On the use of dark stores in omnichannel last-mile distribution

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Abstract

Retailers need to adapt their operations to follow the evolution of e-commerce. Additionally, customers rising expectations for fast deliveries and shopping experience underline the need for improvement in order fulfillment. Omnichannel is a potential solution that by integrating online and offline channels reduces friction for customers. However, it is challenging to implement by retailers.

In this work, we investigate the use of dark stores as a potential fulfillment node to support online demand in an omnichannel retail network. This research studies the performance of three channel designs: SFSW - ship from store and warehouse; SFDSW - ship from dark store and warehouse; and SFSDSW - ship from store, dark store and warehouse. The fulfillment problem is formulated as a profit maximizing mixed integer linear programming (MILP). Warehouses and dark stores' locations can be decided through the model solution, whereas stores are given as retailer assets that can be maintained or closed. Product flows between all echelons are also determined. The demand is treated in an endogenous and elastic manner, as a function of distance and levels of competition in online and store markets. Three different product categories are separately treated in the model: electronics, fashion and food.

Results evaluate the operational efficiency and profitability of different network configurations, showing that the SFSW and SFSDSW generate greater profit than the SFDSW. The electronics category is the most profitable of all. Dark stores do not leverage profit as the traditional facilities do, however, there is space for optimization and growth.

Keywords: Dark store, Omnichannel, E-commerce, Facility location optimization

1. Introduction

At the same time consumers' expectations on retailers are growing, products are expected to be delivered in shorter times and a wide range of product-offering is wanted. Retailers should proceed not only to the digitalization of operations, but also to the automation of those. The ultimate goal is to extend automation to the entire value chain, increasing operational agility and improving customer experience. Traditional retailers are used to eliminate redundancy in order to reduce costs and increase efficiency.

Another consumption shift the pandemic brought was the boost in online demand. Online sales increased dynamically in April and May 2020, due to the closure of physical shops. Taking this into consideration, retailers have to adapt to the online channel, building an omnichannel experience. According to Verhoef et al. [11], omnichannel is defined as the synergetic management of the various channels available and customer contact points, in order to optimize the customer experience and the performance of the chain along all channels. The emergence of omnichannel has completely revolutionized the traditional e-commerce by integrating all customer touch-points into an integrated holistic experience. However, it is quite a challenge to implement by retailers.

With the rise of e-commerce, new delivery modes emerged to make products available to consumers. A recent fulfilment option that has been boosted by the pandemic is the dark store. The literature on this new delivery mode is quite scarce, since it is a recent phenomenon. According to Bryson J. R., (2021) [3], a dark store is a small logistic hub located in high-density urban centre that only serves online customers in a short period of time.

The literature covers also the order fulfillment problem. According to Croxton, (2003) [4], order fulfillment is not only about filling customers' orders efficiently and effectively, but it is also about designing a network and a process that allows a firm to meet customer requests while minimizing the total delivered cost. Order fulfillment includes the generation, the filling, the delivery, and the service of customer orders. In this context, there is a gap in the use of dark stores as a potential fulfillment node to support online demand in an omnichannel retail network, assisting decision-makers in the order fulfillment process.

Retailers have the responsibility to make strategic decisions on how to implement omnichannel distribution. This decision covers whether to fulfill online orders from warehouses, stores, dark stores, or a combination of all. While some research has been done on this decision (Li & Jia, (2019) [9]; Aouad & Ganapathi, (2020); and Millstein, (2022) [10]), more insights are needed. Our study includes three channel designs: (i) ship from store and warehouse (SFSW), (ii) ship from dark store and warehouse (SFDSW), and ship from store, dark store, and warehouse (SFSDSW). These configurations are illustrated in figure 1. Circles represent the markets, including online and store demand. Suppliers are represented with rhombus. The warehouses in the illustration are shown as squares, stores as triangles, and dark stores as rectangles.

The SFSW channel design has been modeled in the Millstein, (2022) [10] study. The SFDSW and SFSDSW were adpated from this original channel design with the incorporation of dark stores.

The main objective of this paper is to study the advantages of a network configuration with dark stores, versus a traditional configuration with stores and warehouses ensuring the last-mile distribution. To deepen our understanding of omnichannel designs, the study was extended to different product categories: Electronics, Fashion and Food. Depending on the product characteristics, and for different levels of online and store demand, this research will give the best configuration with the optimal quantities, locations and capacities for the omnichannel facilities, to maximize profit.

The rest of the paper is structured as follows. Section 2 will summarize the literature reviewed, identifying a research gap for this study. The formalization of the topic for our study is discussed in section 3 along with key modeling concerns. Section 4 presents the results and its analysis. Conclusions and limitations regarding this paper are presented in section 5, along with some recommendations for future work.

2. Literature Review

For a better understanding of the problem in hands and to look for a new searching direction, all operational models reviewed in the literature were categorized into seven parameters in table 1. This first column is the reference and the next four columns indicate the channel designs considered. Columns six and seven indicate whether the model considers home delivery or collection and delivery points for the last-mile. The next two columns show whether the model considers online or store demand (or both). Columns ten to fourteen present the type of decision the models incorporate. The type of objective function is also evaluated, followed by the assessment of channel approach. Finally, the last three columns describe the formulation and solving methods, indicating the type of model, in which kind of problem is the model formulated, and the solver, algorithms and heuristics used.

The review of literature presented exhibits that research on order fulfillment in an omnichannel retail setting remains scarce. Many authors have highlighted this gap in literature and identified the assessment of different order fulfillment configurations as a future area of research. [6] The aim of this study is to fill this gap (see Table 1) through an optimization model that: (i) compares three different network configurations (including shipping from dark store); (ii) includes the last-mile delivery; (iii) responds to both online and store demand; (iv) accounts the following decisions: Facilities' locations, capacity and closure, and Order fulfilment; (v) maximizes profit; (vi) has an omnichannel approach; and finally (vii) uses CPLEX as the solution solver.

