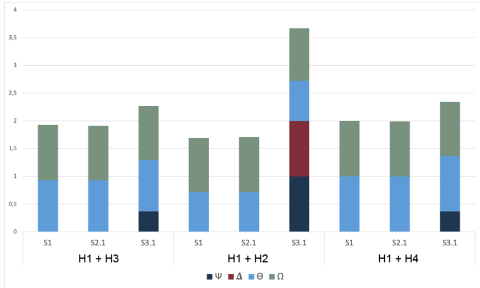

Figure 1: Comparison of districting outcomes between potential pairs of HH units.

When looking at the overcapacity values, they grow in tandem with the increase in demand. Again, as it is assumed that the Hospital 2 unit has about half the capacity of the others, the value of Ψ worsens when this unit is considered open. Considering scenario S2.2, for all patients to be served, it would be necessary to treat three more patients per day and per hospital when opening H2. For the case of opening Hospital 4, increasing the capacity by only one patient in one of the hospitals would be sufficient. In addition, the workload balance value in the higher demand scenario is substantially worse for the trio that includes Hospital 2.

This analysis shows that opening H4 is increasingly beneficial as demand rises. However, this decision is based on a higher overcapacity and worse workload balance for high-demand scenarios, which both derive from Hospital 2's capacity limitation.

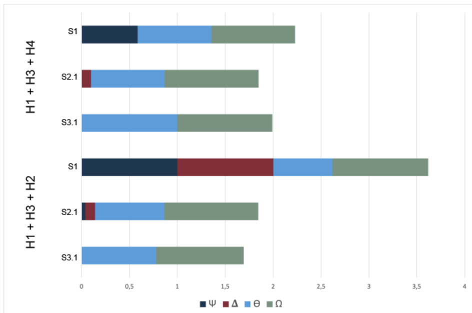

Figure 2: Comparison of districting outcomes between potential trios of HH units.

4.2.2 Districting solution for four HH units

After studying the sequential opening of three Provider X units, the districting decisions were analysed considering all units were open. For scenario S2.3, despite the annual workload value of the HH units varying between 80% and 90%, there were months of under-capacity. It would be necessary to increase capacity by two patients in the Hospital 3 and Hospital 4 units and one in the remaining units to meet this demand scenario. In scenario S3.3, all four units were under-capacity in most months, with more than double the capacity at Hospital 1 and Hospital 3 needed to meet demand. Given the high demand and the reduced capacity at Hospital 2, it was more challenging to ensure workload balance: the other three units had a 123% capacity, while the annual workload at Hospital 2 was around 145%. Regarding compactness, the results suggest that it is nearly indifferent if 3 or 4 units are open since the values of Θ and Ω are not better than those observed in the districting for three units.

It is prominent how different the districting decisions for these two scenarios are: very few clusters are served by the same unit. The model does not promote stable allocation areas, which is important at the strategic planning level. Compared to the districting models found in literature, of the 11 studied papers, only two explicitly mentioned incorporating capacity into the constraints. Furthermore, although most try to ensure workload balance, this is done annually. Incorporating a monthly capacity constraint is a novelty considering the literature studied and justifies that the solution is sensitive to fluctuations in demand.

Because the proposed districts in S2.3 and S3.3 were not congruent with each other and were not representative of the distribution of effective demand at Provider X, few conclusions could be drawn regarding the optimal catchment areas for each HH unit. To address these shortcomings, scenario S4 was created, and the model was tested. The objectives related to compactness again show similar values, as the geographic configuration of the demand points was maintained. It is important to note that this scenario contemplates a significantly lower demand in the municipality of Sintra and that it is, therefore, possible to obtain quite balanced districts in terms of workload, with the value of Δ being around 3%. Again, there are few similarities between the newly obtained districts and the two previous district sets.

Since S4 reflects the current effective demand and this study only covers a three-year horizon where significant changes in demand are not expected, the districting outcomes for this scenario are the most valuable to Provider X.

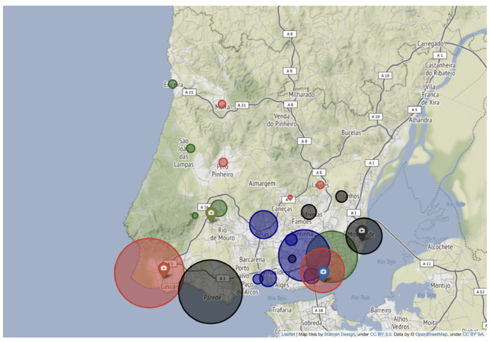

Figure 3: Districting solution considering all four HH units are open for scenario S4.

