On the use of dark stores in omnichannel last-mile distribution

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Declaration

I declare that this document is an original work of my own authorship and that it fulfils all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Declaração

Declaro que o presente documento é um trabalho original da minha autoria e que cumpre todos os requisitos do Código de Conduta e Boas Práticas da Universidade de Lisboa.

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Abstract

Retailers need to adapt their operations to follow the evolution of e-commerce. Additionally, customers rising expectations for fast deliveries and shopping experience underline the need for improvement in order fulfillment. Omnichannel is a potential solution that by integrating online and offline channels reduces friction for customers. However, it is challenging to implement by retailers.

In this work, we investigate the use of dark stores as a potential fulfillment node to support online demand in an omnichannel retail network. This research studies the performance of three channel designs: SFSW - ship from store and warehouse; SFDSW - ship from dark store and warehouse; and SFSDSW - ship from store, dark store and warehouse. The fulfillment problem is formulated as a profit maximizing mixed integer linear programming (MILP). Warehouses and dark stores' locations can be decided through the model solution, whereas stores are given as retailer assets that can be maintained or closed. Product flows between all echelons are also determined. The demand is treated in an endogenous and elastic manner, as a function of distance and levels of competition in online and store markets. Three different product categories are separately treated in the model: electronics, fashion and food.

Results evaluate the operational efficiency and profitability of different network configurations, showing that the SFSW and SFSDSW generate greater profit than the SFDSW. The electronics category is the most profitable of all. Dark stores do not leverage profit as the traditional facilities do, however, there is space for optimization and growth.

Keywords: Dark store, Omnichannel, E-commerce, Facility location optimization

Resumo

Os retalhistas devem adaptar as operações para acompanharem a evolução do e-commerce. Adicionalmente, o aumento das expectativas dos clientes quanto à rapidez das entregas e experiência de compra sublinham a necessidade de melhoria no cumprimento das encomendas. O omnicanal é uma solução com potencial que, integrando canais online e offline, reduz o atrito para os clientes, contudo, é um desafio operacional para os retalhistas.

Neste trabalho, investigamos o uso de dark stores como potencial ponto de abastecimento para satisfazer a procura online numa rede omnicanal. É estudado o desempenho de três formulações: SFSW - envio da loja e armazém; SFDSW - envio da dark store e armazém; e SFSDSW - envio da loja, dark store e armazém. Este problema é formulado como um MILP, com maximização do lucro. A solução dos modelos determina a localização de armazéns e dark stores, enquanto que as lojas são ativos que podem ser mantidos ou fechados. O modelo determina ainda o fluxo de produtos entre os pontos da cadeia. A procura é tratada de forma endógena e elástica, em função da distância e dos níveis de concorrência nos mercados online e offline. Este problema é aplicado a três categorias de produtos: electrónicos, moda e alimentares.

Os resultados avaliam a eficiência operacional e a rentabilidade das diferentes formulações. Os modelos SFSW e SFSDSW geram, em média, maior lucro que o SFDSW. Os produtos electrónicos são os que apresentam maior rentabilidade. As dark stores não alavancam a rentabilidade como as infraestruturas tradicionais, no entanto, há espaço para optimização e crescimento.

Palavras-chave: Dark store, Omnicanal, E-commerce, Optmização da localização dos pontos de abastecimento

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Abbreviations

SC	Supply Chain
AI	Artificial Intelligence
IoT	Internet of Things
BDA	Big Data Analytics
AR	Augmented Reality
MC	Multi-channel
OC	Omnichannel
IA	Intelligent Automation
GDP	Gross Domestic Product
E-GDP	E-commerce Gross Domestic Product
SKU	Stock Keeping Unit
DC	Distribution Centre
CDP	Collection and Delivery Point
ME-LRP	Multi-echelon Location-routing Problem
MILP	Mixed Integer Linear Programming
LRP	Location-routing Problem
TSMP	Time Slot Management Problem
MIP	Mixed Integer Programming
CLRP	Capacitated Location-routing Problem
SS-CFLP	Single-source Capacitated Facility Location Problem
MDVRP	Multi-depot Vehicle Routing Problem
CA	Continuum Approximation
RCE	Routing Cost Estimations
GAMS	General Algebraic Modeling System
OPL	Optimization Programming Language
SFSW	Ship from store and warehouse
SFDSW	Ship from dark store and warehouse
SFSDSW	Ship from store, dark store and warehouse
DS	Dark Store
KPI	Key Performance Indicator

Chapter 1

Introduction

This chapter introduces the context and motivation of the dissertation. Section 1.1 provides an overview of the relevance and the context of the problem at study. Section 1.2 presents the objectives to be achieved with the development of the dissertation. Next, section 1.3 introduces the adopted research methodology. Finally, section 1.4 presents the outline of the document.

