A districting approach for a home hospitalization service network under uncertain demand

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Declaração

Declaro que o presente documento é um trabalho original da minha autoria e que cumpre todos os requisitos do Código de Conduta e Boas Práticas da Universidade de Lisboa.

Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Resumo

Os benefícios da hospitalização domiciliária estão bem estabelecidos na literatura e justificam a sua crescente procura. Juntamente com a sua rápida expansão, este modelo de assistência hospitalar está também sujeito a uma maior incerteza e variabilidade em comparação com outros serviços domiciliários. Dadas as especificidades logísticas de uma rede de serviços de hospitalização domiciliária, instrumentos de investigação operacional que apoiam a tomada de decisões tornam-se imperativos, nomeadamente no que diz respeito à localização e alocação de recursos.

Este documento aborda um problema de distritamento motivado pelo estudo de um caso real de uma rede de serviços de hospitalização domiciliária no distrito de Lisboa, Portugal. O problema é formulado como um modelo de programação linear mista multi-objectivo, considerando critérios de atribuição completa e exclusiva, compatibilidade, limitações de distância e capacidade, compacidade, e equilíbrio da carga de trabalho. A função objectivo combina os três últimos critérios utilizando o método lexicográfico, procurando minimizar o desvio da carga de trabalho média, a distância entre unidades básicas e a utilização de sub-capacidade.

Foram apresentados resultados computacionais para várias instâncias geradas de forma a representar cenários de procura crescente. O modelo foi explorado para três situações: a abertura gradual de potenciais novas unidades de hospitalização domiciliária, o plano de distritamento quando todas as unidades da rede estão abertas e a alocação de múltiplas equipas dentro de uma única unidade. Foram realizadas análises de sensibilidade sobre os principais parâmetros para obter *insights* gerenciais, concluindo-se que é possível obter distritos tanto equilibrados a nível de carga de trabalho como compactos sem compromisso expressivo entre os dois objectivos.

Palavras-chave: Hospitalização Domiciliária, Problema de Distritamento, Optimização Multi-objectivo, Programação Linear Inteira Mista, Investigação Operacional

Abstract

The benefits of home hospitalization are well established in the literature and justify its growing demand. Along with its rapid expansion, this model of care is also subject to higher uncertainty and variability compared to other home-based services. Given the logistical specificities of a home hospitalization service network, operational research tools that support decision-making become imperative, notably regarding the location and allocation of resources.

This paper addresses a districting problem motivated by a real-world case study of a home hospitalization service network in the district of Lisbon, Portugal. The problem is formulated as a multi-objective mixed integer linear programming model by considering criteria of complete and exclusive assignment, compatibility, distance and capacity limitations, compactness, and workload balance. The objective function combines the last three criteria using the lexicographic method, seeking to minimize the deviation from the average workload, the distance between basic units, and the use of under-capacity.

Computational results were presented for several generated instances representing increasing demand scenarios. The model was further explored for three situations: the gradual opening of potential new home hospitalization units, the districting plan when all units in the network are open, and the allocation of multiple teams within a single unit. Sensitivity analyses were performed on the main parameters to obtain managerial insights, concluding that it is possible to obtain both workload-balanced and compact districts with no expressive trade-off between the two objectives.

Keywords: Home Hospitalization, Districting Problem, Multi-objective Optimization, Mixed-Integer Linear Programming, Operational Research

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List of Acronyms

D Deterministic

GA Genetic Algorithm

GS Geographical Simulation

H1 Hospital 1

H2 Hospital 2

H3 Hospital 3

H4 Hospital 4

HH Home Hospitalization

HHC Home Health Care

LA Location-Allocation

LTV Lisbon and Tagus Valley

MH Metaheuristic

MILP Mixed-integer linear programming

MINLP Mixed-integer nonlinear programming

MIP Mixed-Integer Programming

NHS National Health Service

NLP Nonlinear programming

OR Operational Research

OR/MS Operations Research/Management Science

S Stochastic

SP Set Partitioning

TS Tabu Search

WHO World Health Organization

Chapter 1

Introduction

This chapter introduces and motivates the reader to the problem under study and specifies the objectives and methodology used. To this end, Section 1.1 presents the districting problem in a home hospitalization setting and the relevancy of conducting this research. Section 1.2 defines the specific objectives of this dissertation. Section 1.3 summarizes the methodology used to address the problem under study, and finally, Section 1.4 introduces the document's structure.

1.1 Topic Overview and Motivation

Home hospitalization provides acute health care in a patient's home as an alternative to traditional inpatient hospital care. It can replace hospital care entirely or reduce hospital length of stay through early discharge. This model of care is an especially suitable solution given the growing global need for hospital beds, rising healthcare costs, and the aging population. The clinical results observed so far are very favorable, with an overall reduction in hospital days and a decrease in the risk of hospital readmission. In addition, HH has increased patient and family satisfaction while reducing hospitalization costs (Chua et al., 2022).

Home hospitalization is included in the spectrum of home care. Despite sharing several attributes with other home care services, such as the need to travel between patients' homes and the multidisciplinarity of the teams, the specific features of home hospitalization bring new logistical challenges. Some of these challenges are the need for greater proximity to the hospital, the possibility of sharing staff with other hospital services, and the increased uncertainty experienced due to the acute illness profile of patients and consequent shorter lengths of stay .

The environment of extreme uncertainty, variability, and ongoing change in which health services operate makes detailed information for decision-making strongly desirable (Addis et al., 2015). Analysis of the geographic organization and distribution of health service capacity, from the local hospital level to services provided throughout a region or country as a whole, is often required. In this set, *districting* is a strategic-tactical planning decision that involves clustering a set of demand points, i.e., a group of patients aggregated according to their location, into districts that satisfy relevant criteria. Adopting a districting approach in a home hospitalization setting leads to increased reactivity and efficiency of caregivers. It also facilitates human resource management, improving the quality of care and increasing patient and provider satisfaction (Benzarti et al., 2013). Despite the clear benefits of mathematical decision support tools such as districting, these issues have seldom been considered in the home hospitalization literature.

Operational Research is increasingly used as a critical instrument to improve home care programs' outcomes and make them more efficient and effective. The number of articles published in the Web of Science database over the past 25 years proves the growing interest in this topic. Three searches are shown in Figure 1.1: the broader one used the keyword "home care", the second combined "home care" and" operations research", and the last search used "home hospitalization". In all cases, the number of published articles grew continuously, with more than 11 000 articles published in 2021 covering home health care and nearly half of those considering operational research methods. The home hospitalization search showed fewer results than the others, which is reasonable given the specificity and novelty of the topic. Furthermore, combining the search words "home hospitalization" and "operational research" yielded only two results proving that articles covering the intersection of these two topics are residual to date. The shortage of literature regarding home hospitalization is a key matter for this dissertation. This paper seeks to fill the existing gap regarding the literature on home hospitalization, particularly at its convergence with the districting problem.

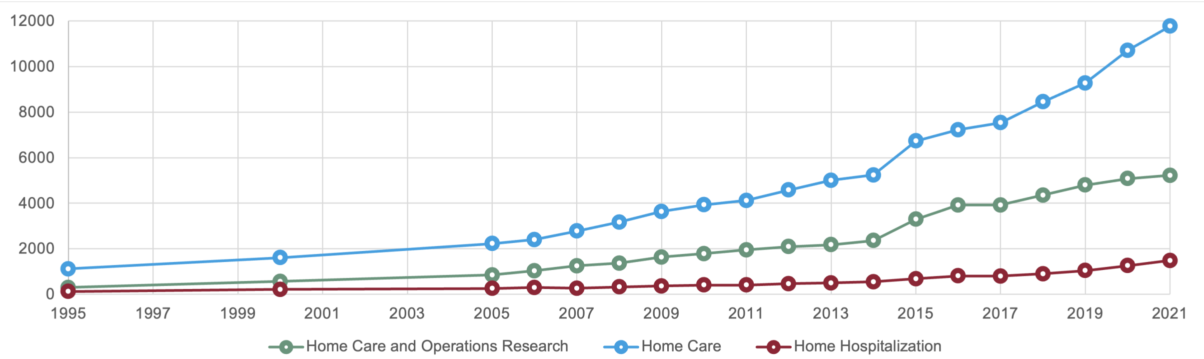

Figure 1.1: Evolution of the number of published articles in the Web of Science database with Home Care, Home Hospitalization, and Operational Research as keywords. The search was done using the topic box that searches for keywords in the title, abstract, author keywords, and keywords plus.

1.2 Objectives and Deliverables

The main objective of this dissertation is to use OR/MS techniques to develop a tailor-made approach for the districting problem in the home hospitalization context. The model should be adjusted to the evidence retrieved from the case study but be developed generically, making it applicable to any other HH service network. In addition, the design and implementation of the model should consider the uncertainty in demand and assess how it affects districting decisions.

To achieve the main objective, below are the milestones to be accomplished throughout the dissertation:

1. Characterizing the case study, a Portuguese private home hospitalization services network, its organization, and operational procedures, with particular regard to the logistical characteristics relevant to the districting plan;

2. Reviewing prior research in the field, analyzing the contributions of operational research to the districting problem, detailing models and approaches to the resolution used so far;

3. Identifying potential issues where existing literature does not meet the needs of the case study and proposing rectifications or alternative modeling approaches;

4. Formulating a mathematical model that captures the main features of the case study and implementing the developed model to generated or real data that certifies the model's applicability to the problem under study;

5. Analyzing the model results for different demand scenarios and providing recommendations that support the districting decisions of the home hospitalization services network under study.

In particular, taking into account the expanding HH service of Provider X that will serve as a case study, the developed model should be able to provide insights that support two deliverables:

- The optimal launch order for the future home hospitalization units in the Provider X network and the districting approach at each implementation phase;
- The districting plan when all units in the network are open, considering varying demand scenarios.

1.3 Research methodology

The methodology used in this dissertation consists of five stages outlined in Figure 1.2. The first phase describes the context of home hospitalization, particularly for the healthcare provider under study; it should also introduce the main applications of OR/MS in this context and define the districting problem. Then, it is necessary to conduct a literature review to comprehensively understand the districting problem and provide a solid theoretical basis for building the following phases. The mathematical formulation of the model takes place in the third phase of the methodology, starting with the statement of assumptions followed by the definition of variables, constraints, and objective functions. Finally, the best solution approach is discussed. The fourth stage ensures that the model produces valid and consistent results and that the assumptions made in the previous stage do not significantly impact the model's outputs. For model validation, it is first necessary to collect and process data to generate the datasets that will serve as input variables for the model. The model is then tested and iterated until it accurately describes the reality of HH. The model is applied to the case study in the fifth phase, and the results for different constraints and scenarios are discussed. Conclusions and recommendations are drawn about the problem presented at the beginning, and future research topics are suggested.

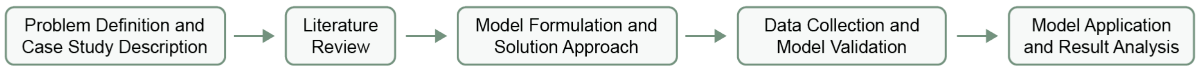

Figure 1.2: Proposed methodology.

1.4 Thesis Outline

The dissertation is structured into six chapters. All chapters begin with a short description of their content and finish with a closing section that synthesizes their main takeaways. The six chapters are outlined as follows:

- **Introduction:** The present chapter introduces the dissertation subject and its importance. It also includes the definition of the objectives, the methodology, and the document's structure.
- **Case Study:** The second chapter aims to introduce the main characteristics of home hospitalization in general, then contextualize the Portuguese case and, finally, the private organization under study.
- **Literature Review:** This chapter reviews the main scientific articles covering OR applications in home care, emphasizing the districting problem. It surveys the existing modeling and solution approaches, dividing them into deterministic or stochastic models.
- **Model Formulation:** The fourth chapter employs the insights from the literature review and case study description to refine the problem statement, highlighting the considered assumptions and criteria. It presents a multi-objective mixed-integer linear programming formulation to solve a districting problem in a home hospitalization setting.
- **Model Implementation:** This chapter describes the procedure for generating the input data applied to the model, specifying the characteristics of each demand scenario. The computational results that result from the model implementation are analyzed and validated. The main findings and recommendations derived from this discussion are presented.
- **Conclusions:** The last chapter summarizes the dissertation's main conclusions, achievements, and limitations, presenting prospects for future work.

Chapter 2

Case Study

This chapter aims to characterize the problem under study, starting with the general definition of Home Hospitalization, then specifying the state of implementation in Portugal, and concluding with the particular case of Provider X.

Section 2.1 presents an overview of HH and its framing in home health care services. The concerns of home care managers are briefly addressed and linked to the importance of proper districting decisions. In Section 2.1, there is also an analysis of the global implementation state of HH services, including observed positive results, possible pain points, and catalyzing factors. Section 2.2 looks at the Portuguese context. The functioning of the Portuguese health system is broadly characterized, along with the country's implementation state of HH services. A brief description of the Portuguese administrative divisions is also presented in this section, with a special focus on the Greater Lisbon region. Finally, population demographics and indicators of the nation's state of health underpinning the growing demand for home hospitalization services are discussed. Section 2.3 describes the Provider X HH unit, the primary stakeholder with whom this dissertation was developed. Section 2.4 features a statement on the research problem addressed in this dissertation and the specific terms of Provider X's collaboration. Lastly, Section 2.5 presents the chapter's conclusions.

2.1 Home Hospitalization

The healthcare landscape is transforming rapidly alongside technological, economic, and demographic shifts. New challenges come from these developments, but also disruptive ways of solving them. This Section discusses home hospitalization, a service that, by leveraging the advent of new technologies and the new needs of the population, has been growing significantly in recent years.

From the general to the specific, Subsections 2.1.1 and 2.1.2 present the definition of Home Health Care, the concerns felt by decision-makers in this setting, and how districting decisions can address these concerns. Subsection 2.1.3 frames home hospitalization within Home Health Care services. It also presents the advantages of HH, possible setbacks, and the impact of the COVID-19 pandemic.

2.1.1 Home Health Care

Home Health Care (HHC), also known as domiciliary care, social care, in-home care, or simply home care, consists of visiting and nursing patients in their homes. Its goal is to allow individuals, even with an illness or injury, to live independently for as long as possible and delay or avoid the need to relocate people from their homes to nursing facilities. This service focuses on elderly people with chronic illnesses or other maladies of aging; accordingly, its demand is closely tied to the aging of the population. HHC is provided by licensed personnel such as nurses, therapists, and home health aids; some of the services covered are basic assistance care, companionship, occupational and physical therapy, speech therapy, home-delivered meals, nutritional support, pharmaceutical services, medical social services, doctor care, and skilled nursing (Hopkins, 2022).

2.1.2 Operations Management in Home Care services

When designing and planning an HHC network, decision-makers encounter several new challenges compared to conventional healthcare delivery. In conventional care, patients move to the care facility. In contrast, there are several decentralized care points in HHC delivery, making coordination between practitioners' activities an essential aspect of managing an HHC network (Matta et al., 2014). Managers need to integrate the patients' homes into the care supply chain and move different flows of human and material resources toward the patients' homes. It is also essential to ensure continuity of care, especially to preserve the quality of service perceived by the patient and increase their trust in the service (Benzarti et al., 2013). To this end, ensuring patients' assignment to a single care provider is necessary. Ultimately, any health service's goal is to respond promptly to patient demand and maximize the number of treated patients without neglecting to provide the best quality service possible. Ensuring this is particularly challenging given a healthcare system's extreme uncertainty and variability (Darmian et al., 2021).

There are several strategic and tactical planning contexts where decision-makers have to divide a geographical area into clusters or districts composed of basic units (Darmian et al., 2021). Districting addresses this problem: it seeks to optimally group basic units into districts, subject to some criteria related to the basic units' activity, demography, or geographical characteristics. In a healthcare system, districting can positively impact accessibility and availability in all residential areas and facilitate health network management, medical logistics planning, and equipment allocation (Matta et al., 2014, Darmian et al., 2021). Section 3.1 will address a more thorough description of the impact of districting on healthcare management decisions.

2.1.3 Home Hospitalization: a home-based care service

Home hospitalization, often referred to as hospital-at-home, consists of treating acute illnesses at home rather than in an in-hospital stay, allowing for early discharge or even total avoidance of hospital admission (Chua et al., 2022). Some of the services included in HH are remote monitoring, presential clinical care from nurses and physicians, diagnostic testing such as electrocardiograms or radiography, and intravenous medication in one's home (Qaddoura et al., 2015).

Home hospitalization addresses several growing needs in the healthcare sector, namely the need for more hospital beds, rising health-related costs, and increasing disease burden. Hospital inpatient admissions represent the most considerable portion of public health costs (Barcala et al., 2006). It comprises approximately one-third of total medical expenditures in the US, according to Levine et al. (2020). The recent development of technologies such as telemedicine and nanotechnology also favors the increase of HH units.

Home hospitalization falls under the umbrella of home care. Nevertheless, it is important to highlight where these two terms intersect and where they differ. The remaining subsection addresses this distinction, summarized in Table 2.1.

Home Hospitalization *versus* Home Care: Similarities

Generally speaking, home care and home hospitalization function similarly, given that both are an alternative to traditional hospitalization. Both services are administered by healthcare professionals who come to the patient's home.

The growth in demand for HHC follows the same trend as the demand for home hospitalization, accelerated by government pressure to reduce healthcare costs and the increasing number of people with chronic diseases. Both services benefit from the developments in new technologies (Benzarti et al., 2010).

In both types of services, several sources of uncertainty have been addressed in the literature. The primary source of uncertainty on which this dissertation will focus most strongly is uncertainty in demand, that is, in the number of people requiring this type of care. The second uncertainty factor relates to the delivery process, where travel and treatment times are somewhat variable and unpredictable. Finally, there is uncertainty related to material and human resources, either by depletion of consumable resources or by the lack of workers due to illness or vacation, among others. Even the progression of the patient's condition brings some uncertainty to decision-making (Benzarti et al., 2010).

Home Hospitalization *versus* Home Care: Differences

While home care focuses on chronically ill patients, hospital-at-home is aimed toward acute-level care. It is distinguished from home care by providing intensive and highly specialized care in acute and complex disease states. The main differences resulting from this distinction are listed below.

- **Treatment time:** By definition, HH is a modality that occurs over a punctual period of time, and this is not usually the case with home care, which is more continuous and extended. For this reason, a home hospitalization service has more patient turnover and presents even greater demand uncertainty. Moreover, trends associated with an aging population and chronic illnesses are more predictable than trends related to acute illnesses.

- **Type of care provider:** Care provided in home care, although provided by licensed personnel, requires less skilled labor. For this reason, unlike HHC providers who mostly only work for that purpose, the medical staff required for HH is the same as would work in other wards of a hospital. This may indicate that the staff allocated to the HH unit may have to be shared by several services in the hospital.

- **Type of equipment:** Care provided in home care might require very little to no equipment. In contrast, a hospital-at-home unit requires a wide range of equipment, both in quantity and type.

- **Need for greater proximity to the hospital:** When dealing with acute diseases, there is a much higher risk of complications and a consequent need for closer monitoring in case the patient has to be brought to the hospital. For these reasons, patients need to be at a relatively short distance from the hospital, which reduces the catchment area of each home hospital unit compared to that of a HHC unit.

Table 2.1: Home Hospitalization *versus* Home Care.

Home Care	**Home Hospitalization**
Encompasses all medical and paramedical services delivered to patients at home. Commonly associated with palliative and end-of-life care.	Solely treats acute or aggravated chronic illness, substituting conventional hospitalization.
Service provided for a widely varying length of time; may be provided continuously over a long time span.	Service typically provided for a short time span.
Assistance is normally delivered by dedicated institutions.	Delivered by hospital facilities, usually through a distinct Home Care Unit.
Provided by licensed personnel.	Provided by specialized physicians and nurses.
Involves little equipment to no equipment.	Involves some equipment.
Does not require proximity to a hospital.	Requires proximity to the hospital.

2.1.4 Outcomes

To date, several studies have analyzed the outcomes of home hospitalization in chronic and acute diseases such as heart failure and chronic obstructive pulmonary disease (Qaddoura et al., 2015, Hernandez et al., 2003). According to these studies, adopting HH positively affects the stakeholders involved in healthcare services provision, including the patient, their families, and the healthcare providers. There is a decreased risk of hospital readmission, higher patient and caregiver satisfaction, and improved health-related quality of life for patients (Qaddoura et al., 2015). At the same time, the mortality rate remains the same as in conventional hospitalization (Barcala et al., 2006, Arsenault-Lapierre et al., 2021). Furthermore, HH frees up hospital beds, allowing for an increase in hospital capacity.

The perceptions of home hospitalization among stakeholders were synthesized by (Chua et al., 2022). The review article reports HH as a step toward more patient-centered care, allowing for a more comfortable experience and consequent observable reduction in anxiety in patients. HH also grants a greater and facilitated interaction with relatives and loved ones, supporting the treatment itself and removing the need to travel to visit such patients. These factors are known to raise patient morale and confidence and thus speed up the recovery process.

The fact that patients are seen in the privacy and comfort of their homes enables healthcare professionals to provide a more accurate and uncluttered analysis, so clinical outcomes also tend to be better in home hospitalization (Chua et al., 2022). The proximity of the HH team to each patient's reality also creates an opportunity to educate and promote healthier habits for both the patient and relatives; adjustments in the control of comorbidities are often visible (Azevedo, 2020). Moreover, the review article published by Arsenault-Lapierre et al. (2021) suggests a lesser risk of hospital readmission and long-term care admission for HH patients.

Regarding cost savings, even though they vary depending on the HH's financing mechanism, a reduction of up to 38 % was reported in Chua et al. (2022). Cryer et al. (2017) registered savings of 19% in their HH unit in New Mexico, USA. The cost reductions were reportedly generated from a shorter average duration of stay and the usage of fewer lab and diagnostic procedures when compared to similar patients in hospital acute care. Additionally, in the recent randomized trial led by Levine et al. (2020), it was observed that the median direct cost for acute care in patients hospitalized at home was 52% lower than in usual care. The savings are even more remarkable when comparing the costs of acute care plus the 30 days of post-discharge, reaching 67%.

2.1.5 Possible setbacks

Despite the benefits presented above, there are also certain challenges to a home hospitalization scheme. The transfer of care responsibility from the hospitals to caregivers increases the burden on them, especially for those who do not live with the patients and those taking care of mentally ill patients. These caregivers report that it is difficult and stressful to care for their family members 24/7, negatively affecting them physically and emotionally (Chua et al., 2022).

A problem of intimacy is also reported, both by the staff, who can be intrusive in their presence in the patient's home, and by the family carers, who may be forced into closer contact with the patient than they would otherwise prefer (Rossinot et al., 2019).

