Propagation of electromagnetic waves in non-hermitian materials

Rodrigo Nuno Diógenes Moreira da Silva
rodrigo.moreira.da.silva@tecnico.ulisboa.pt

Instituto Superior Técnico, Lisboa, Portugal

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Abstract

Metamaterials are materials which exotic properties allow electromagnetic waves to propagate in unnatural ways. In this sense, non-Hermitian materials arise, which, not necessarily being composed of metamaterials, allow the absorption/amplification of the energy of the waves that pass through these media. Precisely because they present matrices of their constituent parameters that are non-Hermitian, these materials allow situations that transcend the mere compensation of losses that would naturally be observed in any medium. In this work, a theoretical model of a non-Hermitian metamaterial, is studied and developed, governed by the operating principle of a transistor, to build an implementation of the same, based on a system of transmission lines periodically charged by transistors. To this end, a few models are built, equivalent to each other, and several comparisons are made to confirm the validity of the respective models and analogies between them.

Keywords: Metamaterial, Wave, Non-Hermitian, Transmission line

1. Introduction

1.1. Background

Metamaterials have been widely analysed in recent years, but their concept had already been discovered longer ago, more precisely, in the late 19th century. Furthermore, the field skyrocketed with great names like Victor Veselago and John Pendry [1, 2]. Non-Hermitian materials, on the other hand, started to be explored more recently and, in a first stage, the concept was introduced in quantum physics, having later discovered its possible applications in the field of optics. Also of great interest are non-reciprocal materials, which guarantee a different response depending on the direction, in a given direction. Their current applications, in terms of electromagnetic solutions, involve the construction of non-reciprocal elements in microwave networks. More specifically, these systems have been used to implement isolators and electromagnetic circulators, based on ferrites [3].

In addition to these applications, several other topologies have been analysed by researchers still regarding materials with non-Hermitian characteristics, resorting to metamaterials and their exotic properties. In [4], a theoretical model inspired by the operation of MOSFETs is used. After establishing the permittivity matrix of the theoretical model of the metamaterial, the wave constants and the electric fields of the supported modes are calculated, and subsequently the mean value of the Poynting vector, thus verifying the non-hermiticity of the system, by the "sinusoidal" oscillation that occurs as a function of position.

Finally, an implementation of an electromagnetic isolator is proposed. This is based on the idealized metamaterial arranged between orthogonal (to each other) linear polarizers. Thus, if a wave passes through the isolator in one direction, another wave will not pass in the opposite direction, due to its anisotropic response. In addition, this paper looks at the levels of unidirectional transmission as a function of the thickness of the metamaterial, which, as noted above, directly influences the balance between loss and gain present in it. Like the power beat, the transmission coefficient will also vary periodically with thickness.

In addiction to this, there are a number of other ways that one could break the reciprocity like with a static magnetic field bias [5, 6], by exploiting nonlinear effects [7, 8] and PT-symmetry [9, 10]

Whatever the branch of the scope of the application investigated, one of the interesting conclusions of these studies is the fact that it is possible, through the dimensioning of the constituent materials of the proposals presented, and furthermore, through their own properties to achieve balances between gain and loss that can translate the transcendence of the mere compensation of losses, i.e., reaching the amplification of the signals.

1.2. Motivation

The propagation of electromagnetic waves is, in general, subject to Lorentz's Law of Reciprocity when it comes to conventional dielectric materials, which are reciprocal linear systems. This states that propagation in these media is obligatorily bidirectional. However, this principle can be transgressed if the characteristics of the wave propagation media, i.e. the materials themselves, can be changed.

Metamaterials are artificial materials whose optical properties are not normally found in nature (such as having a negative refractive index), exhibiting unconventional behaviour and responses. Non-Hermitian systems are media that can locally amplify or absorb the electromagnetic waves propagating in them, depending on the structure of the materials (which may or may not be metamaterials) that constitute them, which is carefully chosen during their synthesis, to obtain certain responses. These may be, for example, non-reciprocal responses, in which case it is possible to circumvent the Law of Lorentz, thus allowing the propagation of electromagnetic waves to occur in a unidirectional manner, which is useful for the construction of non-reciprocal elements such as insulators and circulators.

Having said that, these materials (both non-Hermitian materials and meta-materials in general) have a great potential for exploration and their development may have strong impacts in the fields of optics and photonics, as well as computing and information transmission, and may be used in several applications both at military level (for example in the production of sensors such as radar and camouflage devices) and in telecommunications (for example in the production of antennas with high gain within the fifth generation of technology).

2. MOSFET-metamaterial

2.1. Concept and model of the system under consideration

As already mentioned, the MOSFET behaves as a non-reciprocal, non-linear medium. The voltage (electrical field) applied to its gate terminal influences the width of the channel formed between its drain and source terminals (thus, the impedance of the same), allowing the passage of more or less current.