3. Problem and model formulation

This section formulates the omnichannel warehouse location problem as a profit maximizing mixed-integer programming (MILP) model incorporating costs for transportation to serve online and store demands, costs for warehouses (inventory, handling and fixed costs), costs for stores (inventory, handling), and costs for dark stores (handling and fixed). We model this on a network where markets are represented by nodes that include online demand and may also include stores with retail demand.

Three different channel designs were modeled based on the following characteristics:

- **Ship from store and warehouse (SFSW)** This channel design is the most traditional. With three echelons, the network starts in the suppliers that provide products to the warehouses. Here, products either follow two directions: 1) are shipped to the stores, or 2) are shipped directly to customers of any market to fulfill online demand. The products that end in the stores can be purchased by customers physically or can also be shipped to any market to fulfill online demand. This channel design was built upon the Millstein et al. (2022) [10] model.

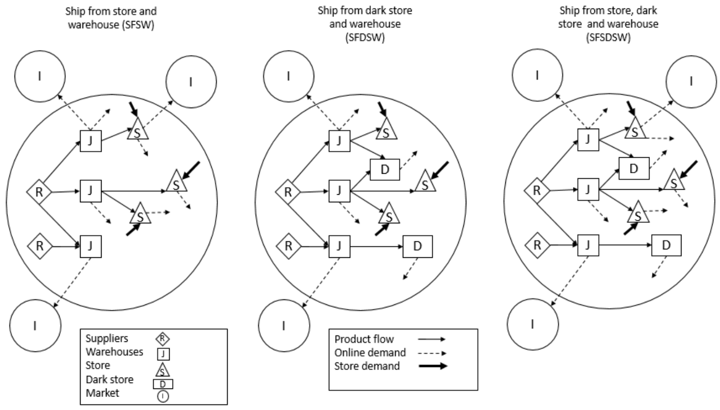

Figure 1: Channel designs illustration.

Table 1: Literature review and research contributions.

Article	Network configuration				Last-mile delivery		Demand		Decision					Objective function		Approach		Formulation and solving		
	Ship from store	Ship from warehouse	Ship from store and warehouse	Ship from dark store	Home	CDP	Online	Store	Facilities' location	Facilities' capacity	Facilities' closure	Vehicle routing	Order fulfilment	Min cost	Max profit	MC	OC	Model	Problem formulation	Solution method
Aksen and Altinkemer (2008)[1]	X				X		X	X	X	X	X	X		X		X		Discrete Optimization	LRP	Lagrangian relaxation method (LR)
Janjevic et al. (2021)[7]		X			X	X	X		X		X	X		X			X	CA Optimization	LRP	Gurobi
Janjevic et al. (2019)[8]		X			X	X	X		X					X		X		CA Optimization	2E-CLRP	Gurobi; Heuristic
Li & Jia (2019)[9]		X			X		X						X	X			X	Optimization	MILP	CPLEX; Benders decomposition algorithm
Ishfaq & Bajwa (2019)[6]			X		X		X						X		X	X		Optimization	MINLP	KNITRO; Outer Approximation (AO) technique
Aouad & Ganapathi (2020)[2]		X		X	X	X	X			X	X		X	X			X	Optimization	MILP	Gurobi
Millstein et al. (2022)[10]	X	X	X		X		X	X	X	X	X		X		X		X	Optimization	MILP	CPLEX
This paper	X	X	X	X	X		X	X	X	X	X		X		X		X	Optimization	MILP	CPLEX

- **Ship from dark store and warehouse (SFDSW)** This channel design adds the dark store to the previous network. It is analogous to the SFSW model until the warehouses echelon. Here, products either follow three directions: 1) are shipped to the stores, 2) are shipped to dark stores, or 3) are shipped directly to customers of any market to fulfill online demand. The products that end in the stores can only be purchased by customers physically, unlike the SFSW formulation. In the dark stores, products are shipped to the same market to fulfill online demand.

- and **Ship from store, dark store and warehouse (SFSDSW)** This channel design is the combination of the SFSW and SFDSW models. It is analogous to the SFDSW model until the distribution from the warehouses to the stores and dark stores. The difference lies in the fact that both stores and dark stores can satisfy the online demand. With the slight distinction that stores can ship products to all markets whereas dark stores ship to the same market only.

The three channel designs are represented in figure 1, for a better understanding of the differences.

3.1. Sets

Let I be the set of markets and let J be the set of candidate (potential) warehouse locations, where $J \in I$. Let R be the set representing suppliers, with $R \in I$. Let S be the set of store locations, where $S \in I$, and D the set of dark store locations, where $D \in I$. Let K be the set of warehouse capacities. The list of sets is present on table 2.

3.2. Functions and parameters

All input parameters for the functions are listed in table 2. Cwl_j, Crl_s, and Cdl_d are the unitary handling cost of an online order fulfilled from a warehouse j, a store s, and a dark store d, respectively. Let Crh_s be the unitary inventory holding cost of a store located at s. The total demand in market i is $Demand_i$. The demand is exogenous, and corresponds to the total average units. This parameter includes both the online and store demand, that is why $OLpct$ is needed to indicate the percentage of demand available to the online channel. G indicates the percentage of store demand available to the store channel. WDD is the maximum distance that can be covered during a work day. Let F be a demand adjustment coefficient. The parameter $Distance$ represents the distance in kilometers between every pair of facilities/entities. $profit$ represents the gross profit (sell-price of purchase) for the retailer for both the online and store channel. Parameters Cwl_j, Crh_s, Crl_s, $Demand_i$, F, G, $OLpct$, $profit$ were adapted from Millstein et al. (2022) model, and parameters Cdl_d, $Distance$, WDD were originally defined.