Further related to staffing, studies report high staff turnover rates, which seem to be linked to the added challenge of the high variability of schedules and the intricate coordination between care and travel time. Added to this problem is the extra workload for the remaining healthcare providers (Chua et al., 2022, Rossinot et al., 2019).

Despite the emergence of a growing number of frameworks with inclusion and exclusion criteria for screening patients for home hospitalization, the patient selection remains challenging. It is pointed out that diagnosis and clinical criteria have too much weight in these decisions, while the patient's environment should be considered with equal importance (Chua et al., 2022). Furthermore, some providers and physicians are hesitant to refer their patients to home hospitalization as they may not be informed of the benefits of this type of care or feel that their patients may not qualify clinically (Gavin, 2022).

2.1.6 The effect of the COVID-19 pandemic

Adding to the factors mentioned above, the COVID-19 pandemic also served as a catalyzer for Care at Home services (Bestsennyy et al., 2022). When comparing virtual care tools, the US article by Singhal et al. (2022) states that the use of telehealth was 38 times higher than pre-pandemic levels, reaching 150 million telehealth claims in less than two years. The pandemic also accelerated the development and implementation of remote monitoring devices and other new technologies: one in every five American healthcare directors said that their practice offered remote patient monitoring in a poll from April 2021. Investment in the digital health market in the United States has also risen exponentially in the last years, from $8.2 billion in 2019 to $14.9 billion in 2020 and $29.1 billion in 2021 (Bestsennyy et al., 2022).

In 2020, HH was the largest growing service in the Portuguese National Health Service (NHS), and the COVID-19 pandemic was closely related to this growth. During this period, three goals had to be achieved: protect patients and families from hospital contact, preserve health professionals, and free up hospital beds for the most critical needs (Gaspar, 2020).

Azevedo (2020) described the role of home hospitalization in tackling the pandemic, stating that it could be segmented into three moments. In the first moment, HH could be an alternative for patients not infected by SARS-CoV-2, allowing the hospitals to have more capacity in conventional hospitalization for situations that required greater vigilance. In the second moment, HH could treat SARS-CoV-2 infected patients: in Portugal, patients could shift to HH after at least 7 days of conventional hospitalization, if having clinical stability and favorable evolution, among other conditions. Finally, if the health system's capacity was exhausted, HH could cover primary care of SARS-CoV-2 patients.

Hospital Doutor Fernando Fonseca (Amadora-Sintra), located in the municipality of Amadora, in Lisbon, launched the HH service in the middle of the pandemic, being the latest unit of this sort. The launch of the Amadora-Sintra HH unit contributed significantly to the hospital's capacity increase, with the unit providing treatment for 159 patients during its first year of existence (HFF, 2021). Currently, they have an inpatient capacity of 5 simultaneous users, with a capacity utilization rate above 90% and the short-term objective of expanding the installed capacity to 15 patients. With the launch of Amadora-Sintra's unit in 2020, the Portuguese NHS could treat 200 patients simultaneously through HH. During that year alone, the number of patients treated at home in Portugal increased by 800% (Gaspar, 2020).

Hospital Garcia de Orta (HGO) has the oldest HH unit in the country and has treated over 2 000 patients in the last 5 years. The unit integrates the Internal Medicine service, with a multidisciplinary team composed of doctors, nurses, a social worker, a pharmacist, a technical assistant, and a hospital administrator. Four doctors and eleven nurses are available full-time. Since its creation, HGO's HH unit has multiplied its response capacity, increasing its average capacity from 5 patients in 2015 to 30 in 2020 (five vacancies are exclusive to COVID-19 patients) (Healthnews, 2020). Between March 2020 and July 2021, 150 patients with COVID-19 were treated and more than 1 200 teleconsultations were held. Since respiratory rehabilitation plays a fundamental role in COVID-19 recovery, the HH unit teams at Hospital Garcia de Horta, composed of a physician and a nurse, presented proposals for exercises that patients could do independently. A telemonitoring system also supervised these patients, allowing remote monitoring while reducing contact (Cov, 2021). The HGO is an example of good HH practices in

Portugal, having been responsible for establishing the groundwork so that, in October 2018, 23 hospitals signed the launching protocol for this service with the National Health Service (Healthnews, 2020, SPMI, 2021). Despite the successful example of this and other units, it is worth noticing that most decisions on HH are made at the hospital level, not being centralized in the decision-making chain. More on the implementation of home hospitalization in Portugal is provided in subsection 2.2.4.

2.2 The Portuguese context

The present section serves to contextualize the reader on the Portuguese reality, in the hope that it will be relevant for a better understanding of the following chapters. Subsection 2.2.1 presents a brief description of the Portuguese health system, followed by an outline of Portuguese administrative boundaries, a particularly relevant subject given the inherent geographical nature of the districting problem. Some sociodemographic insights are presented in subsection 2.2.3, with a focus on characterizing the aging of the Portuguese population felt in the last decades. The last subsection portrays the state of the art of HH in Portugal.

The present section serves to contextualize the reader the Portuguese reality, a framing that will be relevant for a better understanding of the following chapters. Subsection 2.2.1 presents a brief description of the Portuguese health system, followed by an outline of Portuguese administrative boundaries, a particularly relevant subject given the inherent geographical nature of the districting problem. Some sociodemographic insights are presented in subsection 2.2.3, which focuses on characterizing the aging of the Portuguese population in the last decades. The last subsection portrays the state of the art of HH in Portugal.

2.2.1 Portuguese Health System

The Portuguese Health System is composed of three coexisting systems:

1. The National Health Service offers universal and mostly free-of-charge nationwide coverage, with taxation as its primary funding source. The NHS also maintains some agreements with private entities for the complementary provision of health care to its users.

2. Health subsystems are special health insurance schemes that provide coverage for specific sectors, either public or private (e.g., ADSE is the scheme for civil servants and SAMS for the banking sector).

3. Voluntary private health insurances represent a complementary activity to the previous systems. Yet, the number of people with private health insurance exceeds 3 million (Observador, 2022), covering almost a third of the Portuguese population.

The Ministry of Health is responsible for conducting, implementing, evaluating the national health policy and managing the NHS. It conducts most of the planning and regulation at the national level, while the five regional health administrations (North, Center, Lisbon and Tagus Valley, Alentejo, and Algarve) are responsible for managing the NHS at the local level. The Health Regulatory Authority (ERS) is the independent public entity responsible for regulating the activity of all health providers, whether public, private, or social.

All levels of care, from primary to tertiary, are provided by both the public and private sectors. Private providers are mostly concentrated in the Greater Lisbon and Porto metropolitan areas and along the coast between those two cities, with the population in rural and interior regions having more limited access to GPs (OECD et al., 2019). The most significant share of private hospital users is health subsystems and private health insurance scheme beneficiaries.

2.2.2 Administrative divisions

There are 18 Districts and 2 Autonomous Regions in Portugal. The 18 districts are divided into 308 municipalities and then into 3092 civil parishes. The Lisbon and Tagus Valley regional health administration covers 50 municipalities, encompassing the totality of the Lisbon district and part of the Santarém, Setúbal, and Leiria districts. The Metropolitan Area of Lisbon (AML) is a NUT II region [1] and NUT III sub-region, with the capital located in the city of Lisbon. It represents a large portion of the Lisbon and Tagus Valley health region, accounting for 2,870,770 inhabitants in 2021 (Instituto Nacional de Estatística, 2021). AML is one of the only NUTS II regions with a recorded growth in population since 2011 (Instituto Nacional de Estatística, 2021). The part of AML located on the northern bank of the Tagus River is a former NUTS III sub-region referred to as Greater Lisbon. The studies undertaken in this dissertation will predominantly target the Lisbon district, which mainly falls within the AML. The district's breakdown by its' 16 municipalities is shown in Figure 2.1.

Figure 2.1: Map of the Lisbon district and its 16 constituent municipalities. The Greater Lisbon sub-region is also represented.

[1]NUTS - Territorial Units for Statistical Purposes, developed by Eurostat. The nomenclature is subdivided into three levels (NUTS I, NUTS II, NUTS III), defined according to population, administrative and geographical criteria (PORDATA, 2018).

2.2.3 Portuguese sociodemography

The general improvement in living conditions and access to health care over the last decade allowed for a significant increase in life expectancy in Portugal, reaching 81.6 years in 2017, a slightly higher value than the EU average (OECD et al., 2019, Almeida et al., 2017). Combined with the decline in fertility rates and the decrease of people aged 15-64, these factors are causing a “double aging” effect in Portugal (Almeida et al., 2017, Duarte and Gil, 2019). The provisional data from the 2021 census (Instituto Nacional de Estatística, 2021) corroborate the steady growth rate of elderly people: more than one-fifth of the Portuguese population, around 2.4 million people, is older than 65, a number that increased by 20.6% in the last decade. This situation will lead to substantial challenges for the Portuguese health system in the upcoming years.

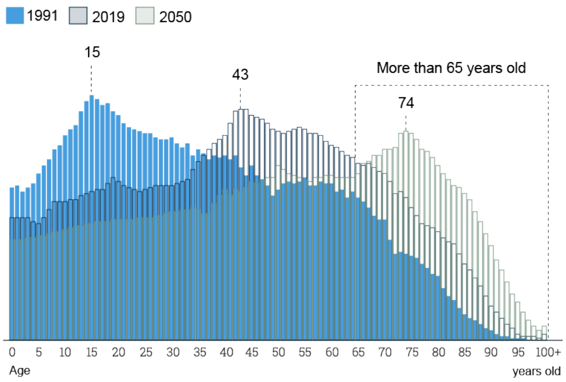

Figure 2.2: Distribution of population by age in the years 1991, 2019 and forecast for 2050. Adapted from Moreira (2020).

While it is true that Portugal has had remarkable success in improving the levels of mortality and life expectancy, the situation is not so favorable in terms of the healthy life years indicator. According to OECD et al. (2019), in 2017, people aged 65 could expect to live about 20 years longer, in line with European figures. However, about 13 out of these 20 years were likely to be lived with some disability, one of the worst results in the EU. Around half of the elderly (53 %) report having at least one chronic disease, with many reporting two or more chronic illnesses (OECD et al., 2019).

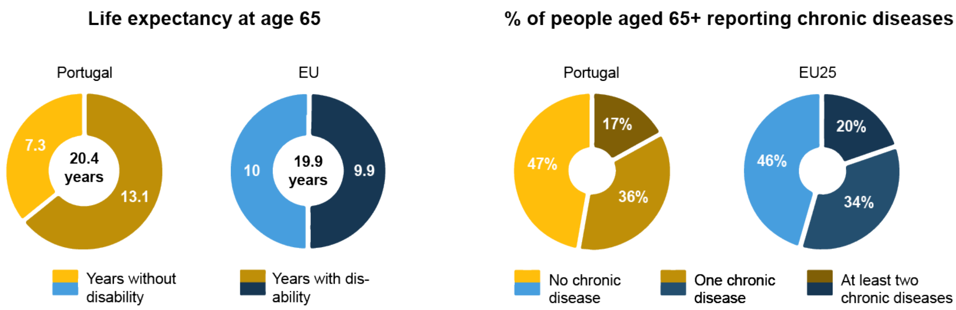

Figure 2.3: Life expectancy and healthy life years at age 65: comparison between Portugal and the EU. Adapted from OECD et al. (2019).

2.2.4 Home Hospitalization in Portugal

Home hospitalization can be adopted through two models, either additive or substitutive (Tay, 2021). The first model completely replaces patient admission by directing patients to a HH unit straight from the emergency department (ER). If a hospital reaches total capacity, patients who might not have received therapies can be treated at home: the HH unit works as an extension of the hospital's capacity. The second model redirects patients after an initial stabilization period in recovery rooms. In Portugal, a hybrid system combining these two models is used, depending on hospital needs (SPMI, 2021).

In the last years, the Portuguese government has been investing in the growth of the HH program: in 2018, the national strategy for the implementation of home hospitalization units in the National Health System was defined (MS, 2019), and in 2019 the Minister of Health established the extension of this care delivery model to all NHS hospital facilities (MS, 2020).

These endeavors resulted in 28 HH units distributed across 5 Portuguese regions at the end of 2020. Most HH units are in the regions of LTV (10 units), Norte (9 units), and Centro (7 units). Between the years 2019 and 2020, the number of patients treated at home doubled reaching a total of 4,830 people, 61,383 visits, and 12,305 remote contacts. In 2020, most users discharged from HH came from the Emergency service (2,283 patients) followed by inpatient care (1,859 patients) (MS, 2020).

The outcomes observed in Portugal are consistent with the ones mentioned in the previous sections. There is a considerable decrease in average hospitalization times from up to 3 days less than an equivalent hospital-admitted patient, a decrease of 24% in the mortality rate and 30% in the readmission rate for older patients. There are no hospital infections and, due to more mobility than in a hospital setting, there is less loss of muscle mass. Greater team unity is also reported as well as high satisfaction rates among patients, families and health professionals, of between 95% and 100%. Beyond the health benefits, it is possible to reduce the costs (direct and indirect) by 40% to 50% in some cases (Gaspar, 2020).

Guidelines for Home Hospitalization in Portugal

For the admission of a patient to HH, some clinical criteria must be met, namely the voluntary acceptance of treatment, the existence of a clinical diagnosis that despite requiring hospitalization presents clinical stability and the possibility of treatment at home, the existence of a caregiver for non-autonomous patients, the existence of basic hygienic-sanitary conditions in the patient's home, the existence of a cell phone for contact with caregivers, and living within a travel distance/time of the hospital, to be defined according to safety requirements for timely intervention. In addition to the inclusion criteria, candidates with alcohol or drug dependence, patients with suicidal thoughts or acute psychosis, as well as patients and caregivers who are mentally or physically unable to understand the care and treatment prescribed and collaborate in the application of such treatment are excluded (Direção-Geral da Saúde, 2018).

Regarding patient typology, only candidates with acute or acute chronic pathologies are treated in HH. Patients in the palliative stage who transitorily require complex care and therapeutic procedures may also be admitted. Moreover, the Portuguese regulation outlines the four categories of pathologies that can be treated in a home hospitalization setting (Direção-Geral da Saúde, 2018):

1. Acute infectious pathology requiring parenteral antibiotic treatment: urinary tract infection, respiratory infection, skin and soft tissue infection, acute cholecystitis, acute diverticulitis, endocarditis, spondylodiscitis, and others manageable at home;

2. Acute chronic pathology: chronic obstructive pulmonary disease, heart failure, renal failure, liver cirrhosis, and other home-controlled pathologies;

3. Post-operative care as part of a transition of care protocol, or in the treatment of chronic decompensated medical pathology in the post-surgical setting;

4. Incurable, advanced, and progressive disease (oncologic or non-oncologic) or organ degenerative process in a terminal situation, requiring intensive and/or specialized palliative care, in close articulation with the intra-hospital palliative care support team.

For treatment of the pathologies listed above, a variety of diagnostic and therapeutic procedures can be performed in home hospitalization, such as (Direção-Geral da Saúde, 2018):

- Diagnostic tests including myelogram, bone biopsy, lumbar puncture, paracentesis, biological sample collection, but also techniques such as blood gas analysis, electrocardiogram, echocardiogram, echography, pulsometry, or oximetry can be used;

- Both peripheral and central vascular access devices can be used, with central access devices being previously placed in a hospital environment;

- Non-invasive mechanical ventilation, enteral and/or parenteral artificial nutritional support, transfusion of blood products, IV therapy of drugs for exclusive hospital use, and short-term home oxygen therapy are also applied;

- Finally, antimicrobial treatment can be carried out with endovenous home antibiotic therapy as well as the treatment of complex wounds.

The following section introduces the hospital network under study in this dissertation, specifically its home hospitalization service.

2.3 Provider X: Case Study Description

This dissertation was developed in collaboration with Provider X, Portugal's largest private healthcare network. This section starts by presenting Provider X and its vital presence in the Portuguese health scene. Following that is a description of Provider X's home hospitalization unit's origin, current operation, and expansion plans.

2.3.1 Overview of Provider X

Provider X is Portugal's leading private healthcare provider. It was founded in 1945, with the inauguration of the first Provider X hospital in Lisbon, created to meet the needs of more than 80,000 employees and family members of the Provider X group, which at the time was a large conglomerate of companies, especially in the chemical sector. More than 76 years later, Provider X now totals 19 health units spread throughout the country: nine hospitals, nine clinics, and one institute with a potential cover of almost 6 million people. The reach of Provider X's network is reflected in their main indicators, described in Provider X's most recent integrated report (2021) and represented in Figure 2.4.

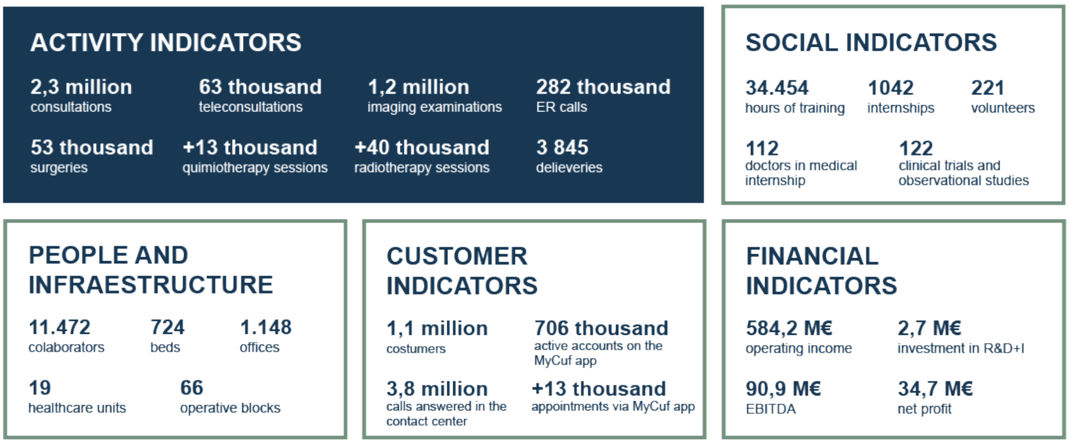

Figure 2.4: The main indicators adapted from Provider X's Integrated Report 2021.

Provider X has innovation as a determinant vector of its past, present, and future. This is evidenced by the investment made in 2021 of 2.7 million euros in new state-of-the-art equipment and technology for Provider X hospitals and clinics, an investment that tripled compared to that made in 2020. This investment aims at guaranteeing the best solutions for patients, not only from the clinical standpoint but also in terms of customer experience, process organization, and even human resources management.

Focused on the future of healthcare delivery, which will surely be a hybrid system between face-to-face and remote contacts, Provider X Digital was created at the end of 2020. Driven by the COVID-19 pandemic, they developed several tools that ensure, even at a distance, that patients continue to have monitoring and access to safe and reliable information. In 2021, they launched a teleconsultation service, having performed more than 63,000 consultations that year. Provider X also implemented a new digital symptom evaluator, a pioneer in Portugal, combining artificial intelligence with medical knowledge and scientific evidence, allowing patients to obtain recommendations on the most appropriate clinical follow-up, free of charge and integrated into their care network.

2.3.2 Provider X's Home Hospitalization Unit

Recognizing the clear advantages for the client and their family, Provider X has a well-consolidated home care network. The offer is extensive and includes rehabilitation, elderly care, palliative care, oncology, and home consultations, among others. It provides an integrated solution of clinical and operational support at home, ensuring the Provider X healthcare experience in the comfort of one's home. Given the growing demand for this type of service and Provider X's strategic commitment to digitalization, one of the goals for 2022 is to extend home care to the entire Provider X network.

In June 2020, prompted by the pandemic, Provider X created the first private home hospitalization unit in the Iberian Peninsula. This unit follows the guidelines set out in Section 2.2.4, providing an alternative to conventional hospitalization, with continuous and coordinated medical and nursing care. Initially, this unit could admit 10 patients simultaneously. Agreements with major voluntary insurance companies in Portugal (Médis, Multicare, Future Healthcare, and Allianz) democratized access to this service and allowed Provider X to gain traction in this new market. To this date, 272 patients have been treated by this HH unit, which now has a capacity for 12 patients simultaneously. In less than two years, Provider X became the 4th largest HH unit in the country.

Currently, this unit is located in Hospital 1 (Alcântara, Lisbon) and covers the subregion of Greater Lisbon. More specifically, the unit serves a 30-kilometer radius, a distance that allows the HH teams to assist patients in about 30 minutes in case their conditions deteriorate. The Greater Lisbon subregion has four Provider X hospitals: Hospital 1 (H1), where the HH unit that serves all four hospitals is currently located, Hospital 3 (H3), Hospital 2 (H2), and Hospital 4 (H4). The company's goal, in three years' time, is to have a unit at Hospital 4, another serving the Hospital 3 and Hospital 2 hospitals, and another in the Porto district, with a planned opening for the end of this year.

The existing service is composed of internal medicine doctors and home care specialist nurses, operating 24 hours a day, 7 days a week, 365 days a year. The teams follow the patients through two daily visits: one in the morning with an internal medicine physician and nurse, and a nursing consultation in the afternoon. On average, each team sees 6 patients, with consultations lasting approximately 40 minutes. Taking into consideration the needs of the patients and their families, the number of visits and their duration may vary, never below 20 minutes per visit.

Although the program is rather recent and there are no published results yet, positive outcomes are already observable. Hospital readmission cases are rare, and the average number of hospitalization days is also lower than conventional hospitalization. This type of treatment promotes autonomy in disease management and health education while reinforcing the doctor-patient relationship and ensuring close family involvement. Accordingly, the satisfaction indexes of both patients and their caregivers are also much higher, reaching the maximum value in most of the parameters evaluated in the satisfaction surveys. Regarding costs, there is a reduction between 25% and 36% compared to traditional hospitalization.

Home hospitalization is particularly relevant given the aging population, increasing chronic comorbidities, and the imperative need for continuity of care. It, therefore, has an excellent perspective to grow along with these trends. Moreover, the development of innovative tools that Provider X has endorsed, viz., in the scope of telemedicine, will increase the number of patients that each HH team can manage simultaneously, thus increasing the capacity of these units and their potential for expansion.

After providing the reader with an overview of home hospitalization, its framework in the Portuguese health system, and the home hospitalization network under study, the following section defines the problem against the presented case study.

2.4 Problem Definition

The main goal of this dissertation is to develop and apply a generic mathematical model that facilitates districting decisions in the home hospitalization setting, taking into consideration some restrictions inherent to this type of care, such as proximity to the hospital or workload balance between HH units. Given a HH network, the model should partition the residential area affected by this network into a set of districts, thus improving care provision. If supply does not meet demand, the model must identify and quantify the periods of under-capacity.