1.1 Motivation and Problem Context

From the early 21st century on-wards, technologies such as Artificial Intelligence (AI), Internet of Things (IoT), Cloud Computing, Big Data Analytics (BDA) and Augmented Reality (AR), came to integrate goods with people through the internet. These technologies have been incorporated in retailing throughout the years. With this operational transition, human capabilities are leveraged and human errors are reduced, building efficiencies while enabling digital operations and innovations. However, the technological shift requires great investment and is not immediate. At the same time, consumers' expectations on retailers are growing, products are expected to be delivered in shorter times and a wide range of product-offering is wanted. Companies are suffering great pressure on supply chains trying to respond to the new market requirements. Retailers should proceed not only to the digitalization of operations, but also to the automation of those. [1] The ultimate goal is to extend automation to the entire value chain, increasing operational agility and improving customer experience. [2]

The covid-19 pandemic brought huge retail implications, as demand patterns changed dramatically and only resilient supply chains could cope with the uncertainty. Traditional retailers are used to eliminate redundancy in order to reduce costs and increase efficiency. However, the lack of flexibility leads to bigger losses once an unexpected situation occurs. [3] Another consumption shift the pandemic brought was the boost in online demand. Although internet sales have been growing for several years, the covid-19 pandemic boosted this growth. Online sales increased dynamically in April and May 2020, due to the closure of physical shops. Although shops have reopened and sales in this channel have dropped, internet sales have remained on a rather high level since summer 2021, with a growth of more than 30% in comparison to the pre-pandemic times, according to Eurostat. Taking this into consideration, retailers have to adapt to the online channel, building an omnichannel experience.

According to Verhoef et al. [4], omnichannel is defined as the synergetic management of the various channels available and customer contact points, in order to optimize the customer experience and the performance of the chain along all channels. Companies that already vertically integrated both online and offline channels, are trying to create a connection between the two. The emergence of omnichannel has completely revolutionized the traditional e-commerce by integrating all customer touch-points into an integrated holistic experience. [5]

Nowadays, the majority of e-commerce orders are concentrated in urban areas. These areas have limited accessibility and scarce logistics infrastructure, which result in decreased efficiency and quality of retailing operations. Retailers have to adapt their networks in order to respond to these changes. In the literature, this problem has been mostly addressed in a multi-channel point of view (MC), and more recently, in a omnichannel approach (OC). The distribution network design problem has been studied in parallel with the delivery planning problem. With the rise of e-commerce, new delivery modes emerged to make products available to consumers. A recent fulfilment option that has been boosted by the pandemic is dark stores. The literature on this new delivery mode is quite scarce, since it is a recent phenomenon. According to Bryson J. R., (2021) [6], a dark store is a small logistic hub located in high-density urban centre that only serves online customers in a short period of time. The literature covers also the order fulfillment problem. According to Croxton [7], order fulfillment is not only about filling customers' orders efficiently and effectively, but it is also about designing a network and a process that allows a firm to meet customer requests while minimizing the total delivered cost. Order fulfillment includes the generation, the filling, the delivery, and the service of customer orders.

In this context, there is a gap in the use of dark stores as a potential fulfillment node to support online demand in an omnichannel retail network, assisting decision-makers in the order fulfillment process.

1.2 Problem Statement and Objectives

After conducting an in-depth literature review on order fulfillment in an omnichannel retail, we found that research on this topic remains scarce. Many authors have highlighted this gap in literature and identified the assessment of different order fulfillment configurations as a future area of research. [8]

The aim of this study is to fill this gap through an optimization model that compares different network configurations (including shipping from dark store), responding to both online and store demand, with an omnichannel approach.

