Preventing Anti – Competitive Behavior: Employing Game Theory to detect Cartels and Collusion

Cartel da Banca case

João Manuel Gonçalves Cohen de Lemos

Thesis to obtain the Master of Science Degree in

Engineering and Industrial Management

Supervisor: Profª. Maria Margarida Martelo Catalão Lopes de Oliveira Pires Pina

Examination Committee

Chairperson: Profª. Ana Isabel Carvalho

Supervisor: Profª. Margarida Catalão Lopes

Member of the Committee: Profª. Inês Carrilho Nunes

November 2022

Acknowledgments

The work done during these last months to be able to present this Dissertation, even though they have not always been easy, have become very gratifying and, above all, satisfying because I arrive here with the sense of duty accomplished. I owe it mainly to Professor Margarida Catalão Lopes who has shown herself to be an excellent supervisor with an impressive work and dedication, always present to solve any problem that might arise. Truly tireless, I am very grateful for everything.

However, this dissertation was only the culmination of a long, laborious, but nevertheless rewarding journey, a journey that, fortunately, I never had to travel alone. For this I must, above all, thank my family and friends. When it comes to family, first of all thanks to my parents for always supporting me, believing in me and giving me the privilege of having all the necessary tools to get this far, and then to my sister, who always puts up with me and will do so for the rest of my life. To you all, my most sincere thanks.

Speaking now to my friends, throughout these years I have had the privilege of making countless friendships that will last for a long time, and with this, I have to particularize 4 important groups that have become very strong foundations not only in these years but will continue to be for the rest of my life. They are, all the friends made inside the course and more in general from Tagus, my friends from my hometown Covilhã, especially the ones studying in Lisbon too, my friends from what was my home for the last 6 years, Colégio Universitário Pio XII and lastly my friends made in Erasmus in Krakow.

The ones from the course and from Tagus, I hope you know that I appreciate everything you did for me all these years. To my lifelong friends from Covilhã, always present on all occasions, always very loyal to our origins, thank you for your support and preoccupation. From Colégio Pio XII I take friendships for the rest of my life, created from the moment I arrived in Lisbon to live alone for the first time. Thank you for all the great moments lived, for the company in the big study room, and above all for knowing that I can count on you for whatever I need. Finally, to my Erasmus friends, some now scattered throughout the rest of Europe, thank you for the best months of my life. It was a period that for sure passed too fast, but the intensity and creation of memories made up for it, to them, thank you for sharing this experience with me and for showing me, by making me leave my comfort zone, that there is a whole world out there full of possibilities for the future. Thank you all so much, I love you from the bottom of my heart.

Abstract

The objective of this Dissertation is to understand whether if it would be possible to predict and prevent the formation of cartels and collusive behavior. The literature review shows that even when the best market conditions are met, it is not always certain that cartel formation will occur. This gave rise to the possibility of focusing the study on a specific type of cartels: mixed cartels. The literature on this, as it is not very common, is still scarce, so it is to contribute to the increase of available information on this topic that this Dissertation is carried out. To this end, a model of a game à la Cournot was developed and later applied to the case of the Cartel da Banca in Portugal, in order to explain its formation and then to be used in the future for prevention. This model is an extension of Mota et al. (2020), where the players are considered symmetric in terms of costs and where there is an increase in the scope of application of the model. It was concluded that without a cartel, given its concern for consumer surplus, the public firm always produces more. With the cartel, the excess production is transferred to the private firms, mainly for the ones the cartel. However, the major result and implications of this model are that, in addition to being able to be used for predicting cartel formation, the creation of mixed cartels can contribute to the increase in total and consumer surplus.

Keywords: Game theory, mixed cartels, collusion, oligopoly, Cartel da Banca

Resumo

O objetivo desta Dissertação é compreender se seria possível prever e evitar a formação de cartéis e comportamentos colusivos. A revisão da literatura mostra que mesmo quando estão reunidas as melhores condições de mercado, nem sempre é certo que a formação de cartéis ocorra. Isto deu origem à possibilidade de centrar o estudo num tipo de cartéis específico: os cartéis mistos. A literatura sobre isto, como não é muito comum, ainda é escassa, pelo que é para contribuir para o aumento da informação disponível sobre este tópico que esta Dissertação é realizada. Para tal, foi desenvolvido um modelo de jogo à la Cournot que foi posteriormente aplicado ao caso do Cartel da Banca em Portugal, a fim de explicar a sua formação e depois ser utilizado no futuro para a prevenção. Este modelo é uma extensão de Mota et al. (2020), onde os jogadores são considerados simétricos em termos de custos e onde se verifica um aumento do campo de aplicação do modelo. Concluiu-se que sem um cartel, dada a sua preocupação com o excedente do consumidor, a empresa pública produz sempre mais. Com o cartel, o excesso de produção é transferido para as empresas privadas, principalmente para as que participam no cartel. Contudo, o principal resultado e as implicações deste modelo são que, para além de poder ser utilizado para prever a formação do cartel, a criação de cartéis mistos pode contribuir para o aumento do excedente total e do consumidor.

Palavras-chave: Teoria de jogos, carteis mistos, colusões, oligopólio, Cartel da Banca

Table of Contents

List of Tables

List of Figures

List of Acronyms

AdC	Autoridade da Concorrência
APB	Associação Portuguesa de Bancos
BBVA	Banco Bilbao Vizcaya Argentaria
BCP	Banco Comercial Português
BES	Banco Espírito Santo
BPI	Banco Português de Investimentos
CEO	Chief Executive Officer
CGD	Caixa Geral de Depósitos
CS	Consumer surplus
FOC	First order conditions
MC	Marginal cost
NE	Nash Equilibrium
SQ	Status Quo
SRF	Société Française du Radiotéléphone (França)
TC	Total cost
TS	Total surplus
UCI	União de Créditos Imobiliários

1. INTRODUCTION

Game Theory presents itself as being a field of study of applied math with applications in several other fields. It is used to define the optimal strategy in order to succeed in imperfectly competitive situations where there may be incomplete information, which is the case of almost all real-life scenarios. It is the mathematical study of decision making and modelling situations of conflict found in day-to-day life and across all industries.

The first person to make Game Theory a true unique field of study was John von Neumann with the publication of the paper "*On the Theory of Games of Strategy"* in 1928, in which its most relevant contribution is the proposed fundamental theorem contained within: It states that in a broad category of two-person games it is always possible to find an equilibrium from which neither player should deviate unilaterally. A few years later, John von Neumann and Oskar Morgenstern, with the publication of another groundbreaking book, *"Theory of Games and Economic Behavior"*, in 1944, established game theory as an interdisciplinary research field.

Around 1950, John Nash introduced the concept of Nash Equilibrium which is essentially a criterion for mutual consistency of players' strategies. With this, Nash proved that every finite n-player, non-zero-sum (not just two-player zero-sum) non-cooperative game has what is now known as a Nash equilibrium in mixed strategies. In addition, Nash was very important for the analysis of the Prisoner's Dilemma, which is highly observable in competitive environments, becoming a major focus of study.

Delving a little deeper into Game Theory itself, we may essentially divide it into 3 major groups: Games of Skill, Games against Nature and Strategic Games.

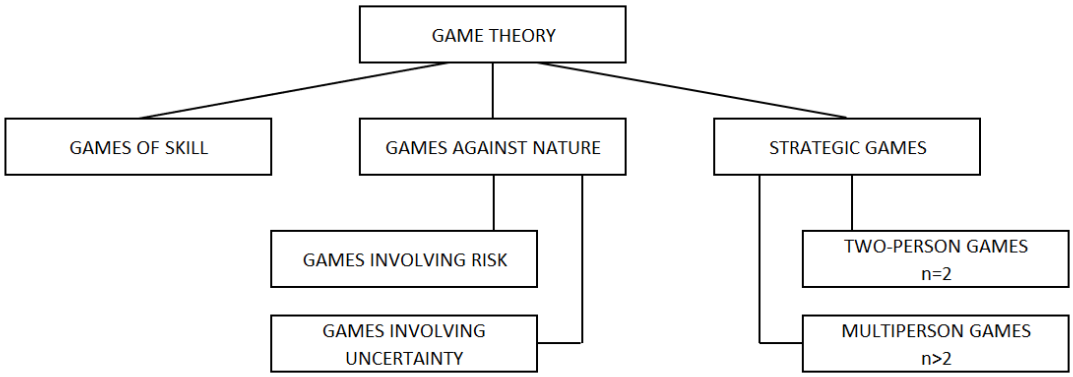

Figure 1: Types of Games

Looking at strategic games, more specifically at multi-person games (n-person games), the goal of this dissertation is, through the creation of a new theoretical model, to understand if it is possible to predict and prevent the formation of cartels, more specifically, mixed cartels. This

will contribute to the existing literature on mixed cartels that, so far, is still rather scarce, given the rarity of this type of cartel.

The model developed, à la Cournot, consists essentially in the characterization of two scenarios (competition and cartel) and the consequent comparison of both. It was then applied to the specific case of the Cartel da Banca.

1.1. Game Theory Applications

Over time Game Theory has been used in a huge range of studies belonging to many different fields. Originally developed for Economics to understand firm, market and consumer behavior. It has also been used in the social sciences, with applications in political and sociological fields, for example.

The main fields of study and application of game theory are economics and business, in which it can be addressed auctions, acquisitions and mergers, price structures, oligopolies, voting systems, and with applications in industrial organization.

Other areas in which game theory is visibly applied are project management, political science, biology, computer science and logics, and one of the most widely covered topics during recent times, derived from the pandemic that has occurred, the field of epidemiology.

1.1.1. Applications in various sectors

Project management, where wise and thoughtful decision making is essential to the success of projects, game theory is used to model the decision-making process of the players (e.g. the investors, project managers and consumers). Here there are essentially 4 types of games used: government-private sector games (to model public-private partnerships), contractor-contractor games, contractor-subcontractor games, subcontractor-subcontractor games.

Regarding political science, the use in this area focuses mainly on topics such as political economy, public choice, and war negotiation where the players are often states, politicians or voters. It can also be used to test the reliability of a government's political system and also to predict the response and level of acceptance by the people of a nation to a new law or rule established by the government

In biology, and contrary to what is common in economic games, payoffs are mostly interpreted as physical fitness in relation to the problem at hand. The focus is also not so much on the notion of equilibrium, which falls heavily on the question of the rationality of the players, and as it is known this is not a characteristic of animals, but rather on outcomes that best explain and are maintained by forces of evolution. Game Theory is used to explain and analyze the emergence of animal communication, fighting behavior, and territorialism and, more broadly, evolutionary theories.

Game theory has been playing an increasingly important and critical role in the fields of logic and computational science and has been the basis for numerous models and studies. It has also been used in the development of online algorithms, algorithms that, with the advent of the Internet, have been specifically designed to find equilibria in games and markets.

Certainly, the most talked about issue of the last years was the pandemic of COVID-19, and in this field of epidemiology, game theory also played an important role, in that it was used to model and predict the opinion and acceptance of vaccination by society. It can also play a very important role when it comes to future pandemic prevention and control.

1.1.2. Applications in Industrial Organization

Focusing now on applications in Industrial Organization, this is a field belonging to economics that deals with the strategic behavior of firms, regulatory policies, competition in markets, and antitrust policies, so it makes perfect sense that game theory is strongly applied in this field. Its main areas of study are the market power originating from the study of the different types of markets, product differentiation, the study of various pricing policies, relationships between secondary and primary markets, antitrust and competition, and mergers and acquisitions.

This dissertation focuses essentially on the competition and antitrust part, more precisely on anti-competitive behaviors. These anti-competitive behavior practices are mainly observed through the formation of cartels or through collusive behaviors.

A cartel is an agreement of cooperation formed between competitors in a specific industry. A cartel will get together to set prices and control levels of production with the aim of gaining mutual benefit. Cartels are made up of companies in the same industry that traditionally compete against each other, but who have realized that it is mutually profitable for all players in the marketplace to work in cooperation to control market conditions. Members of a cartel will restrict levels of production and output, thereby creating high demand for the product and pushing prices higher beyond the equilibrium prices.

As far as collusion goes, it is a secretive agreement between two or more organizations, formed with the aim of gaining illegal mutual benefits. An example of collusion would be two companies that operate in the same industry secretively agreeing on a scheme to fix prices, thereby eliminating competition between them. Collusion will be mutually beneficial to the firms that form the alliance as it will allow them to exercise control over a large share of the market and thereby inflate prices, control supply, and earn large profits.

Over the last 20 years there were many cases of conviction for anti-competitive practices in Portugal, more precisely 93 cases, coming from the most diverse sectors of activity, from commerce and services, distribution and the food sector, with major supermarket chains being involved in numerous cases of price fixing, energy and fuel, with EDP and Galp being the most

convicted companies in this sector, banking and financial markets with several banks being convicted for fixing certain rates, insurance and also the health and pharmaceutical sector.

Table 1: Cases of anticompetitive practices in Portugal in the last 20 years

Sector	Number of cases
Environment and Waste Management	2
Banking, Financial Markets and Insurance	5
Commerce and Services	20
Construction	1
Public Procurement	4
Digital & Information Technologies	1
Distribution and Food	21
Energy and Fuels	3
Education	1
Extractive and Manufacturing Industries	1
Postal	1
Liberal Professions	6
Health & Pharmaceutical	5
Telecommunications & Media	12
Transportes & infraestruturas	7
Tourism & Leisure Activities	2

Note: Author's elaboration

Source: AdC

This dissertation is organized as follows. Chapter 2 provides a literature review, first to understand the positions and opinions of the various authors with studies in the topics relevant to this dissertation. Then, still within the literature review, we move on to the topics with more incidence in the case of the Cartel da Banca, namely the matter of mixed cartels and then banking activity. In Chapter 3, we conduct a very detailed presentation and analysis of the case of the Cartel da Banca. In Chapter 4 we present the most relevant information on the banking market in Portugal. In Chapter 5 the methodology applied for the development of the rest of the dissertation culminates, in Chapter 6, with the introduction and development of the theoretical model that in Chapter 7 will be applied to the Cartel da Banca. In Chapter 8, the main comparisons and initial discussion of the results obtained will be presented, so that in Chapter 9 the main conclusions reached in this dissertation will be presented.

2. LITERATURE REVIEW

The most relevant theory to the subject of this dissertation is mainly found in the Strategic Games and will make it possible to explain the characterization of the game in which the Cartel da Banca is inserted.

2.1. Strategic games

Following Crampton (2019), within this category of games are incorporated two-person games (n = 2) and n-person games (n ≥ 2).

2.1.1. Two-person games

The most usual and simplest way to represent this type of games is through a matrix, in which the players' payoffs are placed.

Some assumptions must be taken into account, namely that, in fact, there are only two players, and that each player has a finite number of strategies to choose from. Each player knows which strategies are possible for both but does not know which strategy will be chosen by the opponent, since this choice must be made secretly and simultaneously. Only after this choice is it known which strategy has been chosen by both and then the respective payoffs will be won.

There are also some considerations about rationality on behalf of the players, meaning that all players are rational, and this is common sense among all players and finally, a dominated strategy will never be chosen by a rational player (these either strictly or weakly dominated strategies, can be eliminated, making the solution of the game simpler).

Zero-Sum games

These types of games were the first to be considered by mathematicians, and in particular by John von Neumann, and were the basis for the development of the game theory developed by him and Oskar Morgenstern.

Theorem of John von Neumann, explained by Mérő (1998):

For any finite two-person zero-sum game, the maximum value of the minimum expected gain for one player is equal to the minimum value of the maximum expected loss for the other player

Zero-sum games are essentially characterized by being games in which the value of the gains of one player equals the value of the losses of the other player, that is, the sum of the outcomes is 0. Some examples are, checkers, two-person card games and rock-paper-scissors.

Since these types of games always have a solution, the best strategy for each player can be pure or mixed.

Considering that both players have the same number of strategies available, when the solution to a zero-sum game is reached through a pair of pure strategies, this solution is called a saddle point. The saddle point principle dictates that if in a game matrix there is a saddle point, both players must choose the strategy to which it refers. The value of the saddle point guarantees the result for both players. No player has an incentive to switch to another strategy and leave the saddle point because that would lead to a worse outcome.

When there are no saddle points, meaning that, it is not possible to solve the game using pure strategies, players have to resort to mixed strategies, which means that each player will play each strategy available for a certain period of time, which is the same as saying with a certain probability.

With this being sad, according to Ferguson (2020), there are some particular types of games that fall into the zero-sum game category and, they are special cases because they have a specific configuration and a particular way of solving.

- Latin Square games

  An $n \times n$ matrix, filled with n different symbols, each occurring only once in each column and in each row. For this type of game there is a very simple solution in which the value, V, of the game is given as the row (or column, since it contains the same numbers) average and the optimal strategy for both players is to play each strategy with the same probability, that is, $\frac{1}{n}$.

- Diagonal games

  Matrix of $n \times n$, in which all numbers outside the main diagonal are equal to 0 and the numbers on the main diagonal are greater than 0. The value, V, of this type of games is given by the expression $V = \left(\sum_{i=1}^{n} \frac{1}{d_i}\right)^{-1}$ and the optimal strategy for both players is to play each strategy with probability equal to $\frac{V}{d_i}$.

- Symmetric games

  The symmetry mentioned refers to symmetry in the payoffs of the matrix. Both players have the same strategies, this indicates that the matrix is square and that $a_{ij} = -b_{ij}$ for all i and j. The value, V, of this game is equal to 0 and the optimal strategy for one player is also optimal for the other player.

Non-zero-sum games

In this type of games, the gains of one player do not necessarily correspond to the losses of the other.

In most simultaneous games each player chooses an individual strategy and only then are the payoffs shown. It is called a one-shot game and, in general, the players' interests are not strictly opposite nor strictly coincident, therefore this type of game combines competitive aspects with some opportunities for collaboration.

Knowing that there is a certain possibility of cooperation, it is important to state that, for it to happen, there may exist communication between the players. So, when analyzing this type of games, it is necessary to make some assumptions about the degree of communication between them.

a) No communication

The players communicate nothing with each other and choose their strategies simultaneously, and their choices are not known to the other player. When this happens, the concept of Nash Equilibrium arises.

Nash Equilibrium

Similar to the saddle point explained for zero-sum games. Nash Equilibrium is a strategic profile from which none of the players wants to deviate given the strategies of the other players, that is, each player's strategy is the best response to the other players' strategies.

Therefore, Nash equilibrium is an outcome (sometimes unique, sometimes not) in which all players are acting rationally, in their best interest, and seeking optimality. To be sure that a certain outcome is a Nash equilibrium it is enough to verify that none of the players can improve their payoff by changing strategy

On this topic, John Nash formulated 2 theorems. The first was an extension of an older theorem by von Neumann, and the second was an extended version of the first.

Theorem 1 (1950): Any two-player non-zero-sum game has an equilibrium in pure or mixed strategies.

Theorem 2 (1951): Any n-player k-action game in normal form has an equilibrium in mixed strategies.

To find the Nash Equilibrium in pure strategies there are different methods that can be followed:

- Underlining method

  Identify each player best response to every strategy of the other player and underline them. The Nash Equilibria will be the cell or cells with both entries underlined

- Movement diagram method

  Starting at an entry in the matrix, check to see if any of the players can improve their payoff by switching to the adjacent strategy. If so, turn your attention to that new entry and repeat the process. Do this with all entries in the matrix, and all entries that only have incoming arrows will be Nash equilibria

- Iterated elimination of dominated strategies

To find the Nash equilibrium with mixed strategies the process of equalizing strategies is used. If it is necessary to adopt this method, it ensures that neither player can benefit by switching to another strategy, that is to say, it removes the choice factor from the other player's consideration. Each player considers the opponent's "half" of the game and determines a mixed strategy just as in the zero-sum case. When a player considers his own "half" of the game, this process is called prudential strategies and is classified as the safety level at which the player guarantees that he will receive at least that payoff.

Looking at an evaluation perspective of Nash Equilibrium, it is possible to quantify some strong points of this concept and some less strong points where it perhaps falls a little short.

The great key strengths of NE are its simplicity coupled with the easiness and power to analyze real world events, and in addition it is very straightforward to calculate. In firms in oligopoly situations where there are large amounts of money involved and intense competition, Nash equilibrium is the perfect notion to apply. It should also be added that this is a solid basis for other more complex models where for example repeated games and games with asymmetric information are incorporated.