Taking this behaviour into account, and through some algebraic manipulation, it is possible to infer a permittivity matrix for a hypothetical metamaterial. One can conclude that the above matrix is non-Hermitian, which in practice means that the material can absorb or generate energy.

2.2. Propagation of supported modes

Let us define the plane wave modes supported by the system. By manipulating Maxwell's equations, one can obtain a three-dimensional ellipsoid, which represents the iso-frequency surface of the medium, containing all possible solutions of the wave equation, considering the whole space. From this, one can see that, for each direction of space, the medium supports two distinct modes, to which two polarizations will be associated.

2.3. Characterization of the energy flow

Consider now, in order to explore the interaction of the two waves and how it can influence the response of the analysed materials, the superposition of the two waves. To study the existing energy flow, we now use the Poynting vector generated by this superposition, or more precisely, we calculate its average value. Then, one can observe its behaviour, for all the directions of the space.

In the cases of propagation in x and z we observe that the components of the Poynting vector parallel to the direction of propagation are constant. This is because the fields of the two waves are in quadrature. It is also observed that in the case of the Poynting vector for propagation along x, there are 3 non-zero space components, which implies that the Poynting vector is not parallel to the direction of propagation, a case that can only occur in anisotropic media. In this case, the x and z components are constant but there is a "sinusoidal" oscillation in the y component. This suggests that not only is the Poynting vector not parallel to the direction of propagation, but its direction still depends on x and does not have the same direction at all positions.

For the case of the y propagation, on the other hand, an interesting situation is observed. The Poynting vector periodically increases and decreases with an approximately sinusoidal behaviour. This shows that such a material can generate or absorb energy in certain segments.

One can then see which polarizations lead to the different gain/loss values in the considered medium. This is done by analysing the power dissipated by the material, which corresponds to the symmetric of the gain verified in each position. By analysing the extremes of the dissipated power, one can infer that for a left circular polarization the gain is maximum. For right-handed circular polarization, it is the losses that are maximum. As for linear polarization, there is no gain nor losses.

3. Analogy with transmission line theory

3.1. Definition of the transmission lines to be analyzed

To study the proposed analogy with the MOSFET-metamaterial problem formerly described, one begins by manipulating the Maxwell's equations, establishing a correspondence between these equations and the equations of the transmission lines.

Restricting the problem to the y direction and considering that the electric permittivity is not a scalar but a matrix, one can find the relation between the voltage and the current (and even the inductance and capacitance parameters) at a given point of two transmission lines, which are arranged in parallel.

Afterwards, by assembling the found expressions, we can deduce the transmission matrix of the system, which is interconnected with the modes discovered previously.

Furthermore, computing the Poynting vector in the line system and observing its behaviour, one can conclude that because it is similar to the one observed before, the analogy that was made is valid.

3.2. Concretization of the studied model

In order to realize the theoretical concept of the set of unilaterally coupled transmission lines presented, we use a set of conventional (decoupled) lines periodically loaded by MOSFET's, in a structure of blocks with length d, as the solution of the exact problem. For this, we proceed to the study of the transmission matrices of the various constituent blocks of the proposed structure, to afterwards multiply them to find the global transmission matrix. The values that were used for the transistor parameters were taken from [3].

4. Validation of the studied model

4.1. Solution to the effective problem

The same problem is then analysed in a more practical and intuitive way, generalizing the concept behind the transmission line system studied earlier. The effect of the transistors is distributed over the two lines to discard the periodic structure loaded by them and, instead, obtain a homogeneous, simpler system.

4.2. Problem comparison and frequency response

To compare the two mentioned problems, we intend to observe the frequency responses of each of the *scattering* parameters obtained for each case. For this purpose, consider the two systems with the lines terminated with adapted loads at one of the ports (these ports being on opposite sides between the two lines). In this way, the system is now interpreted as a two-port network. Once one defines the parameters of each system, namely, inductance and capacitance parameters and length, we proceed to calculate the *scattering* matrix of the two-port networks, by computing first the transmission matrices of the same, which are identical, thus confirming the validity of the model of the effective problem.

Finally, having each of the parameters of the *scattering* matrix, it is possible to represent their behaviour depending on the frequency and depending on whether it is the exact problem or the effective problem.

As can be seen, the graphs for the exact and effective problems are similar with only slight differences, thus once again demonstrating the validity of the effective model used.

Regarding the graphs themselves, it can be said that a deviating behavior is observed after a certain (high) frequency, which can be explained by the effect caused by the capacitors of the MOSFET. Namely, the reduction of their impedances, which makes the propagation no longer occur only in one direction inside the transistors themselves, makes the propagation no longer occur only in one direction in the whole structure.