Ultimately, research will be done aiming to incorporate a mathematical formulation into an optimization model to design a retailer distribution network. After finding a mathematical formulation that realistically represents a retailer network, this project will add up the dark store echelon to the formulation.

Secondary research goals include:

- Reviewing previous research on the field to identify potential research gaps in literature;

- Contributing to the existing literature by adding the dark store entity to an existing network distribution formulation;
- Proposing a mathematical formulation capable to solve the order fulfillment problem;
- Defining the product categories that make more sense to analyse;
- Characterizing the different channel designs that the model will incorporate to understand the whole picture;
- Analysing the results and understand how the new way of delivering is adding to retailing;
- Producing recommendations for omnichannel retail network decision.

In order to accomplish these objectives, we propose to answer to the following research questions:

1. Which channel design should be adopted in different retailing contexts in order to improve profitability?
2. How are profits and costs distributed among the different entities?
3. Is it beneficial to include dark stores as new fulfillment options in a retailer supply chain?

1.3 Dissertation Methodology

In order to achieve the objectives stated in the previous section, the proposed methodology of the present master thesis is presented in Figure 1.1.

Figure 1.1: Steps of the proposed research methodology.

The proposed methodology to address the problem comprises the six distinct steps detailed below:

- **Step 1: Context and objectives definition**

  General introduction to the main concepts and challenges in the retail industry, covering the evolution of retail along the years, the main consumer trends and the most important sectors.

- **Step 2: Literature review**

  Sound review of the state-of-the-art practices and studies applied to order fulfilment management and last-mile distribution in retailing. Focus on the literature regarding distribution network design and delivery planning, from both theoretical and operational sides. The order fulfilment topic is also deeply revised, along with the order fulfilment models.

- **Step 3: Model formulation**

  Considering the problem definition and the relevant models in the literature, three optimization model are selected and developed.

- **Step 4: Data collection**

  Collection of the relevant data for the implementation of the final model. Information such as distances, profit, costs, among others, are collected from sources available in the literature.

- **Step 5: Model validation**

  Validation of the developed model through comparison of the performance of the developed model and existing literature models.

- **Step 6: Results analysis**

  Discussion on the obtained results and recommendations. Analysis of the limitations and assumptions of the model and future work suggestions.

1.4 Dissertation Structure

The project is composed of eight major chapters, each of them organized as follows:

The **first chapter** explains the background and context of the problem to be solved. It also presents the purpose and goals of the work as well as the structure of the document.

In the **second chapter**, a description of the problem under study is provided, along with an overview of the current retail state. Here, the evolution of retail is covered, with its state in the last few years, its current situation, and a future challenge prevision. The most relevant trends of the industry and consumption are also described and examined.

The **third chapter** provides a literature review on distribution network design, from a theoretical and an operational approach. Here, the state-of-the-art practices and studies applied to order fulfilment management are presented. The differences between multi-channel and omnichannel concepts are explored. It will also be studied the advantages and challenges of many delivery modes by reviewing the literature on delivery planning. related to the delivering routing problem. The order fulfilment topic is also covered, along with models for both the network design and order fulfilment models. In the latter, an overview of the key challenges retailers need to address is done. This chapter ends with a summary of the literature reviewed, identifying a research gap for this study.

The **fourth chapter** describes the research methodology developed and employed in this dissertation to attain the problem under study. Here, a clear definition of the problem under study is done, along with the objectives definition. It is in this chapter that the developed mathematical formulation is presented, alongside with all assumptions, indexes, sets, parameters, constraints, and objective functions. Three different channel design formulations are illustrated in this chapter.

In the **fifth chapter**, the main results are presented and discussed. To begin with, the data collection process is explained, followed by the process to validate the models. The main model results are categorized into two performance indicators: channel design operational performance, and economic performance. A sensitivity analysis is conducted. In the end of the chapter there is a general discussion of the results and of the most relevant model limitations.

The **sixth and last chapter** presents the main conclusions of this dissertation as well as further thoughts and considerations for the development of future work.