In less strong points, Nash equilibrium may not always describe the actual behavior of players, mainly because it does not involve the concept of risk. Also, it assumes that players have unlimited reasoning ability and logic, which means that players can solve infinite loops that come from logic of reason, and this is most often not true.

b) Commitments, Threats and Promises

Players can talk to each other before choosing their strategies, and this paves the way for strategic conversation from which compromises, threats, and promises result.

c) Cooperative solutions

Since in the case of the Cartel da Banca the communication was available, it is from here that the presentation of the most relevant concepts for this case begins.

When communication between players is available, the concept of Nash Arbitration Scheme, also called Nash Bargaining Solution, arises. The players communicate in order to decide what will be the best and fairest outcome of the game and agree to implement that outcome (they can turn to an impartial negotiator to determine what would be the fairest outcome).

Nash Bargaining Solution

Proposed by Nash in 1950, further explained by Binmore et al. (1986), it consists of producing a point, N, on the polygon of payoffs, S, with a status quo point, SQ, on that polygon[1]. Nash proposed 4 axioms that he believed a reasonable bargaining solution should respect.

- Axiom 1: Rationality

  The solution point must be contained in the negotiation set

- Axiom 2: Linear invariance

  If players utilities are transformed by a positive linear function, the solution point should be transformed by the same function.

- Axiom 3: Symmetry

  If the polygon happens to be symmetric about the line $y = x + a$ with $a > 1$through SQ, then the solution point should be on this line.

- Axiom 4: Independence of irrelevant alternatives

  Suppose N is a solution point for a polygon S with status quo point SQ. Suppose S' is another polygon which contains both SQ and N, and is totally contained in S. Then N should also be the solution point for S' with status quo point SQ.

There are 2 types of game representation:

- Normal form (matrix form)
- Extensive form

[1] S is compact and the objective function is continuous so, optimal and unique solution exists for this problem

When the game is presented in normal form, it is assumed that the players play at the same time. When players have some information about other players' choices and actions, the game is usually presented in extensive form.

The extensive, finite form formally consists of a finite number, N, of players and a finite number, X, of nodes, which correspond to decision nodes and together form a game tree. Each level of the tree corresponds to the possible decisions of only one player, so the choices are alternating. Here, the concept of information set is introduced.

Imperfect information games

The information set essentially represents that the player to whom the information set refers only knows the possible choices that result from that information and does not know at which specific node the game progress.

To solve these types of games, the perfect Bayesian Nash equilibrium is used, and it is obtained through the backward induction method.

Backward induction

- Start at the end nodes and eliminate the dominated strategies.
- Move up to the nodes above and do the same process.
- Do this until you reach the original node (on each level the elimination of strategies is to be done for the player referring to the level).
- The remaining strategies are the perfect equilibria subgame set.

Perfect information games

All the players always know what node they are on in the game and know all the choices made previously by everyone.

To solve these types of games, the backwards induction with the subgame perfect Nash equilibrium concept is used.

Definition of subgame in extensive form: smaller part of the whole game that starts at any node and continues until the end of the game with the special condition that no set of information is subdivided.

2.1.2. N-Person games

By increasing the number of players in the game, we move on to n-person games. However, we will move directly to the type of game that represents the topic in study, which, as stated before are the cooperative games.

Cooperative games

Following Serna (2016), an n-person cooperative game is a pair (N, v), in which N is a finite set of n players ($N = \{1, 2, \dots, n\}$) and v is the real-valued function ($v: 2^n \ni S \rightarrow v(s) \in R$). Each set $S \in 2^n$ is called a coalition.

The value of the coalition is given by $v(S)$ and is characterized as the utility that the coalition S can allocate among its members and as the maximum value that the members of S can get for cooperating with each other without the help of other players outside the coalition $(N \backslash S)$

The most relevant question that is asked regarding cooperative game theory is how the payoff of the larger coalition is divided. The focus is on the grand coalition N because in most of the times it is expected that it will be formed because it is the coalition with the highest payoff. From this, there are some considerations to be taken into account, namely that all players are interested in maximizing their payoff (and this is the only reason why the player decides to join the coalition) and that if for the player it is more profitable to leave the coalition, he will do so. In the attempt of answer this question, 2 methods were developed: Core and Shapley Value.

Cartels and collusive behavior, being the main manifestations of anti-competitive practices, directly affect the functioning of the markets in which they are present. This literature review aims to analyze and then understand how the various authors have dealt with these events and see their opinions and conclusions. To make it easier to understand, this literature review will be done in parts and its structure is as follows:

- First, the focus will be on the literature concerning cartel formation.
- Second, address the literature on collusion, talking about the types, factors that facilitate collusion and even show some examples present in the literature.
- Third, address what is present in the literature about success of cartels formed and what are the determinants of success.
- Forth, talk about a topic that will always be adjacent to the topic of cartels and collusion, which are the anti-trust laws, the fines and sanctions, basically the legal part of prosecution when it comes to cartels.
- To conclude, the last two topics will focus on issues closely related to the Cartel da Banca, first, it being a mixed cartel, the existing literature on mixed cartels will be addressed and then, since it was a cartel in the banking sector, some of the existing literature on the banking activity itself will be addressed.

2.2. Cartel Formation

When it comes to cartel formation, or more precisely the decision to join a cartel or not, there are differences if this decision is made by just one individual or by a group, Kerr et al. (1999) and Kerr and Tindale (2004) explained these differences from a psychological point of view, however, in the context of Industrial Organization, Insko et al. (1998) found that groups are more competitive than individuals, given that groups more often choose more competitive options than individuals (in Prisoner's Dilemma situations) which they called the Discontinuity Effect. The explanation for this is two-fold: first, groups are less trustworthy than individuals and on the other hand, they are also more selfish in situations where they expect cooperation from others, which would lead to fewer cartels and lower prices. One of the big points is that decisions that exploit others violate norms and principles regarding equality and reciprocity are easier to make in the group context than individuality. Gillet et al. (2011) compared decisions made by groups and by individual people in a Bertrand-type game, that is, in a game where the variable is price, and they found no evidence to support fewer cartels and lower prices when decisions were made by groups. They even found that there was no difference in the tendency for cartels to form whether decisions were made by individuals or groups, and that there were cases where prices were higher when decisions were made by groups. However, prices would always be higher if there was a cartel than if there was not. Gillet et al. (2011) have defined that when a group makes the decision, that decision is either made by the CEO, Majority, or Consensus, and without a cartel, the prices set are higher if they are chosen by the CEO or Majority compared to Consensus and Individual choice; on the other hand, with a cartel the prices set by the CEO tend to be higher than when it is by Majority or Consensus, with Individual choice prices being somewhere in between. Therefore, CEO pricing decisions tend to be the highest whether or not there is a cartel.

The main challenge in measuring the impact of competition policies or antitrust laws on cartels and collusion is that the total population of cartels is unobservable and only the cartels that have already been discovered are observed. Harrington Jr and Chang (2006) in order to be able to model the birth and death of cartels and to evaluate competition policies, built a model in which in some industries there would be cartels and in others not, in some cases the cartels would collapse and in others not, and in some cases, they would be discovered and in others not, using a Prisoner's Dilemma formulation. They found that competition policies are effective in reducing the rate of cartel formation and that there is an increase in the number of cartels discovered. They also noted that in markets without significant entry barriers, an important constraint to collusion is the prospect of entry or expansion of small non-cartel members.

Feinberg (2016) grouped the main determinants of the stability and consequent duration of cartels into three categories. First, the factors that affect the profitability of collusive activities relative to the individual behavior of each member. Second, the factors affecting the monitoring and ease of organization of cartel members, and finally the actions of the government in terms of detection and prosecution.

In the first category falls inelastic demand, which implies a large gap between collusion and monopoly prices and consequently profits and also on the levels of competitiveness and independence among the various firms in the market. What the author drew from this first category is that it is expected that there will be differences across industries in the incentives for cartel formation and that there will be expected to be categories of industries with respect to how easy it is to form and maintain the cartel. In the second category, the number of actual and potential cartel members is quite high, which makes it very difficult to monitor these cartels, and therefore Feinberg (2016) says it is necessary to identify characteristics of certain members that can help to force an agreement to denounce cartels. Finally, as the government is responsible for prosecuting cartels, it is expected that its actions will have an influence on the stability of cartels. Feinberg (2016) defends it is necessary to evaluate the level of fines and jail time that are imposed as well as the evolution of leniency programs.

2.3. Collusive types

Under the law of competition there is a clear and important distinction between explicit or tacit collusion. Explicit collusion is when a group of firms communicate directly with each other, with the intention of coordinating and/or monitoring their actions to raise profits above competitive levels. Tacit collusion occurs when firms coordinate and monitor their actions but without direct communication. The problem with the latter is that this behavior is usually not considered illegal so firms guilty of tacit collusion face no penalties despite the fact that their conduct leads to similar economic effects as explicit collusion. In studying this, Garrod and Olczak (2017) developed a simple framework that captures the incentives for explicit collusion when firms can alternatively collude tacitly. It is common sense that cartels are more likely to happen in markets with fewer firms (no more than three or four) that are relatively symmetric. Davies and Olczak (2008) found that the medium number of firms in a cartel was five and that the asymmetries among the cartel members were so large that it “calls into question whether symmetry of market shares is a pervasive feature of real-world cartels”. There are two explanations for this issue. First is the so-called sample selection bias in which cartels with fewer relatively symmetric firms may be harder to detect and consequently are underrepresented in any sample of accused cartels; the other explanation was given by Hay and Kelley (1974) who says that "firms acting in highly concentrated industries often do not need to collude explicitly and just rely on tacit collusion". In their study, Garrod and Olczak (2017) found that the incentives for firms to form illegal cartels are the lowest in markets with few relatively symmetric firms.

Byford and Gans (2014) argue that there is still another mechanism by which a collusive arrangement can arise, and it is called collusion by the extensive margin, where firms collude by avoiding entering each other's markets or business territories. So, this mechanism is seen as an arrangement where each cartel member receives monopoly rights over a certain territory. It is further said that the responsibility for implementing this collusive arrangement falls on the members of management at the highest level who are in the position to set and direct the

course of the firm's activities, so the manager of the firm has to be part of the arrangement or at least be aware of its existence. Byford and Gans (2014) conclude that this type of collusion by the extensive margin can arise in an uncoordinated fashion.

In an oligopoly context where an infinite game is played, van den Berg and Bos (2016) say that price-matching cartels constantly set those prices below the monopoly price to mitigate overproduction. To solve this problem, firms agree on a market share arrangement for each firm that will eliminate overproduction and allow them to practice monopoly prices. van den Berg and Bos (2016) determined that firms distinguish between three types of collusive structures: pricing structures, allocation structures, and enforcement structures. With regard to price structures, they concern how price increases and quantity reductions are implemented, allocation structures are designed to divide the excess from collusion among all members and finally enforcement structures are required to ensure detection and punishment of firms that want to deviate from the collusive agreement intentionally. They also identified a number of potential reasons why potential collusion participants would prefer to implement an allocation structure, these being mostly relative to the type of demand that firms face. If demand is relatively elastic, then when setting the price, optimally it will be the monopoly price, so in this case the allocation structure may be necessary to deal with coordination and incentive problems. If demand is relatively inelastic, then cartel members face a trade-off between increasing prices and the amount of costly over-production. In terms of policy implications, van den Berg and Bos (2016) findings indicate a clear collusive marker for industries where production precedes sales.

2.4. Factors that facilitate collusion

Harrington Jr and Ye (2019) developed a theory to explain how sellers can effectively collude by coordinating on a price list, leaving each seller to choose their final price. By coordinating on a high list price, indicating high costs, sellers produce supra-competitive prices, leading buyers to be less aggressive. They also show that collusion can lead to increased social welfare with the examples of some collusive agreements that have been studied by other authors before, such as collusion among retail gasoline stations (Clark and Houde (2013)), retail pharmacies (Chilet (2016)), and fine arts auction houses (Mason (2004)). The theory developed by Harrington Jr and Ye (2019) is that these collusive practices work not because they influence which prices sellers propose to buyers, but because they influence which prices buyers propose to sellers. They came to the conclusion that coordinating a price list makes buyers less aggressive in negotiations and coordinating on final prices makes seller rivals less aggressive so if the pressure for a seller to set a price comes directly from buyers' bargaining power, then coordinating the price list will suffice, on the other hand if that pressure comes from rival sellers, the best option will be to resort to the traditional way and coordinate the final price between sellers.

The factors that facilitate collusion[2] cannot always predict the occurrence of cartels. Examples of such factors are the concentration of the industry under study, the homogeneity of the product, the symmetry of the firms present in that market, and regular orders. The impact of these factors is quite clear and can easily be explained and derived from analyses of simple repeated games. However, the power of the factors that facilitate collusion to effectively help in predicting cartel formation is quite limited. Several authors such as Posner (1970), Hay and Kelley (1974), Grout and Sonderegger (2005) and Levenstein and Suslow (2006), when studying the correlation between these factors and the frequency of cartel detection led to empirical results that do not report clear results.

The existing literature on this topic gives the idea that the fewer firms there are, the more explicit the collusion will be expected to be. This conclusion is consistent with the idea that fewer firms make collusion easier, however, it ignores the fact that explicit collusion, that is, cartel formation, is expensive. The costs of cartel formation include the opportunity cost from coordinating tacitly, organizational costs, and cartel fines. Fonseca et al. (2018) argue that while more firms may benefit more strongly from explicitly talking, the gain from talking might eventually decline such that a medium number of firms benefit the most from colluding. Fonseca et al. (2018) focused on a specific facilitating factor, the number of firms, and showed that for the data they had available, this factor was not suitable for cartel detection. Evidence of the existence of communication makes a key difference in real cartel cases, yet economic theory is currently not well prepared to justify and explain how communication facilitates the coordination of cartels.

They then developed 3 hypotheses:

- Hypothesis 1: The fewer the firms, the easier they find it to collude both tacitly and explicitly

Experimental evidence as well as antitrust practice suggests that firms benefit from talking. So, they formally hypothesize that the gain from talking is positive.

- Hypothesis 2: Communication has a collusive effect.

In order to get more structure into this problem, they assume that without communication firms do not manage to sustain collusive output levels at all whereas firms perfectly collude on the monopoly output when communication is allowed.

- Hypothesis 3: The gain from talking, ΔΠ:
    - (i) increases monotonically in n, and (ii) it does so at a decreasing rate.

The minimum discount factor indeed suggests that fewer firms find it easier to collude. The gain from explicit chat, however, is higher for four and six Cournot firms than for duopolies.

[2] Any facilitating factor should apply to either explicit cartel-like arrangements or implicit arrangements. The existence of an incentive for repeated play is a necessary condition for coordination to emerge as a perfect equilibrium subgame in both scenarios.

Therefore, even though fewer firms may find it easier to collude, this does by no means imply that there will be more cartels with fewer firms.

The main research question of Fonseca et al. (2018) is to quantify firms' additional profit from talking explicitly for Cournot oligopolies. They observed an increasing and concave relationship between the number of firms and this gain. In other words, markets with more firms find it more profitable to talk than markets with fewer firms. This shows that strategic substitutes vs. complements may matter (see also Mermer et al. (2016), for duopolies without talk) regarding the incentive to talk, as may asymmetries between firms (see Harrington Jr et al. (2016)). Communication leads to a reduction in average quantities, and lower dispersion, suggesting that it facilitates coordination. The dynamics of output choices shows there are differences in the way communication helps collusion as the number of firms increases.

2.4.1. Examples of collusive behavior

Turning to concrete examples of studied collusive behaviors, Ciliberto et al. (2019) studied price collusion in the US airline industry. Building on the work of Werden and Froeb (1994) and Athey et al. (2004), Ciliberto et al. (2019) performed two empirical tests regarding collusive behavior, where the first predicted that collusive firms will reduce within-market price differences if demand satisfies certain properties, so if greater multi-market contact between the same firms facilitates collusion, it is expected to observe a reduction in the pair-wise price difference when multi-market contact increases. The second predicted that colluding firms will sacrifice production efficiency by increasing price rigidity to avoid informational costs. If multi-market contact facilitates collusion, then it is expected to observe a reduction in the pair-wise price variance when such contact increases.

Using USA airline data Ciliberto et al. (2019) found that increased multimarket[3] contact between carriers leads to price patterns consistent with both empirical tests.

Another market that was studied to see whether or not there was evidence of cartelization was the mobile telecommunications market in France and Germany. During the investigation, in France, 3 cell phone operators (Orange, SFR and Bouygues) admitted to sharing confidential sales information, however, they later argued but without success that, such information sharing did not disrupt competition and that they did not intend to secure a larger and more stable market share.

The decision of the price indexes at the firm level are calculated based on a consumer who uses mobile phone services infrequently, this is the customer who values mobile services the least and his decision to subscribe to the packages offered will determine the basis of market demand. This implies that in France operators may have to compete against operators providing fixed line services, resulting in lower market prices.

[3] Refers to the different routes that various carriers have in common.

On the other hand, in Germany, lower fixed line prices stimulate demand for mobile phones without imposing competitive pressure on mobile phone providers. The analysis by Grzybowsky and Karamti (2019) led to results showing that the elasticity of demand for mobile services is significantly higher in France than in Germany and that consumers seem to recognize mobile services as substitutes for fixed-line services in France but as complementary in Germany, so in conclusion, the results of their empirical analysis suggest that the French mobile telecommunications industry may be relatively competitive, if indeed consumers perceive mobile and fixed-line services as substitutes and demand for mobile subscription is more elastic than in Germany.

The last example is the Chinese automobile industry. The structure of this market is quite complex because, first, it consists of 117 manufacturers and second, the most dominant manufacturers belong to corporations organized by state-owned enterprises.

As noted earlier, collusion is less likely to happen when there are a large number of firms in the market, however, in this case a large number of these manufacturers within the same state-owned business group have the same stakeholders, which can lead to facilitating tacit collusion since, common ownership leads to easier information sharing and reduces the incentive to compete in order to increase total profit. Hu et al. (2014) applying Berry et al. (1995) model concluded that there was no statistical evidence supporting within-group collusion in China's auto industry, however, other forms of coordination may exist outside the scope of the models they tested, for example, in the study Hu et al. (2014) assumed that collusive patterns are applied to all corporate groups, but partial collusion is also possible.

2.5. Success of Cartels and Determinants of Success

Having seen the relevant literature on cartels and collusion and moving on to the concrete examples studied, the next part is whether information sharing is harmful to consumers. Harrington Jr (2021) taking convicted cartel cases, such as in the market of high fructose corn syrup, urethane, cement, air freight, airlines, and railroads, has developed research that answers the questions whether in fact it is harmful to consumers when firms share information and if it is harmful, what should these information exchanges be subject to.

Private exchanges of price information by competitors are harmful to consumers when the cost of adjusting the price is neither too low nor too high, which can be seen as the executives responsible for the collusion having some but not total control over the final price. When there is an arrangement to share prices, the price setters will set the price so that it is supra-competitive because, the price setters know that if by exchanging information, the other firms have set a lower price, they will no longer have an opportunity to respond by lowering prices. Harrington Jr (2021) concluded that it is, in fact, the agreement on information exchanges that causes the harm since it is the anticipation of sharing prices that causes firms to set high prices. To solve this, repeated interaction may be required to induce truthful sharing of information, for there

could be an incentive for a firm to misreport its price at the information exchange. As long as the true price is revealed in a timely manner and firms interact sufficiently frequently, the argument of repeated games can be applied to incentivize firms to truthfully report their prices, making sure that not so much harm is done.

According to Levenstein and Suslow (2006), cartels to be successful have to solve 3 problems: coordination, cheating, and potential market entry. The most successful cartels create organizations to be able to deal with these three problems simultaneously. It is already known that market concentration undoubtedly favors the stability and success of the cartel (increases profits and reduces coordination problems). Instability of demand, even more so if it is not expected, creates problems for cartel stability.

Harrington Jr and Wei (2016) focused on being able to answer the question about what the duration of discovered cartels says about the duration of all cartels in general. Defining that the seriousness of the problem of cartels to the economy depends on three factors, those being how many cartels there are, how much they overcharge or the elasticity of demand, and how long cartels last. The consensus measure of the average duration of cartels is the average duration of discovered cartels, which, given most studies is 5 to 7 years.

The finding of cartel duration is not only relevant to assess the severity of the cartel problem but also to assess whether the measures and levels of enforcement is sufficient to deter cartel formation.