This coupled with the fact that the length of the systems (in particular, the lines) has been dimensioned for a maximum inferior to the limit of the scale which was considered and therefore, the validity of the models is compromised from then on, it was chosen not to represent the *scattering* parameters beyond a certain point (in this case, 30 GHz).

Note also that, as expected, the modulus of the parameter that describes the transmission in the "forbidden" direction is approximately zero, such behavior being due to the non-reciprocal response of both systems, and that the modulus of the parameter that describes the transmission in the opposite direction, for low frequencies is greater than 1, which proves that both systems are amplifiers. In contrast, the parameters that describe the reflection in both ports cannot be (and are not) greater than 1, since the signal that is observed in these measurements does not pass through the elements responsible for amplification (the transistors). More specifically, due to the non-reciprocal behavior, if the signal passes through the transistors, it does not return to the source port, if it does not pass through the transistors, it is not amplified.

5. Conclusions

5.1. Final Remarks

In short, non-Hermitian materials are media that have the peculiarity of presenting a non-Hermitian matrix of electrical permittivity (in the particular case of this work), and therefore can generate or absorb energy from the waves that propagate through them.

There are also non-reciprocal materials, where the propagation of electromagnetic waves is not the same in both ways, in a given direction. These can be natural materials such as ferrites, which form the basis for many of today's applications, as non-reciprocal elements in microwave networks.

Alternatively, either set of materials can be implemented using meta-materials (artificial materials with non-natural optical properties), opening space for the exploration of more exotic solutions.

The study and development of these materials can greatly benefit areas such as optics, computing, information transmission, and military and telecommunications applications.

In this work, we start from a theoretical model of a metamaterial, based on the operation of a transistor, and we evaluate its optical properties, namely its non-hermiticity and, on the other hand, its non-reciprocity and anisotropy.

Namely, starting from the basic principle of a MOSFET, we define the electric permittivity matrix and then the wave constants of the propagation modes supported in each propagation direction. From this, one can also derive the electric fields of the modes themselves, and from a superposition of these, for each of the directions, one obtains the average value of the Poynting vector as a function of position. Here we conclude that, in the y direction, the Poynting vector both increases and decreases, in an approximately sinusoidal fashion, and as such, the energy of the waves passing through the metamaterial is both amplified and absorbed, respectively, confirming its non-hermiticity. It is also observed that, for the x direction, the Poynting vector is not parallel to the propagation direction, having components in all three directions of space, and, particularly in the y direction, the same variable behaviour is observed (but with a magnitude of lower order) as observed for the Poynting vector in the case of the propagation direction according to y. It is also concluded, using the principle of conservation of energy for a given system, that, depending on the polarization of the wave crossing the metamaterial (which is variable through the weighting variables introduced during the previous mode superposition), the derivative of the Poynting vector curve varies for the same position, which implies that the level of energy absorption/amplification (loss/gain) of the medium for that same position, also varies. Namely, for a linear, left circular and right circular polarization the observed gain is null, maximum and minimum, respectively (at the $y = 0$ position).

Next, and assuming the propagation direction according to y, we use Maxwell's equations and the "telegraph equations" to find an analogy between the electric and magnetic fields of the model studied so far and the voltages and currents of a set of two coupled transmission lines arranged in parallel. With this analogy, a transmission matrix of this system is obtained and a comparison between the two models is established. After obtaining the Poynting vector it is verified that it presents the same behaviour as a function of position (now on the transmission line) as it does for the metamaterial, concluding then on the validity of the analogy between the two models.

Then, we assume a physical system based on the same two transmission lines arranged in parallel, but now periodically coupled by transistors. By defining the transmission matrix of each block of the system, it is also possible to define the global matrix of the system. Next, we define the equivalent effective problem, which consists of a homogeneous system of two transmission lines coupled by the distributed effect of a MOSFET. The transmission matrix of the new system is established, and this is compared with that of the exact system, and in the end, it is concluded that the effective model is valid, for the matrices are similar.

Next, the **S** matrix of the two systems is calculated and the behaviours of the *scattering* parameters are compared, finally confirming, by the proximity of the former, the validity of the effective model.

5.2. Future Work

Fortunately, the goal of this work was concluded. However, it could and would be interesting to complete it with the additional study of other directions of space, namely the study of a system built in the x direction, and more than observing the behavior of the parameters of *scattering* of the same, now in this final stage of the work, explore all the implications that the fact that the Poynting vector is not parallel to the propagation direction has.

It would also be interesting to complete this work by actually building, in the lab, the transistor-laden transmission line system used for the exact problem. Study different scalings (as opposed to this paper, where only one was studied) and conclude on the relative relevance that the previous findings. Namely, to understand to what extent the parameters of *scattering* were more affected by the validity of the system sizing than by the shorted capacitors inside the transistors due to the increased frequency.

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