The application of this model to the Provider X case will seek to develop a decision support tool that allows the classification and comparison of different combinations of potential HH units, by testing the different districting outcomes for these different scenarios. Hence, the first main deliverable will be an ordered list of which HH units should have priority of being launched. The second deliverable will consider that after three years, all potential units are open and analyze the districting solution for this new setting. The model will be tested for different demand scenarios, and the differences in districting results will be analyzed.

To test the model, supply and demand data from Hospital 1's HH unit for the year 2021 and the first four months of 2022 will be used, namely capacity, location, and the number of hospitalization days per patient. Characterizing past demand also enables the prediction of future needs. In addition to the demand distribution by municipalities, the distribution of patients by age stands out, demonstrating the greater need for this service in seniors, with more than 70% of the patients treated in this period being over 70 years old.

The model will be applied to the Greater Lisbon sub-region, as it comprehends the four Provider X units under study and their surrounding area. Nevertheless, the developed model may be applied to other realities to define the optimal allocation of patients by HH units of any hospital network.

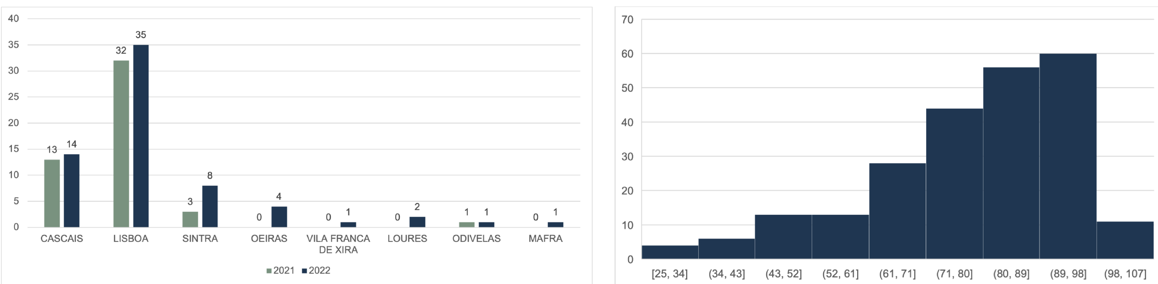

Figure 2.5: On the left: number of treated patients by municipality. On the right: Number of treated patients by age. Both plots refer to Hospital 1's HH unit during the year 2021 and the first quarter of 2022.

2.5 Chapter conclusions

In order to address the growing healthcare demand and rising costs, technological and scientific innovation is becoming more and more crucial. Catalyzed by the COVID-19 pandemic and aligned with today's demographic and economic trends, home hospitalization presents itself as an effective alternative to traditional hospital inpatient care. Among its more notorious benefits, HH improves clinical outcomes and patient satisfaction whilst increasing hospital bed capacity and reducing healthcare spending.

Home hospitalization falls within the scope of home-based care. Some characteristics of HH are, however, unique to this service. These include specialized equipment and personnel, a generally shorter treatment time, and the need to be closer to the hospital, given the possibility of deterioration of the patient's condition.

In Portugal, the Ministry of Health developed a strategy aimed at expanding the delivery of HH services in the NHS, reaching 28 units at the end of 2020. In the same year, home hospitalization also became a reality in the Portuguese private health sector with the opening of Provider X's HH unit.

This dissertation aims to create a mathematical model capable of ensuring the optimal allocation of patients by a set of HH units, taking into account some criteria that will be discussed in the following chapters. As a case study, the model will be applied to candidate Provider X HH units in the Greater Lisbon region.

At this point, a review of the existing literature on districting in the home care setting will be sought to understand what approaches can be used to address this problem. The literature review will also briefly cover the importance of Operational Research in healthcare and clarify how districting decisions impact a hospital network. Once the theoretical background on the problem under study and the literature review has been completed, the chosen methodology will be introduced.

Chapter 3

Literature Review

The following chapter presents a comprehensive literature review on the topic of districting in the healthcare setting, with a particular focus on home healthcare. Section 3.1 illustrates the importance of Operations Research applications in the health sector. It also presents an overview of health management problems where the employment of OR tools is deemed valuable. In Section 3.2, the districting problem is introduced, highlighting the advantages of proper districting decisions. Section 3.3 discusses modeling approaches for the districting problem, with the Location-Allocation and Set Partitioning formulations being the most common ones. Following that, a deeper analysis of the solution methodologies used to solve this problem is conducted in Section 3.4. Section 3.5 presents the state-of-the-art districting in healthcare and HHC. Finally, Section 3.6 presents the main findings of the chapter.

The literature review was conducted between March and May, 2022, through the Web of Science database. Combinations of the keywords ”districting”, ”home care”, ”healthcare” and ”operations management” were used as well as terms of similar meaning. After this first selection, the citations of the articles were analyzed, selecting from this new list the relevant papers. Only articles published in English between the years of 2003 and 2021 and subjected to peer review were taken into consideration.

3.1 Operations research applied to healthcare challenges

The origins of Operational Research (OR) date back to World War II when it was used by the British military forces to achieve better results with a lesser expenditure of ammunition (Kunwar and Srivastava, 2019). The World Health Organization (WHO) was founded years after, in 1948, and research was established as one of its core functions (Kunwar and Srivastava, 2019). To this day, OR/MS applied to healthcare is an increasingly popular topic and promised to improve the iron triangle of health (cost, access, and quality), countering the iron triangle paradigm, which states that any improvement in one dimension will be harmful to the performance of at least one of the remaining two (Diw, 2020). Dai and Tayur (2019) reviewed the recent healthcare operations management literature in terms of research thrusts and methodological tools. Regarding thrusts, the top five most frequently published concern delivery design, emergency care, organization design, inpatient care, and ambulatory care. These five thrusts represent 68% of the reviewed articles. Regarding methodological tools, the most employed classical OR methods for delivery design were queueing theory and queueing games, deterministic programming, and Markov decision process. Queueing theory was also largely used to treat emergency care problems.

3.1.1 OR applications in the decision-making process

Planning decisions are made at three levels: strategic, tactical, and operational (Hulshof et al., 2012). Districting can be classified as both a strategical (Matta et al., 2014, Hulshof et al., 2012) and a tactical decision (Hall, 2012, Gutiérrez-Gutiérrez and Vidal, 2015), as it influences the allocation, planning, and control of patients and resources, both regionally and locally. The classification of OR decisions in home care according to the three decision levels is presented in Table 3.1.

Strategic decisions are done on a 1-to-5-year horizon and must be clearly defined to drive other decision-making activities at lower levels (Matta et al., 2014). This type of planning relies on highly aggregated information and forecasts (Hulshof et al., 2012). On the strategical level, districting aims at distributing a certain territory's existing home hospitalization units by their geographic service areas. It dictates the district and health care facility to which each patient is assigned according to their geographical location (Matta et al., 2014), which corresponds to an assignment problem (Yalçındağ et al., 2016).

Tactical decisions have a shorter horizon of 6 to 12 months and deal with the implementation of strategic decisions, thus being much more concrete than the previous planning level (Matta et al., 2014). Furthermore, demand has to be partly forecasted based on seasonality, waiting list information, and the demand in care pathways of patients that are currently under treatment (Hulshof et al., 2012). Given the effect of districting on the allocation and dimensioning of resources by districts, this decision has a tactical component. Moreover, dividing the service area of each HH agency into regions to be served by teams of caregivers is also a tactical decision (Hall, 2012).

The third level of planning will have less relevance in this dissertation. Operational decisions are highly specific with an emphasis on short-term objectives (Hulshof et al., 2012). Their horizon is very short and varies between hours and months, depending on the level of detail (Matta et al., 2014). These decisions concern the management of flows, planning, and coordination of day-to-day activities.

3.1.2 Decision-making in home health care

The efficient use of resources is seen by Grieco et al. (2021) as critical to the long-term viability of health and social care systems around the world. Operational research tools are extremely useful to support home health care decision-making and can be found at all planning levels. Figure 3.1 offers a graphical representation of the flow of decisions in an HHC service, from the definition of the offered services to the delivery of care. According to the literature review done by Benzarti et al. (2010), the most investigated problems in OR apart from districting are resource dimensioning, allocation of resources to districts, assignment of care providers to patients or visits, and routing. The largest number of articles

Table 3.1: Hierarchy of Operations Research decisions in Home Care organizations, adapted from Hulshof et al. (2012) and Matta et al. (2014).

Strategic level	**Tactical level**	**Operational level**
Capacity dimensioning	Admission control	Assessment and intake
Case and service mix	Capacity allocation	Inventory management
Districting	Resource dimensioning	Operator to patient assignment
Facility location	Skill management	Routing
Panel size		Scheduling
Placement policy		Staff rostering
Regional coverage		

deal with operational planning and concern scheduling, allocation, and routing (Benzarti et al., 2010, Grieco et al., 2021).

The systematic review conducted by Grieco et al. (2021) points out that no comprehensive set of tools comprises all three decision levels and allows the organization to apply an approach at one level without undermining the others. This review also found that there was insufficient literature on some strategic and tactical decisions such as the coordination of care across professions and organizations and role definitions within the workforce.

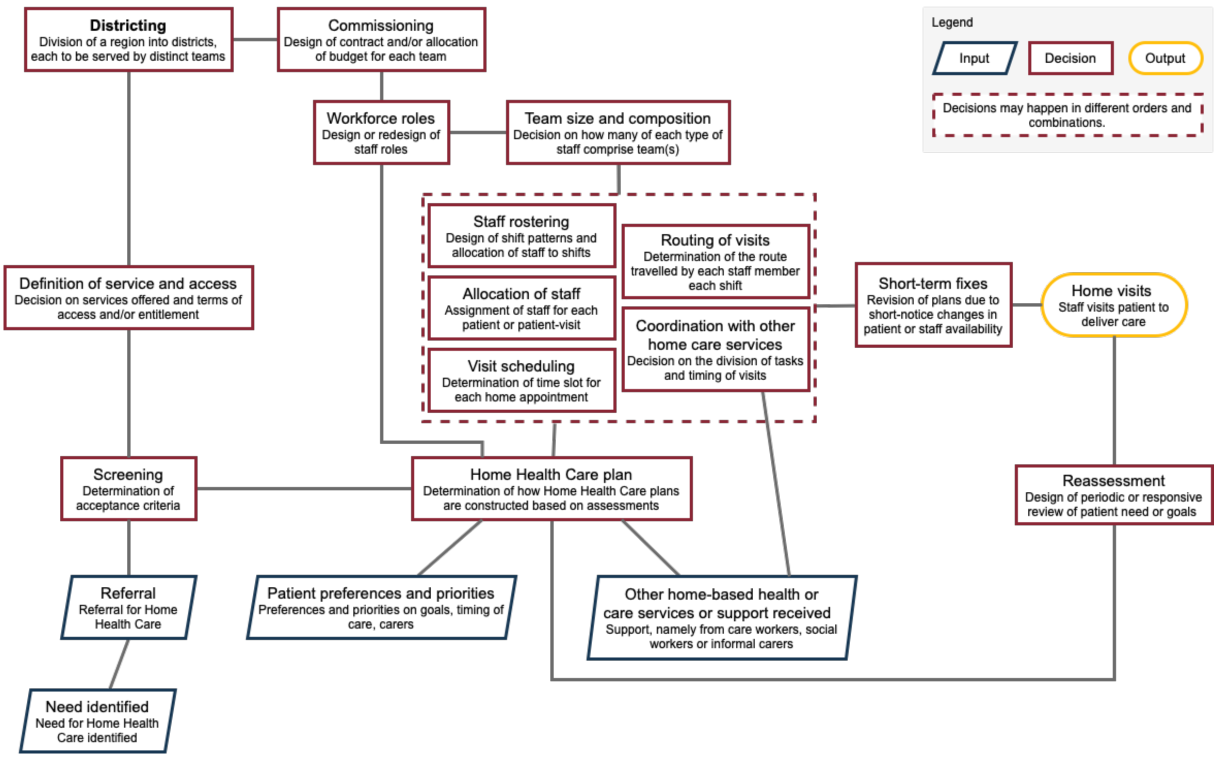

Figure 3.1: Graphical representation of the flow of decision problems associated with the design and operation of an HHC service adapted from Grieco et al. (2021)

3.2 The Districting problem

Within location science, a research field of which districting is part, most problems focus on defining the location of facilities. The second aspect of location problems is often overlooked but plays an equally important role: the allocation of users to those facilities. Districting problems prioritize this second aspect, aiming to determine which customers should be served together. Only then, if necessary, facilities are located (Kalcsics and Ríos-Mercado, 2019).

Districting aims at dividing a large geographical region into sub-areas, referred to as districts, for organizational or administrative purposes. It is the process of grouping small geographic areas, called basic units, into larger geographic clusters (districts) to optimize certain criteria and subject to some constraints (Darmian et al., 2021, Kalcsics and Ríos-Mercado, 2019) and then assigning each cluster to a set of resources (Cissé et al., 2017). According to Cissé et al. (2017), districting encompasses three distinct strategic OR problems: partitioning, assignment, and classification.

Districting problems are motivated by various applications, ranging from political districting to waste collection, school district design, or sales and service territory design. Districting is rarely approached in literature without a practical background (Kalcsics and Ríos-Mercado, 2019).

Healthcare should be accessible and easily available over a geographical area with different population densities and characteristics (Harper et al., 2005). For that reason, proper districting decisions are a powerful tool to improve patient access to the healthcare system in all residential areas. This is especially important given the extremely uncertain and variable environment a healthcare system operates. These decisions also improve and facilitate the healthcare system management at the district level instead of handling a large geographical area (Darmian et al., 2021). Maximizing the balance among districts based on their capacities and demands improves coordination between different care providers, allows for more efficient resource management and alleviates uneven workload (Blais et al., 2003). Districting is also done to promote long-term relationships between providers and patients since normally each district is under the responsibility of a single multidisciplinary care team (Hulshof et al., 2012).

Following the characterization of the districting problem, the next two sections outline the mathematical approaches for modeling and solving this problem.

3.3 Modeling Approaches

Mathematically, Kalcsics and Ríos-Mercado (2019) describe districting as a partitioning problem. Given a set of items, it determines how to separate them into smaller subsets to optimize certain criteria or objectives while satisfying some side constraints. All items in the original set must be contained in one and only one partition. After partitioning a service territory into clusters of patients, each cluster is assigned to a set of resources. An assignment or allocation problem in home care determines which caregivers will provide care for which patients. Besides the geographical locations of patients, Yalçındağ et al. (2016) points out that the main factors to consider are the visiting and traveling times and the required professional skills to deliver the service to each patient. Nonetheless, many other characteristics, related to patient attributes or the geographical aspects of the territory, for example, impact the assignment decision too. In addition to set partitioning and assignment, districting starts out as a classification problem, in that it seeks to categorize a population of basic units into a certain number of classes or types, according to some similarity criteria (Freeman and Frisina, 2010).

There is a consensus in the field as to model districting as a mixed-integer programming (MIP) problem. MIP models deal with problems where some of the decision variables are constrained to be integer values at the optimal solution. When the models do not have any quadratic characteristics, they are referred to as Mixed-Integer Linear Programming (MILP) problems. Capturing the discrete nature of some decisions greatly expands the scope of useful optimization problems that can be defined and solved (Gurobi Optimization, 2022). Ideally, it would be possible to solve these models using exact methods. However, most of the districting problems are NP-hard thus large-scale instances are intractable by exact optimization algorithms (Kalcsics and Ríos-Mercado, 2019). For that reason, a significant number of papers in this field developed heuristics and metaheuristics (Darmian et al., 2021). These approaches have the flexibility of including almost any practical criterion and are able to handle complex constraints (Kalcsics and Ríos-Mercado, 2019).

Like many other optimization problems, many modeling approaches for districting have been suggested in the literature throughout the years. Depending on the specific application, location-allocation and set partitioning are two common formulations for districting problems (Hall, 2012). The following subsections provide a brief explanation of both approaches.

3.3.1 Location-Allocation Formulation

A location-allocation (LA) model takes a fixed set of district centers and assigns each basic unit to exactly one district center. The objective is to minimize the total cost of assigning those units to district centers whilst being subject to certain constraints. This formulation is beneficial in situations where the district center acts as a depot, being the starting and finishing point for all routes within the district (Hall, 2012). This formulation was first developed by Hess et al. (1965) for political redistricting. The LA formulation may be linear or nonlinear, depending on the methods used to evaluate the cost of assigning a unit to a district and on the calculation of activity measures, such as workload (Hall, 2012).

A variation of this formulation, called facility-location, does not require a predefined set of district centers. In that case, an extra decision variable is added to indicate which district centers should be open (Gutiérrez-Gutiérrez and Vidal, 2015).

3.3.2 Set Partitioning Formulation

In the Set Partitioning (SP) formulation, a set of potentially feasible districts are heuristically generated and then selected to optimize the overall balance of the district plan (Kalcsics and Ríos-Mercado, 2019). The objective function minimizes the total cost of all selected districts while ensuring that each unit is assigned to a single district and that a chosen number of districts is generated (Gutiérrez-Gutiérrez and Vidal, 2015).

SP formulation enables the modeler to design and evaluate the cost of complex district restrictions within an auxiliary problem, outside of the core optimization problem. According to Kalcsics and Ríos-Mercado (2019), this confers an advantage to the Set Partitioning formulation compared to LA, since almost any criterion can be applied to the generation of candidate districts. Nonetheless, in SP, an increase in the number of basic units generates an exponential increase in the number of feasible districts and consequently greater computational complexity, not making this formulation suitable for large instances (Hall, 2012).

3.4 Solution Approaches

Many different solution approaches have been proposed in the literature over the years. These approaches can be roughly divided into two categories: those using exact algorithms through mathematical programming models and those that rely on heuristics or meta-heuristics. The main challenge in selecting the proper approach is finding a balance between the available processing time, the size of the problem, the quality of the approximated solution, or the need to find an exact solution. The following section will present and compare the main algorithms used in the reviewed articles.

3.4.1 Exact methods

Contrary to heuristic approaches, exact optimization schemes are guaranteed to find an optimal solution to an optimization problem. These solution approaches are generally not appropriate for NP-hard problems due to their computational complexity (Khodabandeh, 2017). Also, integer programs are much harder to solve than linear programs. However, exact methods may be suitable for NP-hard problems with small instances. Furthermore, given the strategic-tactical nature of the districting problem, it is not expected that these kinds of decisions will be made with high periodicity. Therefore, the speed with which results are obtained is not a preponderant factor when choosing a solution algorithm.

In recent years, major advances have taken place in exact solvers for integer programming problems, with the leading commercial MIP solvers XPRESS, CPLEX, and GUROBI achieving widely acknowledged results (Hvattum et al., 2012). CPLEX, in particular, uses sophisticated mathematical techniques such as *branch-and-cut* search to solve hard integer programs (IBM, a). These methods systematically examine all possible combinations of the discrete decision variables while computing bounds on the value of the best solution using linear programming relaxations. Additionally, they compute linear constraints that exclude potential solutions that violate the discreteness restrictions (IBM, b). Among the analyzed articles that used commercial solvers, the used methods are usually not mentioned.

3.4.2 Heuristics and Metaheuristics

A heuristic approach to solving an optimization problem does not guarantee to obtain the optimal solution but finds an approximate solution in an acceptable time when classical methods are too slow or cannot find an exact solution. Heuristics are specific and problem-dependent techniques. They are usually adapted to the problem at hand and incorporate its specificities. Because these are quite greedy approaches, they usually get trapped in a local optimum and thus fail, in general, to obtain the global optimum solution. Metaheuristics, by contrast, are high-level problem-independent techniques. These are an improvement of classical heuristics with an emphasis on a deep exploration of the solution space. They usually combine advanced neighborhood search rules and recombination of solutions and may even accept a temporary deterioration of the solution (see, for example, Simulated Annealing), toward getting a better result. The quality of the solutions produced by metaheuristics is typically much higher than that obtained by classical heuristics techniques but with a price of increased computing time. Since metaheuristics do not take advantage of any specificity of the problem, they can be used as black boxes. Even though metaheuristics are problem-independent, it is still necessary to fine-tune their parameters to the given context (Blocho, 2020).

Tabu Search

Tabu Search (TS) is one of the most popular local search metaheuristics to solve districting problems. Local search methods tend to be stuck in suboptimal regions. TS enhances the performance of these techniques by forbidding already visited solutions and their neighborhoods. It constructs a list of the last n variable-value assignments and, when picking the next assignment, those on the list are tabu and cannot be chosen (Zhou et al., 2013). It, therefore, explores the solution space by constantly replacing recent solutions with the strictly better non-visited neighboring solutions until finding the best local optimum (Ricca and Simeone, 2008).

Simulated Annealing

Simulated Annealing (SA) is a heuristic method inspired by the annealing process of metals, i.e., the gradual cooling of metals after they have been exposed to high heat treatment and resultant alteration of their molecular arrangements into a uniform crystalline state (Eren et al., 2017). SA algorithm adopts an iterative movement according to a variable temperature parameter. It generates random points in the neighborhood of the current best point and evaluates the cost function value there. If the cost function value is lower than its current best value, then the point is accepted, and the best function value is updated. If the function value is higher than the best value known so far, the point may be accepted according to a probability density function. This function gradually decreases to zero as the temperature variable is reduced, meaning that the method accepts worse designs in the initial stages while almost always rejecting worse designs at the final stages. This way, getting stuck at a local minimum point is avoided (Sahab et al., 2013). SA can be a preferred strategy among the other heuristic approaches because it incorporates randomization into the search to prevent local extreme points. On the other hand, SA has a trade-off between computing time and solution sensitivity (Eren et al., 2017).

Genetic Algorithm

The Genetic Algorithm (GA) is the most widely used metaheuristics method to solve stochastic optimization problems (Cramer et al., 2009). It is an evolutionary algorithm that is inspired by Charles Darwin's theory of natural evolution. In this algorithm, the fittest individuals are selected for reproduction in order to produce offspring for the next generation and the weakest are eliminated. The solution is coded as an array, called a chromosome, and a set of solutions, called a population (generation), evolves toward better solutions. The GA is an iterative process that begins with a population of randomly generated chromosomes. Genetic operators such as crossover and mutation are used in this evolution process to generate a new set of solutions. The chromosomes with higher fitness values are more likely to survive to the next generation, and the algorithm eventually converges to an optimal or near-optimal solution (Darmian et al., 2021). When compared with SA, the genetic algorithm has a greater standard deviation (wider spread) but the average error is smaller. This is somewhat expected, as the genetic approach will search for a wider array of possible solutions, some better and some worse (Wilson and Mantooth, 2013).

Having portrayed the leading modeling and solution approaches for a districting problem in the healthcare setting, a detailed description of the literature will follow, identifying the used criteria, the inclusion of uncertainty, and the primary case study findings from each paper.