All functions are listed in table 2. The first six formulas concern the shipping costs between two entities. The shipping cost is achieved by multiplying the distance by a cost factor depending on the route taken.

Regarding demand, Dr_i is the demand available at the retailer's stores at the physical channel in market i. Don_i is the total online demand in market i. $Donl_{i(j,s)}$ represents the total online demand for market i available to the retailer from a specific facility (warehouse or store) at location j or s. This formula reflects the influence of the delivery time in the demand. F_i is a demand adjustment exogenous coefficient for market i that reflects demand elasticity based on delivery time and the level of competition in the online market. The fraction $\frac{1}{T_{i(j,s)}*F}$ captures the benefits of fast delivery and the level of online competition. $Donl_{id}$ is analogous to $Donl_{i(j,s)}$ for online ordered served from a dark store at d. $T_{i(j,s,d)}$ represents the number of delivery days needed to ship a product from a fulfillment facility to an online market i. This is achieved by dividing the distance by the maximum distance that can be covered during a work day WDD.

$pcow_{ij}$ represents the profit achieved for an online order in market i fulfilled from a warehouse located at j, by subtracting the warehouse handling cost and the shipping cost from the gross profit. pcr_s represents the unitary profit when a product is purchased physically at a store at s, subtracting the inventory holding cost from the gross profit. $pcos_{is}$ is the profit per unit for an online order in market i fulfilled from a store at s, obtained by subtracting the handling cost and the shipping cost from the gross profit. Finally, the last profit formula ($pcod_{id}$) is for online orders fulfilled from dark stores at d, resulting from a subtraction of both handling and shipping costs from the gross profit. Parameters $Csow_{ij}$, $Csos_{is}$, $Csrw_{sj}$, $Csrr_{jr}$, Dr_i, Don_i, $T_{i(j,s,d)}$, $pcow_{ij}$, pcr_s, $pcos_{is}$ were adapted from the Millstein et al. (2022) model, and parameters $Cdrw_{dj}$,$Csod_{id}$, $Donl$, $pcod_{id}$ were originally defined.

3.3. Decision Variables

There are two types of decision variables in this problem: binary ones, which indicate if a given entity or facility is opened or not, and continuous variables. All decision variables are summarized in table 2. There are three binary variables concerning the use of three facilities. Y_{jk} denotes if warehouse of size k is opened at location j, St_s indicates if a store is open at s, and Dt_d that shows if a dark store is to be opened at location d. The remaining decision variables are non-negative and continuous. The first is X_{ij}, indicating the number of units shipped to online market i from a warehouse located at j, then RS_{jr} is the number of units shipped to a warehouse located at j from a supplier located at r, then V_{sj} the total number of units shipped to a store located at s from a warehouse located at j, and P_{dj} the number of units shipped to a dark store located at d from a warehouse located at j. Qs_s is the quantity of units sold to store customers at a store located at s and U_{is} the quantity of the ones shipped to online market i. Finally, W_{id} indicates the number of units shipped to online market i from a dark store located at d.

3.4. Constraints for the Ship from Store and Warehouse (SFSW) model

The objective function for SFSW model is stated in equation 1. The first three terms are the gross profit earned from store and online sales (for orders fulfilled by warehouses and by stores), respectively. The fourth term is the warehouses fixed costs. The fifth term is the warehouse inventory holding cost, which depends on the number of units shipped to fulfill online orders (X_{ij}) and the number of units shipped to stores (V_{sj}). This term is nonlinear as the binary warehouse size and location variable (Y_{jk}) is multiplied by the continuous flow variables (X_{ij} and V_{sj}). To linearize the term, Millstein et al. (2022) [10] defined the positive variable WHC_{jk} as the warehouse holding cost and replaced the nonlinear term with $\sum_{j\in J}\sum_{k\in K} WHC_{jk}$, by adding the constraint 3. In this constraint, the constant M is set to be higher than the sum of the maximum possible inventory holding cost of a warehouse. Note that if Y_{jk} is equal to 1, then equation 3 requires the warehouse holding cost in the objective to be at least as large as the sum of order flows times the holding cost per unit. Because of the profit maximization, this cost will be no higher than the minimum required. If Y_{jk} equals 0, then the warehouse holding cost will be zero to maximize the profit. The sixth and seventh terms represent the shipping costs of the product flow between warehouses and stores, and between suppliers and warehouses, respectively.

Table 2: Input parameters for the calculation formulas used in the models.