Chapter 2

Retail Context Overview

This chapter provides the description of the problem under study, addressing an overview of the current retail situation. Section 2.1 presents the state of retail in the last few years, and the previsions for the current and future challenges. Next, in section 2.2, the most relevant trends of the industry and consumption will be examined. Section 2.3 will cover the retail sectors most relevant for the study. Finally, section 2.4 will summarize the chapter, defining the problem under study.

2.1 State of retail

According to Newman and Cullen [9], the retail concept is defined as the activities that put products and services on the market to serve final for their own personal or household use. Retail does this by organizing the product/service availability on a relatively large scale and supplying them to consumers on a relatively small scale. It is basically a transaction of goods or services, between the seller and the end-user, to satisfy the needs of the consumer. This section will cover the history of retail during the four industrial revolutions (2.1.1), the changes that occur during the Covid-19 pandemic (2.1.2), and finally a prevision for next years (2.1.3).

2.1.1 Evolution of retail during the industrial revolution

Throughout the history, the four industrial revolutions since 1760 allowed significant changes to the retail industry. [10]

In the first industrial revolution, the invention of steam engines reduced the need for human labour and increased the efficiency of workflows. On the retail side, this resulted in mass manufacturing and the emergence of department stores, as the most prosperous consumers started developing broader preferences. [10]

In the second industrial revolution, from the beginning of the 20th century on, electrification appeared, allowing the development of mass production assembly lines. From the retail point of view, products started to have better quality at lower prices. With the wider availability of automobiles, more people started commuting from suburbs to urban areas, which resulted in the clustering of suburban shopping plazas and malls. [10]

In the third industrial revolution, from the late 20th century on, electronics, telecommunications, and computers have popped up. This resulted in online shopping and globalization on the retail side. The creation of the internet allowed global production, marketing, and the global use of digital technologies. It was in the 90s that digital competitors, as eBay, Amazon, and Alibaba, appeared. It was also in this period that big retailers, like Auchan and Walmart, appeared to serve broader geographic areas. [10]

The fourth and last industrial revolution, from the early 21st century on-wards, leveraged new technologies. These technologies, like Artificial Intelligence (AI), Internet of Things (IoT), Cloud Computing, Big Data Analytics (BDA) and Augmented Reality (AR), came to integrate goods with people through the internet. [10] Retail is facing a new technological era with Intelligent Automation (IA). From an Institute for Business Value (IBM) report, AI is defined as the capability in machines to reason. AI can remember information, learn, and identify new insights through data discovery. IA is guided by AI tools that need minimal manual routine interventions. This operational shift augments and assists human capabilities, reduces human errors, and builds efficiencies while enabling digital operations and innovations. [2] The growth of consumers' expectations on organizations, demanding a wider variety of products delivered in shorter time windows, creates great pressure on supply chains. Thus, companies have to find new tools to respond to market requirements, resorting not only to the digitalization of operations, but also to the automation of those operations. [1] Companies are increasingly adopting IA to improve efficiency and reduce costs. The ultimate goal is to extend automation to the entire value chain, increasing operational agility and improving customer experience. [1, 2]

2.1.2 Retailing during Covid-19 pandemic

In the last few years, a huge challenge came across retailers from all over the world. The covid-19 pandemic started in Europe in February 2020, three months after the disease's first report in Wuhan, China.

Streamlined with all the global health implications, covid-19 has brought huge retail implications. Of all the problems brought about by the pandemic, here is an overview of the three most relevant to the upcoming work. [11]

- **Demand fluctuations**

  Wherever the virus was spreading, almost every physical store closed doors to consumers. Only essential goods stores, such as grocery stores and pharmacies, kept their doors open. This has caused unprecedented changes in demand. While some retailers saw demand fall as customers switch channels, others faced unexpected peaks in demand. This was the case of grocery retailers, which had many stock-outs of products that consumers have unexpectedly deemed as essentials. [11]

- **Safety plans**

  During the pandemic, every company had to had a plan to ensure the safety of employees and customers. These plans were mandatory and had to enable retailers to maintain their usual activities. These emergency plans should not be seen as a sporadic event, but rather as a necessity for companies to respond to unexpected situations. [11]