3.5 Districting models: state-of-the-art

In spite of the significant amount of literature on the topic and its different applications, there is still no consensus on which criteria are most important nor which mathematical model best describes districting problems (Kalcsics and Ríos-Mercado, 2019). Additionally, no literature is found that directly addresses the problem of districting applied to home hospitalization. As HH is framed within home care and despite the differences mentioned in the Subsection 2.1.3, it is to be assumed that the districting models for these two types of care are overall similar. Thus, this section will look at districting solutions for healthcare and home care contexts.

Table 3.2 provides an overview of the reviewed articles, summarizing which models have been used, whether they are deterministic or stochastic, whether demand uncertainty was considered, and which solution approaches have been adopted. A more detailed table is found in the Appendix Section, containing information regarding the level of engagement with the current practice of each study.

3.5.1 Deterministic models

Mathematical models represent a simplified version of a real system and should be able to explain previous observations, integrate current data, and anticipate the system's response to planned stresses (Renard et al., 2013). A deterministic model is one in which state variables are determined solely by model parameters and sets of previous states of these variables. As a result, for a given set of parameters and initial conditions, deterministic models always perform the same way, and their solution

Table 3.2: Key findings from the literature review on the districting problem.

Publication	Application	Uncertainty	Model	Approach	Solution	Case Study	Location	Stakeholders
(Blais et al., 2003)	Home Care	D	NLP	MH	TS	√	Canada	√
(Harper et al., 2005)	Healthcare	S	-	-	GS	√	England	√
(Lahrichi et al., 2006)	Home Care	D	NLP	MH	TS	√	Canada	
(Sahin et al., 2010)	Home Care	D	MIP	E	-		-	
(Benzarti et al., 2013)	Home Care	D	MIP	E	CPLEX		-	
(Datta et al., 2013)	Healthcare	D	MIP	MH	GA	√	England	
(Steiner et al., 2015)	Healthcare	D	MIP	MH	GA	√	Brazil	
(Gutiérrez-Gutiérrez and Vidal, 2015)	Home Care	D	MIP	E	Xpress-IVE	√	Colombia	√
(Lin et al., 2017)	Home Care	D	MIP	E, H	Gurobi, Greedy	√	China	
(Yanik et al., 2019)	Healthcare	D	MIP	E	CPLEX	√	Istambul	
(Lin et al., 2020)	Home Care	D	MINLP	MH	GA	√	China	√
(Darmian et al., 2021)	Healthcare	S	MIP	MH	GA	√	Iran	√

is unique. The deterministic districting problem has a comprehensive literature base. Its limitation is not accounting for uncertainty, hence neglecting the effect of unpredictable variables in the solution (Renard et al., 2013).

The first article to address the problem of districting in the context of home care was (Blais et al., 2003) in the region of Côte-des-Neiges, Canada. The authors' objective was to break the region into five districts using a Tabu Search heuristic. They used an objective function that optimized visiting personnel mobility (ease of travel by public transportation) and workload equilibrium simultaneously. Restrictions related to district connectivity, the indivisibility of basic units, and respect for borough boundaries were also considered. The work of (Lahrichi et al., 2006) identified two flaws in the work of (Blais et al., 2003). On the one hand, the solution found cannot predict the fluctuation of demand within each district leading to an imbalance of workload and inequity in the quality of services provided among the districts. On the other hand, it is not flexible enough in terms of care providers thereby failing to encourage collaboration between them. The identification of the shortcomings in (Blais et al., 2003) was supported by an analysis of historical data from patient visits from the years 1998-1999 and 2002-2003. In the latter period analyzed, an in-depth analysis of the home care services provided was also performed, aiming to understand whether the districting kept up with the changing needs of the inhabitants. Since this was not in fact the case, the authors suggested two more dynamic approaches to the problem. The first proposes that patients were allocated to a district but that the districts of each nurse were not fixed. When a request for service came in, the team manager of the district would choose a nurse to attend that patient taking into account his or her location and workload. This would however imply a hyper-efficient information system, which would contribute to transparency and uniformity of nursing practice. The authors' second suggestion would be to divide the home health care team into two groups: the first group would be made up of staff allocated to a fixed district while the second group could work in the whole territory or in a group of fixed districts. This second solution would require a structural change in the distribution of HHC services but could have good results in bridging the (Blais et al., 2003) gaps.

The works of Sahin et al. (2010) and Benzarti et al. (2013) are similar in that they are mainly theoretical and have not been tested on real instances or put into practice, unlike the other articles under study. In (Sahin et al., 2010), the authors first describe extensively which criteria have been used so far for districting problems, dividing them into 4 categories: geographical aspects, activity measures, comparison between different territory partitions, and organizational criteria. Among the criteria identified, they consider for their model the compactness, indivisibility of the basic units, respect for administrative boundaries, the accessibility of the districts, and workload balance, strongly linked to the travel time of caregivers. Similarly to Blais et al. (2003), the authors proposed an integer linear programming formulation, with the objective function being the minimization of travel time, workload balance, and the weighted sum of these two factors. They created several scenarios where the weight of these two variables in the final solution was varied. The authors also added a classification of patients by profile. The profiles depend primarily on the pathology from which the patient suffers, the regularity of necessary visits, the average visit duration, and the types of care provided. As in the aforementioned paper(s), these authors' work did not cover demand fluctuation, leading to workload imbalance, as pointed out by Lahrichi et al. (2006).

Building on the work of Blais et al. (2003) , Benzarti et al. (2013), proposed two MIP models to deal with districting in HHC, solved using CPLEX11.1. The first model aims at minimizing the imbalance of workload between districts while defining a maximum average waiting time for the patients. It is possible to guarantee that by creating a compactness criterion that measures the distance allowed between two basic units in the same district and fixating its upper bound. The second model can be used if a decision-maker prefers to reduce the waiting time of patients as much as possible by minimizing the compactness measure. In this case, the HHC manager also needs to define a tolerance interval that ensures that each district's care workload does not deviate from the average care workload by more than a pre-specified percentage. In both models, criteria such as accessibility, conformity to administrative boundaries, and the indivisibility of basic units are also taken into consideration. Unlike the previous models, these do not consider minimizing travel time as this reduction is guaranteed by the compactness criteria. Also, unlike the other articles, (Benzarti et al., 2013) did not use real data: instances with different sizes were randomly generated. The numerical analysis with different scenarios and instances enabled the authors to evaluate the impact of the key parameters on the workload balance and compactness criteria.

With an adapted Location-Allocation (LA) model, Gutiérrez-Gutiérrez and Vidal (2015) explored home care districting in the context of rapid-growing cities, taking as a case study the urban area of Cali, Colombia. They used a bi-objective mathematical model that minimizes both workload components: travel workload as well as care workload deviation, i.e. the range between minimum and maximum workload values. They further identified the efficient frontier and trade-offs between these two variables. Thanks to strong stakeholder involvement, from managers to patients and practitioners, the authors could develop a lexicographic solution approach and find that a 10% deterioration in travel time improved workload deviations by more than 80%. The criteria considered included contiguity and compactness, equity, continuity of care, respect for natural boundaries, and socioeconomic homogeneity. Given the Colombian context, some factors such as safety conditions for medical staff, the geographic disposition of the population, and the increase in diversity of demand had to be accommodated. For this, the model included sets that differentiated patient types, medical staff, medical activities, and security levels allowing for a more accurate demand estimation.

Another modeling approach for districting problems is through undirected and planar graphs, which are particularly effective for measuring and ensuring connectivity and contiguity (Kalcsics and Ríos-Mercado, 2019). In these models, the studied regions are represented as graphs in which the municipalities or basic units are the nodes and the roads connecting them are the edges. Both Datta et al. (2013)and Steiner et al. (2015) considered graph partitioning problems.

Having the English territory as their case study, the optimization criteria chosen by Datta et al. (2013) were homogeneity (in size, age, and economic), compactness, and respect for the local authority's boundaries. The authors solved the model with a Genetic Algorithm obtaining solutions that approximated the Pareto front. After, they run two-dimensional subproblems with different bi-criteria in order to visualize the approximate Pareto front and identify possible sharp trade-offs. Indeed, they found that compact solutions are also co-extensive with local authorities but that assuring co-extensiveness is incompatible with size homogeneity: both these findings are coherent with the characteristics of the geographical divisions of the East of England. Moreover, good performance on age homogeneity is associated with good outcomes in size homogeneity but conflicts with economic homogeneity performance. The fact that sharp trade-offs do occur proves that a multi-objective approach is the most appropriate for this problem.

On the other hand, the goal of Steiner et al. (2015)'s work was to reorganize the micro-regions of the State of Parana in Brazil to optimize the management of public health services in this area. As in the previous literature, the objective function looked for the minimization of inter-region travel distances and the homogenization of population size. The third objective, exclusive to this work, is the maximization of the type of services offered per micro-region. Contiguity, integrity, size, number, and non-embedding of micro-regions were also included in the model as constraints. An integer-coded genetic algorithm was used to solve the problem, and 92 Pareto-efficient solutions were found. When the number of micro-regions of the solution was fixed to be equal to the current one to facilitate the comparison (83 microregions), they concluded that the solutions found were much improved over the current distribution in Parana and could thus contribute to more homogeneous and higher quality services for the population.

The Tai Po integrated care service structure of the Salvation Army in Hong Kong, China, has been subject to several OR studies over the past few years (Lin et al., 2017, 2020). They offer numerous types of services having the elderly living in Tai Po as their prime customers.

The paper by Lin et al. (2017) covered districting for their Meals-on-Wheels service, a specific HHC service where providers deliver meals to people who are at home and unable to purchase or cook their meals. Contrary to all other reviewed literature, the objective function had a single goal: to minimize the total number of districts created, leading to the minimum number of care workers able to satisfy all customer demand. Constraints related to capacity and time window limitation, accessibility, compactness, and the indivisibility of locations were further included. The authors started by solving the model using the Gurobi Optimizer, which proved to be feasible for this problem. However, they noted that the solver could be very time-consuming and therefore suggested a greedy heuristic method. When comparing the results obtained with the exact and heuristic approaches, they concluded that the greedy heuristic method could achieve a solution as good as that obtained with the Gurobi Optimizer but within a shorter computation time. They then compared the resulting districts with existing districts recorded in historical data, proving that both approaches met the criteria and could improve Hong Kong's current district design. Despite the positive results, there is no mention of their implementation.

The recent paper by Lin et al. (2020) treated the districting of another branch of this Chinese integrated elderly care service. This particular service consists of sending caregivers to the homes of the elderly or picking them up and taking them to care centers, depending on the customer's preference. The optimization model used was formulated as a multi-objective mixed integer nonlinear programming (MINLP), solved with a nondominated sorting Genetic Algorithm II. As in all the papers mentioned in this section, the authors considered as criteria in the objective function the workload balance and compactness. Compactness seeks to ensure that the delivery of health services is done in less than 30 minutes, decreasing the travel time of providers, and increasing the efficiency of the service. They further added a third goal: minimizing the total cost of hiring care workers. Although several other criteria were considered such as the number of care workers in charge per district and the indivisibility of basic units as well as the diversity of service types, elderly, and worker profiles, the authors point out that more criteria and constraints could be introduced.

It is logical to present the work by Yanik et al. (2019) at the end of this subsection. Although the authors' approach is not stochastic, using a multi-period model allows for better planning adjustment according to demand and supply changes and potentially improves the efficiency and quality of the health service. The authors' districting model was applied to the primary care scheme of Istanbul, Turkey. They used a multi-period multi-criteria MIP model with an objective function that maximizes compactness, income equity, and district similarity, obtaining sub-optimal solutions. First, a single-period model was tested with the first two objectives. The criteria of workload balance, capacity, and accessibility were also included in the model as constraints. The approach by Yanik et al. (2019) is more relaxed than previous models, allowing patients from one district to be allocated to two or more district centers (through gradual assignment). In reality, this relaxation represents the indifference of patients to choose two or more general practices if they are more or less at the same distance from them. The multi-period component was later added. However, a dissimilarity measure that seeks to minimize the number of patients changing their GP also had to be implemented since the addition of multi-period causes district changes in each timeframe. The third objective, district similarity, should guarantee continuity of care and improve the patient–GP relationship.

3.5.2 Stochastic models

Stochastic models are characterized by having some inherent randomness to them. Every time a stochastic model is run, given the same set of parameters and initial conditions, different outputs might be obtained. Stochastic model parameters and state variables can be random or described by probability distributions rather than single values. Therefore, stochastic models allow the modeler to evaluate the inherent uncertainty of the HH setting (Renard et al., 2013), better reflecting the real-world complexity and variability. The literature using stochastic models is scarce. However, as stated in the previous chapter, uncertainty is a key aspect of healthcare districting and will be taken into consideration in this dissertation.

(Harper et al., 2005) used a stochastic approach to LA through a geographic simulation model using Delphi programming. Some of the elements of the geographic model were the number and location of health centers and services offered at each, the distribution of population and demand for each service, the means of patient transportation, travel time and distance as well as the critical mass which represents the minimum number of patients per service per time interval that guarantees the viability of a given center. The fact that they incorporated stochasticity in their model allows on the one hand a better fit of variable factors such as patient flows or transportation time, but also makes the model more generic and easier to use by different users, a fact that the authors emphasized as being preponderant in their choice of using a location-allocation model. In fact, their model was used in two case studies, both at the local district level for the provision of dental services in London and at the regional level for the provision of coronary artery by-pass graft (CABG) services across Eastern England. In this work, there is no objective function and an optimal solution is not necessarily found. It does, however, encourage discussions between stakeholders and allows for the rapid configuration of new scenarios, where health centers are easily relocated, deleted or added as well as the services in each of the centers.

(Darmian et al., 2021) use a mixed programming model (MIP) to optimize districting for healthcare by considering 4 main criteria: district contiguity, size balance, exclusive assignment, and maximum travel time between districts. Since it is an NP-hard problem and of graph nature, their solution was an improved genetic algorithm. This is the first paper in this review of the literature that estimates demand, doing so through hedonic models. They use two different methods to deal with uncertainty: the Bertsimas & Sim RO approach (BSRO) and weighted protection against worst-case condition (WWRO). The use of these approaches allows the decision-makers to adjust their results according to their attitude towards risk, obtaining higher values for the original objective when the worst-case scenario is not considered and lower values when protection against it is increased. This work was applied to residential areas in Iran, leading to positive results, such as improving the equilibrium criterion by 32% compared to existing districting decisions and reducing the number of districts in which the capacity of health services was less than the demand.

3.6 Chapter conclusions

Districting is a strategic-tactical planning decision that strongly impacts the performance of a health system, namely in terms of accessibility, resource management, and workload balance. Contrary to other optimization issues, districting problems lack a consensual mathematical formulation. Nonetheless, a variety of models have been proposed to address this issue throughout the last 20 years. Furthermore, given the NP-hardness of these models, several solution techniques have been implemented, ranging from exact procedures to heuristics and meta-heuristics.

To date, no article addresses the problem of districting considering the specific characteristics of home hospitalization. Most of the papers reviewed used real healthcare case studies but less than half of the authors explicitly refer to the inclusion of stakeholders in the decision-making process. There is also a shortage of models that encompass the uncertainty inherent to the healthcare setting. This dissertation will tackle these three shortcomings. The proposed optimization model is presented in the following chapter, combining the methodologies discussed in the previous chapter with the case-study aspects of Chapter 2.

Chapter 4

Model Formulation

The following chapter proposes a mixed-integer linear programming formulation to address the districting problem in a home hospitalization setting. Although the model was developed considering the case study's specificities, it is aimed to be generic and applicable to any home hospitalization unit network. The problem statement can be found in Section 2.4 along with the underlying assumptions and used criteria. The mathematical formulation, including the notation used, the objective function, and the constraints, can be consulted in Section 4.2. Section 4.3 proposes a solution methodology and the chapter's conclusions are presented in Section 4.4.

4.1 Problem statement

Given a particular service territory and a set of patients, the districting problem consists of grouping the patients' locations into good districts according to relevant criteria. The goal is to minimize the travel distances and travel times within districts, thus making districts as compact as possible while assuring workload balance between the region's HH units. The districting plan must also guarantee that care is provided according to the different capacities of the HH units. Thus, minimizing the periods in which the units cannot satisfy demand is also part of the objective function. The time horizon of the districting plan is an input of the model; for this dissertation, one year will be considered.

It is presumed that the region of interest can be modeled through basic units, i.e., points located in a two-dimensional geometrical space by its centroidal coordinates. The basic units can be split into two groups: a set of $d \in D$ demand points and a set of $u \in U$ home hospitalization units or supply points. The geodesic distance between two demand points is expressed by $\delta_{dd'}$, while δ_{du} denotes the distance between a demand point and a HH unit. Each demand point represents the aggregate demand of a particular civil parish or parish cluster. It is assumed that these demand points have been defined *a priori*. Note that the demand could be aggregated in other manners as appropriate to the case study.

Demand points are described by a certain number of patients, about whom the month they were admitted to HH care and the number of days of hospitalization is known. The number of daily visits per month needed to treat the patients of each demand point (given by h_d^m) is an input of the problem. To obtain this number, the hospitalization days of each month's patients are summed. Assume that a daily visit represents the complete daily care provided to a patient and may, in practice, entail two trips to the patient's home.

HH units are characterized by their capacity, meaning the number of patients each unit can treat daily. In other words, the capacity of each unit is specified by the number of daily visits available. From the daily capacity, the monthly and annual supply can be calculated. Of the supply points, only those that are open, denoted by $u \in U^{open}$, will be considered in the districting decision.

In districting problems, the district number is often subject to planning. In this formulation, the number of districts to design is predetermined and equal to the number of HH open facilities. In districting nomenclature, each HH unit can be considered a district center. The model identifies the optimal partitioning of basic units, grouping demand points into a set of U non-empty and disjoint partitions, with a supply point each. Each demand point is assigned to precisely one district.

The decisions to be made include the allocation of demand points to HH units, given by the decision variable x_{du}, the maximum workload deviation, the longest distance between two demand points, the longest distance between a demand point and its assigned HH unit, and the maximum under-capacity on which each HH unit would operate on, denoted by Δ, Ω, Θ, and Ψ, respectively. This formulation also quantifies each supply point's monthly and annual workload (wl_u^m and wl_u), that is, what percentage of its capacity is deployed in those periods. In addition, the model identifies units in under-capacity and in which months that occurs; the auxiliary variable $UnderCap_u^m$ represents the monthly value of under-capacity for each HH unit.

4.1.1 Assumptions

This subsection summarizes the assumptions that were made in the model formulation. Without losing generality, it is assumed that:

- A.1 The districting is done once for a relatively long time. The period of a common calendar year is considered, totaling 365 days.
- A.2 The HH structure can treat all patients, and all the demand points are covered, meaning that all the patients admitted to the HH must be assigned to a district.
- A.3 Each open HH unit serves exactly one district.
- A.4 The number of patients admitted to the HH structure is known in advance and does not change while considering the districting problem.
- A.5 As there are no unallocated demand points and supply is limited, under-capacity may occur in specific units in certain months.
- A.6 Each demand point has a predefined monthly number of required daily visits, given by the sum of the hospitalization days of the aggregate demand.
- A.7 Patients admitted on any day of a particular month are accounted as entering on the first day of that month.

A.8 The coordinates of each demand point correspond to the midpoint location of each parish obtained through Nominatim, a geocoding software that uses open data from OpenStreetMap.

A.9 Given that the road networks are dense in urban locations, geodesic distances can be utilized to approximate road distances and journey durations.

A.10 All patients are homogeneous in terms of care requirements and service demand.

A.11 HH units and their teams are homogeneous regarding skills, contracts, and workload capacity.

4.1.2 Considered criteria

A districting approach within the home hospitalization context should satisfy both patients and care providers. From the patient's perspective, such a decision should assure the best quality of care, with reduced waiting times and equivalent quality service in all districts. From the providers' point of view, proper districting should achieve a fair distribution of workload between districts as well as efficient usage of both material and human resources, notably through reduced travel times. The criteria described hereinafter are precisely intended to address the concerns of the stakeholders.

Through the literature review, it can be noted that the most used criteria in the objective function are the workload balance between districts, followed by the compactness, travel time and district similarity. The Appendix Table A.1 summarizes the criteria used so far both in the objective function and as soft and hard constraints.

Capacity limitation

Given that each district only has one supply point with a specified daily capacity, this criterion seeks to respect the capacity limitations of each HH unit, ensuring that supply meets demand.

Compactness

A geographically compact district is somewhat round-shaped, undistorted, and without holes (Darmian et al., 2021, Kalcsics and Ríos-Mercado, 2019). This widely considered criterion strives for short travel distances and travel times thus improving a provider's efficiency. Different approaches can be used to measure compactness since its' definition is strongly dependent on the geometric representation of basic units. In this work, a distance-based measure will be implemented.

Complete and Exclusive Assignment

Also referred to as the*indivisibility of basic units* or *integrity*, this criterion states that each basic unit must be assigned to one and only one district, allowing the establishment of long-term relationships with patients and avoiding overlapping caregiver responsibilities.

Contiguity

A contiguity or *connectivity* criterion aims at assuring that all basic units within a certain district are connected. In other words, it guarantees that it is possible to travel between any two points of a certain district without having to go through any other district. It is a desirable property not only for administrative reasons but also because it facilitates the reduction of travel distances. This criterion will not be considered in an explicit manner since it is implied through compactness (Sahin et al., 2010) and respect for administrative boundaries.

Distance limit

To assure efficient health network management through a district, the maximum distance between residential areas of a district should be less than a predetermined limit. For home hospitalization, greater proximity between the supply and the demand points must be assured given the potential need to assist the patient promptly.

Respect for administrative boundaries

The districts designed must conform to the administrative boundaries, either municipalities or civil parishes. This not only simplifies the organization of health care delivery procedures but also indirectly assures district contiguity.

Workload balance

Balance describes the desire for districts of similar size with respect to some performance measure, in this context concerning workload. The workload, usually expressed in hours per year, corresponds to the sum of the service time, or "care" workload, and of the average travel time between the district center and the demand points (Blais et al., 2003, Sahin et al., 2010). For this formulation, the focus will be on the care workload since the travel time is encompassed in other criteria. Workload balance is considered essential in district design, hence being mentioned in most districting literature. It was also pondered as the most important factor to optimize by the stakeholders consulted during this thesis.

The subsequent section presents the model's mathematical formulation under the above assumptions and criteria.

4.2 Numerical Model

In this section, the mathematical formulation of the districting model is presented. The model is classified as MILP since, on the one hand, some variables are restricted to integers, and on the other hand, the objective functions and constraints are linear. Before describing the model itself, the notation used is clarified. The sets and subsets, parameters, and decision variables are introduced. Afterward, the objective functions and the model's constraints are introduced.