Sensitivity analysis: The potential trade-offs when optimizing one objective over the others were evaluated. It was assumed that the objective of minimizing under-capacity should always be solved first: only the six possible permutations among the remaining three objectives were evaluated. In the case of scenario S2.3, by varying the order in which the objectives are solved, the same solution is always obtained, indicating that there is a global optimal solution and that there is no compromise in minimizing one objective before any other. The same does not apply to scenarios S3.3 and S4, though the improvement of a given objective over the detriment of the others is minimal. Regarding Δ, the differences are negligible: in S4, the value does not vary, and in S3.3, it varies less than 1%. The only notable trade-off is between Θ and Ω. Still, the variation of the values of these two objectives on the Pareto frontier is minimal. By minimizing Ω first, the value of Θ increases by less than 2 kilometers for both scenarios. In the case of optimizing Θ first, the value of Ω increases by about 3.5 kilometers in scenario S3.3 and 2.7 kilometers in scenario S4. Overall, there are no notable trade-offs between the optimized variables.

The compatibility constraint, which imposes respect for administrative boundaries and also forces contiguity, was not used so far because it was not considered necessary for this case study. Note that this constraint is adaptable to various circumstances and the needs of the decision-makers since it starts from a list of compatible parishes and imposes that these are attributed to the same district. The results of districting for scenario S4 were tested, considering that demand points from the same municipality had to be allocated to the same HH unit. It was verified that imposing this constraint worsens the optimization significantly, leading to a workload imbalance of 50.8% and an increase in under-capacity from 234 to 3025.

4.2.3 Districting solution for two HH teams

The model's volatility regarding demand makes it more useful at the tactical level than the strategic level. It is possible, for example, to use the model considering only one unit at a time but dividing it into different teams. Consider the *as is* demand scenario and the sole operation of the Hospital 1 unit. Since each team treats an average of 6 patients daily, the supply was separated into two teams departing from the same location.

Solving the model obtains the optimal patient allocation between the two teams, represented in Figure 4. It is possible to balance the workload (Δ is approximately 0), guaranteeing that within the same unit, no team is overloaded. Furthermore, it is possible to ensure greater intra-district compactness, reducing the travel distances of each team. If one prioritizes the optimization of Ω over Δ, it is possible to reduce the intra-district distance by about 3 kilometers without causing a significant imbalance in workload, with Δ increasing to 3.3%.

Not straying from the tactical realm, this decision precedes and facilitates routing problems since it helps establish which area each team should cover. Since this analysis represents a novel application for a districting model, the decision-makers examined these results and validated their pertinence and importance.

Table 3: Districting results considering all four HH units open in different demand scenarios. Outcomes for the tactical districting of the Hospital 1 unit.

	S1	S2.3	S3.3	S4	S4 (C)	S1 (T)
Ψ	0.000	35.000	1264.000	234.000	3025.000	0.000
Δ	0.011	6.425	15.125	3.149	50.804	0.023
Θ	28.609	28.583	26.744	26.744	25.503	41.785
Ω	37.110	34.601	34.601	34.237	31.597	36.366
CPU time	0:00:01.90	0:00:01.06	0:03:50.37	0:00:00.63	0:00:00.17	0:00:00.88

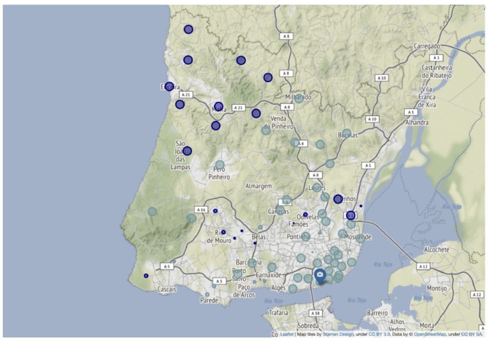

Figure 4: Tactical districting solution with two HH teams at the Hospital 1 facility for scenario S1.

4.3. General recommendations

Note that, contrary to other studies, it is impossible to compare the obtained results with previous district configurations since these do not exist for this setting. Also, when assuming full coverage of the seven assessed municipalities, it was not always possible to comply with the maximum distance restriction, where all patients should be less than 30 kilometers from the hospital. Even for the scenario *as is* and considering all hospitals were open, the most distant demand point was 31.5 km away.

With concerns to the sequential launch of the new Provider X HH units, the expansion should begin with the opening of the Hospital 3 unit, followed by Hospital 4, and lastly, Hospital 2. Nonetheless, according to the districting objectives, the results for the different units' combinations are similar, meaning that if a different order were to be adopted, comparable outcomes could be obtained.