The assessment of enforcement to see whether or not it is being effective is done through the likelihood and subsequently conviction, the penalties that cartels face in the event of discovery and in the profit gains that cartels have made from collusion. Using data on the duration of discovered cartels, Harrington Jr and Wei (2016) used an estimate of a 15% probability per year that a cartel will be discovered and convicted.

2.6. Antitrust, Fines and Sanctions

Cosnita-Langlais and Tropeano (2022) identified two ways that competition agencies have at their disposal to address an antitrust violation through anticompetitive practices, and they are either to negotiate a settlement with the infringing firm or to decide to pursue formal litigation and decide on the infringement that way. Going the formal litigation route can be favorable because it leads to the creation of legal precedents that serve as a useful stock of knowledge for evaluating future cases. Over 60% of antitrust cases, excluding cartels, did not formally sanction the violation (in the case of cartels the violation is always sanctioned) and over 70% of abuse of power cases were resolved with commitments.

Because enforcement is too flexible, Competition Authorities resolve to propose commitments too often and this leads to neglecting the greatest deterrent effect of litigation, worrying only about saving the costs of trial.

Cosnita-Langlais and Tropeano (2022) also argue that proper detection of anticompetitive practices makes litigation an extremely valuable tool by imposing fines and consequently effectively deterring anticompetitive practices. They also concluded that if the deterrence effect of litigation is high enough, better case assessment reinforces the effectiveness of strict enforcement.

Seeing how often formal litigation is used in addressing antitrust violations, it is necessary to know if there is some sort of criteria for which cases are chosen to have litigation.

Gual and Mas (2011) studying and taking evidence from the European Commission's decisions regarding the fight against anticompetitive behavior came to the conclusion that, the selection of cases is not random, which somehow makes the process quite efficient but not very significant.

In the European Union, competition policies are based on three main pillars: antitrust, merger control and state aid monitoring. The study by Gual and Mas (2011) had two main objectives. First, to understand the process that is followed by the European Commission in its selection decisions (to determine whether selection bias exists) and second, to determine what industry characteristics led the Commission to decide against a company investigated for antitrust reasons (to know whether the background of competitive conditions in the corresponding industry are likely to lead to anticompetitive behavior).

They came to the result that the selection of cases is not done randomly, thus making the fight more efficient because the European Commission is more likely to follow cases involving industries with characteristics that economic theory predicts are more likely to encourage anticompetitive behavior (in industries with high concentration or where firms are more symmetrical they are more likely to be examined more thoroughly by the Commission) or in cases where the infringement has a more negative effect on consumer surplus.

However, the results of Gual and Mas (2011) show that the sample selection problem is not important, and this is a really important and positive finding as it proves that there is no need to worry about a possible bias that might affect the empirical results achieved so far in the literature.

All else equal, the Commission is more likely to rule against cases involving a potential horizontal practice without efficiency justifications and less likely to rule against cases involving more than one firm in practices that may have an efficiency justification.

Regarding antitrust prohibiting excessive pricing, Gilo and Spiegel (2018) examined the competitive implications of two benchmarks. The first a retrospective benchmark where the price that prevails after a rival enters the market is used to assess whether the dominant firm's pre-entry price was excessive and the second, a contemporaneous benchmark, where the dominant firm's price is compared to the price that the firm charges at the same time in another market. Their results show that the two benchmarks constrain the dominant firm's behavior

when it acts as a monopolist but mitigate competition when the dominant firm competes with a rival.

The antitrust policy of prohibiting excessive pricing is highly controversial, with opponents and supports. Opponents argue that the policy has a dampening effect on firms' incentive to invest and that it creates considerable legal uncertainty due to the difficulty of determining whether or not prices are excessive. Supporters of the policy argue that many antitrust laws create legal uncertainty, and that excessive pricing is not a problem that can be fixed by itself and since price regulation itself is inefficient, the antitrust policy of prohibiting excessive pricing should complement price regulation.

Antitrust laws can result in the application of fines and sanctions, which can be public or reputational. Mariuzzo et al. (2020) found that in the context of cartelization, public and reputational sanctions act as substitutes since, when there is a reputational sanction, increasing this penalty reduces the effect of the public sanction and, on the other hand, the absence of a reputational sanction reduces the cartel effect, that is, the public sanction (the firm is frowned upon for having participated in cartelization) comes into question.

The public sanctions are fines imposed directly by an administrative body or court and the reputational sanctions are the damage to the firm's image which will then be negatively reflected in the market mechanisms. The credibility of reputational sanctioning is verified by the change in purchasing patterns that accompanies a variation in beliefs about the infraction and, Mariuzzo et al. (2020) argue that these reputational sanctions should be sufficient in the case where the only injured party of the cartelization event is the consumer and therefore, the total damages are internalized by the infringing firm. When the Competition Authority discovers a cartel, this information is not automatically distributed to all related parties. Various information channels come into play to convey the news about the cartel conduct to the public, and because without this information there would be no reputational impact, Mariuzzo et al. (2020) postulate that the reputation effect is directly related to the sentiment and intensity of the information transmitted.

Once the most relevant literature in more theoretical terms on the subject of cartelization and collusion, passing also to antitrust law application and law enforcement efficiency has been seen, it is time to move on to the central theme of the most practical application of this dissertation, which is the "Cartel da Banca", which was a well-known and highly talked about Portuguese case recently. The interesting thing about this cartel is that it involved private but also public banks, thus speaking of a mixed cartel. Mixed cartels are not very often discussed since, when thinking about collusion it is only thought that it is practiced by private entities, because the main goal of collusion is to increase the final profits, and this is precisely the goal of private entities in their normal activity.

Having said this, the next step is to present the most relevant literature on mixed cartels.

2.7. Mixed Cartels

Focusing on mixed cartels, Correia-da-Silva and Pinho (2017) tried to understand what are the effects of a potential privatization of the public entities participating in the collusion so that the cartel becomes more private.

One of the acquired data and also one of the bases in the treatment of mixed cartel cases is the difference in the objective functions of the entities participating in the collusion. Private entities have, as already stated, the objective of maximizing profits and public entities have the objective of maximizing total surplus.

Privatization, by increasing the number of private firms, makes collusion easier to sustain, and it becomes socially detrimental when firms are only able to collude after privatization (if this happens there is a loss of total surplus, which is greater the lower the slope of the marginal cost function or the greater the number of firms). This notion goes against the traditional belief that privatization is socially desirable if there are too many firms in the same industry.

Correia-da-Silva and Pinho (2017) concluded that if the number of private firms is small or the slope of the marginal cost function is small, private sector profits are lower if private firms maximize their joint profit (in collusion) than if they maximize their individual profit (in competition). In this case, collusion is not desired. In mixed oligopolies, collusion becomes easier to sustain as the number of firms colluding increases, specifically when there is only a single public firm and few (four or fewer) private firms, increasing the number of private firms makes collusion easier.

They also studied three scenarios and found that privatization is only able to contribute to the increase in total surplus in the first scenario which is when firms behave in a competitive manner.

By studying the presence of a public firm in a collusion, Mota et al. (2020) tried to understand what was the impact of that firm's preference for the consumer surplus. Using a simple model of only one private firm and one public firm, they characterized the collusion outcome that resulted from the Nash bargaining power (equally distributed) between the two firms and compared it with the competitive outcome and evaluated the sustainability of that same collusion.

They defined that the public firm's preference for consumer surplus can be mild or strong and in case there is a deviation from the collusive agreement, both firms assume grim trigger punishments, permanently reverting to the static Nash outcome.

Mota et al. (2020) concluded that without collusion, the public firm, by having a preference for consumer surplus produces more than the private firm, leading to productive inefficiencies. Collusion reduces or even eliminates these inefficiencies since it makes possible the transfer of outputs from the public firm to the private firm, which will accept to increase its output as long as the final profit increases as well (this result contrasts with the traditional idea that firms in collusion reduce total output to force a price increase in the market).

The major impact of this study is that collusion can increase total surplus and with-it consumer surplus as well.

Mota et al. (2022) introduced the possibility that the preference for consumer surplus by the public firm could be intermediate. In this new study, they reached a new conclusion relative to what had been said before regarding the preference for consumer surplus by the public firm being strong. In this new conclusion, they said that the expansion of the private firm's output does not offset the contraction of the public firm's output, which leads to a reduction in total output. They also argue that there are types of cartels exempt from antitrust laws because they clearly benefit society, such as export cartels and agricultural cartels.

Under our assumption of symmetric and convex cost functions, productive efficiency is attained if firms produce the same output. Since the public firm produces more than the private firm in the Cournot-Nash equilibrium, decreasing the quantity produced by the public firm and increasing the quantity produced by the private firm increases the total output. This suggests a beneficial effect of collusion. In this new study it also emerged the possibility that the bargaining power of the firms is not equivalent, that is, asymmetric (the private firm has more power) resulting in the collusive agreement moving along the contract curve from the agreement that the public firm prefers to the agreement that the private firm prefers.

2.8. Banking activity

This being a problem about competition in the financial system it is necessary to go a little deeper into the topic of bank competition and how it affects the consumer and the economy overall. So now let's move on to the existing literature focused on this subject.

First the focus is on the cost structure of a bank, which like any business entity has fixed costs and variable costs. Within the fixed costs that a bank faces are included staff costs, marketing expenses, taxes, among others.

It is clear that the cost structure of a bank has an influence in several aspects and at several levels on the business performance.

For the banking industry specifically, competition influences and determines each bank's strategic decisions, market positioning and selection, risk appetite and management, product development and daily operation management, and ultimately determines the bank’s sustainability, stability, and prospect.

The financial system is an important part of the economic stability of a country; its stability is directly related to the security and stability of the financial system. So, it is essential to study the impact that banking competition has on stability.

Yuan et al. (2022), identified three ways in which bank competition can affect stability: franchise value, borrowing costs and, operational behavior.

- The franchise value refers to its own value of the franchise license, which must be achieved through the bank's ongoing operations. To some extent, it reflects the present value of the bank's future earnings and represents the reputation and status of a bank. With the intensification of bank competition, bank's market power declines and profits gradually decrease, which reduces the franchise value; for shareholders, when the value of the bank's franchise declines, the potential losses caused by the bankruptcy will be reduced, which will increase the bank's motivation to take risks in the operation, resulting in increased risks and reduced stability.

- The borrowing costs refers to the cost incurred by the bank during the credit process, including both the bank (necessary expenses incurred by banks to absorb deposits - reflected in the interest rates on deposits) and the borrower (expenses paid by the borrower to obtain bank loans - reflected in the interest rates on loans). When bank competition in the bank deposit market declines, the market power in the deposit market rises. This will lead to the falling of deposit interest rates, which means that the cost of capital decreases and the bank's profit rises, so the risk-taking motivation decreases accordingly, thereby increasing stability. However, increased competition in the deposit market may weaken the prudence of bank behavior, which does harm the stability of banks. In turn, when competition in the loan market declines, the market power of banks in the loan market rises, which means that the financing costs faced by borrowers will increase, and then the probability of bankruptcy will rise. Due to the risk transfer effect, the bank's credit risk will rise, and stability will decline.

- The operating behavior refers to activities the bank engages in for profits. With the intensification of competition, the bank's market power gradually declines, and it will adjust its operating behaviors to protect survival and development. According to Berger and Hannan (1998), banks with high market power will lack the motivation of cost control, and the intensified competition can encourage them to reduce their operating costs or increase their income by improving their management ability, thereby increasing stability; banks can capture more customers, which helps increase its revenue and thus enhances bank stability.

Specifying an appropriate measure of the output of a banking firm, that is, what a bank produces and how this output can be measured, is one of the unresolved theoretical debates in the banking economics literature. Unlike an industrial firm, a bank's output cannot be measured through physical quantities, and adding to this difficulty, a banking firm is characterized by being multi-product.

All this problematic of the definition of a bank's production has as theoretical support the economic definition of bank enterprise that brought to the literature two approaches: that of production and that of intermediation (Humphrey (1985)).

- Production approach: banking firms are producers of credit and deposit services, and the production factors are labor and capital. For costing purposes, all services are considered as separate outputs, and the number of deposit and loan accounts is generally the unit of measurement adopted to measure the output.
- Intermediation approach: the banking firm is seen as a simple intermediary in the financial markets, that is, the production process requires the collecting or borrowing of funds that will then be applied or lent (here deposits are also considered inputs). In total costs are included the operating or production costs and the financial costs.

3. THE CASE OF "CARTEL DA BANCA"

Cartel da Banca, according to the accusations of the Autoridade da Concorrência (AdC) (Portuguese competition authority), involved 14 banking entities operating in Portugal and together hold about 95% of market share of the Portuguese banking market, that between 2002 and 2013 communicated with each other sharing confidential information about housing loans.

The banks accused were Caixa Geral de Depósitos (CGD), Banco Comercial Português (BCP), Santander Totta, Banco Português de Investimentos (BPI), Banco Montepio, Banco BBVA, Banco Espírito Santo (BES), Banco BIC, Crédito Agrícola, Banco UCI, Barclays, Banif, ABANCA and Deutsche Bank (which ended up having its case dismissed).

The AdC sentenced the 14 banks to pay fines for exchanging sensitive and private commercial information. In this scheme the banks provided information about their commercial offers indicating, for example, the spreads to be applied in the near future on housing loans or the amounts of credit they had granted in the previous month. The AdC also argued that this illicit exchange of information favored banks by reducing risks and uncertainty about the actions of direct competitors, thus harming the competitive environment in the financial market and directly affecting consumers. Each bank knew, with particular detail, accuracy and timeliness, the characteristics of the offer of other banks, which discouraged the targeted banks to offer better conditions to customers, eliminating competitive pressure, beneficial to consumers.

"What the AdC found was more or less this: an employee of one bank sent to competing banks sensitive, non-public and strategic information about commercial conditions on housing loans, following a rule of reciprocity where he shared information on the assumption that he would receive identical information from his competitors." (Alberto Teixeira, SAPO ECO 2020)

The AdC's case against the Cartel da Banca essentially rests on 7 charges. It also noted that these illicit practices were deeply institutionalized and rooted within the banks and that the highest levels of the banks' hierarchy were aware of everything. These charges are as follows:

1. Banks exchanged spreads grids.

   The information that the banks exchanged with each other was sensitive, strategic, and of a non-public nature or difficult to access or systematize.

   The AdC further adds that “It is unacceptable for a group of banks to exchange information among themselves about spreads, in particular about intentions to change their values in the near future, thereby artificially altering normal competitive conditions in the market and clearly reducing each bank's risk and uncertainty about the strategic behavior of its direct competitors.”

2. Banks exchanged information about the cost of transferring credit.

   Banks exchanged sensitive information on other commercial conditions associated with housing credit, such as costs associated with the transfer of housing credit.

   The exchange of information allowed the banks to “understand the positioning of each listed bank with respect to limits on costs incurred when transferring housing loans between banks”.

3. Alignment in the commissions.

   The banks also exchanged information on the interpretation of legislation applicable to their activity, particularly with regard to commissions on housing credit, with the AdC stating that “the content of the emails analyzed clearly suggests a purpose of commercial alignment in the face of doubts raised by the application of that legislation on commissions”.

   The oldest piece of evidence, dating back to 2002, concerns the exchange of a spreadsheet prepared by CGD, with an analysis of the commissions charged by BCP, Santander, BPI, BES and Montepio.

4. Illicit practices were part of the workers' day-to-day life.

   Exchanging secret information with other banks was normal practice, part of everyday life for those who worked in the marketing departments or in the commercial departments of the banks. “Part of the tasks of the employees assigned to the departments involved in the exchange of information (usually marketing or product management) necessarily comprised liaising with competitors to obtain information about their offers and commercial conditions”, says the AdC.

   There was a stable and institutionalized network of contacts, which ensured that the flow of information was not interrupted even when an employee left the bank.

5. High levels of management had knowledge.

   The hierarchies were aware of the sharing of information, including directors and administrators in some cases, which is proven in several emails in the investigation.

   The exchange of information with competitors persisted and continued even when there were changes in employees, “which shows that it was an institutionalized activity within the respective departments”.

   Considering the characteristics of the exchange of information, namely its detail and intensity, it is also "not credible that this practice could have occurred over ten years without the directorships and administrations of the employees who participated in it being aware”.

6. These information exchange practices restricted competition among banks.

   Banks restricted competition in the credit market. Knowing what the other banks were going to do as regards pricing, for example, an institution was “able to define its individual commercial policy, adapting and adjusting it to the conduct - ascertained through the exchange of information - of its competitors”.

   This adaptation and adjustment were “susceptible” of occurring due to access to detailed non-public information of competitors, "directly requested and made available" by the banks.

7. Put up impediments to the entry of new banks.

   The banks' illegal practices affected the structure of the market, as they reinforced the barriers to the entry of new operators, “that is, those whose services could potentially respect the offer of banking products and services in Portugal”.

Table 2: Explanation of the fines throughout the process

Bank	Initial fine (in millions)	Fine asked by Ministério Público (in millions)	Fine asked by Adc (in millions)	
CGD	82	82	More than 82	(1)
BCP	60	Less than 60	More than 60	(2)
Santander Totta	35.65	35.65	35.65	(3)
BPI	30	Less than 30	30	(4)
Montepio	26 [1]	4.4 [1]	26 [1]	(5)
BBVA	2.5	2.5	2.5	(6)
BES	0.700	---	0.700	(7)
BIC	0.500	---	0.500	(8)
Crédito Agrícula	0.350	0.350	0.350	(9)
UCI	0.150	Less than 0.150	0.150	(10)
Barclays	8 [2]	0.500 [1]	8 [2]	(11)

The last two columns refer to the fines asked by both parties in the final arguments of the trial

Note: Author’s elaboration

(1) The prosecutor who accompanied the trial, Paulo Vieira considered that the fine, although high, represents only 7 million for each year of infringement, and that it is less than the gains that the bank would have had in that period with the exchange of information. He requested that the fine should be increased based on the lack of regret and the light manner in which the bank handled the case, being a wholly publicly owned bank.

(2) The fine seems “adequate”, but the MP is not sure, because it depends on the results of 2021, so they asked for the fine to be lower if the 2021 results impaired capital. The lack of repentance and censure on the facts lead the AdC to ask the court to consider increasing the penalty to BCP.

(3) The prosecutor justified that the fines initially calculated by the AdC were appropriate and that they could be maintained in full. The AdC disagreed and defended that the fine should be increased because, like CGD, they were very criticized in court for their lack of cooperation and, also, there was no evidence of regret.

(4) The representative of the Ministério Público said that the initial fine was adequate but that, given the correct attitude and assumption of responsibility in court, the fine could be reduced marginally. The AdC maintained its initial stance.

(5) [1] Waiver of half of the amount because of the leniency program.
The bank joined the AdC's leniency program (second bank to join after Barclays), so it was entitled to a waiver of up to 50% of the stipulated fine. In relation to the fine to be applied, the prosecutor recalled the “social nature” of Montepio, as well as its fragile economic situation and the fact that the capital belongs to savers (associates of the mutualist). The AdC rejected the idea that there could be a reduction of the fine based on the economic situation of the bank, saying that “it would be to reach the conclusion that crime pays off, especially in the banking sector”.

(6) The fine was adequate, and it should be maintained in full, because the bank really had extraordinary profits, explained the head of the Ministério Público. The AdC maintained its initial position.

(7) The Ministério Público said that there were no preventive reasons that would justify a high fine, so it asked for a fine of "merely symbolic value". The AdC maintained its initial position.

(8) The prosecutor considered that there should be a marginal reduction in the fine, praising the work of the bank's defense. The AdC maintained its initial position

(9) The fine proved appropriate for the Ministério Público and the AdC maintained its initial position.

(10) The fine imposed was the lowest among all the banks involved in the case, however, for the Ministério Público, the fine imposed to this bank should have been even lower. The AdC maintained its initial position.

(11) It was Barclays that originated the process, by denouncing the existence of information exchange, at the end of 2012. The AdC's fine, for its violations, was 8 million, but with

total waiver, because of the leniency program. The prosecutor said that the calculation should point the fines to 500 thousand euros, but given the "conduct", even these 500 thousand euros should be reduced. [2] In any case, this amount will not have to be paid due to leniency. The AdC maintained its initial position.

In the face of these charges by the AdC, the argument used by the major banks that have contested the charges in court has been based on saying that the shared data "was public" and accessible and that the decisions made by the banks "helped customers".

To defend itself from the accusations, CGD assures that it "has always excluded any sharing of elements that support its commercial strategy" and that the elements of information shared "essentially constitute public" and "accessible" information. BCP says that "there is a difference between sharing information with 'coordination effect' and sharing information with 'monitoring effect'", arguing that the latter scenario is "very unlikely to have a necessarily competition-restricting effect".