4.2.1 Sets and subsets

Sets

$d \in D$: Set of demand points, $d = 1, 2, ..., |D|$, based on zip-codes

$u \in U$: Set of home hospitalization units , $u = 1, 2, ..., |U|$

$m \in M$: Set of months in the planning horizon, $m = 1, 2, ..., 12$

Subsets

$u \in U^{open}$: Potential new home hospitalization units

$(d, d') \in E$: Set of demand points pairs $(d, d') \in E$ where $(d, d') \in E$ if and only if $e_{dd'} = 1$

4.2.2 Parameters

$\delta_{dd'}$: Distance between demand points d and d'

δ_{du} : Distance between demand points d and home hospitalization unit u

δ_{Dmax} : Maximum distance allowed between 2 demand points $d, d' \in D$ assigned to the same district $u \in U$

δ_{Umax} : Maximum distance allowed between a demand point $d \in D$ and its assigned home hospitalization unit $u \in U$

τ : Time frame considered in the districting decision, expressed in days

γ_m: Number of days of month $m \in M$

h_d^m : Number of hospitalization days of demand point $d \in D$ in month $m \in M$

Cap_u: Daily capacity of each HH unit $u \in U$, expressed in number of patients

$MonthSupply_u^m$: Capacity of each HH unit $u \in U$ for month $m \in M$

$$MonthSupply_u^m = Cap_u * \gamma_m \tag{4.1}$$

$Supply_u$: Yearly capacity of each HH unit $u \in U$

$$Supply_u = Cap_u * \tau \tag{4.2}$$

$e_{dd'}$: Compatibility index

$$e_{dd'} = \begin{cases} 1 & \text{if demand points } d \text{ and } d' \text{ are compatible} \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

Reasons for incompatibility:

- existence of geographical obstacles between them
- difficulty or impossibility to travel from one basic unit to another
- not belonging to the same municipality or civil parish (depending on the decision-maker's interest).

$open_u$: Potential open home hospitalization units

$$open_u = \begin{cases} 1 & \text{if } u \in U^{open} \\ 0 & \text{otherwise} \end{cases} \tag{4.4}$$

4.2.3 Decision variables

Primary decision variables

x_{du} : Assignment of demand point d to district u.

$$x_{du} = \begin{cases} 1 & \text{if demand point } d \text{ is assigned to district } u \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

Δ : The maximum deviation, expressed as a percentage, between the care workload associated to each HH unit $u \in U$ and the average care workload among all districts

Ω : The maximum distance between two demand points $d, d' \in D$ assigned to the same unit $u \in U$

Θ : The maximum distance between a demand point $d \in D$ and it's assigned HH unit $u \in U$

Ψ : The maximum undercapacity on which a HH unit $u \in U$ would operate

Auxiliary variables

wl_u : Percentage of utilized care workload out of the total annual supply of district u

wl_u^m : Percentage of utilized care workload out of the total supply of district u in month m

$\overline{wl}$: Average workload among all districts

$MonthDemand_u^m$: The number of required daily visits attributed to district $u \in U$ during month $m \in M$

$Demand_u$: The total number of required daily visits attributed to district $u \in U$

$UnderCap_u^m$: The number of extra daily visits that unit $u \in U$ would have to operate for there to be no under-capacity in month $m \in M$

4.2.4 Objective function

Objective 1: Workload balance

The most common measure to quantify imbalance is based on the relative deviation of the district workload from the mean district workload. Minimizing the deviation seeks to ensure that each of the HH units' teams is as close as possible to the average care workload.

$$\min \big(\max_{u=1,\dots,U} |wl_u - \overline{wl}| \big) \tag{4.6}$$

Where wl_u and $\overline{wl}$ are defined as follows:

$$Demand_u = \sum_{d=1}^{D} \sum_{m=1}^{M} x_{du} * h_d^m \tag{4.7}$$

$$wl_u = \frac{Demand_u}{Supply_u} \tag{4.8}$$

$$\overline{wl} = \sum_{u=1}^{U} \frac{wl_u}{U} \tag{4.9}$$

Given the nonlinearity of expression 4.6, there is the need to introduce another decision variable Δ, and add two hard constraints that link Δ to wl_u and $\overline{wl}$.

$$\min \Delta \tag{4.10}$$

$$\Delta \geq wl_u - \overline{wl}, \forall u = 1, ..., U \tag{4.11}$$

$$\Delta \geq \overline{wl} - wl_u, \forall u = 1, ..., U \tag{4.12}$$

Objective 2: Compactness

Objectives 2.1 and 2.2 seek to minimize the compactness measure and therefore assure geographically closely packed districts. For that, two distance-based measures were formulated.

The goal is to minimize the maximum distance between demand points and their assigned HH unit (4.13) as well as the maximum distance between any two points assigned to the same HH unit (4.14). This formulation is, however, not linear, and therefore difficult to solve. For that reason, the linearization and implementation of both objective functions is presented below.

$$\min \big(\max_{u=1,\ldots,U} \delta_{du} * x_{du} \big) \tag{4.13}$$

$$\min_{u=1,\ldots,U} \left[\max_{d=1,\ldots,D d'=1,\ldots,D} (\delta_{dd'} * x_{du} * x_{d'u}) \right] \tag{4.14}$$

Objective 2.1: Compactness between supply and demand

Objective 2.1 seeks to assure the minimal distance between a given HH unit and its' assigned demand points.

$$\min \Theta \tag{4.15}$$

$$\Theta \geq \delta_{du} * x_{du}, \forall d = 1, ..., D, u = 1, ..., U \tag{4.16}$$

Objective 2.2: Intra-district compactness

Objective 2.2 minimizes the maximum distance between two demand points assigned to the same HH unit (intra-district distance) .

$$\min \Omega \tag{4.17}$$

$$\Omega \geq \delta_{dd'}(x_{du} + x_{d'u} - 1), \forall d, d' = 1, ..., D, u = 1, ..., U \tag{4.18}$$

Objective 3: Under-capacity

Ideally, every demand point's needs should be met, which is achievable by constraining the supply in a certain HH unit to always be higher than the demand for that unit. However, for high levels of demand, the units' installed capacity may not be sufficient. As all patients must be allocated, this constraint must be soft: the variable Ψ, representing under-capacity, allows the violation of the maximum installed capacity. It is however necessary to penalise the use of extra capacity through an additional cost in the objective function. The fourth objective seeks to ensure that the allocation is carried out with minimum values of under-capacity.

$$\min \Psi \tag{4.19}$$

$$Supply_u + \Psi \geq Demand_u \tag{4.20}$$

Whilst districting is mostly a strategic problem and should therefore encompass a long time span, assuring that yearly supply satisfies yearly demand is insufficient since seasonality and other demand fluctuations are not being pondered. For that reason, an additional constraint reflecting monthly capacity was added.

$$MonthDemand_u^m = \sum_{d=0}^{D} x_{du} * h_d^m \tag{4.21}$$

$$MonthSupply_u^m + \Psi \geq MonthDemand_u^m, \forall u = 1, ..., U \tag{4.22}$$

4.2.5 Constraints

Selection of open units

Allocation must only happen between open units. This constraint ensures that if a unit is closed, $u \notin U^{open}$, then no patient $d \in D$ is allocated to it.

$$x_{du} \leq open_u, \forall d = 1, ..., D, \forall u = 1, ..., U \tag{4.23}$$

Complete and Exclusive Assignment

Every demand point must be assigned to one and only one district.

$$\sum_{u=1}^{U} x_{du} = 1, \forall d = 1, ..., D \tag{4.24}$$

$$x_{du} \in \{0, 1\}, \forall d = 1, ..., D, u = 1, ..., U \tag{4.25}$$

Compatibility

This compatibility constraint seeks to intersect the criteria of respect for administrative boundaries and contiguity to the extent that, for the Portuguese case, the former implies the latter. Although different graphical and geometrical measures have been used in the literature, obtaining a rigorous and concise mathematical formulation of contiguity is difficult. For this formulation, contiguity can be ensured by guaranteeing that demand points belonging to the same parish are assigned to the same HH unit. This restriction is based on the premise that these parishes are themselves contiguous.

To this end, making use of the set $(d, d') \in E$ containing all pairs of demand points considered compatible, the constraint 4.26 ensures that if the demand point d is assigned to the district $u \in U$, then the point d' will also be. The notion of compatibility is adaptable and can represent points that belong to the same civil parish but also points in the same city or any pair of demand points that, by some reasoning of the decision maker, must necessarily be allocated to the same district.

For the case under study, Provider X did not establish the use of existing Portuguese administrative boundaries as a prerequisite. Hence, the model will be tested with and without this restriction.

$$x_{d'u} \geq x_{du}, \forall (d, d') \in E, \forall u \in U \tag{4.26}$$

Distance limit

A distance limit is imposed between demand points and their associated HH unit (4.27). Also, there is a maximum distance allowed between demand points assigned to the same district (4.28).

Respecting the limits of each unit's operating radius is imperative in the context of home hospitalization, so distance restrictions are modeled as hard constraints.

$$\delta_{du} x_{du} \leq \delta_{Umax}, \forall d \in D, u \in U \tag{4.27}$$

$$\delta_{dd'}(x_{du} + x_{d'u} - 1) \leq \delta_{Dmax}, \forall d, d' = 1, ..., D, u = 1, ..., U \tag{4.28}$$

The coming section discusses the chosen solution approach, briefly comparing it to other optimization methods.

4.3 Solution approach

Mathematically, there can be infinitely many Pareto optimal solutions in multi-objective optimization. Hence, there is often a need to decide which solution is preferred. A multi-objective optimization method should ideally reflect the user's preferences if they are known, meaning that it should consider how the decision-maker perceives different solution points (Arora, 2017). Several approaches to expressing preferences over different objective functions can be articulated *a priori* or *a posteriori*. Some of the most common approaches are the *weighted sum method*, where a single scalar function with a single objective combines the multiple objectives; the *min-max weighted sum method*, which minimizes the "distance" to an ideal utopia point; and the *lexicographic method* (Chang, 2015). The latter was the one employed in this dissertation.

In the lexicographic method, preferences are imposed by ordering the objective functions according to their importance and not by assigning weights. After ordering the objective functions, the most important objective is solved first as a single objective problem. When solving the second objective, an additional constraint concerning the optimal solution of the first objective function is added. The process is repeated, in which the optimal solution obtained in the previous step is added as a new constraint, and the sequence of single objective optimization problems is iteratively solved. In addition to not requiring that the objective functions be normalized, Arora (2017) points out that the lexicographic method always provides a Pareto optimal solution.

4.4 Chapter conclusions

The above chapter presents a mathematical model that aims to partition the potential patients of a given territory into districts by optimizing four objective functions subject to general conditions and problem-specific constraints. The multi-objective formulation minimizes average care workload deviation, the distance between demand points and their assigned supply point, the distance between demand points allocated to the same unit, and the values of under-capacity throughout all supply points. In addition to the restrictions embedded in the objective functions, the model establishes maximum distances between basic units, seeks to respect administrative divisions, and guarantees the complete and exclusive assignment of demand points.

The suggested approach to solving the problem is using the lexicographic method to obtain an exact solution. This approach considers the stakeholders' preferences, guarantees Pareto-efficient solutions, and does not require normalization of the objective functions.

The model intends to be generic. Its parameters can be adapted to the user's needs, such as the districting period, the maximum distance allowed to the hospital, and the demand and supply data frames.

The next chapter addresses the implementation of the formulated model to the case study, detailing the data collection and preprocessing procedures and the districting results for various demand scenarios.

Chapter 5

Case Study Results

This chapter presents the application of the proposed model to Provider X's case study. The developed model was applied to various instances that simulated different demand scenarios. The features of these instances are gathered in Section 5.1. Section 5.2 begins by analyzing Provider X's current HH unit and its three potential new supply points. After analyzing the districting scores for various instances, the launching order that best meets current and future demand is proposed. After that, the focus becomes the study of districting when the four units under study are open. A sensitivity analysis is presented in Section 5.3. Following the analysis of the results, a set of recommendations for Provider X was compiled and can be consulted in Section 5.4. The chapter's conclusions are presented in Section 5.5.

5.1 Growing demand scenarios and test instances

Three demand scenarios were created to study the possible combinations between the upcoming HH units. Later, a fourth scenario was created to analyze districting with all units open. For each scenario, two data frames were set: one for demand and one for supply. The supply data frame contains the assessed hospitals' list, location, daily capacity, and whether or not their HH unit is open. The last two parameters were varied for the different scenarios and discussed later in this section.

The demand data frame must contain each demand point's coordinates and monthly demand, given by the number of patients treated each month multiplied by the number of hospitalization days for each patient. Both the number of patients per parish per month and the number of hospitalization days were randomly generated following a triangular distribution ($X \sim \mathrm{triangular}(a, c, b)$). The parameters a, c, and b define the minimum, most likely, and maximum values, respectively. Equation 5.1 presents the cumulative distribution function of a triangular random variable X. The inverse cumulative distribution function can be used to generate a set of random variates (5.2), being u a probability value between 0 and 1.

$$F(x) = P(X \leq x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & a < x < c \\ 1 - \frac{(x-b)^2}{(b-a)(b-c)} & c \leq x < b \end{cases} \tag{5.1}$$

$$F^{-1}(u) = \begin{cases} a + \sqrt{(b-a)(c-a)u} & 0 < u < \frac{c-a}{b-a} \\ b - \sqrt{(b-a)(b-c)(1-u)} & \frac{c-a}{b-a} \leq u < 1. \end{cases} \tag{5.2}$$

The number of hospital days per patient followed $X \sim \mathrm{triangular}(1, 7, 70)$, attempting to represent the distribution observed in Provider X's patients, which is illustrated in Figure 5.1. In turn, the number of patients per month per parish was calculated based on the number of inhabitants in each parish and an arbitrary monthly service utilization rate. A symmetric distribution was considered, given by $X \sim \mathrm{triangular}(\frac{2p}{3}, p, \frac{4p}{3})$ where p is the product of a monthly utilization rate by the number of inhabitants in each parish. The same utilization rate was used throughout all the municipalities. This simplification is not representative of reality and was corrected in scenario 4, as discussed later.

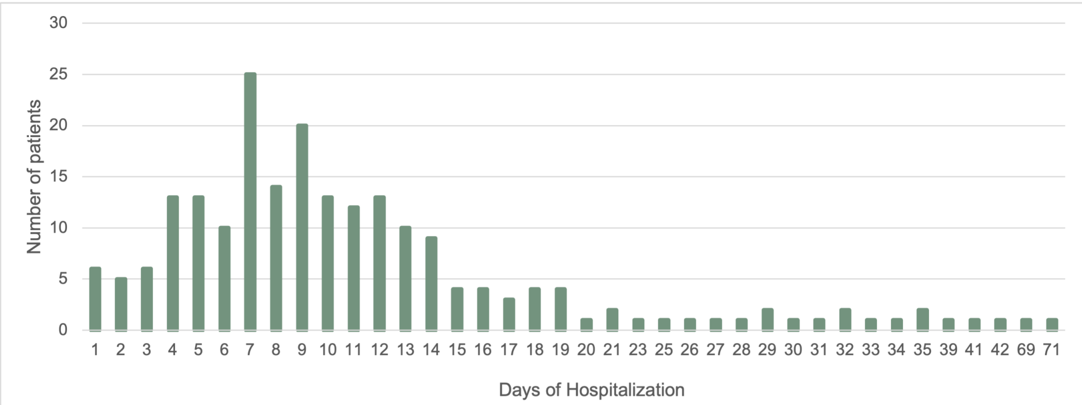

Figure 5.1: Distribution of days of hospitalization at the Provider X HH unit in 2021 and 2022.

The distance limit and time horizon parameters were defined and did not differ between scenarios. The current Provider X hospitalization services' catchment area was studied for a one-year period. It should be noted that ideally, patients should not be more than 30 kilometers away from the hospital. However, Hospital 1 has already treated patients at greater distances. This indicates that the 30 kilometers should be a recommendation rather than an obligation and that this value could be flexible considering the assessment of the patient in question. The value of parameter δ_{Umax} strongly impacts the feasible region of the districting problem. Therefore, as the feasible region should cover all demand points, the maximum distance between the patients and their attributed hospital was increased to 45 kilometers for this case study.

Ideally, all scenarios would consider one demand point per parish, adding up to 75 points. However, when testing the instances for districting with three or more HH units, finding an optimal solution did not prove feasible in less than 3 hours, the established time limit for each run. Even after applying a relative optimality gap tolerance of 0.1, which instructs CPLEX to stop as soon as it finds a feasible solution within ten percent of the optimum, computational times remained significantly high. The high computational times may not be a problem since districting is primarily a strategic decision. However, given that to validate the model it was necessary to run it for several instances and simulate multiple scenarios, and considering the limited time frame of this dissertation, the number of demand points was reduced. Neighboring parishes belonging to the same municipality were aggregated to make the clusters compact. A detailed description of the clusters can be found in the Appendix Table B.2. Scenarios S2.2, S2.3, S3.2, S3.3 and S4 were tested with this new dataframe, which has 22 demand points.

Finally, notice that the constraint concerning compatibility (4.2.5) was not used as it was not considered appropriate in the context of this case study. The model was tested with this constraint to verify this assumption, which worsened the optimization results. Please consult Section 5.3 for more details on this assessment.

The remaining section will detail each demand scenario. Table 5.1 and the Appendix Table B.1 offer an overview of the scenarios and test instances.

Table 5.1: Characterization of the scenarios and instances used for the case study districting.

		H1	H1 + H3	H1 + H2	H1 + H4	H1 + H3 + H4	H1 + H3 + H2	H1 + H3 + H2 + H4
S1								
Capacity		H1=12	H1=H3=12	T=12, H2=6	H1= H4=12	H1=H3=H4=12	H1=H3=12, H2=6	H1=H3=H4=12, H2=6
Utilization rate		0.014%	0.014%	0.014%	0.014%	0.014%	0.014%	0.014%
Patients		147	147	147	147	147	147	147
Variables	Binary	276	276	276	276	276	276	276
	Continuous	109	109	109	109	109	109	109
Constraints		39144	39146	39146	39146	39148	39148	39150
S2				**S2.1**		**S2.2**		**S2.3**
Capacity			H1=H3=15	H1=15, H2=8	H1=H4=15	H1=H3=H4=15	H1=H3=15, H2=8	H1=H3=H4=15, H2=8
Utilization rate			0.018%	0.018%	0.018%	0.027%	0.027%	0.036%
Patients			273	273	273	513	513	677
Variables	Binary		300	300	300	88	88	88
	Continuous		109	109	109	109	109	109
Constraints			46136	46136	46136	4321	4321	4323
S3				**S3.1**		**S3.2**		**S3.3**
Capacity			H1=H3=15	H1=15, H2=8	H1=H4=15	H1=H3=H4=15	H1=H3=15, H2=8	H1=H3=H4=15, H2=8
Utilization rate			0.028%	0.028%	0.028%	0.042%	0.042%	0.056%
Patients			486	486	486	803	803	1068
Variables	Binary		300	300	300	88	88	88
	Continuous		109	109	109	109	109	109
Constraints			46136	46136	46136	4321	4321	4323

Scenario 1 – *As Is*

Scenario 1 (S1) is based on the demand for HH services in the Hospital 1 unit in 2021 and the first quarter of 2022. 145 patients were admitted in 2021 and 66 until April 2022, spread across Lisbon, Cascais, Sintra, Odivelas, Loures, Mafra, Oeiras, and Vila Franca de Xira. It was not possible to access more detailed information on the patient location. Therefore, the population living in all the parishes of these municipalities was considered in this scenario. The municipality of Vila Franca de Xira was excluded because it was considered that most of the population of this municipality lived outside the catchment area of the four Provider X hospitals under consideration.

From the number of patients treated in 2021, a service utilization rate was calculated to generate approximately the same effective demand as Provider X when applied to the inhabitants of the municipalities under study. Drawing on this rate, an arbitrary number of patients per parish was then generated, totaling 147 patients throughout 69 civil parishes. The distribution is, thus, proportional to each parish's number of inhabitants in 2021, obtained from Instituto Nacional de Estatística (2021). The generation of the number of days of hospitalization per patient was also stochastic, as previously mentioned.

Regarding supply, each hospital can treat 12 patients simultaneously except Hospital 2, where the capacity is expected to be 6 since the decision-makers reported that the unit would always be smaller than the remainder.

Scenario 2 – Demand meets supply

Scenario 2 (S2) assumes that demand will equal supply if the latter increases. Therefore, this scenario represents a demand proportional to that in Scenario 1, where the number of patients in S2 roughly matches the number of patients in S1 multiplied by the number of open HH units. This scenario's effective demand extends to the Amadora municipality, the only municipality in the region that has not yet obtained Provider X home hospitalization services, thus adding up to 75 encompassed parishes. There are three sub-scenarios within S2 whose utilization rates are commensurate to the number of units opened and adjusted considering the coverage of an additional city (Amadora). By stakeholders' request, the units' capacity was increased to 15 patients per day for all hospitals except Sintra, where the capacity is assumed to be 8.

Scenario 3 – High-demand

Scenario 3 (S3) forecasts a 3-year high-demand scenario. It considers the sub-scenarios described in S2 and Portugal's historical growth of home hospitalization services. A growth rate of 16.5% per year was considered. This value accounted for the growth experienced between August 2021 and the same month in 2022, as documented by Santos (2022). When running this scenario, the same supply as in S2 was considered. In the discussion of the results, it was evaluated how much additional capacity was needed to meet the new demand figures.

Scenario 4 - Tailored demand to Provider X's reality

So far, the number of patients per parish has been based solely on the number of inhabitants and considering a common utilization rate for all parishes. However, both potential and effective demand reflect several other factors. To obtain a more accurate estimate of the number of patients, it would be necessary to comprehensively characterize the patient profiles for this type of service and quantify them for each parish. One could, for example, incorporate economic factors or break down the demand by age group, given that senior citizens are the most in need of HH services. For a more accurate estimate of hospitalization days, the correlation between days of hospitalization and patients' primary diagnoses could also be studied.

Scenario 4 seeks to approximate the proportion of patients per parish felt in Provider X's reality observed up to now to obtain districting solutions that are of practical use for decision-makers. To this end, different rates were considered for each parish, seeking to scale patients according to equation 5.3 instead of considering a single utilization rate. These rates reflect a high-demand scenario, with a total of 773 patients per year, an intermediate number between S2.3 and S3.3. Scenario S4 was only tested with the four units open and an equal capacity to the last two scenarios.

$$demand[Lisboa] \simeq \left\{ \begin{matrix} 2 * demand[Cascais] \\ 4 * demand[Sintra] \\ 8 * demand[Oeiras] \\ 10 * demand[Amadora, Loures, Mafra, Odivelas] \end{matrix} \right\} \tag{5.3}$$

Having described the demand scenarios and the different instances with which the model was tested, the next section will present the computational results of the model implementation.