Sets	Description
I	The set of nodes representing markets
R	The set of nodes representing the suppliers, $R \subseteq I$
J	The set of potential warehouse locations, which belong to the set of markets, $J \subseteq I$
K	The set of warehouse sizes (capacities)
S	The set of store locations, which belong to the set of markets, $S \subseteq I$
D	The set of dark store locations, which belong to the set of markets, $D \subseteq I$
Parameters	Description
Cdl_d	Dark store handling cost per unit for an online order fulfilled from a dark store located at d
Cwl_j	Warehouse handling cost per unit for an online order fulfilled from a warehouse located at j
Crh_s	Inventory holding cost per unit for store located at s
Crl_s	Handling cost per unit of online orders fulfilled from a store located at s
$Demand_i$	Total demand in market i
$Distance_{(i,j,s,d)(r,j,s,d)}$	Kilometers to ship to every online market, warehouse, store or dark store located at (i, j, s, d) from every fulfillment facility node (supplier, warehouse, store or dark store) located at (j, s, d)
F	Demand adjustment coefficient
G	Percentage of store demand available to the store channel
$OLpct$	Percentage of total demand available to the online channel
$profit$	Gross profit per unit for both channels (online and store)
WDD	Distance covered during a work day
Cap_{jk}	Capacity in number of units of a warehouse of size k located at j
Cdo_d	Fixed cost of a dark store located at d
$Caps_s$	Capacity in number of units of a store located at s
$Capds_d$	Capacity in number of units of dark store located at d
Cwh_{jk}	Inventory holding cost per unit for a warehouse of size k located at j
Cwo_{jk}	Warehouse fixed cost for warehouse of size k located at j
HD	Minimum number of orders to keep the dark store open
HS	Minimum number of orders to keep the store open
M	A very large number, M=M_1, M_2, M_3, M_4, M_5
Functions	Description
$Cdrw_{dj} = factor * Distance_{dj}$	Shipping cost per unit to fulfil a dark store located at d from a warehouse located at j
$Csow_{ij} = factor * Distance_{ij}$	Shipping cost per unit for an online order in market i from a warehouse located at j
$Csos_{is} = factor * Distance_{is}$	Shipping cost per unit for an online order in market i from a store located at s
$Csod_{id} = factor * Distance_{id}$	Shipping cost per unit for an online order in market i from a dark store located at d
$Csrw_{sj} = factor * Distance_{sj}$	Shipping cost per unit to fulfil a store located at s from a warehouse located at j
$Csrr_{jr} = factor * Distance_{jr}$	Shipping cost per unit to fulfil a warehouse located at j from a supplier located at r
$Don_i = Demand_i * OLpct$	Total online demand in market i (online market size)
$Donl_{i(j,s)} = Demand_i * OLpct * \frac{1}{T_{i(j,s)} * F}$	Total online demand for market i available to the retailer when the market is served from a facility (warehouse or store) at location j or s
$Donl_{id} = Demand_i * \frac{OLpct}{20} * \frac{1}{T_{id} * F}$	Total online demand for market i available to the retailer when the market is served from a dark store at location d
$Dr_i = Demand_i * (1 - OLpct) * G$	Retail demand for the retailer at store(s) in market i (total retail market size)
$pcow_{ij} = profit - Cwl_j - Csow_{ij}$	Profit per unit for an online order in market i fulfilled from warehouse located at j
$pcr_s = profit - Crh_s$	Profit per unit for a store order located at s
$pcos_{is} = profit - Crl_s - Csos_{is}$	Profit per unit for online order located in market i fulfilled from a store located at s
$pcod_{id} = profit - Cdl_d - Csod_{id}$	Profit per unit for online order located in market i fulfilled from a dark store located at d
$T_{i(j,s,d)} = Distance_{i(j,s,d)}/800$	Number of delivery days to ship to every online market $i \in I$ from the fulfillment facility node (warehouse,store or dark store) located at j or s or d
Decision variables	Description
Y_{jk}	Binary variable that indicates if a warehouse of size k is open ($Y_{jk} = 1$) or closed ($Y_{jk} = 0$) at location j
St_s	Binary variable that indicates if a store is open ($St_s = 1$) or closed ($St_s = 0$) at location s
Dt_d	Binary variable that indicates if a dark store is open ($Dt_d = 1$) or closed ($Dt_d = 0$) at location d
X_{ij}	Total number of units shipped to online market i from a warehouse located at j
RS_{jr}	Total number of units shipped to a warehouse located at j from a supplier located at r
V_{sj}	Total number of units shipped to a store located at s from a warehouse located at j
P_{dj}	Total number of units shipped to a dark store located at d from a warehouse located at j
Qs_s	Total number of units sold to store customers at a store located at s
U_{is}	Total number of units shipped to online market i from a store located at s
W_{id}	Total number of units shipped to online market i from a dark store located at d

$$
\begin{aligned}
Maximize\, profit = & \sum_{s \in S} pcr_s * Q_s + \sum_{i \in I} \sum_{j \in J} pcow_{ij} * X_{ij} \\
& + \sum_{i \in I} \sum_{s \in S} pcos_{is} * U_{is} - \sum_{j \in J} \sum_{k \in K} Cwo_{jk} * Y_{jk} - \\
& \sum_{j \in J} (\sum_{k \in K} Cwh_{jk} Y_{jk} * (sum_{i \in I} X_{ij} + sum_{s \in S} V_{sj})) - \\
& \sum_{s \in S} \sum_{j \in J} Csrw_{sj} * V_{sj} - \sum_{j \in J} \sum_{r \in R} Csrr_{jr} * RS_{jr}
\end{aligned} \tag{1}
$$

The objective function for the SFSW model is now reformulated for the linearization issue in equation 2.

$$
\begin{aligned}
Maximize\, profit = & \sum_{s \in S} pcr_s * Q_s + \sum_{i \in I} \sum_{j \in J} pcow_{ij} * X_{ij} + \\
& \sum_{i \in I} \sum_{s \in S} pcos_{is} * U_{is} - \sum_{j \in J} \sum_{k \in K} Cwo_{jk} * Y_{jk} - \sum_{j \in J} \sum_{k \in K} WHC_{jk} - \\
& \sum_{s \in S} \sum_{j \in J} Csrw_{sj} * V_{sj} - \sum_{j \in J} \sum_{r \in R} Csrr_{jr} * RS_{jr}
\end{aligned} \tag{2}
$$

$$
Cwh_{jk} * (\sum_{i \in I} X_{ij} + \sum_{s \in S} V_{sj}) - M(1 - Y_{jk}) \leq WHC_{jk} \tag{3}
$$