4. BANKING MARKET IN PORTUGAL

In this section, the situation of the banking market in Portugal will be briefly discussed. First, an overview of the chronology of Portuguese banking. Then, a little bit on the economies of scale and scope, in a more general way, since the banking market is well known for taking advantage of these two economies and, at the end, classify what type of market the Portuguese banking market is, taking into account the available information.

4.1. Chronology

Regarding the history and chronology of Portuguese banking, starting from the beginning, the second half of the 19th century was marked by a profound dynamism in the practice of financial institutions in Portugal. The first actual issuing bank to exist in Portugal was Banco de Lisboa in 1821, which, in a merger with Companhia Confiança Nacional, created in 1844, would give rise to Banco de Portugal in 1846. In 1887, the Banco de Portugal was granted the exclusive right of nationwide issuance.

In parallel with the policy of dynamizing the national banking system, and with the aim of fostering the economy of the Portuguese colonial lands, the Banco Nacional Ultramarino was founded in 1864.

The expansion and diversification of commercial banking was linked to the need to diversify the sources of financing for the public debt that the Portuguese state needed. It was then that several banks and banking houses appeared: Caixa de Crédito Eborense in 1875, Banco Agrícola Comercial e Industrial de Vila Real and Banco do Douro de Lamego in 1874, Banco de Bragança and Banco de Barcelos in 1875.

In 1876 Caixa Geral de Depósitos was created and, with the creation of a Board of Directors in 1896, consolidated its importance as the state bank of the Portuguese economy.

The banking crisis of 1891 stemmed from the unbridled speculation on transfers from Brazil, which in the meantime had ceased, and the corresponding high amount of credit granted in Portugal. This led to the immediate or gradual disappearance of about one-fifth of the pre-crisis banking institutions.

With the outbreak of World War I in 1914, Portugal experienced an inflationary rise in the cost of living and an abrupt devaluation of the currency. Using its consolidated role as a commercial and issuing bank for the former colonies, Banco Nacional Ultramarino expanded its network of branches in the then Overseas Territories, in Brazil, in the international network and also in the national territory, acquiring all the banks created in 1874 and 1875.

In 1969, the Caixa Geral de Depósitos was transformed into a public company and continued to contribute to the financing of public works during the 1960s and 1970s. Another important movement was the decolonization process of the European possessions which began in the 1960s and in Portugal took place after the 25th of April 1974. This process implied the end of BNU's activity in the former colonies and the passage of its assets and liabilities to the central banks of the new countries. In 1988, CGD took over as majority shareholder of BNU with 99% of the share capital and 1% belonging to the state, which would change in 2001, when the merger by incorporation took place.

The privatization of the Portuguese banking sector, which had been nationalized in 1975, took place between 1988 and 1999, in an effort to comply with market economy rules and address its integration into the European Community in financial symmetry. The only bank that remained public was CGD, thus turning the Portuguese banking market into a mixed oligopoly.

4.2. History of the private banks in the Cartel da Banca

Focusing more on the private banks discussed in the Cartel da Banca, BCP was formed in 1985, founded by a group of more than 200 shareholders and a team of experienced banking professionals with the objective of capitalizing on the opportunity to form an independent financial institution that would serve the then underdeveloped Portuguese financial market more effectively than state-owned banks. In 1995, BCP acquired control of Banco Português do Atlântico S.A., which was until then the largest private bank in Portugal. In 2004, in order to strengthen its focus on its core business of distributing financial products and optimize capital consumption, it sold the insurance companies Império Bonança, Seguro Directo, Impergesto and Servicomercial to the Caixa Geral de Depósitos group. In 2006, the rebranding to Millennium BCP was completed.

In 1844, Caixa Económica de Lisboa[4], then an entity attached to Montepio Geral - Associação Mutualista, opened its doors. The idea of opening branches in the different district capitals was approved in 1943 (later than most other banks), but the project took years to materialize due to the difficulty in obtaining government authorization. Throughout its history there were three acquisitions made by this banking institution: in 1970 there is the acquisition of Caixa Económica Madeirense, in 1995 of Caixa Económica Açoreana, and more recently, in 2010, the acquisition of companies of the Finibanco group.

Currently, Banco Montepio - trading name adopted in 2019 - has its headquarters in Lisbon and has a vast international presence.

The Santander bank, created in 1857, initially operating in the province of Santander in Spain, later spread to the entire Spanish territory. In 1919 the Hispanic Central Bank was created from the merger of eight small banks. Later it acquired several other banks, until it became the main private bank in Spain in the 1980s. The Santander Bank, as it is known today, comes from the merger of Santander Bank and the Hispanic Central Bank in 1999.

Banco Santander in Portugal was founded in 1988 and is currently the third largest bank. Banco Santander Totta arose from the merger in 2004 between Banco Santander, Banco Totta e Açores and Banco Crédito Predial Português, acquired in 2000. In this merger Banco Santander remained the majority shareholder. The three brands were maintained until 2006 and were renamed Banco Santander Totta.

In 2018, Banco Santander Totta was renamed Banco Santander Portugal, as part of the strategy to renew the Santander Group's global brand.

The origin of Banco Português de Investimento dates back to 1981 with the creation of Sociedade Portuguesa de Investimentos (SPI), with a diversified shareholder structure. In 1985, SPI was transformed into BPI, the first private bank created after the sector reopened to private initiative following the 1975 nationalizations. In 1986, BPI became the first bank listed on the Portuguese Stock Exchange.

In 1998, through a pioneering merger process, a single bank with a single brand was created: Banco BPI. In 2017, CaixaBank took control of BPI after the completion of a tender offer, in May 2018 CaixaBank agreed to acquire from Allianz the entire 8.425% stake that it held in the capital of Banco BPI, and in 2018 acquired the entire share capital of Banco BPI.

In 1988, after a merger of Banco de Bilbao and Banco Vizcaya, the BBV bank was created. In 1999, BBV merged with Banco Argentaria, thus creating BBVA. Banco Bilbao Vizcaya Argentaria (BBVA), based in Bilbao, is a Spanish banking group with holdings in financial entities in more than thirty countries.

[4] In 1989 it changed its name to Caixa Económica Montepio Geral. In 2017 became a Private Limited Company, with its capital being represented by shares.

In Portugal, BBV was incorporated in June 1991, having integrated in its assets all the assets and liabilities of the Portuguese branches of Lloyds Bank and Bilbao Vizcaya.

Banco Espírito Santo (BES) originates from the activity of José Maria do Espírito Santo e Silva in 1869. When the founder died in 1915, the firm was dissolved to give rise to Casa Bancária Espírito Santo Silva. In the 20's, BES reinforced its position and in 1926 joined the group of the five largest private banking institutions; 10 years later, in 1936, took the leadership of the national private banking.

In 1937, with the merger with Banco Comercial de Lisboa (founded in 1875), the Banco Espírito Santo e Comercial de Lisboa (BESCL) was created.

Finally, by deed in 1999, BESCL adopted the designation of Banco Espírito Santo. In 2014 filed for bankruptcy resulting in an emergency intervention by the Banco de Portugal to save the good assets of the bank, giving rise to Novo Banco

Banco BIC Português was established in 2008 and is headquartered in Lisbon. In 2012, Banco BIC Português added the retail banking dimension to the vectors that were already recognized (corporate banking, correspondent Angolan banks and private banking). With a shareholder structure identical to Banco BIC (Angola), Banco BIC Português includes shareholders of reference in the Portuguese corporate market, namely in the banking sector.

In 2017 Banco BIC Português, S.A. changed its brand to EuroBic.

The historical origin of the Caixas de Crédito Agrícola Mútuo is associated with the Santas Casas da Misericórdia (founded in 1498). In 1778, the Misericórdia of Lisbon was the first to grant loans to farmers. An example followed by other Misericórdias. The real Crédito Agrícola was born a few months after the implantation of the Republic, in 1911. In 1919, finally, the activities of the Caixas de Crédito Agrícola Mútuo were defined. In 1982, the Caixas were no longer under the control of the Caixa Geral de Depósitos, foreseeing the creation of a Central Bank, oriented to regulate the credit activity of its associated Caixas.

The introduction, in 1998, of a single IT platform for the affiliated banks and the Caixa Central corresponds to a reinforcement of Crédito Agrícola's unification and its affirmation in the market as a "complete bank".

UCI is a financial institution that specializes in granting Housing Loans. Belonging to a solid financial group that has been working for over 60 years with the European real estate sector, UCI offers its clients a global solution for their home financing needs.

In Portugal since 1999 and with a consolidated international presence, UCI is the partner of reference for real estate professionals by being close to the financing needs of clients and professionals.

Finally, Barclays is a British multinational bank based in London with operations in more than 50 countries. In 2014 Barclays announced it was leaving Portugal, stating that staying in the country was no longer compatible with its strategy[5].

4.3. Number of banks operating in Portugal in the last 20 years

Table 3: Financial institutions in Portugal over the years

YEAR	National Institutions	Foreign Institutions	Total
2002	-	-	53
2003	-	-	53
2004	-	-	50
2005	-	-	40
2006	-	-	48
2007	-	-	48
2008	-	-	45
2009	22	14	36
2010	22	14	36
2011	22	14	36
2012	21	13	34
2013	22	10	32
2014	18	10	28
2015	18	11	29
2016	19	12	31
2017	17	13	30
2018	18	14	32
2019	17	12	29
2020	19	11	30
2021	19	11	20

Note: From 2002 to 2008 there is information about the number of banks operating, but not whether they are domestic or foreign banks.

Note: Author's elaboration

Source: APB Statistics and Information bulletin

[5] The sale of Barclays was announced, with Bankinter acquiring the operations in Portugal

Having seen the chronology of the Portuguese banking system and the number of banks in activity over the years, we now move on to address, economies of scale and economies of scope.

4.4. Economies of scale and scope

As far as economies of scale are concerned, they are those in which the increase in production results in a fall in the average cost of the product. To increase its production, it is common that the company must also increase the production factors used in the process, so economies of scale occur when the cost of this investment increases less than the production resulting from it. The concept of economies of scale is therefore a non-proportional relationship between average production costs and production volume.

The degree of economies of scale can be interpreted as the elasticity of output with respect to the cost required to produce it (Baumol et al. (1988)).

As for economies of scope, two types of economies of scope can be referred to: internal or production economies, resulting from the bundling of services such as production and marketing, and external or consumption economies, deriving from the possibility for consumers to source various products or services at the same location or from the same firm. In banking, although there are also economies of scale, economies of scope, namely external or consumption economies, are more typical, however, internal economies of scope are also present.

If the costs associated with the joint production of two products are called $TC(Q1, Q2)$ and the cost functions resulting from separate production are represented by $TC(Q1)$ and $TC(Q2)$, then we have economies of scope if and only if:

$$TC(Q1, Q2) < TC(Q1) + TC(Q2)$$

4.5. Classification of the Portuguese banking market

The final topic of this section refers to the definition of the type of market that is the Portuguese banking market. For this, it is necessary to make an assessment of the competition in this market.

An increase has been observed in concentration of the Portuguese banking system since 1990. This development was particularly evident in the 1990s and was reinforced in two periods during which major mergers and acquisitions took place – 1995 and 2000.

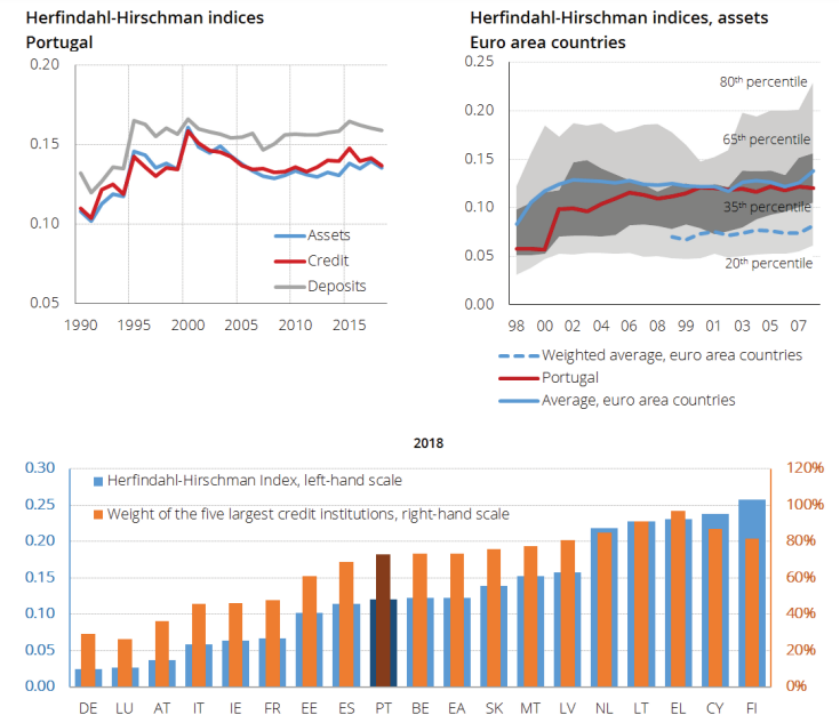

Figure 2: Herfindahl-Hirschman Indices for the Portuguese financial market

Source: Data from APB, Associação Portuguesa de Bancos

According to this chart, the Portuguese banking market is classified as being moderately concentrated, with an oligopolistic market structure, although close to competition.

The rest of this dissertation presents the model, bases and assumptions regarding the model. It is then applied to the Cartel da Banca (pre-cartel situation / competition; incentive to form a cartel / collusive arrangement and lastly sustainability of the collusion).

5. RESEARCH METHODOLOGY

The literature review laid the groundwork as to which points are important to keep in mind regarding cartel formation, and several were covered. Then moving on to the factors that facilitate collusion, it was found that most of the authors speaking on this subject are in agreement, and then there is also a clear distinction between the two types of collusion. In the end, regarding the part about prosecutions, fines and sanctions, this part in the literature is the most difficult to address, because the cases of cartels so far, very often get solved with leniency programs so there is not yet a solid foundation to be based on how to proceed in the prosecution and setting of sanctions for cartel participants. Regarding mixed cartels, it has been shown that the existing literature is still very limited given the rarity of public companies participating in cartels.

The purpose of this Dissertation is to contribute precisely to these last two points, that is, with the creation of a model applicable to potential mixed cartels, that first demonstrates under which conditions the creation of the cartel is preferred and then whether it has conditions to be sustainable. With the results obtained, if it is confirmed that the creation of the cartel has indeed affected competition and consumer welfare, it is possible to calculate which fines are appropriate for each member.

When addressing the topic of preventing anticompetitive behavior in oligopolies where there is the possibility of mixed cartel formation, it is considered, in a perhaps simplistic way, that there is cost symmetry among all players, that is, that all players present an identical cost structure (the same simplification has already been made in several studies in the existing literature). The model is applied to markets where firms are competing à la Cournot, that is, in quantity. By defining the decision variable, we are choosing it for the long term, that is, if we chose price, we were setting it for a long time, and in banking this is not very feasible because interest rates (which are the prices for the banks) change every day. Because the interest rates that banks pay to finance themselves also change every day. Setting Q is typical in cartels and by deciding on Q, P gets approximately determined by the demand curve, given the very high share of the market that the cartel members represent in this case.

Strictly speaking, P would only be fully defined if all the companies in the market participated in the cartel, which rarely happens, but usually includes the largest ones. These characteristics can be applied to the Portuguese banking market.

When doing the literature review, in the specific part concerning mixed cartels 3 candidates emerged to be the starting point for the development of the theoretical model developed in this Dissertation. They were: Correia-da-Silva and Pinho (2017), Mota et al. (2020) and Mota et al. (2022), which have already been addressed in the literature review and the differences between them explained.

It was decided that the best of these 3 to take as a starting point would be Mota et al. (2020). The extension of this model to the one developed in this dissertation involves two important

points, one being a simplification and the other an increase in the scope of application of the model, that is to say, the simplification involves the cost structure were Mota et al. (2020) use $C(q) = \frac{q^2}{2}$ and we use $TC(q) = cq$, which facilitates the expression of marginal costs from $MC(q) = q$ to $MC(q) = c$ (the simplification was discussed when addressing the topic of prevention of anticompetitive behavior) and, the increase in the scope of application of the model is related to going from a game with only one private and one public firm to a game with n private firms and one public firm. The characterization of the Nash equilibrium in the situation under competition and the characterization of the formation of the cartel are maintained.

The next chapters will then have the following structure. After defining the theoretical model and applying it to the Cartel da Banca with the relevant specific data from the Portuguese banking market, we will first compare the quantities produced by both types of banks in the two situations and then, for private banks, we will compare the profit they would obtain under the competition scenario with the profit they would obtain under the cartel or the staying out of the cartel scenario and, for public banks, we will compare their objective function, which is the weighted sum of two factors, also in the competition versus the cartel scenario. Once this is done, the results and conclusions obtained are then discussed in order to determine the incentives of each type of bank to form a cartel and then the explanation of what the consequences of the creation of the cartel were.

At the end, this Dissertation ends with the overall conclusion and main findings made in conducting this study.

6. THEORETICAL MODEL

In the Portuguese financial market, having already been confirmed to be an oligopoly, we can go even further and say that it is a mixed oligopoly. A mixed oligopoly is a market structure characteristic of markets where homogeneous or differentiated goods are provided by a small number of firms where the objective function of at least one of these firms is different from the objective functions of the remaining firms. Here, as there is a public firm (welfare-maximizer) and private firms (profit-maximizer), there is the difference in the objective functions.

Wen and Sasaki (2001) were the first to study collusion between a welfare-maximizing public firm and a profit-maximizing private firm. They found that the public firm may hold excess capacity as a commitment device to sustain a welfare-improving agreement.

In accordance with and based on the model of Mota et al. (2020), we assume that bargaining power, essential in a cartel, is equally distributed among different firms, contrary to the view of Haraguchi and Matsumura (2018), who place all bargaining power in the public firm.

The asymmetry in this model falls on the objective functions and the scope of application.

6.1. Setup

Consider an industry with 1 public firm, p, and n private firms, $i = 1, 2, \dots, n$, producing homogeneous products over an infinite number of periods. In each period, the firms simultaneously choose quantities, q_p and q_i.

Demand is linear and is given by the function $P = 1 - Q$, where Q is the sum of the quantities of all firms, that is, total output, $q_p + \Sigma q_i$.

The total cost of producing q units is the same for the public firm and for the private firms, following the model of De Fraja and Delbono (1989) and Correia-da-Silva and Pinho (2018), resulting in the total cost being given by $TC(q) = cq$. With constant marginal costs, given by $MC(q) = c$. For a purely public firm, the competitive outcome would be one where price equals marginal cost, leading to zero profit and consumer surplus maximization.

As already mentioned, the public firm and the private firms have different objective functions, since private firms aim to maximize own profit and the public firm aims to maximize a weighted sum of consumer surplus and own profit.

Having said that, the profit function for each private firm i, is given by the expression:

$$max_{q_i} \Pi_i = \left(1 - q_i - \sum_{\substack{j \neq i \\ j=1}}^{n} q_j - q_p - c \right) . \, q_i$$

Note: $q_i \rightarrow$ firm i quantity.

$\sum_{\substack{j \neq i \\ j=1}}^{n} q_j \rightarrow$ quantity of all the other private firms.

$q_p \rightarrow$ quantity of the public firm

Note2: general expression for profit is $\Pi = (p - MC).\, q_i$

The objective function of the public firm, p, is given by the expression:

$$max_{q_p} \, \Omega = \mu \mathrm{CS} + (1 - \mu)\pi_p$$

$$= \mu \frac{(\sum_{i=1}^{n} q_i + q_p)^2}{2} + (1 - \mu) \left(1 - q_p - \sum\nolimits_{i=1}^{n} q_i - c \right) q_p$$

Consumer Surplus (CS) with linear demand (slope 1 and intercept at origin 1) is given by $\frac{Q^2}{2} = (\frac{(q_i + q_p)^2}{2})$.

The coefficient μ is the weight that the public firm attaches to consumer's welfare. For example, if the firm is purely profit maximizing, then $\mu = 0$; on the other hand, if the firm gives as much weight to own profit as to consumer welfare, then $\mu = \frac{1}{2}$.

That said, it should be noted that both parameters μ and c must have values between 0 e 1 ($0 < \mu < 1; 0 < c < 1$).