5.2 Computational results

This section presents the computational results of applying the proposed model to Provider X's case study. It starts by describing a preliminary model validation and scalability analysis. After that, Subsection 5.2.2 implements the model to support the decision of the best launch order for Provider X's upcoming HH units. Subsection 5.2.3 then focuses on the districting of the same region, considering that all units under study are open.

The model was implemented in a *Python* script using the library *docplex - IBM Decision Optimization CPLEX*. All tests were run on a Macbook Pro computer with an Apple M1 processor and 16 GB of RAM, running *macOS Monterey* (version 12.2.1). The test instances were created using *MS Excel*. The remaining data was also inserted into this software, and the program was externally initialized. After solving the model, the results were exported back to *MS Exce*l, where they were processed with the support of *Visual Basic for Applications*.

5.2.1 Model validation and scalability analysis

Before examining the results for the test instances described in the previous section, the model was first tested with the patients treated by Provider X in 2021: initially, each demand point described a single patient. The supply and demand data frames had 4 and 145 entries, respectively. As the exact locations of patients were unobtainable, all patients from each municipality were considered to have the same coordinates, corresponding to a central point in the municipality. It stands to reason that this fact simplifies the resolution of the model. Thus, the model ran in a few seconds for all simulated cases, from having only one HH unit open to making all four units part of the districting plan.

In order to test the model's performance with larger instances, an instance with 420 patients and new random locations was created. When testing the new instance, a solution could not be found in less than three hours. In light of this problem, demand was aggregated by civil parishes. Besides not implying any compromise concerning the districting outcomes' quality, this decision is aligned with the strategic-tactical nature of the problem at hand.

Looking at the tests performed on the instances described in 5.1, it is noticeable that there should be an increase of only six entries in the demand data frame between the first and second scenarios, corresponding to the inclusion of Amadora's civil parishes. This slight increase led to the model's inability to develop a solution in the appropriate time frame. Even after applying a relative gap, the computational times remained high.

This shows that the computational effort increases exponentially with the instance size. For relatively small instances, this should not be a problem. However, if one wants to expand the districting region, it may be necessary to use non-exact methods or decomposition techniques that may explore the problem characteristics.

5.2.2 Optimal launch order for the upcoming Provider X HH units

As mentioned in Section 4.3, the lexicographical method was the chosen approach for this multi-objective optimization. The order of preference among the objectives was discussed with the stakeholder and is detailed in Table 5.2. The preference is in descending order, where the highest value corresponds to the objective to be optimized first.

Table 5.2: Order of preferences defined for each objective of the Objective Function.

Objective	**Minimize**	**Preference**
Under-capacity	Ψ	4
Workload balance	Δ	3
Compactness between supply and demand	Θ	2
Intra-district compactness	Ω	1

The possible combinations of 2 and 3 HH units within the four units under study were explored to sustain the best launch order decision. The review considered three scenarios (S1, S2, and S3) to ensure that decisions were appropriate to current and future demand. Table 5.3 presents the objective values for the different districting decisions, while Figure 5.2 provides a comparison between these outcomes. In the figure, the values for each objective were normalized to facilitate comparison.

As is

In order to validate the proposed model and as a starting point for further analysis, instance S1 was run, considering that only Hospital 1 was open. The results of this test are the ones that should most closely resemble Provider X's current circumstances, where a single unit located in H1 treats 147 patients in one year.

After solving the model, the farthest patient would be 42 kilometers from the Hospital 1 hospital. The same approximate distance separates the two farthest demand points. The current installed capacity of this unit would be able to treat all patients, and the annual workload would be 39,11%. Looking at the breakdown of demand by month, it ranges from 124 daily slots in November to 193 in July, leading

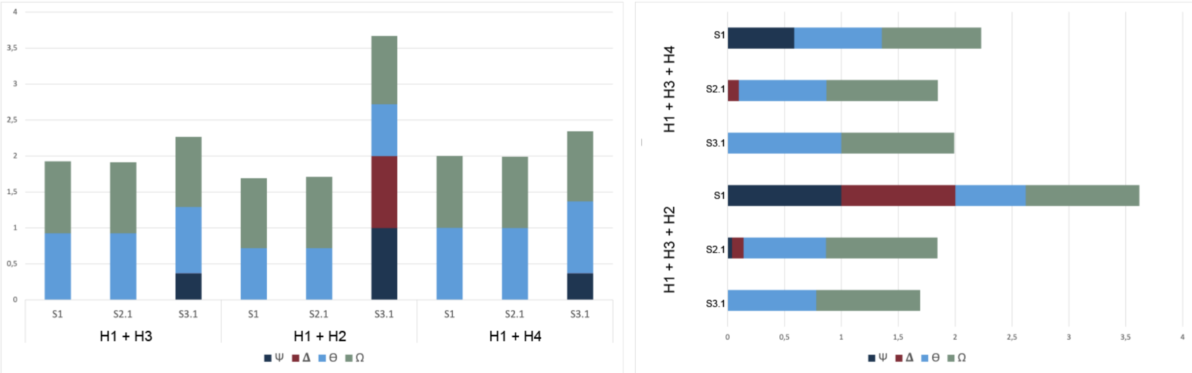

Figure 5.2: Comparison of districting outcomes between potential pairs (left) and trios (right) of HH units.

Table 5.3: Districting outcomes for different HH unit combinations in different demand scenarios.

		H1	H1 + H3	H1 + H2	H1 + H4	H1 + H3 + H2	H1 + H3 + H4
	Ψ	0.000	0.000	0.000	0.000	0.000	0.000
	Δ	0.000	0.011	0.000	0.011	0.015	0.000
S1	Θ	41.785	36.763	28.609	39.752	28.609	36.763
	Ω	42.252	36.366	35.282	36.367	32.599	35.282
	CPU time	0:00:00.07	0:00:00.59	0:00:00.45	0:00:00.56	0:00:01.20	0:00:01.56
	Ψ		0.000	0.000	0.000	76.000	6.000
	Δ		0.009	0.010	0.0009	2.232	2.052
S2	Θ	-	36.763	28.609	39.752	26.643	28.331
	Ω		35.966	35.966	35.966	34.900	34.900
	CPU time		0:00:01.43	0:00:01.55	0:00:00.08	0:00:00.57	0:00:00.83
	Ψ		743.000	2020.000	743.000	2050.000	1199.000
	Δ		0.009	16.142	0.009	21.842	0.012
S3	Θ	-	36.763	28.609	39.752	22.764	28.287
	Ω		35.462	34.441	35.462	35.630	31.079
	CPU time		0:00:00.94	0:00:00.85	0:00:01.00	0:00:01.10	0:00:42.84

to a capacity utilization level that varied between 34% to 52%. As intended, the dataset's variations in demand throughout the year portray seasonality. However, these variations do not accurately represent the actual health services' fluctuations in demand. This scenario presents a considerable capacity underutilization, indicating that it would be possible to treat almost twice as many patients in a year. This number of patients presents an early implementation stage where adherence to this service is expected to be relatively low, which may contribute to the apparent overcapacity. Having a stochastic distribution of hospitalization days also naturally impacts these results.

The *as is* demand scenario was also tested considering the gradual opening of more HH units. For the case that all units are open and assuming the same capacity for all units except for Hospital 2, where half is assumed, it is possible to obtain highly balanced districts with around 11% of capacity utilized in each unit.

The patient furthest from their assigned unit would be less than 37 kilometers from the hospital. The fact that some demand points are in more outlying locations of the four hospitals' catchment areas implies that compactness indicators can never be too low, regardless of the districting decisions. Figure 5.3 portrays this demand point dispersion in the region's northern section. It also represents the districting proposals for the *as is* scenario, with one or four available units.

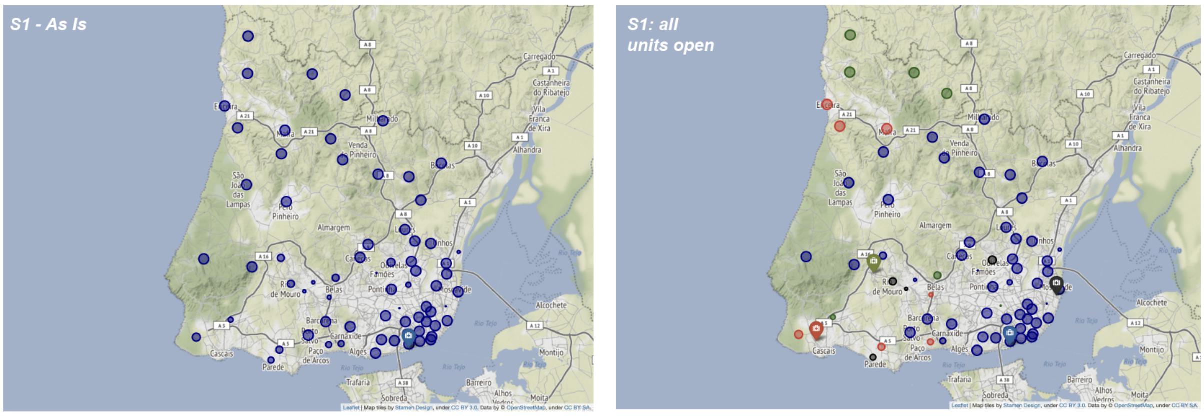

Figure 5.3: Districting solution for scenario S1, with only one HH unit at Hospital 1 (left) and all four units open (right). The colors marking the H3, H4, H1, and H2 units are red, black, blue, and green, respectively. The circles represent the demand points; the larger the radius, the greater the demand. The colour of the circles represents their allocation.

Districting between two HH units

The first decision concerns the location for the second HH unit to be opened. Three pairs of units were analyzed: Hospital 1 and Hospital 3, Hospital 1 and Hospital 2, Hospital 1 and Hospital 4. It is assumed that Hospital 3 and Descobertas units are similar in everything except their location, having the same capacity and no preference on the part of the decision-makers to choose one unit over the other. The Hospital 2 unit, on the other hand, is a smaller unit that historically serves fewer people due, among other variables, to socio-economic factors that are not considered in this analysis. Hence it is assumed that this unit has half the installed capacity of the other facilities. Furthermore, if the districting performance is equally satisfactory for all three cases, it would be preferable to open the Hospital 3 or Descobertas unit rather than Hospital 2.

The objective representing under-capacity only has non-null values in the last scenario. In this case, the model obtains under-capacity values proportional to the capacity of the units, whereby in the case of opening Hospital 2, the value will be higher. It is noticeable that when aggregating Hospital 1 and Hospital 2 in scenario S3, both hospitals are at under-capacity during all months of the year. To meet the demand, it is necessary to increase capacity by 11 patients at Hospital 1 and 14 patients at Hospital 2. On the other hand, by opening Hospital 3, for example, it would be necessary to increase capacity by 10 patients in both hospitals to meet demand. Moreover, the value for Δ is nearly optimal for all three cases, meaning that despite the location choice of the second unit, a very even distribution workload is achievable. This observation does not extend to the last scenario. Since the Hospital 2 unit's capacity is half of the other units' capacity, it is impossible to balance the workload in the scenario of higher demand.

Given that the workload is balanced in most scenarios, assessing compactness is necessary. The value of Ω is relatively homogeneous for all the tests run. However, regarding Θ, representative of the distance between the demand points and the hospitals, Hospital 2 presents the best results, followed by Hospital 3.

Overall, when running the model for the three scenarios, the results for the three combinations are somewhat balanced. Hospital 2 would have some advantage in terms of location for the first two instances run, managing to be more central for patients from Loures, Mafra, and Sintra. There are, however, two reasons to discard the Hospital 1 + Hospital 2 pair. On the one hand, the instances' demand is based solely on population density in the municipalities covered by Provider X. While this is a fair measure to represent potential demand, it does not represent effective demand. The effective demand is impacted by several other drivers that were not considered when creating the test instances. When looking at the actual distribution of Provider X's patients in 2021 and 2022, the number of patients in Sintra's surroundings is less than that observed in, for example, Cascais (see Figure 2.5). On the other hand, the fact that the Hospital 2 hospital is smaller than the others and is equipped to serve fewer people makes it a less evident alternative for high-demand scenarios. It may be relevant for Provider X to analyze ways to capture more patients in Sintra and surrounding municipalities, as there is currently an under-demand for home hospitalization services compared to the population.

When looking at the two other options, Hospital 3 or Descobertas, it is more advantageous to open Hospital 3 first. Although the difference is not substantial, opening Hospital 3 allows the patient furthest from their HH unit to be 3 kilometers closer than if Hospital 4 had opened. The fact that the effective demand for Provider X in 2021/2022 in Cascais was proportionally higher than in the test instances reinforces the choice of opening the Hospital 3 unit first.

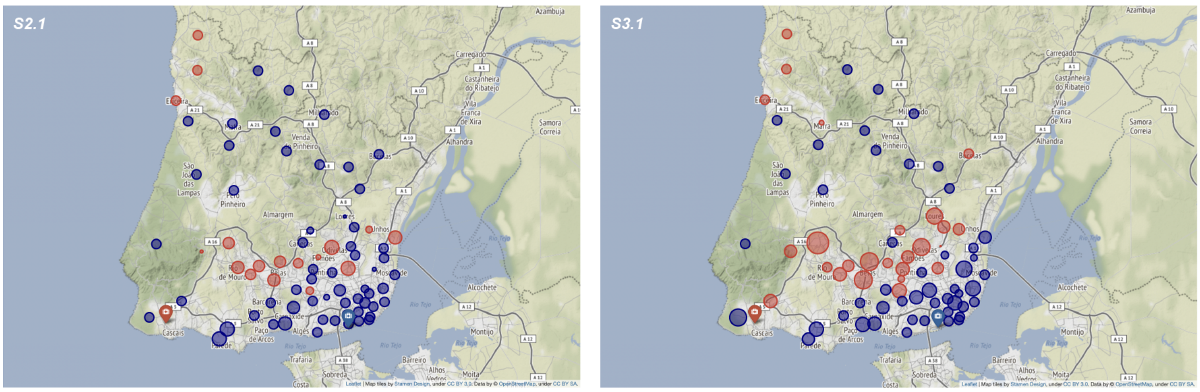

Figure 5.4: Districting solution for the pair Hospital 1 + Hospital 3 in scenarios S2.1 (left) and S3.1 (right). Hospital 3 is marked red and Hospital 1 is marked blue.

Districting between three HH units

The next decision is the choice of which should be the third unit to open. The analysis is similar to that made for the previous case. The trios behave similarly concerning workload balance for the first two scenarios, not exceeding a 2.5% difference between them. The difference in compactness objectives is quite significant in the first scenario, where opening the Hospital 2 unit would be advantageous. However, this difference is attenuated in scenarios S2 and S3, and the Θ value differs in less than 2 kilometres. When looking at the under-capacity values, they grow in tandem with the increase in demand. Again, as it is assumed that the Hospital 2 unit has about half the capacity of the others, the value of Ψ worsens when this unit is considered open. Considering scenario S2.2, for all patients to be served, it would be necessary to treat three more patients per day and per hospital when opening Hospital 2. For the case of opening Hospital 4, increasing the capacity by only one patient in one of the hospitals would be sufficient. In addition, the workload balance value in the higher demand scenario is substantially worse for the trio that includes Hospital 2.

This analysis shows that opening Hospital 4 is increasingly beneficial as demand rises. However, this decision is based on a higher under-capacity and worse workload balance for high-demand scenarios, which both derive from Hospital 2's capacity limitation.

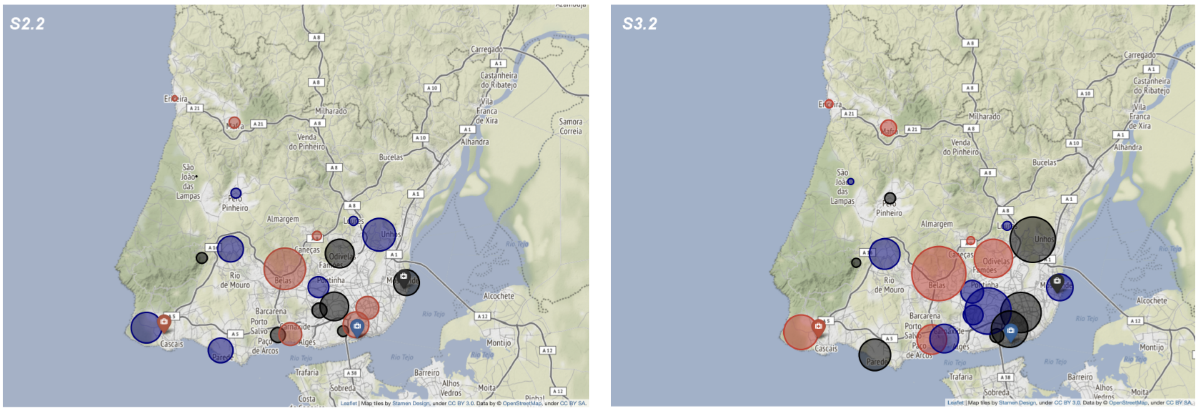

Figure 5.5: Districting solution for the combination Hospital 1 + Hospital 3 + Hospital 4 in scenarios S2.2 (left) and S3.2 (right). Hospital 3 is marked red, Hospital 1 is marked blue, and Hospital 4 is marked black.

5.2.3 Districting solution for four HH units

After studying the sequential opening of three Provider X units, the districting decisions were analysed considering all units were open. The analysis began with scenarios S2.3 and S3.3. For the first one, despite the annual workload value of the HH units varying between 80% and 90%, there were months of under-capacity. It would be necessary to increase capacity by two patients in the Cascais and Descobertas units and one in the remaining units (see Table 5.5) to meet this demand scenario. In scenario S3.3, all four units were under-capacity in most months, with more than double the capacity at Hospital 1 and Cascais needed to meet demand. Given the high demand and the reduced capacity at Hospital 2, it was more challenging to ensure workload balance: the other three units had a 123% capacity, while the annual workload at Hospital 2 was around 145%. Regarding compactness, the results suggest that it is nearly indifferent if 3 or 4 units are open since the values of Θ and Ω are not better than those observed in the districting for three units.

When looking at the allocation in these two scenarios, illustrated in Figure 5.8 and detailed in Appendix Table B.3, it is prominent how different the two districting decisions are: very few clusters are served by the same unit. The model does not promote stable allocation areas, which is important at the strategic planning level. Compared to the districting models found in literature, of the 11 studied papers, only two explicitly mentioned incorporating capacity into the constraints. Furthermore, although most try to ensure workload balance, they are done annually. Incorporating a monthly capacity constraint is a novelty considering the literature studied and justifies that the solution is sensitive to fluctuations in demand.

Because the proposed districts in S2.3 and S3.3 were not congruent with each other and were not representative of the distribution of effective demand at Provider X, few conclusions could be drawn regarding the optimal catchment areas for each HH unit. To address these shortcomings, scenario S4 was created, and the model was tested. The objectives related to compactness again show similar values, as the geographic configuration of the demand points was maintained. It is important to note that this scenario contemplates a significantly lower demand in the municipality of Sintra and that it is, therefore, possible to obtain quite balanced districts in terms of workload, with the value of Δ being around 3%. Again, there are few similarities between the newly obtained districts and the two previous district sets.

Because S4 reflects the current effective demand and since this study only covers a three-year horizon where significant changes in demand are not expected, the districting outcomes for this scenario are the most valuable to Provider X. Therefore, the sensitivity analysis presented in the Section 5.3 will focus on this last scenario.

Table 5.4: Districting results considering all four HH units open in different demand scenarios. Outcomes for the tactical districting of the Hospital 1 unit. Column S4(C) presents the results for scenario S4 considering the compatibility constraint and column S1(T) presents the tactical districting results for two HH teams at the Hospital 1 facility.

	S1	**S2.3**	**S3.3**	**S4**	**S4 (C)**	**S1 (T)**
Ψ	0.000	35.000	1264.000	234.000	3025.000	0.000
Δ	0.011	6.425	15.125	3.149	50.804	0.023
Θ	28.609	28.583	26.744	26.744	25.503	41.785
Ω	37.110	34.601	34.601	34.237	31.597	36.366
CPU time	0:00:01.90	0:00:01.06	0:03:50.37	0:00:00.63	0:00:00.17	0:00:00.88

5.2.4 Districting solution for two HH teams

The model's volatility regarding demand makes it more useful at the tactical level than the strategic level. It is possible, for example, to use the model considering only one unit at a time but dividing it into different teams. Consider the *as is* demand scenario and the sole operation of the Hospital 1 unit. Since each team treats an average of 6 patients daily, the supply was separated into two teams departing from the same location. Solving the model obtains the optimal patient allocation between the two teams, represented in Figure 5.6. It is possible to balance the workload (Δ is approximately 0), guaranteeing that within the same unit, no team is overloaded. Furthermore, it is possible to ensure greater intra-district compactness, reducing the travel distances of each team. If one prioritizes the optimization of Ω over Δ, it is possible to reduce the intra-district distance by about 3 kilometers without causing a significant imbalance in workload, with Δ increasing to 3.3%.

Not straying from the tactical realm, this decision precedes and facilitates routing problems since it helps establish which area each team should cover. Since this analysis represents a novel application for a districting model, the decision-makers examined these results and validated their pertinence and importance.

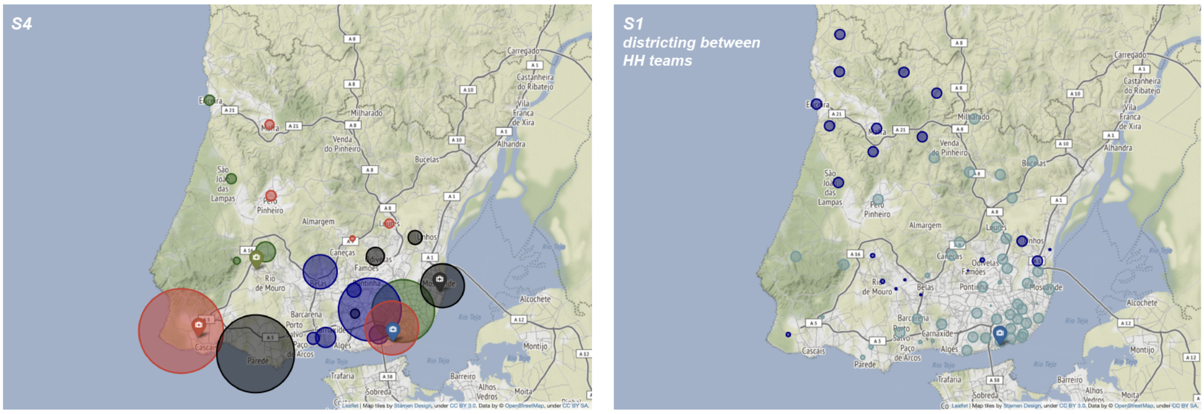

Figure 5.6: Districting solution considering all four HH units are open for scenario S4 (left) and tactical districting solution with two HH teams at the Hospital 1 facility for scenario S1 (right).