$for\ all\ j \in J\ and\ for\ all\ k \in K$

Constraint 4 ensures that if a warehouse of capacity k located at j is closed, then no flow can exist between that warehouse and any market i. Constraint 5 is analogous, but for the flow between the warehouse and all suppliers at r. Constraints 6 and 7 limit the online demand in market i served from warehouse j, and from store s, respectively, to the maximum level assigned in table 2. Constraint 8 limits the total online demand of market i served from all the warehouses and all stores for those that can ship within one day. Constraint 9 is similar, however, this equation limits the online demand for warehouses and stores that ship within a particular number of delivery days ($T_{ij} = \{1, 2, 3, 4, 5, 6, 8\}$). Constraint 10 ensures that the number of units sold at a store at s do not exceed the available store demand for the retailer. Constraint 11 ensures that the total units shipped to the online markets and to stores from warehouse j do not exceed the total units shipped from suppliers to that warehouse. Equation 12 is another shipment balance constraint, but for any store open at s. This constraint ensures that the quantity of units sold at a store s and the quantity shipped from that store to online markets do not surpass the quantity shipped from the warehouses to that store. Constraint 13 ensures that the quantity shipped from warehouse j is limited by the actual capacity of that warehouse. Constraints 14, 15, and 16 ensure that the total units shipped to online markets from store s, sold at store s, and shipped from warehouses to store s, are limited to the capacity of that store, respectively. Constraint 17 makes sure that each warehouse j is opened at only one capacity k. Constraint 18 ensures that if a store at s is closed, then nothing can be sold at that store. Constraint 19 is analogous, if a store at s is closed, then no flow can exist between that store and online markets. Similarly, constraint 20 ensures that a store at s is open if the quantity sold at that store exceeds the predetermined minimum number of orders. The last two constraints, 21 and 22 are variable domain constraints. The majority of the formulation of the SFSW model is based on Millstein et al. (2022) [10].

$$X_{ij} \leq \sum_{k\in K} Y_{jk} * M_1\ for\ all\ i \in I\ and\ for\ all\ j \in J \tag{4}$$

$$RS_{jr} \leq \sum_{k\in K} Y_{jk} * M_2\ for\ all\ j \in J\ and\ for\ all\ r \in R \tag{5}$$

$$X_{ij} \leq Donl_{ij}\ for\ all\ i \in I\ and\ for\ all\ j \in J \tag{6}$$

$$U_{is} \leq Donli_{is}\ for\ all\ i \in I\ and\ for\ all\ s \in S \tag{7}$$

$$\sum_{j\in J} X_{ij} + \sum_{s\in S} U_{is} \leq Demand_i * OLpct * \frac{1}{F}\ for\ all\ i \in I \tag{8}$$

$$\sum_{j\in J|T_{ij}=t} X_{ij} + \sum_{s\in S|T_{ij}} U_{is} \leq Demand_i * OLpct * \frac{1}{F*t}$$
$$for\ all\ i \in I\ and\ T_{ij} = \{1, 2, 3, 4, 5, 6, 8\} \tag{9}$$

$$Q_s \leq Dr_s\ for\ all\ s \in S \tag{10}$$

$$\sum_{i\in I} X_{ij} + \sum_{s\in S} V_{sj} \leq \sum_{r} RS_{jr}\ for\ all\ j \in J \tag{11}$$

$$Qs_s + \sum_{i\in I} U_{is} \leq \sum_{j} V_{sj}\ for\ all\ s \in S \tag{12}$$

$$\sum_{i\in I} X_{ij} + \sum_{s\in S} V_{sj} \leq \sum_{k} Y_{jk}Capjk\ for\ all\ j \in J \tag{13}$$

$$\sum_{i\in I} U_{is} \leq Caps_s\ for\ all\ s \in S \tag{14}$$

$$Qs_s \leq Caps_s\ for\ all\ s \in S \tag{15}$$

$$\sum_{j\in J} V_{sj} \leq Caps_s\ for\ all\ s \in S \tag{16}$$

$$\sum_{k\in K} Y_{jk} \leq 1\ for\ all\ j \in J \tag{17}$$

$$Qs_s \leq St_s * M_3\ for\ all\ s \in S \tag{18}$$

$$U_{is} \leq St_s * M_4\ for\ all\ i \in I\ and\ for\ all\ s \in S \tag{19}$$

$$(HS - Qs_s) \leq (1 - St_s) * M_5\ for\ all\ s \in S \tag{20}$$

$$Y_{jk}, St_s \in \{0, 1\} \tag{21}$$

$$X_{ij}, V_{sj}, Qs_s, RS_{jr} \geq 0 \tag{22}$$

3.5. Constraints for the Ship from Dark Store and Warehouse (SFDSW) model

The SFDSW model is a variation of the SFSW model with a new entity: dark store. The objective function for SFDSW model is stated in equation 23. It is quite similar to the objective function for the SFSW model (equation 2), however with slight changes to accommodate the addition of the dark stores.

$$Maximize\ profit = \sum_{s\in S} pcr_s * Q_s + \sum_{i\in I}\sum_{j\in J} pcow_{ij} * X_{ij} +$$
$$\sum_{i\in I}\sum_{d\in D} pcod_{id} * W_{id} - \sum_{j\in J}\sum_{k\in K} Cwo_{jk} * Y_{jk} -$$
$$\sum_{d} Cdo_d * Dt_d - \sum_{j\in J}\sum_{k\in K} WHC_{jk} - \sum_{s\in S}\sum_{j\in J} Csrw_{sj} * V_{sj}$$
$$- \sum_{d\in D}\sum_{j\in J} Cdrw_{dj} * P_{dj} - \sum_{j\in J}\sum_{r\in R} Csrr_{jr} * RS_{jr} \tag{23}$$