Total surplus, or social welfare is given as the sum of consumer surplus and industry's profit:

$$TS = CS + \sum_{i=1}^{n} \pi_i + \pi_p$$

6.2. Nash Equilibrium (pre-cartel situation)

Under competition, in the Nash equilibrium private firms choose the quantity, q_i^N, that maximizes $\pi_i(q_i, q_p)$ while the public firm chooses the quantity, q_p^N, that maximizes $\Omega\ (q_i, q_p)$.

These quantities are obtained through First Order Conditions (FOC) which are as follows:

- For the private firms:

$$\frac{\partial \pi_i}{\partial q_i} = 0$$

- For the public firm:

$$\frac{\partial \Omega}{\partial q_p} = 0$$

Combining the 2 equations it is possible to determine the quantities of each type of firm in the Nash equilibrium (q_i^N for private and q_p^N for public) and adding everything together gives the total quantity (Q^N). After this, using the inverse demand equation is possible to obtain the price, P^N.

Knowing the individual quantities of the firms, it is possible to obtain the payoff of each firm (π_i^N for private ones and Ω^N for the public one) for the non-cooperative situation.

After knowing all the relevant equations for the characterization of the pre-cartel situation it is necessary to make a study of their behavior according to the impact of the parameters present in the functions.

6.3. Collusive agreement (Explicit collusion / Cartel situation)

Having explained and characterized the model in the pre-cartel situation, we now move on to the most central part, the major focus of analysis in this paper, which is the characterization and subsequent concrete application of the model regarding cartel formation.

In this case, the cartel was formed on the basis of explicit collusion as there was direct information exchange between the cartel members.

In this cartel, not all firms, particularly the private ones, that are active in the market participate. So, this cartel consists of 1 public firm and m private firms ($m < n$).

By assuming that the private firms all produce the same quantity, it is adjacent to assume that these firms are symmetric, however, when comparing the private firms to the public firm this assumption can no longer be made, as the public firm is asymmetric to the private firms.

When asymmetric firms collude, it is not clear what type of agreement should be established. For this model, this problem was solved by creating the following objective functions.

- For the firms inside the cartel:

$$max_{q_i,q_p}\ m\Pi_i + \Omega$$

$$= m\left(1 - q_i - \sum_{\substack{ii=1 \\ ii\neq i}}^{m} q_{ii} - q_p - \sum_{l=m+1}^{n} q_l - c\right)q_i + \mu\frac{\left(\sum_{i=1}^{m} q_i + \sum_{l=m+1}^{n} q_l + q_p\right)^2}{2}$$

$$+ (1-\mu)\left(1 - \sum_{i=1}^{m} q_i - \sum_{l=m+1}^{n} q_l - q_p - c\right)q_p$$

Note: q_i → quantity of the private firm i in study inside of the cartel

$\sum_{\substack{ii=1 \\ ii\neq i}}^{m} q_{ii}$ → quantity of all other private firms inside of the cartel

$\sum_{l=m+1}^{n} q_l$ → quantity of all private firms outside of the cartel

q_p → quantity of the public firm inside the cartel

- For the firms outside the cartel (all private, denominated as l)

$$max_{q_l}\Pi_l = \left(1 - q_l - \sum_{\substack{ll=1 \\ ll\neq l}}^{n} q_{ll} - \sum_{i=1}^{m} q_i - q_p - c\right)q_l$$

Note: q_l → quantity of the private firm l in study outside of the cartel

$\sum_{\substack{ll=1 \\ ll\neq l}}^{n} q_{ll}$ → quantity of all other private firms outside of the cartel

q_p → quantity of the public firm inside the cartel

$\sum_{i=1}^{m} q_i$ → quantity of the m private firms in the cartel

The way to determine the quantities of each type of firm is done in a manner analogous to the pre-cartel situation, using the First Order Conditions:

- For the private firms inside the cartel:

$$\frac{\partial(m\Pi_i+\Omega)}{\partial q_i} = 0$$

- For the public firm inside the cartel:

$$\frac{\partial(m\Pi_i+\Omega)}{\partial q_p} = 0$$

- For the private firm outside the cartel:

$$\frac{\partial \Pi_l}{\partial q_l} = 0$$

Then, in the practical application of this cartel model, by defining n and m, it is possible to obtain the quantities q_i^*, q_p^* and q_l^*, from which the conditions for their positivity will be defined. Adding all these quantities together will give the total cartel quantity, Q^*. Then, again using the inverse demand equation, we obtain the final price, P^*.

Having thus all the necessary data, it will be possible to obtain a value for each of the objective functions depending on the type of firm in question.

7. MODEL APLICATION TO CARTEL DA BANCA

By applying the explained model to the specific case of the Cartel da Banca, it is possible to define the expressions of the banks' payoffs as well.

For private banks, i, the profit maximization function is given by:

$$max_{q_i} \Pi_i = \left(1 - q_i - \sum_{\substack{j \neq i \\ j=1}}^{n} q_j - q_p - c \right) q_i$$

For the public bank, p, the objective function being the maximization of the weighted sum between individual profit and consumer surplus is given by:

$$max_{q_p} \Omega = \mu \frac{(\sum_{i=1}^{n} q_i + q_p)^2}{2} + (1-\mu)\left(1 - q_p - \sum\nolimits_{i=1}^{n} q_i - c\right) q_p$$

7.1. Nash Equilibrium (pre-cartel situation)

In the initial stage of competition in the Portuguese banking market, there were n private banks and only 1 public bank ($initial\ state = n\ public\ banks + 1\ public\ bank$).

The private banks choose the quantity, q_i^N, that maximizes $\pi_i(q_i, q_p)$ while the public bank chooses the quantity, q_p^N, that maximizes $\Omega\ (q_i, q_p)$.

These quantities are obtained through First Order Conditions (FOC) and are as follows:

- For the private bank:

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow\ q_i = \frac{1-q_p-c}{n+1} \tag{1}$$

- For the public bank:

$$\frac{\partial \Omega}{\partial q_p} = 0 \Rightarrow\ q_p = \frac{nq_i(1-2\mu)-(1-c)(1-\mu)}{(3\mu-2)} \tag{2}$$

By combining expressions (1) and (2), the Nash quantities of the pre-cartel equilibrium can be determined. Then the total quantity, price, and payoffs for each type of bank are also obtained.

$$\begin{cases} q_i = \dfrac{1-q_p-c}{n+1} \\ q_p = \dfrac{nq_i(1-2\mu)-(1-c)(1-\mu)}{(3\mu-2)} \end{cases}$$

$$q_i^N = \frac{(1-c)(1-2\mu)}{n(1-\mu)+2-3\mu} \tag{3}$$

$$q_p^N = \frac{(1-c)(1+\mu(n-1))}{n(1-\mu)+2-3\mu} \tag{4}$$

7.1.1. Constraints for μ

Bearing in mind that $c < 1$ and $\mu < 1$.

The next step to take into account is to make sure that the individual quantities of both the private banks and the public bank have to be positive.

Looking first at the private banks' quantity expression, for it to be greater than 0:

$$q_i^N > 0 \Rightarrow \frac{(1-c).(1-2\mu)}{n(1-\mu)+2-3\mu} > 0$$

The following system of equations must be respected:

$$\begin{cases} 1-2\mu > 0 \\ n(1-\mu)+2-3\mu > 0 \end{cases} \vee \begin{cases} 1-2\mu < 0 \\ n(1-\mu)+2-3\mu < 0 \end{cases} \tag{5}$$

This will lead to μ having meet the following conditions:

- $\mu < \frac{1}{2}$
- $\mu > \frac{n+2}{n+3}$

Therefore, joining the 2 conditions, we arrive at the range of possible values that μ can take to make the quantity of private banks positive, and it is:

$$\mu \in \left[0, \frac{1}{2}\right[\cup \left]\frac{n+2}{n+3}, 1\right[\quad (6)$$

Doing the same analysis for the public bank quantity expression, for it to be positive this has to happen:

$$q_p^N > 0 => \frac{(1-c).(1+\mu(n-1))}{n(1-\mu)+2-3\mu} > 0$$

Resulting in the following system of equations that must be respected:

$$\begin{cases} 1+\mu(n-1) > 0 \\ n(1-\mu)+2-3\mu > 0 \end{cases} \vee \begin{cases} 1+\mu(n-1) < 0 \\ n(1-\mu)+2-3\mu < 0 \end{cases} \quad (7)$$

The system leads to μ having to respect the following condition for the public bank quantity to be positive:

- $\mu < \frac{n+2}{n+3}$

Originating the range of possible values:

$$\mu \in \left[0, \frac{n+2}{n+3}\right[\quad (8)$$

Putting the two intervals in (6) and (8) together to ensure the positivity of both quantities, it follows that for the pre-cartel situation, $\mu \in \left[0, \frac{1}{2}\right[$, that is, μ has to be smaller than 0.5.

In all the analysis that proceeds from here, it will be $\mu < \frac{1}{2}$.

7.1.2. Total Quantity, Price and Payoffs

Knowing the individual quantities of each type of bank in equilibrium, the total equilibrium quantity available in the market, Q^N, can be deduced.

$$Q^N = nq_i^N + q_p^N <=> Q^N = \frac{(1-c)(n+1)(1-\mu)}{n(1-\mu)+2-3\mu} \quad (9)$$

Using the inverse demand expression, the market price under competition will be given by the expression:

$$P^N = 1 - Q^N <=> P^N = \frac{c(n+1)(1-\mu)+1-2\mu}{n(1-\mu)+2-3\mu} \quad (10)$$

Finally, to finish the chapter on competition, all that remains is to know the expression of the payoffs of the two types of banks. Everything that is needed to deduce these expressions is already known, so this is given by:

$$\Pi_i^N = (P^N - MC)q_i^N$$

$$\Omega^N = \mu\frac{(nq_i + q_p)^2}{2} + (1-\mu)(P^N - MC)q_p^N$$

$$\Pi_i^N = (P^N - MC)q_i^N = \frac{((1-c)(1-2\mu))^2}{(n(1-\mu)+2-3\mu)^2} \quad (11)$$

$$\Omega^N = \mu\frac{(Q^N)^2}{2} + (1-\mu)(P^N - Mc)q_p^N = \frac{(1-c)^2(1-\mu)(-\mu^2(n^2+6n-3)+\mu(n+5)(n-1)+2)}{2(n(1-\mu)+2-3\mu)^2} \quad (12)$$

Having arrived at the possible values for μ, let us check the impact of the parameters on the equations defined for the pre-cartel stage.

7.1.3. Impact of the parameters n, c and μ

Recalling that $c < 1$ and $\mu < \frac{1}{2}$:

- Regarding the total quantity (Q^N) equation:
  - For n:

$$\frac{\partial Q^N}{\partial n} = \frac{\partial}{\partial n}\left(\frac{(1-c)(n+1)(1-\mu)}{n(1-\mu)+2-3\mu}\right) = \frac{(1-c)(1-2\mu)(1-\mu)}{(n(1-\mu)+2-3\mu)^2} \quad (13)$$

Observing the derivative of the quantity in function of n, we see that the expression obtained (for the values of μ and c established), is always positive.

$$\frac{(1-c)(1-2\mu)(1-\mu)}{(n(1-\mu)+2-3\mu)^2} > 0 \quad \text{for } \mu \in \left[0, \frac{1}{2}\right[\text{ and } c < 1$$

We can then conclude that the higher the n, the higher the total quantity available in the market, which means, the total quantity increases with the number of private banks.

  - For c:

$$\frac{\partial Q^N}{\partial c} = \frac{\partial}{\partial c}\left(\frac{(1-c)(n+1)(1-\mu)}{n(1-\mu)+2-3\mu}\right) = -\frac{(n+1)(1-\mu)}{n(1-\mu)+2-3\mu} \quad (14)$$

Observing the derivative of the quantity in function of c, we see that the expression obtained (for the values of μ), is always negative.

$$-\frac{(n+1)(1-\mu)}{n(1-\mu)+2-3\mu} < 0 \quad \text{for } \mu \in \left[0, \frac{1}{2}\right[$$

Since the derivative is always negative, it follows that c has a negative impact on the total quantity, that is, the higher the c, the lower the total quantity. A relationship that empirically makes a lot of sense.

- For μ:

$$\frac{\partial Q^N}{\partial \mu} = \frac{\partial}{\partial \mu}\left(\frac{(1-c)(n+1)(1-\mu)}{n(1-\mu)+2-3\mu}\right) = \frac{(1-c)(n+1)}{(n(1-\mu)+2-3\mu)^2} \tag{15}$$

Observing the derivative of the quantity in function of μ, we see that the expression obtained (for the values of μ and c established), is always positive.

$$\frac{(1-c)(n+1)}{(n(1-\mu)+2-3\mu)^2} > 0 \quad \text{for } \mu \in \left[0, \frac{1}{2}\right[\text{ and } c < 1$$

Since the derivative is always positive, similarly to what happened with n, the higher the μ, the higher the total quantity will be. Increasing the weight that the public bank assigns to the consumer surplus leads to a higher quantity that must be supplied.

- Regarding the price (P^N) equation:
  The impacts of the parameters on Price will always be symmetric to their impact on Total Quantity (because of the function from which the price expression is obtained). Therefore,
  - For n:

  The higher n is, the lower the price will be.

  - For c:

  The higher the c, the higher the price is set.

  - For μ:

  The higher the μ, the lower will be the price charged. Increasing the weight assigned to consumer surplus will result in a reduction in the price.

- Regarding the private banks profit (Π_i^N) equation:
  - For n:

$$\frac{\partial \Pi_i^N}{\partial n} = \frac{\partial}{\partial n}\left(\frac{(1-c)^2(1-2\mu)^2}{(n(1-\mu)+2-3\mu)^2}\right) = -\frac{2(1-c)^2(1-2\mu)^2(1-\mu)}{(n(1-\mu)+2-3\mu)^3} \tag{16}$$

Seeing the derivative of the profit of the private banks in function of n, we see that the expression obtained (for the values of μ and c established), is always negative.

$$-\frac{2(1-c)^2(1-2\mu)^2(1-\mu)}{(n(1-\mu)+2-3\mu)^3} < 0 \text{ for } \mu \in \left[0,\frac{1}{2}\right[\text{ and } c < 1$$

This means that profit is negatively affected by n, that is, the higher it is, the lower the profit of the private banks will be. This relationship makes sense because, since there is a total quantity available at a certain price, the greater the number of private banks in the market, the lower each bank's individual profit will be.

- For c:

$$\frac{\partial \Pi_i^N}{\partial c} = \frac{\partial}{\partial c}\left(\frac{(1-c)^2)(1-2\mu)^2}{(n(1-\mu)+2-3\mu)^2}\right) = -\frac{2(1-c)(1-2\mu)^2}{(n(1-\mu)+2-3\mu)^2} \quad (17)$$

The derivative of private bank profits in order to c shows us that it will always have a negative value (for the values of μ and c established).

$$-\frac{2(1-c)^2(1-2\mu)^2(1-\mu)}{(n(1-\mu)+2-3\mu)^3} < 0 \text{ for } \mu \in \left[0,\frac{1}{2}\right[\text{ and } c < 1$$

The higher c is, the lower the profit of the private banks. This also makes a lot of sense because, if the selling price is set, the higher the costs, the lower the profits of the private banks will be.

- For μ:

$$\frac{\partial \Pi_i^N}{\partial \mu} = \frac{\partial}{\partial \mu}\left(\frac{(1-c)^2(1-2\mu)^2}{(n(1-\mu)+2-3\mu)^2}\right) = -\frac{2(1-c)^2(n+1)(1-2\mu)}{(n(1-\mu)+2-3\mu)^3} \quad (18)$$

Observing the derivative of the private banks profit in order of μ, we see that the expression obtained (for the values of μ established), is always negative.

$$-\frac{2(1-c)^2(n+1)(1-2\mu)}{(n(1-\mu)+2-3\mu)^3} < 0 \text{ for } \mu \in \left[0,\frac{1}{2}\right[\text{ and } c < 1$$

Again, this means that the profit of the private banks is negatively affected by μ, that is, the higher the μ, the lower the profit of the private banks. The reason is that total quantity increases with μ, as we have already seen, which pushes the price down and reduces profits for the private firms

- Regarding the public bank objective function (Ω^N) equation:
  - For n:

$$\frac{\partial \Omega}{\partial n} = \frac{\partial}{\partial n}\left(\frac{(1-c)^2(1-\mu)\left(-\mu^2\left(n^2+6n-3\right)+\mu(n+5)(n-1)+2\right)}{2(n(1-\mu)+2-3\mu)^2}\right) =$$

$$= \frac{(1-c)^2(1-\mu)(12\mu^3-20\mu^2+11\mu-2)}{(n(1-\mu)+2-3\mu)^3} \quad (19)$$

Although the public bank's objective function gives weight to consumer surplus, it will still be decreasing with n, similarly to what happened with the private banks' profit function. The higher the n, the lower the public bank's objective function will be.

  - For c:

$$\frac{\partial \Omega}{\partial c} = \frac{\partial}{\partial c}\left(\frac{(1-c)^2(1-\mu)(-\mu^2\left(n^2+6n-3\right)+\mu(n+5)(n-1)+2)}{2(n(1-\mu)+2-3\mu)^2}\right) =$$

$$= -\frac{(1-c)(1-\mu)(-\mu^2\left(n^2+6n-3\right)+\mu(n+5)(n-1)+2)}{(n(1-\mu)+2-3\mu)^2} \quad (20)$$

As can be seen in the derivative of the objective function of the public bank in order to c, for the values of μ and c established, it will always present a negative value.

$$-\frac{(1-c)(1-\mu)(-\mu^2\left(n^2+6n-3\right)+\mu(n+5)(n-1)+2)}{(n(1-\mu)+2-3\mu)^2} < 0 \text{ for } \mu \in \left[0,\frac{1}{2}\right[\text{ and } c < 1$$

Which means that, the c negatively affects the objective function, in the sense that the higher the c, the lower the objective function will be. This relationship makes sense, and its explanation is analogous to the one made for the profit of private banks previously.

  - For μ:

$$\frac{\partial \Omega}{\partial \mu} =$$

$$\frac{(1-c)^2(9\mu^3-n^3\mu^3-9n^2\mu^3-15n\mu^3+3n^3\mu^2+24n^2\mu^2+27n\mu^2-18\mu^2+11\mu-3n^3\mu-21n^2\mu-19n\mu+n^3+6n^2+5n-2)}{2(n(1-\mu)+2-3\mu)^3}$$

(21)

Finally, the impact of the parameter μ on the objective function of the public bank is positive. As can be seen, the derivative of the public bank's objective function in order to μ (for the values of μ and c established) always presents positive values.

Which means that the higher the μ, the higher will be the public bank's payoff. The higher μ, one can interpret that the bank is becoming more and more pure public (μ establishes the degree of nationalization of the firm)

After seeing the impact of the parameters on the obtained functions, it is necessary to look at the mark-up of the price over the marginal cost.

The mark-up can be observed in the profit function of private banks given that the mark-up is defined by the equation $\frac{c(1-\mu)(n+1)+1-2\mu}{n(1-\mu)+2-3\mu} - c = \frac{(1-c)(1-2\mu)}{n(1-\mu)+2-3\mu}$.

$$Mark-up = \frac{(1-c)(1-2\mu)}{n(1-\mu)+2-3\mu}$$

The mark-up provides a pretty solid idea of the banks' market power. So, making a quick analysis of the obtained function it is easy to verify and it was already expected that the higher is n, the lower will be the mark-up and consequently the market power (the more banks operate in the market in competition, the lower will be the market power of each individual bank).

As for the impact of the parameter μ, by deriving the mark-up in order to μ, the result is the following:

$$\frac{\partial}{\partial\mu}\left(\frac{(1-c).(1-2\mu)}{n(1-\mu)+2-3}\right) = \frac{(c-1)(n+1)}{(n(1-\mu)+2-3\mu)^2}$$

As it can be seen, the higher the μ, the lower the mark up and also the market power. This also makes sense because the public bank places more weight on consumer surplus.

7.2. Collusive agreement (Explicit collusion / Cartel situation)

In the cartel situation exemplified by the Cartel da Banca in the Portuguese banking market, participated m private banks and 1 public bank ($cartel\ state = m\ private\ banks + 1\ pulic\ bank$).

The private banks and the public bank that participated in the cartel chose their quantities, q_i^* and q_p^*, respectively that maximized $m\Pi_i + \Omega\ (q_i, q_p)$, while the private banks that stayed out of the cartel but were still active chose the quantity, q_l^* which maximizes $\Pi_l\ (q_l)$.