5.3 Sensitivity analysis

As mentioned in Section 4.3, the lexicographical method always finds Pareto-efficient solutions when optimizing multi-objective problems. This section will evaluate potential trade-offs when first optimizing one objective over the others. It is assumed that the objective of minimizing under-capacity should always be solved first: only the six possible permutations among the remaining three objectives were evaluated. This analysis is represented graphically in Figure 5.7. The values for each objective were normalized to facilitate comparison.

In the case of scenario S2.3, by varying the order in which the objectives are solved, the same solution is always obtained, indicating that there is a global optimal solution and that there is no compromise in minimizing one objective before any other.

The same does not apply to scenarios S3.3 and S4, though the improvement of a given objective over the detriment of the others is minimal. Regarding Δ, the differences are negligible: in S4, the value does not vary, and in S3.3, it varies less than 1%. The only notable trade-off is between Θ and Ω. Still, the variation of the values of these two objectives on the Pareto frontier is minimal. By minimizing Ω first, the value of Θ increases by less than 2 kilometers for both scenarios. In the case of optimizing Θ first, the value of Ω increases by about 3.5 kilometers in scenario S3.3 and 2.7 kilometers in scenario S4.

The takeaway from this analysis is that for this case study, there are no notable trade-offs between the optimized variables.

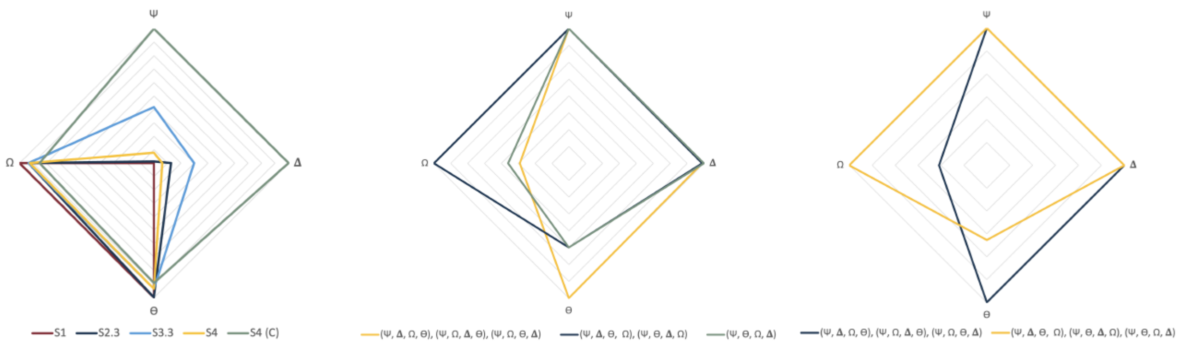

Figure 5.7: Radar chats representing the districting objectives for scenario S4 (left) and the sensitivity analysis for scenarios S3.3 (middle) and S4 (right).

As previously mentioned, the compatibility constraint, which imposes respect for administrative boundaries and also forces contiguity, was not used so far because it was not considered necessary for this case study. Note that this restriction is adaptable to various circumstances since it starts from a list of compatible parishes and imposes that these are attributed to the same district. The definition of compatibility can mean several things according to the needs of the decision-makers. The results of districting for scenario S4 were tested, considering that demand points from the same municipality had to be allocated to the same HH unit. The results are summarized in Table 5.5 and Figure 5.8. It was verified that imposing this constraint worsens the optimization significantly, leading to a workload imbalance of 50.8% and an increase in under-capacity from 234 to 3025.

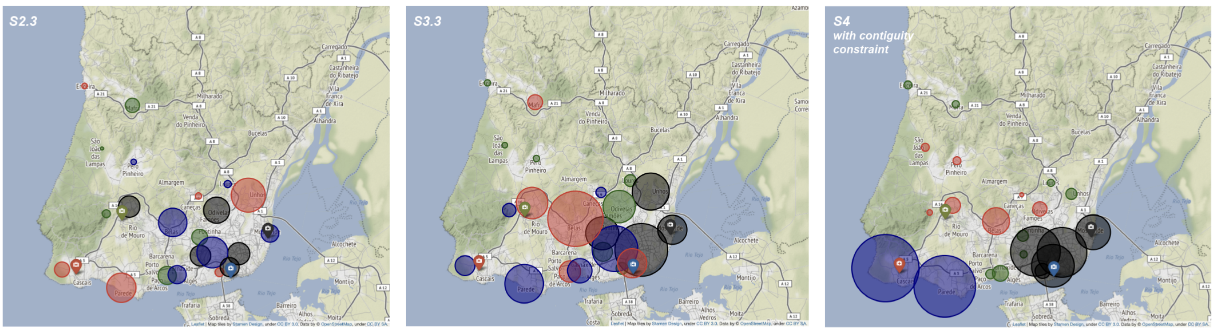

Figure 5.8: Districting solutions considering all four HH units are open for scenarios S2.3, S3.3 and S4 when using the compatibility constraint.

Table 5.5: Annual workload and monthly under-capacity distribution across the four Provider X HH units for the S2.3, S3.3, and S4 scenarios.

	S2.3			S3.3			S4			S4 (C)		
	Anual Workload	Undercapacity		Anual Workload	Undercapacity		Anual Workload	Undercapacity		Anual Workload	Undercapacity	
		# Months	Maximum		# Months	Maximum		# Months	Maximum		# Months	Maximum
Hospital 3	80.055%	5	2	123.087%	10	15	103.671%	4	7	53.644%	0	0
Hospital 4	88.311%	4	2	123.087%	9	14	103.872%	6	8	155.251%	10	21
Hospital 1	87.416%	3	1	123.087%	9	18	103.799%	6	8	104.201%	7	10
Hospital 2	90.137%	5	1	145.253%	11	11	107.979%	6	8	104.692%	6	3

5.4 General recommendations

This section summarizes the main conclusions drawn from the previous discussion. Note that, contrary to other studies, it is impossible to compare the obtained results with previous district configurations since these do not exist for this setting. Also, when assuming full coverage of the seven assessed municipalities, it was not always possible to comply with the maximum distance restriction, where all patients should be less than 30 kilometers from the hospital. Even for the scenario *as is* and considering all hospitals were open, the most distant demand point was 31.5 km away.

With concerns to the sequential launch of the new Provider X HH units, the expansion should begin with the opening of the Cascais unit, followed by Hospital 4, and lastly, Hospital 2. Nonetheless, according to the districting objectives, the results for the different units' combinations are similar, meaning that if a different order were to be adopted, comparable outcomes could be obtained.

The achieved districting decisions are quite sensitive to monthly demand oscillations. Thus, they reflect the specific demands of the scenarios studied and may not accurately represent different realities. For this reason, the model is most useful when run annually with instances generated through accurate demand forecasting. From the examined scenarios, S4 was the one to reflect best Provider X's effective demand over a three-year horizon. For the case of S4, the districting would allow for highly balanced districts in terms of workload. Districts would also be relatively compact: the maximum distance between demand points and their respective units would be 26.7 kilometers, and the longest distance between two points allocated to the same unit would be 34.2 kilometers.

Despite being less explored in this dissertation, another application for this model was discussed. It is possible to run the model more periodically to draw the optimal catchment areas of the multiple teams within an HH unit. The model was applied to the current Provider X paradigm: the H1 unit was divided into two teams, and the proposed districting allowed for minimal intra-district distances and a balanced workload between the two teams.

5.5 Chapter conclusions

The implementation of the mathematical model can be divided into two phases: creating the test instances with varying demand and, afterward, testing the instances and discussing the results.

The starting point was the creation of test instances based on the Provider X case study and its 3-year HH services expansion plan. The geographic distribution of demand for HH services at H1 and their distribution of hospitalization days were taken into account to randomly create four scenarios of increasing demand. The first three scenarios focused exclusively on generating demand proportional to the population density of each municipality. In contrast, the last scenario sought to reproduce Provider X's effective demand, considering different service utilization rates for different municipalities.

After creating and running the test instances, the model implementation results were analyzed for two separate cases. The model performed as expected for both, with outcomes aligned with the objectives in Chapter 4. Initially, a gradual opening of potential Provider X units was simulated to compare the districting results for the various combinations of hospitals. The optimal order of opening was established for the three units under study. Then, all four units were considered open. It was found that the allocation was markedly different for the various scenarios, so this model formulation does not enable robust solutions when faced with uncertainty in demand. Furthermore, it is stressed that the criterion regarding respect for administrative boundaries deteriorated the districting results, so this should only be applied if strictly necessary.

Regarding the responsiveness of the model when facing an increase in the size of the test instances, it was necessary to aggregate the parishes by clusters to reduce computational time. After clustering, it was possible to run all tests in very few minutes. This simplification did not significantly affect the results; however, it indicates that the model may not respond promptly for larger instances.

The following chapter concludes this dissertation by presenting a summary of the main accomplishments and thoughts on the most relevant topics for future research.

Chapter 6

Conclusions

The last chapter of this dissertation summarizes the main conclusions and presents potential directions for future research. It is subdivided into two sections: Section 6.1 concerns the concluding remarks and the study's contribution to the existing literature; Section 6.2 focuses on limitations and resultant opportunities for future work.

6.1 Achievements

Unprecedented health-related demographic changes are taking place worldwide. They include an elderly population expected to outnumber the child population and a growing number of patients with chronic diseases. The increasing disease burden and rising healthcare costs have yielded a trend toward home health services for patients to avoid hospital admissions.

Home hospitalization creates value for all parties, from patients to physicians, providers, technology companies, and investors. In addition to freeing up hospitals in capacity and enabling more patients to be treated, home hospitalization ensures the equivalent quality of care at a lower cost and higher patient satisfaction compared to in-hospital care. The concept is not new and has been increasingly adopted over the past decade. However, like telehealth, home hospitalization became even more necessary when acute care beds were filled during the first outbreak of the COVID-19 pandemic. In addition, advancement in technology is likely to favor the growth of the HH market by allowing care to be more virtualized through progress in areas such as remote patient monitoring.

Employed to improve cost-effectiveness, efficiency, and decision-making, OR is particularly useful for analyzing complex logistic health issues, especially in settings with high disease burdens and limited resources. In particular, a districting approach improves the efficiency and reactivity of care delivery and, as a consequence, satisfies patients more efficiently.

This dissertation proposes a general multi-objective MILP model for a districting problem applied to a home hospitalization service network. The proposed approach considers four objective functions: balancing workload among districts, maximizing the compactness of districts, both between supply points and their allocated demand points and between demand points within the same district, and minimizing the number of months in which HH units are in under-capacity. The objectives are also subject to constraints related to the complete and exclusive assignment of demand points, compatibility between demand points, and the maximum distance between basic units.

The model was applied under the context of Provider X, the leading Portuguese private healthcare provider, and consisted of studying its 3-year prospective home hospitalization network. The model application sought to illustrate the two types of results that can be obtained. The primary solution achieved was the optimal partition of a service region composed of home hospitalization units and aggregate demand points. Although less addressed in the literature, it is also possible to use the same model to evaluate which teams should serve which patients within the same HH unit.

Provider X's three-year business plan foresees opening three more HH units to complement its existing location. The main results were obtained considering that all HH units were already open. The intermediate supply point combinations were also analyzed to establish which units should be prioritized for opening. A planning horizon of one year per districting decision was considered.

To evaluate how the districting would vary for different demand distributions, computational results were presented for several randomly generated instances based on the real-world case study data. The instances incorporated uncertainty at two levels: the number of patients per civil parish and hospitalization days per patient.

The proposed solution approach included a lexicographic ordering that allowed the efficient frontier and trade-offs between objectives to be identified. However, for this case study, no sharp trade-off was identified, as improvements to a particular objective never resulted in significant deterioration of the others.

For most of the tested instances, the compatibility constraint, which ensures respect for municipal boundaries and contiguity of the created districts, was not used. When testing the model with this set of constraints, it was found that it significantly worsened the results, leading to an increase in workload imbalance of more than 47% and an under-capacity value almost 13 times higher. Thus, the compatibility constraints should only be employed to meet specific situations such as past partnerships, historical reasons, or other administrative situations.

To date, this is the first study to address the model in the context of home hospitalization explicitly. Compared to the literature handling districting in home care, there are mainly two noteworthy differences. On the one hand, the parameter values inserted into the model differ, particularly the maximum distance between the hospital and the patient. On the other hand, concerning the model formulation, it is imperative to consider the monthly capacity, given the seasonality of demand, and the scarcity of skilled medical personnel, especially in the Portuguese context.

Ensuring that supply meets demand and, when it does not, minimizing under-capacity is a novelty of this work. It allows the model to provide insights into which months and units there is a capacity shortage and thus can help decision-makers select which HH units are in greatest need of expansion.

This study benefited from contact with HH service managers, namely with the national director of Provider X's Home Care Services. This experience enriched the dissertation, evidencing stakeholders' participation is essential for adequately characterizing the problem and validating the results. In addition, it confirmed that operational research techniques can help healthcare providers improve service delivery.

Lastly, although the proposed model and solution have been implemented and validated for the Portuguese case, particularly for the Provider X's reality, the approach can be easily extended to other HH providers in any territory.

6.2 Future Work

Future research opportunities can take several directions. The current model's computation tractability is limited for small and medium-sized instances. Therefore, future work can explore efficient approaches to multi-objective optimization for bigger cases through exact methods or heuristics and metaheuristics. The performance of these new approaches should be compared in terms of objective function values and computational time.

There is still very diminutive research that addresses the problem of healthcare districting, considering the substantial uncertainty inherent to this sector and its impact on all levels of decision-making. This dissertation focused on uncertainty in demand by simulating the results for varying demand scenarios. It is possible to identify two potential points for improvement in addressing uncertainty. Firstly, very dissimilar districting plans were obtained when testing the model for the different demand scenarios. Getting a single solution that could respond to all scenarios was impossible. It would be relevant to investigate other robust ways of inserting uncertainty in the optimization. This could be obtained through uncertainty intervals with an adjustable worst-case scenario protection parameter, as in the work of Darmian et al. (2021), or through a multi-period model with an additional district similarity criterion, as in the work by Yanik et al. (2019). Secondly, demand forecasting greatly influences districting results, so it would be valuable to have used regression methods that would allow demand to be estimated more accurately, considering the population's demographics, socioeconomic factors, and the seasonality in demand for HH services.

Further objectives can also be considered for the problem. It could be explored, for instance, the insertion of economic criteria to assess and compare expenditures or savings between districting plans. Additionally, it may be relevant to investigate further the use of the proposed model by fixing a single HH unit and addressing the districting of several teams within that unit. It would be necessary to review the specific constraints and objectives of this problem.

The importance of including stakeholders in designing, implementing, and validating OR/MS models has been evidenced previously. This dissertation benefited from the contact with HH managers. Still, it would be interesting to diversify the stakeholders' involvement, complementing the model with inputs from medical staff and other practitioners, patients, and their families. In this process, new constraints and objectives may emerge, as well as a rearrangement of existing objectives. Moreover, the preferences of several decision makers may naturally conflict, so multicriteria decision analysis tools such as the analytic hierarchy process (AHP) or M-MACBETH can be applied.

When presenting the final results to the decision-makers, they showed interest in independently accessing the proposed model, varying its parameters, and using it punctually for districting planning among different HH units and between teams of the same unit. For this, creating a graphical user interface that allows a simplified interaction between users and the model will be necessary.

Finally, it would be insightful to evaluate the impact of districting on the tactical and operational decisions of managing a home hospitalization service network.

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Appendix A

Literature Review

Table A.1: Criteria mentioned in the reviewed articles on the districting problem. *X* marks criteria included in the objective function and *C* marks criteria that were included as hard or soft constraints.

Publication	Acessibility/ Mobility	Administrative Boundaries	Capacity	Compactness (distance)	Compactness (time)	Contiguity	Continuity of care	District number	Hiring costs	Income equity	Indivisibility of basic units	Range of service offer	Size Balance	Workload Balance
(Blais et al., 2003)	C	C				C					C			X
(Lahrichi et al., 2006)	C	C				C					C			X
(Sahin et al., 2010)	C	C		C	X			C			C			X
(Benzarti et al., 2013)	C	C		X, C				C			C			X
(Datta et al., 2013)		X		X		C		C			C		C	
(Steiner et al., 2015)					X	C		C			C	X	X	
(Gutiérrez-Gutiérrez and Vidal, 2015)		C		C	X	C	C	C			C		C	X
(Lin et al., 2017)	C		C	C	C			X			C			
(Yanik et al., 2019)	C		C	X, C			X			X				C
(Lin et al., 2020)				C	X			C	X		C			X
(Darmian et al., 2021)				C	X	X		C			C			
Total X	0	1	0	3	5	1	1	1	1	1	0	1	1	6
Total C	6	5	2	7	1	5	1	7	0	0	9	0	2	1

Appendix B

Model Implementation

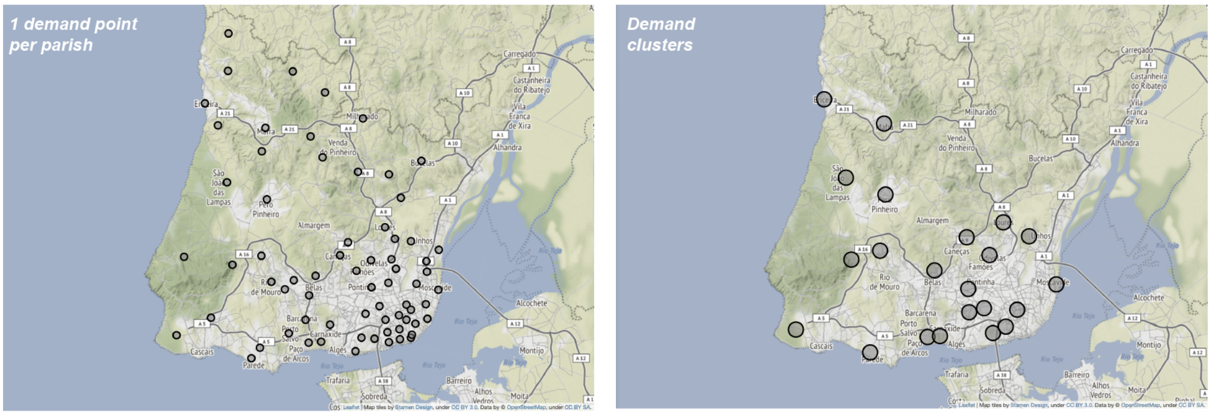

Figure B.1: Geographic representation of the demand points used in the case study instances. On the left, the demand points used for districting with instances S1, S2.1, and S3.1. On the right, the simplified demand clusters used to solve the remaining instances.

Table B.1: Characterization of scenario S4 with and without the contiguity constraints.

		S4	**S4 (with contiguity)**
Capacity		T=C=D=15, S=8	
Utilization rate		Lisboa = 0.065%, Cascais = 0.08%, Sintra = 0.025%, Oeiras = 0.025%, Remaining = 0.018%	
Patients		773	
Variables	**Binary**	88	
	Continuous	109	
Constraints		4323	4619

Table B.2: Features of the demand points used in the case study instances. One demand point per parish was used for districting with one and two open HH units. For more than two HH units districting, the parishes were grouped into clusters to find the solution quicker and more efficiently.

Municipality	Parish Clusters	Number of Inhabitants	Civil Parish	Number of Inhabitants
Amadora	Amadora Sul	80628	Águas Livres	37612
			Alfragide	16840
			Venteira	26176
	Amadora Norte	90872	Encosta do Sol	27115
			Falagueira-Venda Nova	20792
			Mina de Água	42965
Cascais	Cascais Oeste	108378	União das freguesias de Cascais e Estoril	64201
			Alcabideche	44177
	Cascais Este	105780	São Domingos de Rana	59248
			União das freguesias de Carcavelos e Parede	46532
Lisboa	Lisboa Ocidente	44714	Ajuda	14313
			Alcântara	13852
			Belém	16549
	Lisboa Centro	136904	Alvalade	33313
			Areeiro	21167
			Arroios	33307
			Avenidas Novas	23261
			Santo António	11062
			Campolide	14794
	Lisboa Norte	157465	Benfica	35367
			Carnide	18029
			Lumiar	46338
			São Domingos de Benfica	34081
			Santa Clara	23650
	Lisboa Oriente	102233	Beato	12185
			Parque das Nações	22382
			Marvila	35482
			Olivais	32184
	Centro Histórico	104607	Estrela	20308
			Misericórdia	9660
			Penha de França	28485
			Santa Maria Maior	10052
			Campo de Ourique	22146
			São Vicente	13956
Loures	Loures 1	40917	Bucelas	4804
			Fanhões	2639
			Loures	30258
			Lousa	3216
	Loures 2	160715	União das freguesias de Camarate, Unhos e Apelação	33517
			União das freguesias de Moscavide e Portela	20926
			União das freguesias de Sacavém e Prior Velho	24681
			União das freguesias de Santa Iria de Azoia, São João da Talha e Bobadela	44461
			União das freguesias de Santo Antão e São Julião do Tojal	8607
			União das freguesias de Santo António dos Cavaleiros e Frielas	28523
Mafra	Mafra 1	24521	Carvoeira	2848
			Encarnação	4918
			Ericeira	12359
			Santo Isidoro	4396
	Mafra 2	62000	Mafra	20783
			Milharado	7645
			União das freguesias de Azueira e Sobral da Abelheira	4434
			União das freguesias de Enxara do Bispo, Gradil e Vila Franca do Rosário	3979
			União das freguesias de Igreja Nova e Cheleiros	4695
			União das freguesias de Malveira e São Miguel de Alcainça	9648
			União das freguesias de Venda do Pinheiro e Santo Estêvão das Galés	10816
Odivelas	Odivelas 1	113524	Odivelas	59604
			União das freguesias de Pontinha e Famões	35114
			União das freguesias de Póvoa de Santo Adrião e Olival Basto	18806
	Odivelas 2	34534	União das freguesias de Ramada e Caneças	34534
Oeiras	Oeiras 1	73199	União das freguesias de Oeiras e São Julião da Barra, Paço de Arcos e Caxias	58099
			Porto Salvo	15100
	Oeiras 2	98568	União das freguesias de Algés, Linda-a-Velha e Cruz Quebrada-Dafundo	48030
			União das freguesias de Carnaxide e Queijas	36087
			Barcarena	14451
Sintra	Sintra 1	194596	Casal de Cambra	13348
			União das freguesias de Agualva e Mira-Sintra	41327
			União das freguesias de Massamá e Monte Abraão	47811
			União das freguesias de Queluz e Belas	52417
			União das freguesias do Cacém e São Marcos	39693
	Sintra 2	118147	Rio de Mouro	49493
			Algueirão-Mem Martins	68654
	Sintra 3	17994	União das freguesias de São João das Lampas e Terrugem	17994
	Sintra 4	17264	União das freguesias de Almargem do Bispo, Pêro Pinheiro e Montelavar	17264
	Sintra 5	37653	União das freguesias de Sintra (Santa Maria e São Miguel, São Martinho e São Pedro de Penaferrim)	29907
			Colares	7746

Table B.3: Proposed allocation for scenarios S2.3, S3.3, S4, and S4 when considering the contiguity constraint. The gray-colored cells show the common allocations between scenarios.