This objective function is subject to constraints 4, 5, 17, 10, 15, 16, 6, 18, 20, 21, 22 and:

$$Cwh_{jk} * (\sum_{i\in I} X_{ij} + \sum_{s\in S} V_{sj} + \sum_{d\in D} P_{dj}) - M(1 - Y_{jk}) \leq WHC_{jk}$$
$$for\ all\ j \in J\ and\ for\ all\ k \in K \tag{24}$$

$$W_{id} \leq Donlin_{id}\ for\ all\ i \in I\ and\ for\ all\ d \in D \tag{25}$$

$$\sum_{j\in J} X_{ij} + \sum_{d\in D} W_{id} \leq Demand_i * OLpct * \frac{1}{F}\ for\ all\ i \in I \tag{26}$$

$$\sum_{j\in J|T_{ij}=t} X_{ij} + \sum_{d\in D|T_{ij}} W_{id} \leq Demand_i * OLpct * \frac{1}{F*t}$$
$$for\ all\ i \in I\ and\ T_{ij} = \{1, 2, 3, 4, 5, 6, 8\} \tag{27}$$

$$\sum_{i\in I} X_{ij} + \sum_{s\in S} V_{sj} + \sum_{d\in D} P_{dj} \leq \sum_{r} RS_{jr} \ for\ all\ j \in J \quad (28)$$

$$Qs_s \leq \sum_{j} V_{sj} \ for\ all\ s \in S \quad (29)$$

$$\sum_{i\in I} W_{id} \leq \sum_{j} P_{dj} \ for\ all\ d \in D \quad (30)$$

$$\sum_{i\in I} X_{ij} + \sum_{s\in S} V_{sj} + \sum_{d\in D} P_{dj} \leq \sum_{k} Y_{jk} Capjk \ for\ all\ j \in J \quad (31)$$

$$\sum_{i\in I} W_{id} \leq Capds_s \ for\ all\ d \in D \quad (32)$$

$$\sum_{j\in J} P_{dj} \leq Capds_s \ for\ all\ d \in D \quad (33)$$

$$W_{id} \leq Dt_d * M_1 \ for\ all\ i \in I\ and\ for\ all\ d \in D \quad (34)$$

$$(HD - \sum_{i} W_{id}) \leq (1 - Dt_d) * M_2 \ for\ all\ d \in D \quad (35)$$

$$Dt_d \in \{0, 1\} \quad (36)$$

$$P_{dj}, W_{id} \geq 0 \quad (37)$$

Constraint 24 is analogous to 3 including the flow between the warehouse and dark stores. Constraint 25 is analogous to 7 but instead of the flow being between stores and online markets, it is between dark stores and online markets. Constraints 26 and 27 are analogous to constraints 8 and 9. Constraint 28 is analogous to 11. Constraint 29 is analogous to 12 without the flow between the store and online markets. Constraint 30 ensures that the total units shipped to the online markets from dark store at d do not exceed the total units shipped from warehouses to that dark store. Constraint 31 is analogous to 13. Constraint 32 ensures that the quantity shipped from dark store d to online markets is limited by the actual capacity of that dark store. Similarly, constraint 33 ensures that the quantity shipped from warehouses to dark store at d is also limited by dark store capacity. Constraint 34 ensures that if a dark store at d is closed, then no flow can exist between that dark store and online markets. Similarly, constraint 35 ensures that a dark store at d is only open if the quantity sold at that dark store exceeds the predetermined minimum number of orders. Finally, the last two constraints, 36 and 37 are variable domain constraints that complement constraints 21 and 22, respectively.

Equation 29 was adapted from the Millstein et al. (2022) model [10] and the remaining were originally formulated for this model.

3.6. Constraints for the Ship from Store, Dark Store and Warehouse (SFSDSW) model

The SFSDSW model is a combination of the two previous models, SFSW and SFDSW. The objective function for SFSDSW model is stated in equation 38. It is very similar to the objective function for the SFDSW model (equation 23), only with minor changes to accommodate the addition of the shipment to online markets from stores.

The first fourth terms are the gross profit earned from store and online sales (for orders fulfilled by warehouses, stores and dark stores), respectively. The fifth and sixth terms are the warehouses and the dark stores fixed costs, respectively. The seventh term is the linearized warehouse inventory holding cost, already explained in equation 1. The last three terms represent the shipping costs of the product flow between: warehouses and stores, warehouses and dark stores, and suppliers and warehouses, respectively.

$$Maximize\ profit = \sum_{s\in S} pcr_s * Q_s + \sum_{i\in I}\sum_{j\in J} pcow_{ij} * X_{ij} + \sum_{i\in I}\sum_{s\in S} pcos_{is} * U_{is} + \sum_{i\in I}\sum_{d\in D} pcod_{id} * W_{id} - \sum_{j\in J}\sum_{k\in K} Cwo_{jk} * Y_{jk} - \sum_{d} Cdo_d * Dt_d - \sum_{j\in J}\sum_{k\in K} WHC_{jk} - \sum_{s\in S}\sum_{j\in J} Csrw_{sj} * V_{sj} - \sum_{d\in D}\sum_{j\in J} Cdrw_{dj} * P_{dj} - \sum_{j\in J}\sum_{r\in R} Csrr_{jr} * RS_{jr} \quad (38)$$

This objective function is subject to constraints 4, 5, 17, 10, 14, 15, 16, 6, 18, 19, 20 21, 22, 24, 25, 7, 28, 12, 30, 31, 32, 33, 34, 35, 36, 37 and two additional constraints:

$$\sum_{j\in J} X_{ij} + \sum_{d\in D} W_{id} + \sum_{s\in S} U_{is} \leq Demand_i * OLpct * \frac{1}{F} \ for\ all\ i \in I \quad (39)$$

$$\sum_{j\in J|T_{ij}=t} X_{ij} + \sum_{d\in D|T_{ij}} W_{id} + \sum_{s\in S|T_{ij}=t} U_{is} \leq Demand_i * OLpct * \frac{1}{F * t} \ for\ all\ i \in I\ and\ T_{ij} = \{1, 2, 3, 4, 5, 6, 8\} \quad (40)$$

Constraints 39 and 40 are analogous to constraints 26 and 27, accommodating the flow between stores and online markets. All these constraints are original work.