These quantities are obtained through the following FOCs:

- For the private banks inside the cartel:

$$\frac{\partial(m\Pi_i+\Omega)}{\partial q_i} = 0 => \ q_i = \frac{-2mq_p(1-\mu)-m(n-m)(1-\mu)q_l+m(1-c)}{m^2\left((1-\mu)+\frac{1}{m}\right)} \quad (22)$$

- For the public bank inside the cartel:

$$\frac{\partial(m\Pi_i+\Omega)}{\partial q_p} = 0 => \ q_p = \frac{-2mq_i(1-\mu)-(n-m)(1-2\mu)q_l+(1-c)(1-\mu)}{2-3\mu} \quad (23)$$

- For the private bank outside the cartel:

$$\frac{\partial\Pi_l}{\partial q_l} = 0 => \ q_l = \frac{1-q_p-mq_i}{n-m+1} \quad (24)$$

Combining expressions (22), (23) and (24) would give the final quantities of each bank type. However, the system of 3 equations gets somewhat complicated to do and to achieve any plausible result without some kind of simplification at the beginning. This simplification will be to define the n taking into account a specific year of operation of the Cartel da Banca namely, the year of its formation in 2002 when there were 53 banking institutions operating in Portugal and the m which will be equal to 13, as already stated from the introduction to the Cartel.

Considering $n = 52$ and $m = 13$:

$$\begin{cases} q_i = \dfrac{-26(1-\mu)q_p - 507(1-\mu)q_l + 13(1-c)}{169\left((1-\mu) + \dfrac{1}{13}\right)} \\ q_p = \dfrac{-26(1-\mu)q_i - 39(1-2\mu)q_l + (1-c)(1-\mu)}{2-3\mu} \\ q_l = \dfrac{1 - q_p - 13q_i}{40} \end{cases}$$

$$q_i^* = \frac{-26793\mu^3+40512149\mu^2-19395672\mu-402124-4c(6988\mu^2+10309173\mu-5144271)}{(-53+13\mu)(687\mu^2-1060712\mu+542836)} \quad (25)$$

$$q_p^* = \frac{52(66+401\mu+13c(-41+2\mu))}{687\mu^2-1060712\mu+542836} \quad (26)$$

$$q_l^* = \frac{89310\mu^3-135193064\mu^2+79227711\mu-6207473+13c(6988\mu^2+10309173\mu-5144271)}{10(-53+13\mu)(687\mu^2-1060712\mu+542836)} \quad (27)$$

7.2.1. Constraints for c and μ

It is again necessary to define the positivity conditions for the individual quantities described by the equations above, that is, we need to define constraints for the parameters c and μ. The process is similar to the one done for the individual quantities under competition.

The public bank's quantity expression is the simplest, so that may be the starting point. For this quantity to be positive, q_p^*, the following system of equations has to be respected:

$$\begin{cases} 66 + 401\mu + 13c(-41+2\mu) > 0 \\ 687\mu^2 - 1060712\mu + 542836 > 0 \end{cases} \vee \begin{cases} 66 + 401\mu + 13c(-41+2\mu) < 0 \\ 687\mu^2 - 1060712\mu + 542836 < 0 \end{cases} \quad (28)$$

This will lead to the following conditions:

$$\begin{cases} \mu > \dfrac{533c-66}{26c+401} <=> c < -\dfrac{401\mu+66}{26\mu-533} \\ \mu < 0.5119 \end{cases} \vee \begin{cases} \mu < \dfrac{533c-66}{26c+401} <=> c > -\dfrac{401\mu+66}{26\mu-533} \\ \mu > 0.5119 \end{cases}$$

In order to keep the scenarios comparable, it is necessary that it continues to be respected $\mu < 0.5$, so from this system it has to be chosen the constraints that originate from the left side conditions. It can be seen that from there, μ has to be smaller than 0.5119, which is even more comprehensive than $\mu < 0.5$. So, by choosing $\mu < 0.5$ both scenarios are satisfied

Regarding the restriction of c, it can be seen from the equation that it will depend on the value of μ chosen. For $\mu = 0$, the maximum value that c can have is $\frac{66}{533} = 0{,}1238$.

Looking at the expression of the quantity of private banks participating in the cartel, for it to be positive, the following system has to be respected:

$$\begin{cases} -26793\mu^3 + 40512149\mu^2 - 19395672\mu - 402124 - 4c(6988\mu^2 + 10309173\mu - 5144271) > 0 \\ (-53+13\mu)(687\mu^2 - 1060712\mu + 542836) > 0 \end{cases} \vee \begin{cases} -26793\mu^3 + 40512149\mu^2 - 19395672\mu - 402124 - 4c(6988\mu^2 + 10309173\mu - 5144271) < 0 \\ (-53+13\mu)(687\mu^2 - 1060712\mu + 542836) < 0 \end{cases} \quad (29)$$

The denominator of the equation, for the values of $\mu < 0.5$ is always negative, so it is necessary to choose the conditions that come from the bottom system.

$$c < \frac{-26793\mu^3 + 40512149\mu^2 - 19395672\mu - 402124}{4(6988\mu^2 + 10309173\mu - 5144271)}$$

Once again, the value of c depends on the value of μ.

Looking at the expression of the quantity of private banks outside of the cartel, for it to be positive, the following system has to be respected:

$$\begin{cases} 89310\mu^3 - 135193064\mu^2 + 79227711\mu - 6207473 + 13c(6988\mu^2 + 10309173\mu - 5144271) > 0 \\ 10(-53+13\mu)(687\mu^2 - 1060712\mu + 542836) > 0 \end{cases} \vee \begin{cases} 89310\mu^3 - 135193064\mu^2 + 79227711\mu - 6207473 + 13c(6988\mu^2 + 10309173\mu - 5144271) < 0 \\ 10(-53+13\mu)(687\mu^2 - 1060712\mu + 542836) < 0 \end{cases} \quad (30)$$

The denominator of the equation, for the values of $\mu < 0.5$ is also always negative, so it is necessary to choose the conditions that come from the bottom system for the parameter c.

$$c > \frac{-89310\mu^3 + 135193064\mu^2 - 79227711\mu + 6207473}{13(6988\mu^2 + 10309173\mu - 5144271)}$$

The value of c depends on the value of μ.

Looking only at the three restrictions of c separately can lead to complications in the conclusions being drawn.

The following figure shows the three constraints together (the blue line is for the constraint from system (28), the orange line is for the constraint from system (29) and the green line is for the constraint from system (30)) where it is possible to see the admissible region of values (red area) that satisfy all conditions.

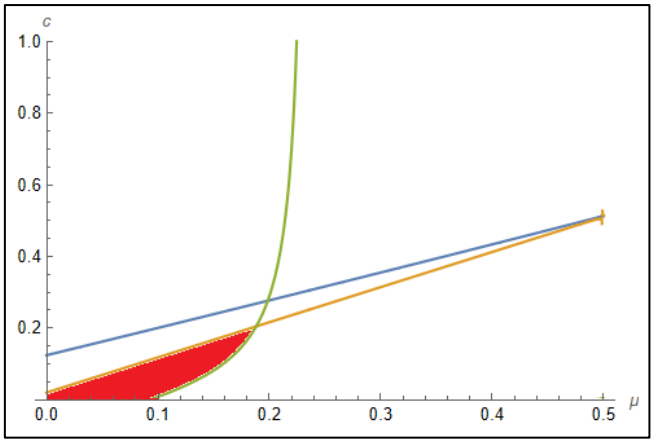

Figure 3: Constraints of c and μ

From here, by looking at the region of admissible values, it can be seen that the parameters have reduced their possible values. By seeing what are the values of the intersection of the green and orange lines it will be possible to define the values that the parameters can assume. The lines intersect at $\mu = 0.1882$ and $c = 0.2043$, which means that is c has a range of values between 0 and 0.2043 ($0 < c < 0.2043$ obtained by $\frac{-89310\mu^3+135193064\mu^2-79227711\mu+6207473}{13(6988\mu^2+10309173\mu-5144271)} < c < \frac{-26793\mu^3+40512149\mu^2-19395672\mu-402124}{4(6988\mu^2+10309173\mu-5144271)}$). These values are low, but still plausible, because generally the financing costs of banks are very low. μ has a range of values between 0 and 0.1882 ($0 < \mu < 0.1882$). Such low values were not expected for μ but we will come back to this later on.

$$\frac{-89310\mu^3+135193064\mu^2-79227711\mu+6207473}{13(6988\mu^2+10309173\mu-5144271)} < c < \frac{-26793\mu^3+40512149\mu^2-19395672\mu-402124}{4(6988\mu^2+10309173\mu-5144271)}$$

7.2.2. Total Quantity, Price and Payoffs

Knowing the individual quantities of each type of bank in the cartel situation, the total quantity available in the market, Q^*, can be deduced.

$$Q^* = 13q_i^* + q_p^* + 39q_l^* \tag{31}$$

$$= \frac{13\left(-249182\mu^2+42910613\mu-22783471+520c(-53+13\mu)(-41+2\mu)-c\left(6988\mu^2+10309173\mu-5144271\right)\right)}{10(-53+13\mu)(687\mu^2-1060712\mu+542836)}$$

Analogously to what was done in the competitive situation, to find the price when the cartel was formed, the inverse demand expression is used.

$$P^* = 1 - Q^* \tag{32}$$

$$= \frac{89310\mu^3-135017304\mu^2+74908071\mu+8482043-13c\left(6532\mu^2-10641453\mu+6274231\right)}{10(-53+13\mu)(687\mu^2-1060712\mu+542836)}$$

To finalize the application of the model in the cartel situation, it is necessary to obtain and analyze the expressions of the banks' objective functions. Everything that is needed to deduce these expressions is already known, so the profit of each type of bank is given by:

$$\Pi_i^* = (P^* - MC)q_i^*$$

$$\Pi_l^* = (P^* - MC)q_l^*$$

$$\Omega^* = \mu\frac{\left(13q_i + 39q_l + q_p\right)^2}{2} + (1-\mu)(P^* - MC)q_p^*$$

$$\Pi_i^* = (P^* - MC)q_i^* \tag{33}$$

$$= \frac{\left(26793\mu^3-40512149\mu^2+19395672\mu+402124+4c\left(6988\mu^2+10309173\mu-5144271\right)\right)}{\left(-89310\mu^3+135017304\mu^2-74908071\mu-8482043+c\left(89310\mu^3-138171754\mu^2+494407151\mu-206138077\right)\right)}$$
$$\frac{}{10(53-13\mu)^2(687\mu^2-10600712\mu+542836)^2}$$

$$\Pi_l^* = (P^* - MC)q_l^* \tag{34}$$

$$= \frac{-\left(89310\mu^3-135193064\mu^2+79227711-6207437+13c\left(6988\mu^2+10309173-5144271\right)\right)}{\left(89310\mu^3+135017304\mu^2-74908071-8482042+c\left(89310\mu^3-138171754\mu^2+494407151\mu-206138077\right)\right)}$$
$$\frac{}{100(53-13\mu)^2(687\mu^2-1060712\mu+542836)^2}$$

$$\Omega^* = \mu\frac{(Q^*)^2}{2} + (1-\mu)(P^* - MC)q_p^* \tag{35}$$

$$= \frac{13\left(\frac{13\mu\left(-249182\mu^2+42910613\mu-22783471+c\left(6532\mu^2-10641453\mu+6274231\right)^2\right)}{(53-13\mu)^2}\right)+}{13\left(\frac{80(-1+\mu)(401\mu+66+13c(-41+2\mu))\left(-89310\mu^3+135017304\mu^2-74908071-8482043+c\left(89310\mu^3-138171754\mu^2+494407151\mu-206138077\right)\right)}{-53+13\mu}\right)}$$
$$\frac{}{200(687\mu^2-1060712\mu+542836)^2}$$

Having defined all the relevant equations regarding the application of the model to the cartel situation it is necessary to again define the positivity conditions for the quantities post-cartel.

8. KEY COMPARISONS AND DISCUSSION

After the characterization of the two scenarios, that is, under competition and under the existence of the cartel, with the discovery of all the relevant expressions in each one of the scenarios, it is necessary to proceed to the comparison of the two. This step is one of the most important steps of this dissertation because it is from here that relevant information is discovered, first about the benefits or harms of the creation of this cartel, then the analysis on the incentives that each type of bank had to join the cartel and lastly on the incentives that they had to use the leniency program.

After this, once the cartel was in fact created, it is possible to draw conclusions about what were the conditions of its formation and after this what were the incentives for the banks to resort to leniency programs.

To study the benefits or harms of the creation of the cartel, it is necessary to see what happened to the total quantity and consumer surplus when comparing the two scenarios.

As far as the total quantity is concerned:

$$Q^N = \frac{(1-c)(n+1)(1-\mu)}{n(1-\mu)+2-3\mu}$$

$$Q^* = 13q_i^* + q_p^* + 39q_l^*$$

$$= \frac{13\left(-249182\mu^2 + 42910613\mu - 22783471 + 520c(-53+13\mu)(-41+2\mu) - c(6988\mu^2 + 10309173\mu - 5144271)\right)}{10(-53+13\mu)(687\mu^2 - 1060712\mu + 542836)}$$

The first action is to apply in the expression of the total quantity under competition the known values relative to banking activity in Portugal in 2002. From which it is already known that 53 banks operated in Portugal.

$$Q^N = \frac{(1-c)53(1-\mu)}{52(1-\mu)+2-3\mu} = \frac{53(1-c)(1-\mu)}{52(1-\mu)+2-3\mu}$$

$$Q^* = 13q_i^* + q_p^* + 39q_l^*$$

$$= \frac{13\left(-249182\mu^2 + 42910613\mu - 22783471 + 520c(-53+13\mu)(-41+2\mu) - c(6988\mu^2 + 10309173\mu - 5144271)\right)}{10(-53+13\mu)(687\mu^2 - 1060712\mu + 542836)}$$

Both expressions are subject to combinations of the parameters c and μ, so, for better visualization with the help of Wolfram Mathematica it is possible to visualize how the functions behave with the different possible parameter combinations in 3D plots[6].

First the graphs will be displayed individually and then the two graphs will be plotted together.

In the plots, the base axes are labeled with the parameters c and μ and show the possible values that both can take in each situation. On the height axis are the values that the function takes at each combination of the parameters.

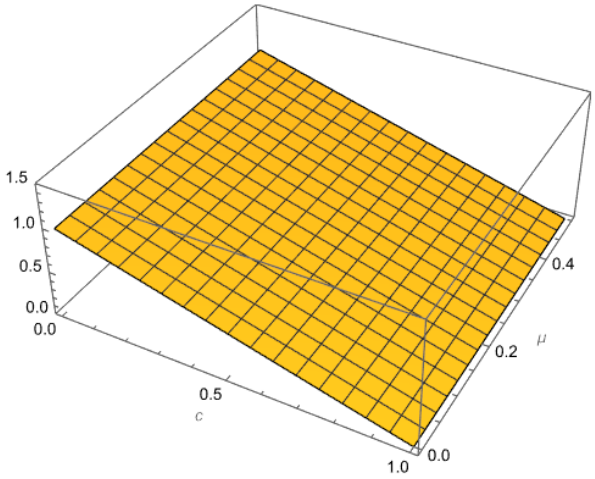

Figure 4: Total quantity under competition

This graph confirms the analysis made earlier regarding the impacts of the parameters, that is, the quantity is higher for smaller values of c and for higher values of μ.

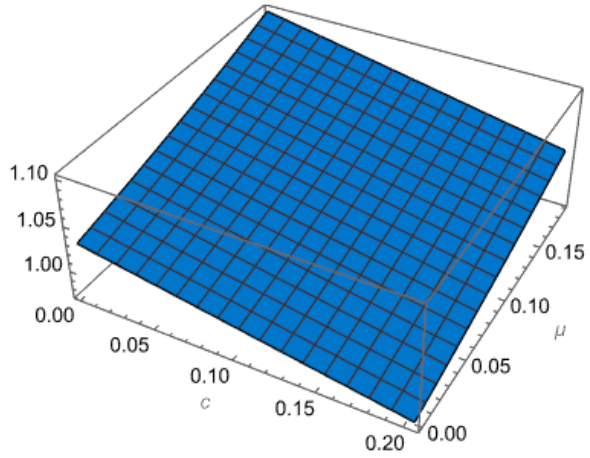

Figure 5: Total quantity under the existence of the cartel

[6] The grey areas at the bottom of some graphs mean that the function represented presents negative values in that area.

By plotting the 2 graphs together, it follows:

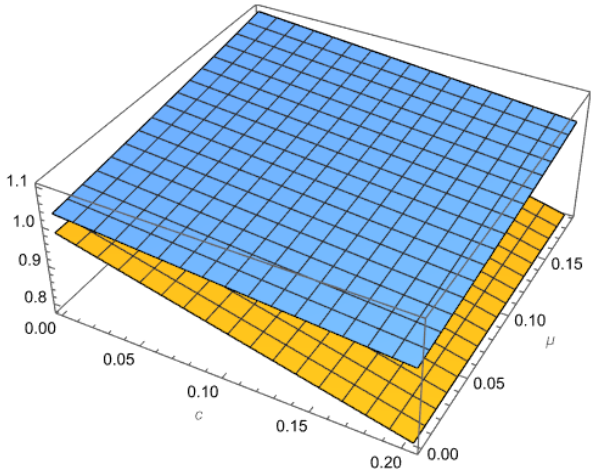

Figure 6: Plot of both quantities together

The scenario under the existence of a cartel, for all combinations of the parameters, shows higher values for the total quantity. In the cartel situation, the impact of c, is much smaller, as can be seen by the slope of the graphs, which means that the quantity reacts less to "good" stimulations to rise, meaning that a smaller c does not lead to a much higher quantity than a higher one.

This can be explained because, although there is a cartel, the profit maximization of private banks is done in combination with the optimization of the public bank's objective function, which values the consumer surplus and causes the quantity to increase.

As far as consumer surplus is concerned, the same will have to be done:

$$CS^N = \frac{(52q_i + q_p)^2}{2} = \frac{(Q^N)^2}{2}$$

$$CS^* = \frac{\left(13q_i + 39q_l + q_p\right)^2}{2} = \frac{(Q^*)^2}{2}$$

As can be seen, the total quantity being part of the expression of consumer surplus, this will also be dependent on combinations of the parameters c and μ.

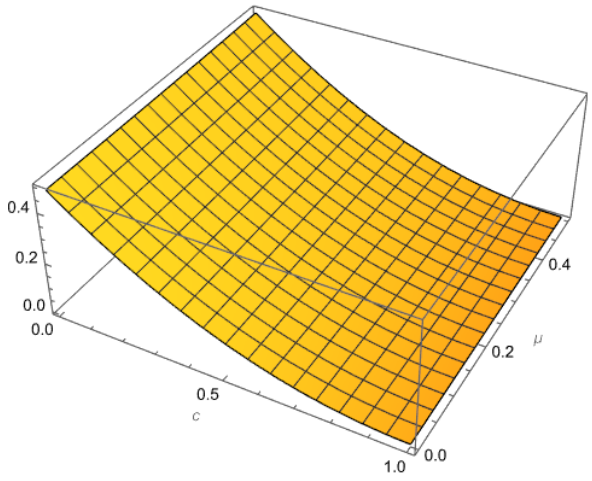

Figure 7: Consumer surplus under competition

Even though it is barely perceptible, there is an increase in consumer surplus as μ increases, which makes sense and corroborates the analysis made earlier; on the other hand, it can clearly be seen that as c increases, the consumer surplus decreases, meaning that, the impact of c is more notable than the impact of μ for the scenario under competition.

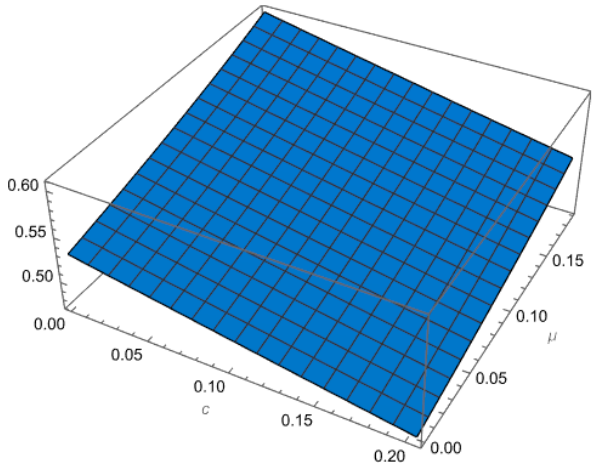

Figure 8: Consumer surplus under the existence of a cartel

Here the opposite happens to what happens in the scenario under competition, since, in this case, the impact of μ is more noticeable than the impact of c. The explanation for this is similar to the one given above for the total quantity in which, under cartel, the consumer surplus also reacts less to "good" stimulations to rise, meaning that a smaller c does not lead to a much total surplus than a higher one.