Cluster	**Municipality**	**S2.3**	**S3.3**	**S4**	**S4 ('C)**
Amadora sul	Amadora	Hospital 4	Hospital 2	Hospital 4	Hospital 2
Amadora norte	Amadora	Hospital 2	Hospital 4	Hospital 1	Hospital 2
Cascais oeste	Cascais	Hospital 3	Hospital 1	Hospital 3	Hospital 1
Cascais este	Cascais	Hospital 3	Hospital 1	Hospital 4	Hospital 1
Lisboa ocidente	Lisboa	Hospital 3	Hospital 2	Hospital 1	Hospital 4
Lisboa centro	Lisboa	Hospital 4	Hospital 4	Hospital 2	Hospital 4
Lisboa norte	Lisboa	Hospital 1	Hospital 1	Hospital 1	Hospital 4
Lisboa oriente	Lisboa	Hospital 1	Hospital 4	Hospital 4	Hospital 4
Centro histórico	Lisboa	Hospital 4	Hospital 3	Hospital 3	Hospital 4
Loures 1	Loures	Hospital 1	Hospital 2	Hospital 3	Hospital 2
Loures 2	Loures	Hospital 3	Hospital 4	Hospital 4	Hospital 2
Mafra 1	Mafra	Hospital 3	Hospital 2	Hospital 2	Hospital 2
Mafra 2	Mafra	Hospital 2	Hospital 3	Hospital 3	Hospital 2
Odivelas 1	Odivelas	Hospital 4	Hospital 2	Hospital 4	Hospital 3
Odivelas 2	Odivelas	Hospital 3	Hospital 1	Hospital 3	Hospital 3
Oeiras 1	Oeiras	Hospital 2	Hospital 3	Hospital 1	Hospital 2
Oeiras 2	Oeiras	Hospital 1	Hospital 1	Hospital 1	Hospital 2
Sintra 1	Sintra	Hospital 1	Hospital 3	Hospital 1	Hospital 3
Sintra 2	Sintra	Hospital 4	Hospital 3	Hospital 2	Hospital 3
Sintra 3	Sintra	Hospital 2	Hospital 2	Hospital 2	Hospital 3
Sintra 4	Sintra	Hospital 1	Hospital 2	Hospital 3	Hospital 3
Sintra 5	Sintra	Hospital 2	Hospital 1	Hospital 2	Hospital 3

Table B.4: Proposed allocation for scenarios S1, S1(T), S2.1, and S3.1.

Civil Parish	Municipality	S1 (H1+H3)	S1 (H1+H3+H4)	S1 (H1+H3+H4+H2)	S1 (T)	S2.1 (H1+H3)	S3.1 (H1+H3)
Águas Livres	Amadora	-	-	-	-	Hospital 3	Hospital 3
Alfragide	Amadora	-	-	-	-	Hospital 1	Hospital 1
Encosta do Sol	Amadora	-	-	-	-	Hospital 1	Hospital 3
Falagueira-Venda Nova	Amadora	-	-	-	-	Hospital 1	Hospital 3
Mina de Água	Amadora	-	-	-	-	Hospital 3	Hospital 3
Venteira	Amadora	-	-	-	-	Hospital 1	Hospital 1
Alcabideche	Cascais	Hospital 1	Hospital 3	Hospital 2	Team 1	Hospital 1	Hospital 3
São Domingos de Rana	Cascais	Hospital 1	Hospital 4	Hospital 3	Team 1	Hospital 1	Hospital 1
União das freguesias (UF) de Carcavelos e Parede	Cascais	Hospital 1	Hospital 1	Hospital 4	Team 1	Hospital 1	Hospital 1
UF de Cascais e Estoril	Cascais	Hospital 3	Hospital 1	Hospital 3	Team 2	Hospital 1	Hospital 1
Ajuda	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Alcântara	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Alvalade	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Areeiro	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Arroios	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Avenidas Novas	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Beato	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Belém	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Benfica	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Campo de Ourique	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Campolide	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Carnide	Lisboa	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Estrela	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Lumiar	Lisboa	Hospital 1	Hospital 3	Hospital 1	Team 1	Hospital 3	Hospital 3
Marvila	Lisboa	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Misericórdia	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Olivais	Lisboa	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Parque das Nações	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Penha de França	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Santa Clara	Lisboa	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Santa Maria Maior	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Santo António	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
São Domingos de Benfica	Lisboa	Hospital 1	Hospital 4	Hospital 2	Team 1	Hospital 1	Hospital 1
São Vicente	Lisboa	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Bucelas	Loures	Hospital 3	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 3
Fanhões	Loures	Hospital 3	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Loures	Loures	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 3
Lousa	Loures	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Camarate, Unhos e Apelação	Loures	Hospital 3	Hospital 1	Hospital 1	Team 2	Hospital 3	Hospital 3
UF de Moscavide e Portela	Loures	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Sacavém e Prior Velho	Loures	Hospital 1	Hospital 1	Hospital 1	Team 2	Hospital 1	Hospital 1
UF de Santa Iria de Azoia, São João da Talha e Bobadela	Loures	Hospital 1	Hospital 4	Hospital 1	Team 2	Hospital 3	Hospital 1
UF de Santo Antão e São Julião do Tojal	Loures	Hospital 3	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Santo António dos Cavaleiros e Frielas	Loures	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 3
Carvoeira	Mafra	Hospital 3	Hospital 3	Hospital 3	Team 2	Hospital 1	Hospital 1
Encarnação	Mafra	Hospital 3	Hospital 3	Hospital 2	Team 2	Hospital 3	Hospital 3
Ericeira	Mafra	Hospital 3	Hospital 3	Hospital 3	Team 2	Hospital 3	Hospital 3
Mafra	Mafra	Hospital 3	Hospital 3	Hospital 3	Team 2	Hospital 1	Hospital 3
Milharado	Mafra	Hospital 3	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Santo Isidoro	Mafra	Hospital 3	Hospital 3	Hospital 2	Team 2	Hospital 3	Hospital 3
UF de Azueira e Sobral da Abelheira	Mafra	Hospital 3	Hospital 3	Hospital 2	Team 2	Hospital 1	Hospital 1
UF de Enxara do Bispo, Gradil e Vila Franca do Rosário	Mafra	Hospital 3	Hospital 3	Hospital 2	Team 2	Hospital 1	Hospital 1
UF de Igreja Nova e Cheleiros	Mafra	Hospital 1	Hospital 1	Hospital 1	Team 2	Hospital 1	Hospital 1
UF de Malveira e São Miguel de Alcainça	Mafra	Hospital 1	Hospital 1	Hospital 1	Team 2	Hospital 1	Hospital 1
UF de Venda do Pinheiro e Santo Estêvão das Galés	Mafra	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Odivelas	Odivelas	Hospital 3	Hospital 3	Hospital 4	Team 2	Hospital 3	Hospital 3
UF de Pontinha e Famões	Odivelas	Hospital 3	Hospital 4	Hospital 1	Team 2	Hospital 3	Hospital 3
UF de Póvoa de Santo Adrião e Olival Basto	Odivelas	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 3
UF de Ramada e Caneças	Odivelas	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 3
Barcarena	Oeiras	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Porto Salvo	Oeiras	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Algés, Linda-a-Velha e Cruz Quebrada-Dafundo	Oeiras	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Carnaxide e Queijas	Oeiras	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Oeiras e São Julião da Barra, Paço de Arcos e Caxias	Oeiras	Hospital 1	Hospital 4	Hospital 3	Team 1	Hospital 1	Hospital 1
Algueirão-Mem Martins	Sintra	Hospital 3	Hospital 1	Hospital 1	Team 2	Hospital 3	Hospital 3
Casal de Cambra	Sintra	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
Colares	Sintra	Hospital 1	Hospital 4	Hospital 1	Team 1	Hospital 1	Hospital 1
Rio de Mouro	Sintra	Hospital 3	Hospital 3	Hospital 4	Team 2	Hospital 3	Hospital 3
UF de Agualva e Mira-Sintra	Sintra	Hospital 3	Hospital 4	Hospital 1	Team 2	Hospital 3	Hospital 3
UF de Almargem do Bispo, Pêro Pinheiro e Montelavar	Sintra	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 1	Hospital 1
UF de Massamá e Monte Abraão	Sintra	Hospital 1	Hospital 3	Hospital 3	Team 2	Hospital 3	Hospital 3
UF de Queluz e Belas	Sintra	Hospital 3	Hospital 1	Hospital 2	Team 1	Hospital 3	Hospital 3
UF de São João das Lampas e Terrugem	Sintra	Hospital 1	Hospital 1	Hospital 1	Team 2	Hospital 1	Hospital 1
UF de Sintra	Sintra	Hospital 1	Hospital 1	Hospital 1	Team 1	Hospital 3	Hospital 3
UF do Cacém e São Marcos	Sintra	Hospital 3	Hospital 4	Hospital 4	Team 2	Hospital 3	Hospital 3

Table B.5: Proposed allocation for scenarios S2.2 and S3.3.

Cluster	**Municipality**	**S2.2 (H1+H3+H4)**	**S3.2 (H1+H3+H4)**
Amadora sul	Amadora	Hospital 4	Hospital 1
Amadora norte	Amadora	Hospital 1	Hospital 1
Cascais oeste	Cascais	Hospital 1	Hospital 3
Cascais este	Cascais	Hospital 1	Hospital 4
Lisboa ocidente	Lisboa	Hospital 4	Hospital 4
Lisboa centro	Lisboa	Hospital 3	Hospital 4
Lisboa norte	Lisboa	Hospital 4	Hospital 1
Lisboa oriente	Lisboa	Hospital 4	Hospital 1
Centro histórico	Lisboa	Hospital 3	Hospital 4
Loures 1	Loures	Hospital 1	Hospital 1
Loures 2	Loures	Hospital 1	Hospital 4
Mafra 1	Mafra	Hospital 3	Hospital 3
Mafra 2	Mafra	Hospital 3	Hospital 3
Odivelas 1	Odivelas	Hospital 4	Hospital 3
Odivelas 2	Odivelas	Hospital 3	Hospital 3
Oeiras 1	Oeiras	Hospital 4	Hospital 3
Oeiras 2	Oeiras	Hospital 3	Hospital 1
Sintra 1	Sintra	Hospital 3	Hospital 3
Sintra 2	Sintra	Hospital 1	Hospital 1
Sintra 3	Sintra	Hospital 4	Hospital 1
Sintra 4	Sintra	Hospital 1	Hospital 4
Sintra 5	Sintra	Hospital 4	Hospital 4

Table B.6: Workload distribution (in percentage) among the proposed HH units for scenario S1.

Month	**S1 (H1 + H3)**		**S1 (H1 + H3 + H4)**			**S1 (H1 + H3 + H4 + H2)**				**S1 (T)**	
	H1	**H3**	**H1**	**H3**	**H4**	**H1**	**H3**	**H4**	**H2**	**Team 1**	**Team 2**
0	14.247	20.699	19.086	8.333	7.527	12.097	6.989	9.140	13.441	39.785	30.108
1	17.262	24.405	14.881	15.476	11.310	11.310	8.631	13.690	16.071	43.452	39.881
2	19.624	15.860	10.215	13.978	11.290	9.946	8.065	9.677	15.591	37.634	33.333
3	26.944	19.722	14.167	18.889	13.611	13.611	13.056	11.944	16.111	52.778	40.556
4	17.204	18.011	8.602	10.753	15.860	6.989	11.022	14.785	4.839	26.344	44.086
5	18.333	19.444	19.444	12.222	6.111	14.722	10.833	7.778	8.889	41.111	34.444
6	29.570	22.312	10.484	17.742	23.656	12.903	20.161	8.333	20.968	60.753	43.011
7	17.473	22.312	10.484	13.172	16.129	11.022	9.677	15.591	6.989	39.247	40.323
8	21.944	15.833	10.278	8.611	18.889	16.111	13.333	5.556	5.556	38.889	36.667
9	11.559	23.656	16.667	9.409	9.140	4.032	12.634	15.591	5.914	29.032	41.398
10	18.611	15.833	9.722	13.611	11.111	7.500	11.111	12.778	6.111	22.778	46.111
11	21.774	16.935	12.634	14.516	11.559	14.247	8.333	9.140	13.978	37.634	39.785
Annual	19.543	19.566	13.037	13.037	13.037	11.187	11.164	11.164	11.187	39.087	39.132

Table B.7: Workload distribution (in percentage) among the proposed HH units for scenario S2. The gray-colored cells indicate periods in which the workload exceeded 100% and would therefore need additional installed capacity.

Month	S2.1 (H1 + H3)		S2.2 (H1 + H3 + H4)			S2.3 (H1 + H3 + H4 + H2)			
	H1	H3	H1	H3	H4	H1	H3	H4	H2
0	75.484	60.000	53.978	57.204	89.462	86.452	49.462	51.828	70.161
1	68.333	77.619	83.333	76.667	85.714	103.571	101.667	104.762	110.714
2	64.731	62.581	79.140	79.785	56.344	76.989	104.516	88.172	68.952
3	63.333	69.556	101.333	101.111	101.333	61.333	57.333	106.000	110.833
4	40.215	49.677	51.828	53.333	48.602	102.581	103.656	107.097	105.645
5	73.333	61.778	94.444	80.889	63.333	89.111	107.333	107.778	109.583
6	50.753	54.624	85.376	69.462	58.495	105.591	85.161	75.699	110.887
7	54.624	47.097	55.914	81.935	66.452	63.226	57.849	92.688	66.532
8	58.667	77.778	76.444	82.667	67.778	93.556	90.444	95.333	91.667
9	63.656	60.000	94.409	46.452	79.570	90.108	40.430	88.172	72.177
10	84.222	52.889	72.000	98.222	90.222	79.111	103.778	71.556	99.167
11	62.151	86.452	100.000	96.559	91.828	98.065	62.366	73.118	68.952
Annual	63.178	63.196	78.904	76.877	74.776	87.416	80.055	88.311	90.137

Table B.8: Workload distribution (in percentage) among the proposed HH units for scenario S3.

Month	S3.1 (H1 + H3)		S3.2 (H1 + H3 + H4)			S3.3 (H1 + H3 + H4 + H2)			
	H1	H3	H1	H3	H4	H1	H3	H4	H2
0	155.699	95.484	106.022	126.882	138.925	85.806	107.957	105.161	136.694
1	121.905	135.476	167.857	152.619	103.095	140.000	117.619	133.810	60.268
2	143.226	110.538	112.473	140.645	161.935	100.645	136.344	109.892	127.823
3	92.667	111.333	153.333	165.111	156.222	112.000	140.222	139.111	156.250
4	127.742	106.667	101.720	79.785	116.344	89.677	58.280	88.602	114.919
5	127.111	109.333	127.111	132.889	126.222	132.889	104.889	116.000	60.833
6	103.656	97.419	146.882	135.269	106.452	89.677	136.344	105.161	107.661
7	101.290	120.860	80.000	112.473	85.806	36.774	99.570	83.011	110.484
8	105.333	88.667	129.333	155.333	138.222	138.444	127.556	95.778	114.583
9	98.925	120.645	64.086	132.258	112.903	98.925	99.355	83.226	52.016
10	95.778	110.000	162.000	65.778	131.111	127.556	39.333	109.333	155.000
11	89.247	157.204	119.140	67.527	85.591	93.333	66.022	72.903	75.000
Annual	113.571	113.553	121.900	121.881	121.881	103.251	102.667	103.123	106.164

Table B.9: Workload distribution (in percentage) among the proposed HH units for scenario S4.

Month	H1	H3	H4	H2
0	112.473	141.505	129.247	51.210
1	115.000	75.000	153.333	30.357
2	150.108	72.473	137.204	194.355
3	144.222	94.000	56.667	72.917
4	71.613	80.860	53.333	26.613
5	95.333	142.000	76.222	187.083
6	88.387	95.699	93.978	99.597
7	146.667	79.570	132.258	76.210
8	61.333	97.333	68.444	107.083
9	75.269	93.763	90.323	142.742
10	104.222	130.444	132.667	154.583
11	81.720	140.215	124.946	148.387
Annual	103.799	103.671	103.872	107.979

Table B.10: Characterization of the months in under-capacity in each HH unit for scenario S3.1 with two open units. The *under-capacity* column indicates how many extra daily visits would be required to meet all demand in that given month.

HH Unit	Month	Workload (%)	Under-capacity	Maximum under-capacity
H1	0	155.699	9	
H1	1	121.905	4	
H1	2	143.226	7	
H1	4	127.742	5	9
H1	5	127.111	5	
H1	6	103.656	1	
H1	7	101.290	1	
H1	8	105.333	1	
H3	1	135.476	6	
H3	2	110.538	2	
H3	3	111.333	2	
H3	4	106.667	1	
H3	5	109.333	2	9
H3	7	120.860	4	
H3	9	120.645	4	
H3	10	110.000	2	
H3	11	157.204	9	

Table B.11: Characterization of the months in under-capacity in each HH unit for scenario S3.2 with three open units.

HH Unit	Month	Workload (%)	Under-capacity	Maximum under-capacity
H1	0	157.204	9	
H1	1	108.810	2	
H1	2	176.559	12	
H1	3	156.222	9	
H1	4	123.011	4	12
H1	5	114.889	3	
H1	6	110.323	2	
H1	8	130.222	5	
H1	10	130.222	5	
H3	0	115.914	3	
H3	1	156.429	9	
H3	2	150.968	8	
H3	3	181.556	13	13
H3	5	147.556	8	
H3	6	125.161	4	
H3	8	138.444	6	
H3	9	134.624	6	
H4	1	158.333	9	
H4	3	136.889	6	
H4	5	123.778	4	
H4	6	153.118	8	10
H4	7	105.161	1	
H4	8	154.222	9	
H4	10	166.667	10	
H4	11	107.312	2	

Table B.12: Characterization of the months in under-capacity in each HH unit for scenario S3.3 with four open units.

HH Unit	Month	Workload (%)	Under-capacity	Maximum under-capacity
H1	1	165.000	10	
H1	2	109.677	2	
H1	3	130.444	5	
H1	4	118.280	3	
H1	5	186.000	13	
H1	6	152.688	8	13
H1	7	167.527	11	
H1	8	124.222	4	
H1	9	143.226	7	
H1	10	148.444	8	
H1	11	166.237	10	
H3	0	110.968	2	
H3	1	118.333	3	
H3	2	151.398	8	
H3	3	142.444	7	
H3	4	138.280	6	
H3	5	163.778	10	12
H3	6	177.419	12	
H3	7	175.269	12	
H3	8	133.111	5	
H3	9	153.118	8	
H3	10	136.889	6	
H2	0	148.848	4	
H2	1	267.347	12	
H2	2	156.221	4	
H2	3	142.381	3	
H2	4	141.935	3	
H2	5	197.619	7	12
H2	6	182.488	6	
H2	7	177.880	6	
H2	8	235.238	10	
H2	9	199.539	7	
H2	10	230.476	10	
H2	11	190.323	7	
H4	0	157.849	9	
H4	1	191.190	14	
H4	2	157.419	9	
H4	3	110.222	2	
H4	5	161.111	10	14
H4	6	130.108	5	
H4	8	156.222	9	
H4	9	158.495	9	
H4	10	128.889	5	
H4	11	154.624	9	

Table B.13: Characterization of the months in under-capacity in each HH unit for scenario S4.

HH Unit	Month	Workload (%)	Under-capacity	Maximum under-capacity
H1	0	112.473	2	8
H1	1	115.000	3	
H1	2	150.108	8	
H1	3	144.222	7	
H1	7	146.667	8	
H1	10	104.222	1	
H3	0	141.505	7	7
H3	5	142.000	7	
H3	10	130.444	5	
H3	11	140.215	7	
H2	2	194.355	8	8
H2	5	187.083	7	
H2	8	107.083	1	
H2	9	142.742	4	
H2	10	154.583	5	
H2	11	148.387	4	
H4	0	129.247	5	8
H4	1	153.333	8	
H4	2	137.204	6	
H4	7	132.258	5	
H4	10	132.667	5	
H4	11	124.946	4	

Table B.14: Outcomes when varying the order in which the objective functions are solved.

Preference	$(\Psi, \Delta, \Omega, \Theta)$	$(\Psi, \Delta, \Theta, \Omega)$	$(\Psi, \Theta, \Delta, \Omega)$	$(\Psi, \Theta, \Omega, \Delta)$	$(\Psi, \Omega, \Delta, \Theta)$	$(\Psi, \Omega, \Theta, \Delta)$
			S2.3			
Ψ	35.000	35.000	35.000	35.000	35.000	35.000
Δ	6.425	6.425	6.425	6.425	6.425	6.425
Θ	28.583	28.583	28.583	28.583	28.583	28.583
Ω	34.601	34.601	34.601	34.601	34.601	34.601
CPU time	0:00:01.06	0:00:01.19	0:00:01.33	0:00:01.52	0:00:00.99	0:00:01.03
			S3.3			
Ψ	1264.000	1264.000	1264.000	1264.000	1264.000	1264.000
Δ	15.125	15.125	15.125	15.155	15.125	15.125
Θ	28.459	26.744	26.744	26.744	28.459	28.459
Ω	31.079	34.601	34.601	31.555	31.079	31.079
CPU time	0:00:28.52	0:00:38.85	0:00:28.44	0:00:24.89	0:04:33.06	0:00:29.77
			S4			
Ψ	234.000	234.000	234.000	234.000	234.000	234.000
Δ	3.149	3.149	3.149	3.149	3.149	3.149
Θ	28.287	26.744	26.744	26.744	28.287	28.287
Ω	31.555	34.237	34.237	34.237	31.555	31.555
CPU time	0:00:00.80	0:00:00.63	0:00:01.07	0:00:00.70	0:00:00.69	0:00:00.72
			S1 (T)			
Ψ	0.000	0.000	0.000	0.000	0.000	0.000
Δ	0.023	0.023	0.023	3.356	3.356	3.356
Θ	41.785	41.785	41.785	41.785	41.785	41.785
Ω	36.366	36.366	36.366	33.070	33.070	33.070
CPU time	0:00:00.75	0:00:00.77	0:00:00.75	0:00:00.60	0:00:00.58	0:00:00.58