The achieved districting decisions are quite sensitive to monthly demand oscillations. Thus, they reflect the specific demands of the scenarios studied and may not accurately represent different realities. For this reason, the model is most useful when run annually with instances generated through accurate demand forecasting. From the examined scenarios, S4 was the one to reflect best Provider X's effective demand over a three-year horizon. For the case of S4, the districting would allow for highly balanced districts in terms of workload. Districts would also be relatively compact: the maximum distance between demand points and their respective units would be 26.7 kilometers, and the longest distance between two points allocated to the same unit would be 34.2 kilometers.

Despite being less explored in this dissertation, another application for this model was discussed. It is possible to run the model more periodically to draw the optimal catchment areas of the multiple teams within an HH unit. The model was applied to the current Provider X paradigm: the H1 unit was divided into two teams, and the proposed districting allowed for minimal intra-district distances and a balanced workload between the two teams.

5. Conclusions and Future Work

This paper proposes a generic multi-objective MILP model for a districting problem applied to a home hospitalization service network. The proposed approach considers four objective functions: balancing workload among districts, maximizing the compactness of districts, both between supply points and their allocated demand points and between demand points within the same district, and minimizing the number of months in which HH units are in under-capacity. The objectives are also subject to constraints related to the complete and exclusive assignment of demand points, compatibility between demand points, and the maximum distance between basic units.

The model application sought to illustrate the two types of results that can be obtained. The primary solution achieved was the optimal partition of a service region composed of home hospitalization units and aggregate demand points. Although less addressed in the literature, it is also possible to use the same model to evaluate which teams should serve which patients within the same HH unit.

To evaluate how the districting would vary for different demand distributions, computational results were presented for several randomly generated instances based on the real-world case study data. The instances incorporated uncertainty at two levels: the number of patients per civil parish and hospitalization days per patient.

The proposed solution approach included a lexicographic ordering that allowed the efficient frontier and trade-offs between objectives to be identified. However, for this case study, no sharp trade-off was identified, as improvements to a particular objective never resulted in significant deterioration of the others. For most of the tested instances, the compatibility constraint, which ensures respect for municipal boundaries and contiguity of the created districts, was not used. When testing the model with this set of constraints, it was found that it significantly worsened the results, leading to an increase in workload imbalance of more than 47% and an under-capacity value almost 13 times higher. Thus, the compatibility constraints should only be employed to meet specific situations such as past partnerships, historical reasons, or other administrative situations.

To date, this is the first study to address the model in the context of home hospitalization explicitly. Compared to the literature handling districting in home care, there are mainly two noteworthy differences. On the one hand, the parameter values inserted into the model differ, particularly the maximum distance between the hospital and the patient. On the other hand, concerning the model formulation, it is imperative to consider the monthly capacity, given the seasonality of demand, and the scarcity of skilled medical personnel, especially in the Portuguese context.

Ensuring that supply meets demand and, when it does not, minimizing overcapacity is a novelty of this work. It allows the model to provide insights into which months and units there is a capacity shortage and thus can help decision-makers select which HH units are in greatest need of expansion.

This study benefited from contact with HH service managers, an essential interaction for adequately characterizing the problem and validating the results. In addition, it confirmed that operational research techniques can help healthcare providers improve service delivery. Lastly, although the proposed model and solution have been implemented and validated for the Portuguese case, the approach can be easily extended to other HH providers in any territory.

Future research opportunities can take several directions. The current model's computation tractability is limited for small and medium-sized instances. Therefore, future work can explore efficient approaches to multi-objective optimization for bigger cases through exact methods or heuristics and meta-heuristics. The performance of these new approaches should be compared in terms of objective function values and computational time.

There is still very diminutive research that addresses the problem of healthcare districting, considering the substantial uncertainty inherent to this sector and its impact on all levels of decision-making. This dissertation focused on uncertainty in demand by simulating the results for varying demand scenarios. It is possible to identify two potential points for improvement in addressing uncertainty. Firstly, very dissimilar districting plans were obtained when testing the model for the different demand scenarios. It would be relevant to investigate other robust ways of inserting uncertainty in the optimization. Secondly, demand forecasting greatly influences districting results, so it would be valuable to have used regression methods that would allow demand to be estimated more accurately.

Further objectives can also be considered for the problem. It could be explored, for instance, the insertion of economic criteria to assess and compare expenditures or savings between districting plans. Additionally, it may be relevant to investigate further the use of the proposed model by fixing a single HH unit and addressing the districting of several teams within that unit. It would be necessary to review the specific constraints and objectives of this problem.

It would be interesting to diversify the stakeholders' involvement, complementing the model with inputs from medical staff and other practitioners, patients, and their families. Creating a graphical user interface that allows a simplified interaction between the decision-makers and the model would also be of added value. Finally, it would be insightful to evaluate the impact of districting on the tactical and operational decisions of managing a home hospitalization service network.

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