4. Results & discussion

This section presents the most relevant results of the different formulations.

4.1. Data Collection and assumptions

To implement and validate the constructed models, input data was needed to assess the feasibility of the solutions. We used Daskin (1995) [5] data set as a reference for our model demand. This data set is based on the US state capitols geographic locations and the distribution of the population. It includes 49 markets (48 state capitols and Washington DC.). Each state demand represents its' population size.

The distances between market nodes were calculated using reference coordinates of each state. The distance between two entities of the same state was assumed to be around 30km. The delivery time (T) between each fulfillment entity and market i is calculated based on the distance (as explained in table 2). The WDD (work day distance) was assumed to be of 800km. The demand adjustment coefficient F_i was assumed to be: 1) $F_i = 0.5$ to benefit short periods of deliveries; 2) $F_i = 2$ to a pessimistic scenario; and 3) $F_i = 1$ for a neutral approach. To model online demand we considered for scenarios with $OLpct = \{0.1, 0.3, 0.5, 0.75\}$. For retailers' physical store market size we considered $G_i = \{0.3, 0.5, 0.75\}$.

In terms of the set definition, we assumed that each state has a store of the retailer, that can be kept open or can be closed depending on the optimization decision. For the dark store study, we assumed that only one dark store can be opened in every state and each dark store served only the market where it is located. The warehouse distribution is only in 15 of the previously defined states, and within that options the model will tell where to open warehouses. Finally, like the warehouse distribution, suppliers are available in 6 states.

The three models were run for three different product categories: electronics, apparel and food, to assess the difference of the benefits of installing dark stores on different product types.

The actual values of the demand, market locations coordinates, profits and costs per product categories that were used during the computational test were adapted from Millstein (2022) [10].

4.2. Discussion of results

In this problem, operational efficiency was not quite easy to study, since we have simplified the solution to a single product type in each retail sector. However, some KPIs were studied to assess the three channel design performances, such as the market coverage and the unitary cost of satisfying online demand in the 3 echelons (warehouse, store and dark store). Regarding the model's economic profitability, other KPIs were studied as the distribution of profit by the facilities, and the evolution of profit in all product categories and partitions considered.

Market coverage

The market coverage was studied to understand the dimension of markets reached. Table 3 shows the average percentage of demand fulfilled in the markets that are reached. Table 4 shows the number of markets covered. This study was performed to the reference partition ($Olpct = 30\%$ and $G = 50\%$) for all model configurations and product categories.

We can see that the greatest market coverage is achieved by the SFDSW configuration for the fashion category. For the SFSDSW and SFSW the electronics category is the one with best coverage, followed by fashion and food categories. We have a service level in terms of fulfilling demand higher than 63% with an average of 75%. It is also interesting to see that the model configuration with only dark store has the highest average level of service.

In terms of number of markets reached, fashion and food categories cover 100% of the markets, in contrast with electronics.

We assume that this analysis is representative to most demand partition configurations.

Table 3: Market coverage per product category for the $Olpct$=30% and G=50% partition.

	$Olpct$=30% G=50%		
	SFSDSW	SFSW	SFDSW
Electronics	87%	90%	55%
Fashion	64%	73%	94%
Food	63%	67%	86%
Average	71%	77%	78%

Table 4: Number of markets covered per product category for the $Olpct$=30% and G=50% partition.

	$Olpct$=30% G=50%		
	SFSDSW	SFSW	SFDSW
Electronics	38	38	49
Fashion	49	49	49
Food	49	49	49

Unitary cost of fulfilling demand

To have a better understanding of the operational efficiency and the cost of fulfillment, we studied the unitary cost of fulfilling online demand in each of the fulfillment facilities. This assessment was done to the reference partition ($O = 30\%$ and $G = 50\%$) and the reference product category (Electronics), for all the three model configurations.

To measure this KPI, we have averaged the facility's operating costs weighted by the actual number of units that fulfill online demand. As it can be seen in table 5. It was already expected that the warehouse had the lowest cost, since it is a facility normally located in less populated areas which corresponds to lower facility costs. The lowest warehouse fulfillment cost is in the SFSDSW and the highest in the SFSW. The highest fulfilment cost is for the store facility. This happens mainly because we are only considering the online demand and these facilities serve mostly store demand. This was also already expected due to the high fixed costs of operating an open-to-public store. In the dark store facility, we can see that it in the SFSDSW is cheaper to fulfill demand through dark stores than in the SFDSW model. Overall, we can see that the SFSDSW model, the one with higher flexibility to ship from between the three entities, is the one with lowest operating costs in terms of fulfilling online demand.

It is interesting to see that that the channel designs that have the highest fulfillment costs are the ones with highest market coverage.

Table 5: Unitary cost of fulfilling demand for each facility, for the $O = 30\%$ and $G = 50\%$ partition and the electronics product category.

	Olpct=30% G=50%		
Unitary fulfillment cost	SFSDSW	SFSW	SFDSW
Warehouse	2.27	3.81	3.70
Store	8.46	8.68	
Dark Store	5.23		6.99
Average	5.32	6.25	5.35

Distribution of profit by the facilities

In order to asses which facilities bring the greatest benefit to the model, we studied the distribution of profit through the three different entities: warehouses, stores and dark stores. In table 6 there are the correspondent percentages of profit divided by product category and the two model configurations with dark stores. This study was carried for the reference partition ($Olpct$=30% and G=50%).