Plotting the two graphs together results in the following:

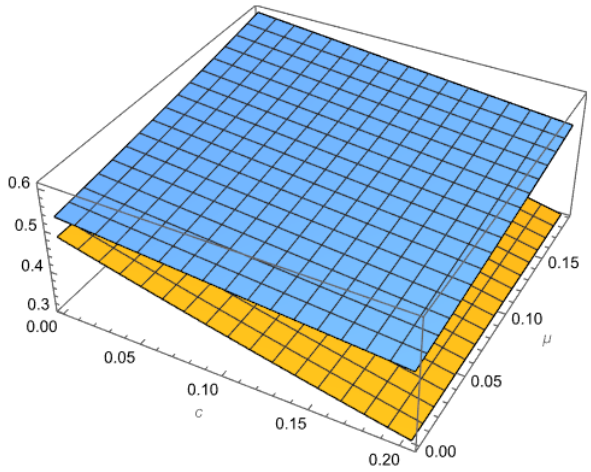

Figure 9: Plot of both consumer surpluses together

By looking at the graphs it can be seen that clearly there is one scenario that always comes out on top, the one under the existence of the cartel.

This result is highly contradictory to the existing theory on the impacts on a market due to the existence of a cartel, namely the fact that theoretically the total quantity and consumer surplus would decrease, but the explanation is the same as the one given for what happened with the total quantity, that the profit maximization of the private banks is done in combination with the optimization of the objective function of the public bank that values the consumer surplus, thus contributing to its increase.

Having said this, now let's look at both bank's expressions of individual quantities and payoffs under both scenarios. By comparing them, the goal is, together with the conclusions that have already been drawn above, to verify their incentive for the formation of the cartel.

8.1. Incentive to form the cartel

- Private bank

$$q_i^N = \frac{(1-c)(1-2\mu)}{52(1-\mu)+2-3\mu}$$

$$q_i^* = \frac{-26793\mu^3+40512149\mu^2-19395672\mu-402124-4c\left(6988\mu^2+10309173\mu-5144271\right)}{(-53+13\mu)(687\mu^2-1060712\mu+542836)}$$

$$q_i^* = \frac{89310\mu^3-135193064\mu^2+79227711\mu-6207473+13c(6988\mu^2+10309173\mu-5144271)}{10(-53+13\mu)(687\mu^2-1060712\mu+542836)}$$

Following the same comparison procedure used before, the quantity of the private bank under competition will be compared with the quantities of the private banks inside and outside the cartel.

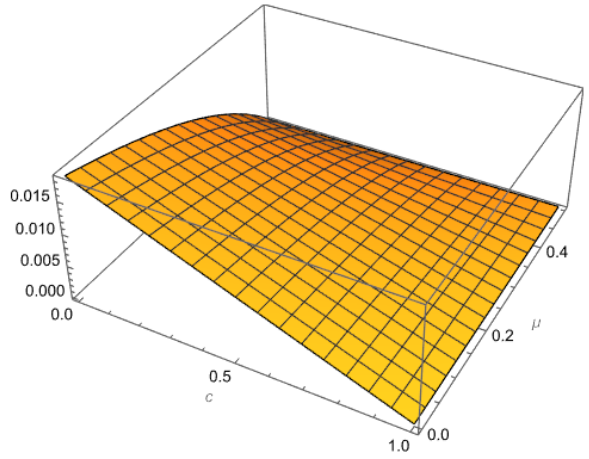

Figure 10: Individual quantity of the private bank under competition

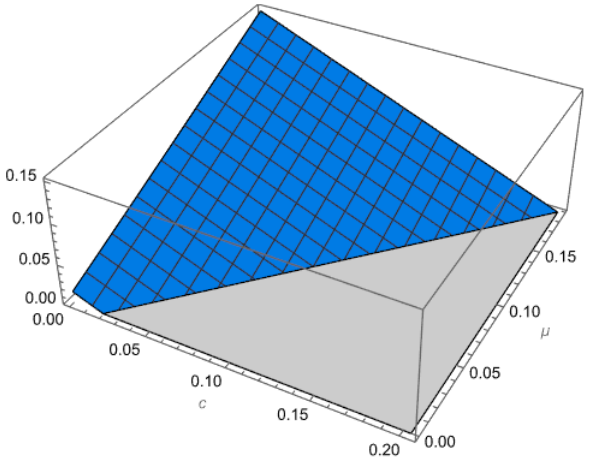

Figure 11: Individual quantity of the private bank inside the cartel

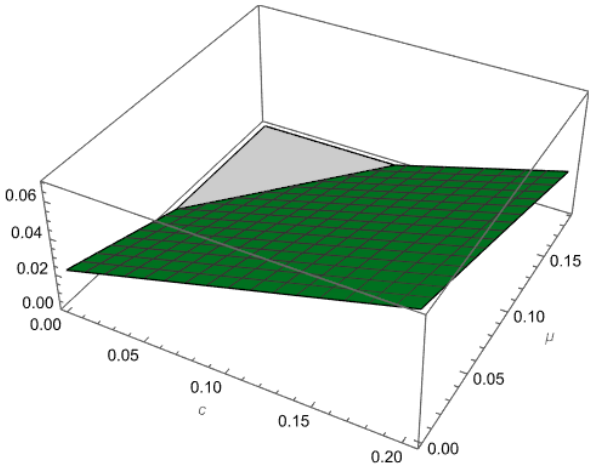

Figure 12: Individual quantity of the private bank outside the cartel

To draw better conclusions, the following plot shows everything together.

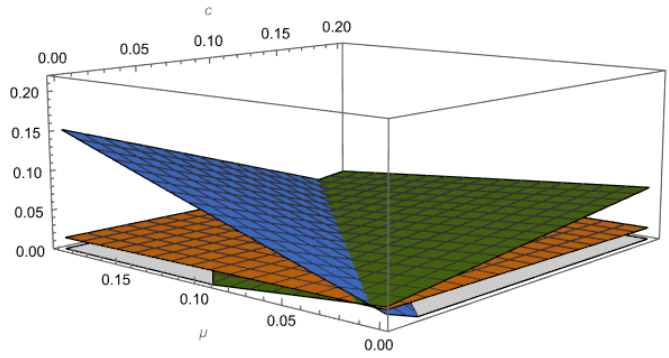

Figure 13: Plot of all quantities of the private banks together

Note: This graph is presented from a different point of view from the others to allow better visualization and to be able to draw better conclusions.

The individual quantity produced by the private banks under competition is always positive for all combinations of the parameters within the set values. However, the same is not true when looking at the quantity produced by these banks under the existence of the cartel, whether they participate in it or not.

From this point of view it is possible to see the region of values that make all three quantities positive.

In this region, two situations happen: on the left side of the intersection of the quantity produced by the private banks inside and outside of the cartel, the lowest quantity produced is by those outside of the cartel, followed by the quantity produced by private banks under competition, and the highest quantity produced is by the banks participating in the cartel.

On the right side of the intersection, the order reverses, meaning that the lowest quantity is produced by the banks participating in the cartel, followed by the quantity produced by the private banks under competition and finally the highest quantity is produced by the banks that stay out of the cartel. Therefore, one conclusion that can be drawn from this is that with the existence of a cartel, depending on the combination of the parameters, there will always be private banks producing more than if they were under competition.

The allowable zone on the left side of the intersection is much larger than the allowable zone on the right side, so it is much more likely to choose a parameter combination that falls on the left side zone, resulting in the private banks inside the cartel being the largest producers.

Stating the conditions of the quantities as a starting point, the important thing is to see what happens to the profit in both scenarios, as it is the most crucial factor for the incentive to form cartel.

$$\Pi_i^N = \frac{((1-c)(1-2\mu))^2}{(52(1-\mu)+2-3\mu)^2}$$

$$\Pi_i^* = \frac{\left(26793\mu^3-40512149\mu^2+19395672\mu+402124+4c\left(6988\mu^2+10309173\mu-5144271\right)\right)}{10(53-13\mu)^2(687\mu^2-10600712\mu+542836)^2}\left(-89310\mu^3+135017304\mu^2-74908071\mu-8482043+c\left(89310\mu^3-138171754\mu^2+494407151\mu-206138077\right)\right)$$

$$\Pi_i^* = \frac{-\left(89310\mu^3-135193064\mu^2+79227711-6207437+13c\left(6988\mu^2+10309173-5144271\right)\right)}{100(53-13\mu)^2(687\mu^2-1060712\mu+542836)^2}\left(89310\mu^3+135017304\mu^2-74908071-8482042+c\left(89310\mu^3-138171754\mu^2+494407151\mu-206138077\right)\right)$$

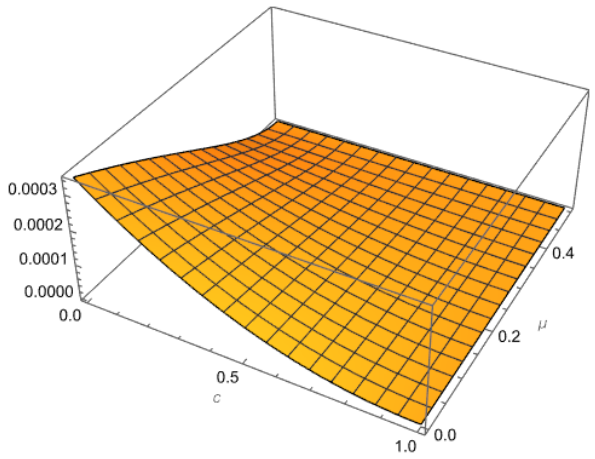

Figure 14: Profit of the private bank under competition

The parameter μ is once again the main responsible for the variance, in this case of the profit; the higher the μ, the lower the profit, and for very high values of μ, the profit is virtually 0.

The same is true for c, in the sense that, with very high values of this parameter, the profit is also virtually 0. However, it is said that μ has more impact, because it has the same effect with a smaller range of values.

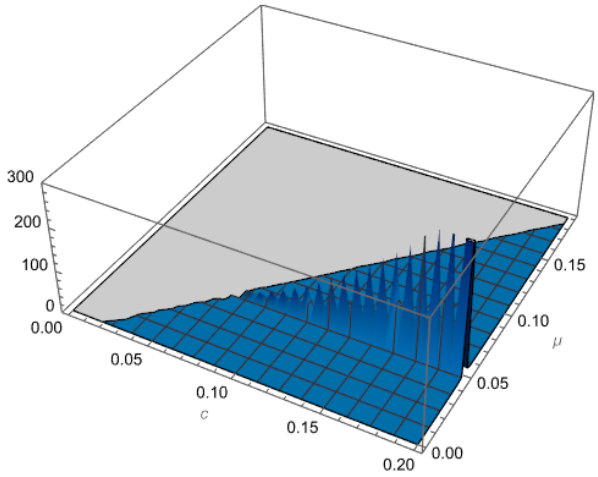

Figure 15: Profit of the private bank inside the cartel

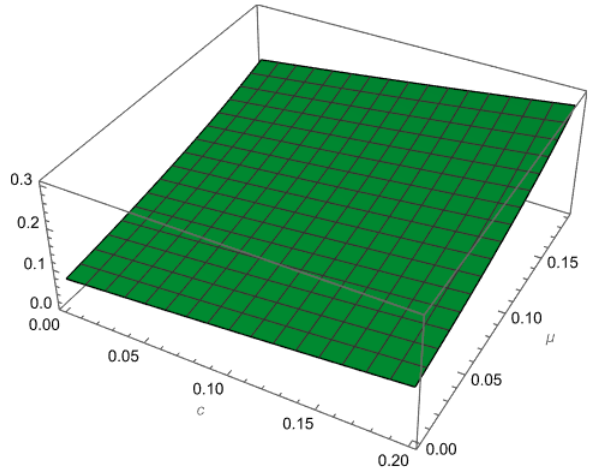

Figure 16: Profit of the private bank outside the cartel

Plotting the two graphs together results in the following[7]:

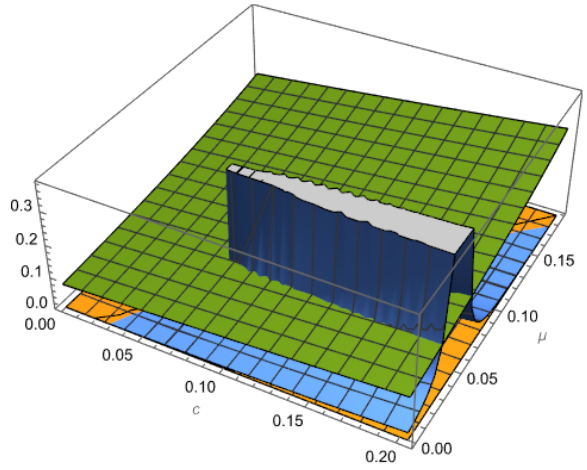

Figure 17: Profits of the private banks in the different scenarios

Observing the graph above, the payoff under competition is the lowest of the three situations, as expected. As for the payoff under the existence of the cartel, for the most part, the payoff is higher when staying out of the cartel, but for the low values of μ, as we can see there is an increase in the banks' payoffs when participating in the cartel, forming that bump, so for the values of μ under this bump, the best option is to join the cartel.

The fact that the payoffs of the ones outside the cartel grow more than the payoff of the ones inside the cartel is a free-riding incentive. The reason for this happening is because by joining in a cartel, the banks became "softer" and internalized the adverse effects of the competition that existed between them, benefiting the ones outside.

For very low μ, it is given little importance to consumer surplus, so it is as if this is almost not a mixed cartel but a normal one where the incentive for free-riding is generalized.

For μ slightly higher, more importance is given to consumer surplus, so the mixed cartel effect is stronger making the private firms inside the cartel benefit little from it, so it is logical that the outsiders are the main beneficiaries.

The bump not existing for low values of c can be explained as follows: for low c, firms inside the cartel being concerned with consumer surplus is irrelevant, in the sense that this concern leads to lower prices, however, with small c, the price would not be so high to begin with. Thus, μ matters little for distinguishing between firms inside and outside the cartel, so the cartel effect predominates, and the incentive for free-riding prevails.

[7] For a better visualization of the three graphs, the scale of the z-axis was reduced, with this a grey section appears on the graph of the banks' profit inside the cartel, which only means that this graph presents values larger than the presented scale

- Public bank

$$q_p^N = \frac{(1-c)(1+51\mu)}{52(1-\mu)+2-3\mu}$$

$$q_p^* = \frac{52(66+401\mu+13c(-41+2\mu))}{687\mu^2-1060712\mu+542836}$$

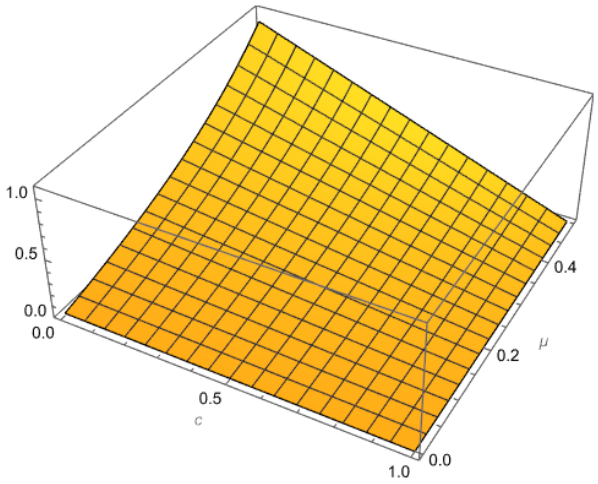

Figure 18: Individual quantity of the public bank under competition

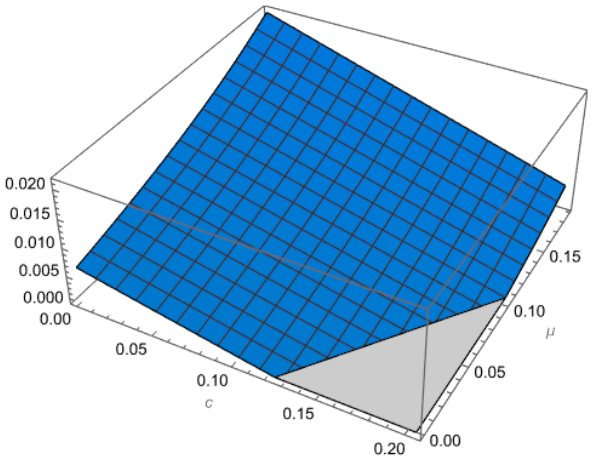

Figure 19: Individual quantity of the public bank under the existence of the cartel

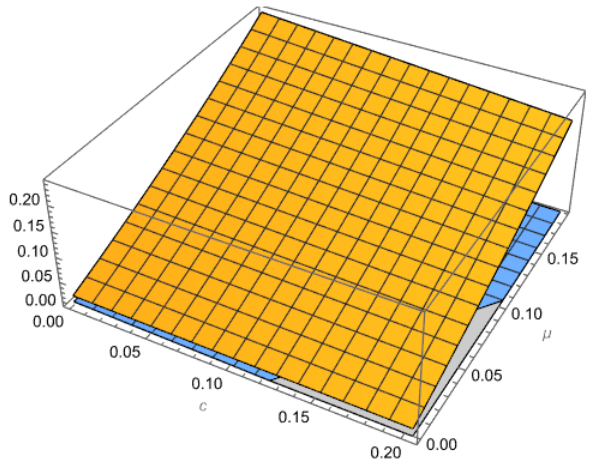

Figure 20: Plot of both individual quantities together

In this graph, it can be seen that the scenario under competition leads the public bank to always produce higher quantity for all combinations of the parameters. One of the typical effects of the creation of a cartel is the reduction of the quantity produced.

For the situation of the public bank, the approach is the same, that is, having seen the scenarios regarding the quantities, now let's see what happens with its objective function.

$$\Omega^N = \frac{(1-c)^2(1-\mu)(-\mu^2(3013)+\mu(57)(51)+2)}{2(52(1-\mu)+2-3\mu)^2} = \frac{3013(-\mu)^2(1-\mu)(1-c)^2+2907\mu+2}{2(52(1-\mu)+2-3)^2}$$

$$\Omega^* = \frac{13\left(\frac{13\mu\left(-249182\mu^2+42910613\mu-22783471+c\left(6532\mu^2-10641453\mu+6274231\right)^2\right)}{(53-13\mu)^2}\right)+13\left(\frac{80(-1+\mu)(401\mu+66+13c(-41+2\mu))\left(-89310\mu^3+135017304\mu^2-74908071-8482043+c\left(89310\mu^3-138171754\mu^2+494407151\mu-206138077\right)\right)}{-53+13\mu}\right)}{200(687\mu^2-1060712\mu+542836)^2}$$

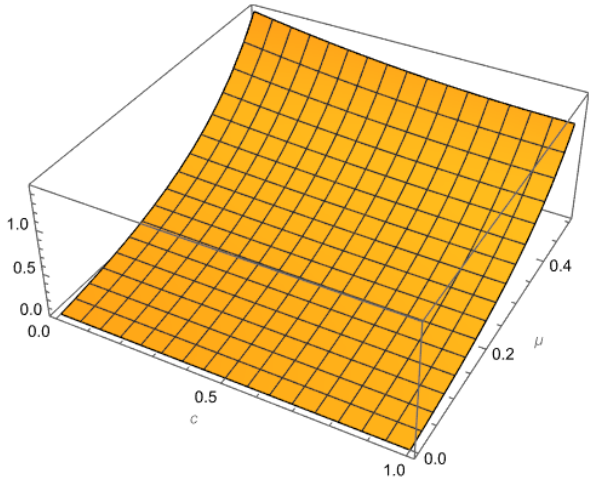

Figure 21: Objective function of the public bank under competition

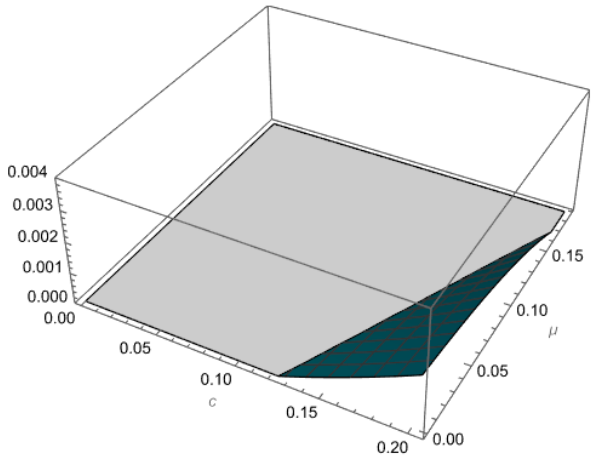

Figure 22: Objective function of the public bank under the existence of the cartel

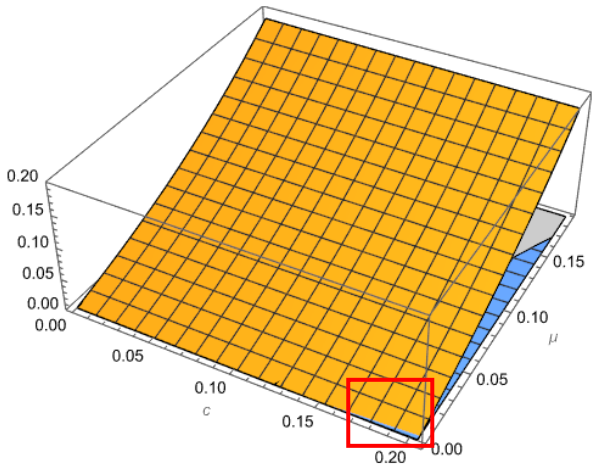

Figure 23: Plot of the objective function of both scenarios together

Putting the two graphs together in the same plot, it is possible to see that what happens with the objective function is very similar to what happens with the individual quantity, that is, in the vast majority of the parameter combinations, the objective function presents higher values in the scenario under competition, however, for values of μ very close to 0, the best option is to join the cartel. In this region, the public firm has practically no regard for consumer surplus, so it is like a normal cartel, in which there is a gain in participating.

It should be noted that a small variation of μ is enough for the public company to prefer to stay out of the mixed cartel.

Developing further the small conclusions drawn when joining all the graphs in the same plot, in the various points of study, now the main objective will be to verify and explain if the different types of banks had motivation and incentive to form the cartel and, in case of a positive response, to make it clear what were the conditions for each one. In the end, since the study is

applied to the specific case of the Cartel da Banca, given that in fact the cartel was formed, it will be possible to take some insights about what was the reality practiced in terms of the definition of the parameters.

As seen earlier, the major target for the private banks is the profit maximization. Regarding this aspect. For private banks, assuming acceptable values for μ, the best alternative for the private banks would be to stay out of the cartel since it was the option that produced the highest profit for most of the parameter combinations. For values of μ very close to 0, the public bank would present a very extreme posture of pure profit maximization however, theoretically in the real world, this situation would not really happen, since the public bank is expected to assign plausible values to μ.

The major target for the public bank is the maximization of the consumer surplus weighted with its own profit. With regard to the consumer surplus, there is a clear advantage from the cartel's existence because it causes its increase, that is, in this case the cartel promoted the increase of the consumer surplus. As far as the objective function of the public bank is concerned, under the cartel scenario, the public bank, for the most part, comes off a bit worse relative to the scenario under competition.

With all this said, each private bank would only have an incentive to join the cartel if they knew that the public bank placed a very low weight on the consumer surplus, since that is the only way that it is the most favorable scenario for them. Regarding the public bank's incentives, the low values of μ would tempt the private bank to join and by doing so, the consumer surplus would be higher, which is a big advantage when considering joining the cartel.

8.2. Consequences of the existence of the cartel

The cartel did exist, and both the public bank and some private banks were part of it. From this statement it is possible to define under what conditions the cartel was formed. So, putting all the previous information together, it is possible to see that for both types of banks to participate in the cartel, the public bank must have assigned a very low value to the parameter μ, from which it can already be concluded that after all the public bank adopted a very extreme position, which is not exactly in accordance with what is expected from a state-owned company.

From the existence of the cartel, it is possible to assess the consequences arising from it, namely in terms of the total quantities available, consumer surplus, individual quantities and in the end the resulting price.

The total quantity available and consumer surplus after the formation of the cartel were higher than when all banks were under competition, which is a positive aspect. This increase in quantity lies mainly in the increase in the individual quantity produced by the private banks, since with the existence of a cartel the quantity produced by the public bank is lower. The biggest contributors to the increase in total quantity are those participating in the cartel because,

as seen before, most combinations of the parameters would lead to these being the ones responsible for the highest production.

This last point, explaining the increase, comes as somewhat unexpected, since it was exactly the opposite that was predicted to happen, which was that the public bank would be the responsible for the increase, since it is responsible for weighting the consumer surplus that is highly related to the total quantity available (higher quantity, higher consumer surplus).

For the price the following happens:

- $P^N = \frac{53c(1-\mu)+1-2\mu}{52(1-\mu)+2-3\mu}$
- $P^* = \frac{89310\mu^3-135017304\mu^2+74908071\mu+8482043-13c(6532\mu^2-10641453\mu+6274231)}{10(-53+13\mu)(687\mu^2-1060712\mu+542836)}$

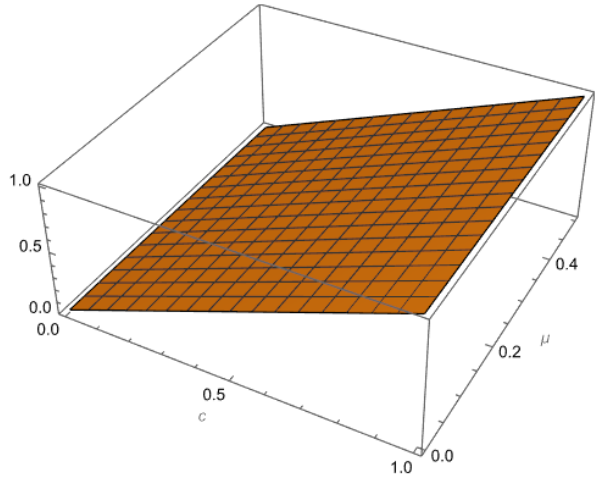

Figure 24: Price under competition

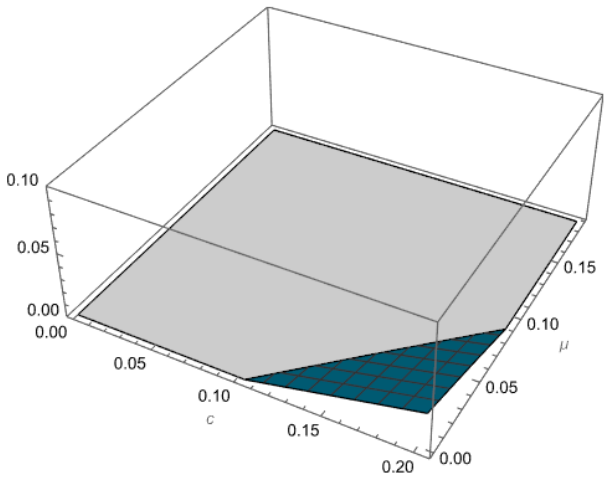

Figure 25: Price under the existence of the cartel

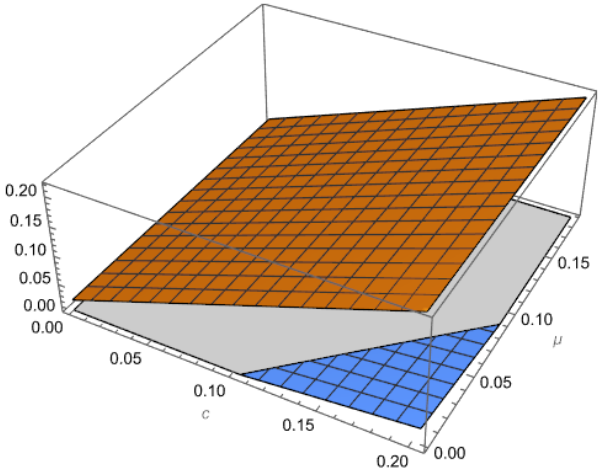

Figure 26: Plot of both prices together

The individual graph of the price under the existence of the cartel has an interesting appearance as it allows to clearly see areas where the combinations of the parameters that result in a negative price, which is not possible, so it is possible to clearly see the areas where banks can operate with the price being positive.

For the consumer, the best option is the one that offers the lowest price, which occurs under cartel (and is consistent with the consumer surplus increasing with the cartel, which also occurs in this case). The reason this happens is exactly the same as the one given for the explanation of why quantity and consumer surplus increase under the existence of the cartel, which is that the private banks inside the cartel also care about the consumer surplus, causing the price to go down. Therefore, as long as the cartel is mixed, it is possible that there are advantages for the consumer.

8.3. Incentive to use the leniency program

From the analysis of the graphs, regarding this part of the incentive to report the cartel, and to resort to the leniency programs proposed by the AdC, for Barclays, knowing that with the cartel all parties involved were better off than when they were competing, the only explanation why it reported the cartel is that after 11 years of cartel activity, it was no longer being able to bear the extra quantity that they needed to produce when compared to the competition scenario that was required by the cartel participation scenario.

On the other hand, surely the bank officials would have known well in advance (certainly during the cartel activity) that in 2014 the bank would exit the Portuguese banking market, perhaps they saw this as an opportunity to leave the cartel without any kind of charges or fine to be paid.

Upon exiting the market, there is incentive to report the cartel as there is nothing more to be gained from it. That said, the game comes to an end. For Barclays this game went from an

infinitely repeated game (this is the case for all the banks in the cartel) to a finite repeated game. In infinitely repeated games, it is found that the preferred strategy is not to play a Nash strategy of the stage game, but to cooperate and play a socially optimum strategy, which means that it can lead to collusion, which can be sustainable under certain conditions, like for example, having an effective trigger strategy as potential punishment ensures that cooperating has more utility to the player than acting selfishly and facing the other player's punishment in the future. However, in finitely repeated games, with a fixed and known number of time periods, applying the backwards induction method talked about previously, the best individual option is always chosen, so there is no more incentive for cooperation, leading Barclays to expose the cartel.

9. CONCLUSION

Having seen all the existing literature on the topics relevant to this dissertation, it was possible to focus the study on a specific type of cartel: mixed cartel, where not only private firms participate but also public firms. The existence of these cartels is very rare since it is not common for public firms to want to participate in cartels because they have objective functions in which they always take consumer surplus into account.

Therefore, by developing the theoretical model and characterizing in detail two scenarios (competition and cartel existence) and then applying it to the specific case of the Cartel da Banca, it was possible to compare these two scenarios in which it was possible to reach some initial conclusions so, given that the cartel was indeed created and was in operation for about 11 years, with all the analysis done it is possible to draw some very relevant conclusions that may lead to the discovery of information pertinent to the case.

According to the results of the model, the creation of the cartel led to the consumer benefiting greatly, both in total quantity available and in the increase of the consumer surplus (it had been one of the main arguments of the banks, the fact that customers benefit).

Taking yet again the conclusion reached above that the only way the cartel could have been created with banks of both types was by assigning very low values to μ, this way, the role of the public bank to optimize the social function becomes distorted, not having much concern for consumer surplus (with μ very low, it becomes just a profit maximizer like the other private banks). By being profit-maximizing, the existence of the cartel caused its profit to increase. So, this suggests that the cartel was created in a highly unconventional manner since the only way the private banks that decided to incorporate the cartel knew about the value of μ, was if it had been the public bank that had told them this information and had taken the initiative to create the cartel or if it had been an arrangement between all of them and there had been consent from the public bank to set the value low (which does not make it any less serious), which is quite unconventional since cartels generally arise from private companies and it is not very common for a public company to be part of a cartel, much less to take the initiative to create one.

This is said because, as seen, the profit of the private banks was higher if they stayed out of the cartel, unless the weight given to the consumer surplus was very low, so the conclusion is that the public bank in attempting to form the cartel informed the low value assignment to the weight given to the consumer surplus to convince the private banks to join.

The biggest beneficiary, whether premeditated or not, with the cartel was the public bank, specifically CGD, because it decreased the quantity produced and for the low values of μ increased its profits.

However, the biggest result with the biggest policy implications regarding what is known about cartel creation is that the creation of mixed cartels can lead to benefits in terms of total quantity available and more importantly, contribute to increased consumer surplus. Therefore, the

prosecuting authorities have to pay attention and take this into consideration when they are dealing with decisions involving cartels with state-owned companies.

For a final note, it must be said that there were some limitations and therefore some assumptions had to be made, namely assuming that all players have the same cost function and produce the same quantity. In reality, it is known that they all produce different quantities and that due to their internal costs their cost functions will also be different. However, this two was essential to keep the model tractable.

Having said that, two suggestions for future work related emerge, since the funding costs of banks are generally very low, redo the model using parameter c equal to 0, making it easier to see the dependence parameter μ; admit different costs in a possibly still tractable way, by considering that companies differ by a common constant to “adjacent” firms when ranked in terms of efficiency.

REFERENCES

Badra, Y. (2022). Price wars in strategical price-setting supergames: Application to the retail oil industry. Australian Economic Papers. https://doi.org/10.1111/1467-8454.12252

Binmore, K., Rubinstein, A., & Wolinsky, A. (1986). The Nash Bargaining Solution in Economic Modelling. The RAND Journal of Economics, 17(2), 176–188. https://doi.org/10.2307/2555382

Byford, M. C., & Gans, J. S. (2014). Collusion at the extensive margin. International Journal of Industrial Organization, 37(1). https://doi.org/10.1016/j.ijindorg.2014.08.001

Choi, J. P., & Gerlach, H. (2012). Global cartels, leniency programs and international antitrust cooperation. International Journal of Industrial Organization, 30(6). https://doi.org/10.1016/j.ijindorg.2012.05.005

Ciliberto, F., Watkins, E., & Williams, J. W. (2019). Collusive pricing patterns in the US airline industry. International Journal of Industrial Organization, 62. https://doi.org/10.1016/j.ijindorg.2018.07.008

CMU School of Computer Science. (n.d.). https://www.cs.cmu.edu/afs/cs/academic/class/15859-f01/www/notes/mat.pdf

Correia-Da-Silva, J., & Pinho, J. (2018). Collusion in mixed oligopolies and the coordinated effects of privatization. Journal of Economics/ Zeitschrift Fur Nationalokonomie, 124(1). https://doi.org/10.1007/s00712-017-0560-6

Cosnita-Langlais, A., & Tropeano, J. P. (2022). Learning by litigating: An application to antitrust commitments. International Journal of Industrial Organization, 80. https://doi.org/10.1016/j.ijindorg.2021.102795

Escrihuela-Villar, M., & Guillén, J. (2014). On the sustainability of collusion in a differentiated oligopoly with a cartel and a fringe. Bulletin of Economic Research, 66(S1). https://doi.org/10.1111/boer.12023

Evenett, S. J., Levenstein, M. C., & Suslow, V. Y. (2001). International cartel enforcement: Lessons from the 1990s. World Economy, 24(9). https://doi.org/10.1111/1467-9701.00408

Feinberg, R. M. (2016). Cartel Stability: Determinants and International Evidence. Review of Industrial Organization, 48(4). https://doi.org/10.1007/s11151-016-9503-6

Fonseca, M. A., Li, Y., & Normann, H. T. (2018). Why factors facilitating collusion may not predict cartel occurrence — experimental evidence. Southern Economic Journal, 85(1). https://doi.org/10.1002/soej.12278

Garrod, L & Olczak, M. (2018). Explicit vs tacit collusion: The effects of firm numbers and asymmetries. International Journal of Industrial Organization, 56, (C), 1-25. https://doi.org/10.25932/publishup-47388

Gillet, J., Schram, A., & Sonnemans, J. (2011). Cartel formation and pricing: The effect of managerial decision-making rules. International Journal of Industrial Organization, 29(1). https://doi.org/10.1016/j.ijindorg.2010.03.015

Gilo, D., & Spiegel, Y. (2018). The antitrust prohibition of excessive pricing. International Journal of Industrial Organization, 61. https://doi.org/10.1016/j.ijindorg.2018.05.003

Gössl, F., & Rasch, A. (2020). Collusion under different pricing schemes. Journal of Economics and Management Strategy, 29(4). https://doi.org/10.1111/jems.12392

Grzybowski, L., & Karamti, C. (2010). Competition in mobile telephony in France and Germany. Manchester School, 78(6). https://doi.org/10.1111/j.1467-9957.2009.02155.x

Gual, J., & Mas, N. (2011). Industry Characteristics and Anti-Competitive Behavior: Evidence from the European Commission's Decisions. Review of Industrial Organization, 39(3). https://doi.org/10.1007/s11151-011-9290-z

Haraguchi, J., & Matsumura, T. (2018). Government-leading welfare-improving collusion. International Review of Economics and Finance, 56. https://doi.org/10.1016/j.iref.2017.11.005

Harrington, J. E. (2022). The Anticompetitiveness of a Private Information Exchange of Prices. International Journal of Industrial Organization, 85. https://doi.org/10.1016/j.ijindorg.2021.102793

Harrington, J. E., & Chang, M. H. (2009). Modeling the birth and death of cartels with an application to evaluating competition policy. Journal of the European Economic Association, 7(6). https://doi.org/10.1162/JEEA.2009.7.6.1400

Harrington, J. E., & Wei, Y. (2017). What Can the Duration of Discovered Cartels Tell Us About the Duration of All Cartels? Economic Journal, 127(604). https://doi.org/10.1111/ecoj.12359

Harrington, J. E., & Ye, L. (2019). Collusion through Coordination of Announcements. Journal of Industrial Economics, 67(2). https://doi.org/10.1111/joie.12199

Hu, W. M., Xiao, J., & Zhou, X. (2014). Collusion or competition? Interfirm relationships in the chinese auto industry. Journal of Industrial Economics, 62(1). https://doi.org/10.1111/joie.12035

John Wooders Distinguished Research Professor of Economics. (2022, September 13). John Nash and his contribution to game theory and economics. https://theconversation.com/john-nash-and-his-contribution-to-game-theory-and-economics-42355

Lefouili, Y., & Roux, C. (2012). Leniency programs for multimarket firms: The effect of Amnesty Plus on cartel formation. International Journal of Industrial Organization, 30(6). https://doi.org/10.1016/j.ijindorg.2012.04.004

Levenstein, M. C., & Suslow, V. Y. (2006). What determines cartel success? In Journal of Economic Literature (Vol. 44, Issue 1). https://doi.org/10.1257/002205106776162681

Mariuzzo, F., Ormosi, P. L., & Majied, Z. (2020). Fines and reputational sanctions: The case of cartels. International Journal of Industrial Organization, 69. https://doi.org/10.1016/j.ijindorg.2020.102584

Mérő, L. (1998). John von Neumann's Game Theory. In: Moral Calculations. Springer, New York, NY. https://doi.org/10.1007/978-1-4612-1654-4_6

MIT libraries logo MIT libraries. (n.d.). https://libraries.mit.edu/150books/2011/04/07/1951/

Mota, F., Correia-da-Silva, J., & Pinho, J. (2020). Welfare-Improving Mixed Collusion. https://www.fep.up.pt/docentes/joao/material/research/mixedcollusion.pdf

Ordóñez-De-Haro, J. M., Borrell, J. R., & Jiménez, J. L. (2018). The European Commission's Fight against Cartels (1962–2014): A Retrospective and Forensic Analysis. Journal of Common Market Studies, 56(5). https://doi.org/10.1111/jcms.12708

Strategic games - UMD. (n.d.). https://cramton.umd.edu/econ414/ch02.pdf

UPC Universitat politècnica de catalunya. (n.d.). https://www.cs.upc.edu/~mjserna/docencia/agt-miri/slides/AGT13-coop-GT.pdf

van den Berg, A., & Bos, I. (2017). Collusion in a price-quantity oligopoly. International Journal of Industrial Organization, 50. https://doi.org/10.1016/j.ijindorg.2016.12.001

von Blanckenburg, K., Geist, A., & Kholodilin, K. A. (2012). The Influence of Collusion on Price Changes: New Evidence from Major Cartel Cases. German Economic Review, 13(3). https://doi.org/10.1111/j.1468-0475.2011.00558.x

Zucchini, L., Claussen, J., & Trüg, M. (2013). Tariff-mediated network effects versus strategic discounting: Evidence from German mobile telecommunications. International Journal of Industrial Organization, 31(6). https://doi.org/10.1016/j.ijindorg.2013.04.002