First, we calculated the part of profit that each entity adds to the model. For example, for the store facility in the electronics SFSDSW model, the profit was the product of the gross profit of the product (sales price) and the number of units sold:

$$Storeprofit = (profit * Qs_s) + (profit * U_i, s)$$

Then, we calculated the percentage of total profit aggregated by the facility. In the example, the percentage that the store profit has from the total. In this case it is more than 100%, which means that the store itself aggregates more than the total profit and it is the facility that leverages the model profitability.

$$Storeprofit = 3749414000$$
$$Totalprofit = 2956349700$$
$$percentageofprofit = 3749414000/2956349700$$
$$percentageofprofit = 126\%$$

Of course not every facility can aggregate such a significant part of profitability. It is clear that the dark store is the facility that brings less value to the network. This was already expected since we considered the dark store space to be subcontracted and strategical located in highly populated areas (with high fixed costs). However we can see that in a configuration with both stores and dark stores (SFSDSW), the fashion category is the one were dark stores add greater benefit. In the SFDSW configuration, it is in the food category were dark stores are most valuable.

Overall, the facility that aggregates more profit in the SFSDSW and SFSW models is the store and in the SFDSW model is the warehouse. It is worth to note that despite the low percentage of aggregated profit in the dark store facility, the net profit of dark stores is positive in all scenarios.

Table 6: Distribution of profit by the three facilities for the three product categories($Olpct$=30% and G=50% partition).

	Percentage of profit		
	Store	Warehouse	Dark Store
Electronics SFSDSW	126%	15%	3.04%
Fashion SFSDSW	75%	49%	3.43%
Food SFSDSW	115%	57%	2.82%
Electronics SFDSW	76%	77%	3.63%
Fashion SFDSW	64%	79%	2.78%
Food SFDSW	75%	86%	3.81%
Electronics SFSW	68%	16%	
Fashion SFSW	60%	56%	
Food SFSW	67%	60%	

Evolution of profit in all product categories and partitions considered

Studying the evolution of profit with the different partitions of the online and physical demand ($Olpct$ and G), for the three product categories, we concluded that profit increases with the increase in both the online and physical demand fraction.

In this study the most profitable product category is the Electronics, followed by Fashion and Food. The food category is mostly affected by to the requirement of special shipping or warehousing for perishable products, leading to increased holding and handling costs. The Fashion category has also additional costs when compared to the electronics, due to the size of the products and the assortment.

The most profitable configuration varies accordingly to the product category. In the electronics category the most profitable configuration is the SFSW, however it is only 0.2% more profitable than the SFSDSW. In the fashion category the most profitable configuration is the one that incorporates more entities (SFSDSW). In the food products it is preferable to implement the ship from store and warehouse (SFSW) design. Overall we can conclude that the SFSW design is the safest option in the short term. However, the SFSDSW configuration has only 0.09% less profit, which can be negligible and we can state that both SFSW and SFSDSW offer the same level of profit.

5. Conclusions

The three models were developed to account the interaction of different product categories, variable online demand percentages, the elasticity of online demand based on the delivery time, and the level of competition in the market. In addition, this study highlights several important considerations for omnichannel firms deciding which channel design to utilize. First, by optimizing the number, location and size of omnichannel warehouses, as online demand increases, the firm can increase profit compared to using fixed warehouse locations and sizes. Second, the unit profitability for a product category influences which channel design is optimal, resulting in different optimal channel designs for different product categories. Third, with increasing levels of competition in a market, the different levels of market share result in different solutions in different product categories.

Results show that optimizing the number, location and capacities of warehouses benefits SFSW and SFSDSW more than SFDSW for most product categories, because of the store presence in all markets where demand can be fulfilled quickly with lower inventory and handling costs, thereby increasing profitable market share. In this study the highly profitable product category is electronics, followed by fashion and food. The warehouse is the facility with lower cost of fulfilling demand and the store is the one with the highest. In terms of channel designs, the most traditional configuration (SFSW) is the one with higher unitary cost of fulfilling demand, followed by the model with only dark stores (SFDSW) and the most flexible channel (SFSDSW). In terms of profit, it is the store and warehouse facilities that aggregate more profit with a large margin of difference from the dark store.

The low contribution of dark stores to the profit is due to the higher scale of number and capacity of stores and warehouses to respond to the demand. Dark stores do not leverage the profit of a retailer as the traditional facilities do. However, since they are a recent phenomenon, there is a great space for optimization and for growth. Our recommendation is for retailers to start investing in this type of facilities to respond to more demanding customers.

The great amount of uncertainty associated with the parameters used to design the model experiment makes it challenging to draw conclusions with a high degree of confidence. Thus, it would be recommended to invest more in data collection. We based most parameters in the ones used in Millstein et al. (2022) [10] work, but these were not enough for the construction of new network configurations.

Our results suggest some directions for further work. While we varied the online demand market share, retail demand market share available to the retailer was held constant, and vice-versa. In the future, these parameters could be dynamically incorporated into the models. In addition, demand was treated in a static and single period manner. In a future research, demand could be modeled stochastically to incorporate population changing patterns across various geographies while maintaining elasticity. In addition, a time variable could also be incorporated to give dynamism to the model, by changing online demand levels over time to study the evolution of an optimal omnichannel network. For example, this changes could be applied to simulate the demand patterns during the Covid-19 pandemic, for example, or to simulate unforeseen events. Another suggestion for further work would be to explore the opportunity of not only closing physical stores, but to convert these retail facilities into dark stores for a greater support to omnichannel distribution.

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