- **Supply chain resilience**

  While companies were still adapting to the new ways of non-physical working, retailers had to keep their supply chain working to deliver goods to people. Consumers started panic buying, leading to shelves' stock-outs. This was a wake-up call for supply chain managers. Companies are used to eliminate redundancy to reduce costs and increase efficiency. However, these attempts lead to decreased flexibility, resulting in bigger losses when an unexpected situation occurs. [3] Retailers should redefine key suppliers, diversifying the portfolio politically and geographically, assessing risks and developing contingency plans. [11]

Economic impact of Covid-19

According to estimates from Eurostat, the statistical office of the European Union, from 2015 until March 2020 the retail trade volume in EU and in the euro area followed a growth trend. Just as the retailers of the world felt the effects of the pandemic, Europe was no exception. In March 2020 there was a huge drop on trade volume, that was only fully overcame in September 2020.

In December 2021, the retail trade volume fell again by 2.7% in the euro area and by 2.6% in the EU, due to another wave of the covid-19 virus (figure 2.1).

In January 2022, the calendar adjusted retail sales index increased by 7.8% in the euro area and by 8.3% in the EU, compared with January 2021 (figure 2.1).

According to estimates from Eurostat, the total EU retail trade volume in January 2022 is 4.1% higher than in February 2020, the month preceding the Covid-19 crisis (see figure 2.2).

In March and April 2020, the total retail trade volume dropped by 9.2% and 11.2%, respectively. Considerable differences in food and non-food products were found in this period. Food products sales increased in March while the sale of non-food products decreased.

The levels of internet sales are present on the right-hand axis of figure 2.2. Internet sales have been growing for several years, however, the covid-19 pandemic boosted this growth. Online sales increased dynamically in April and May 2020, due to the closure of physical shops. Although shops have reopened and sales in this channel have dropped, internet sales have remained on a rather high level since summer 2021, with a growth of more than 30% in comparison to the pre-pandemic times (figure 2.2).

In January 2022, total EU retail sales increased by 0.6% compared with December 2021; sales of food increased by 0.2%, sales of non-food by 1.0%, while sales of automotive fuels decreased by 1.1%.

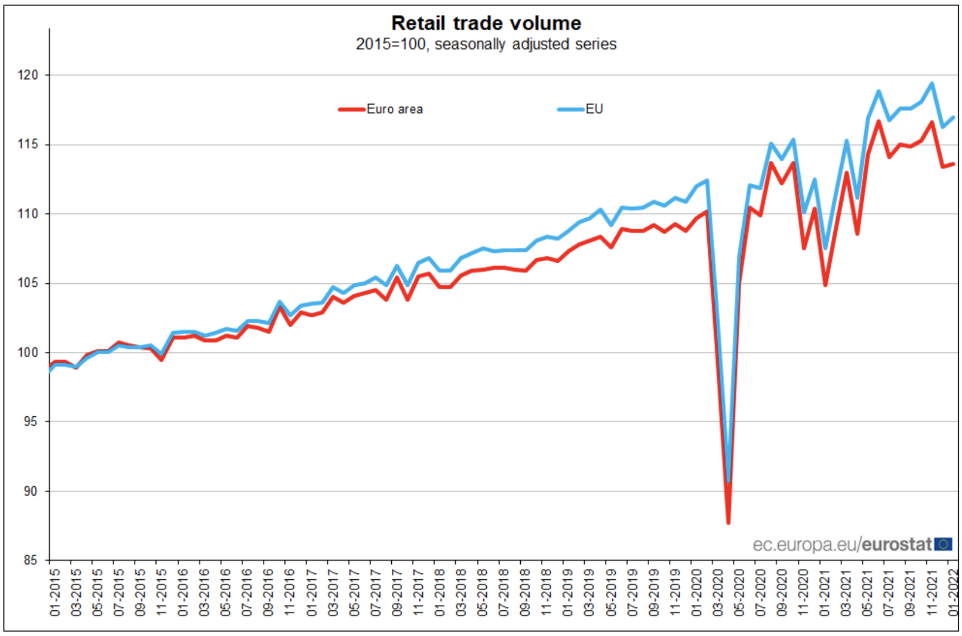


Figure 2.1: Retail trade volume - seasonally adjusted series. Source: Eurostat, March 2022

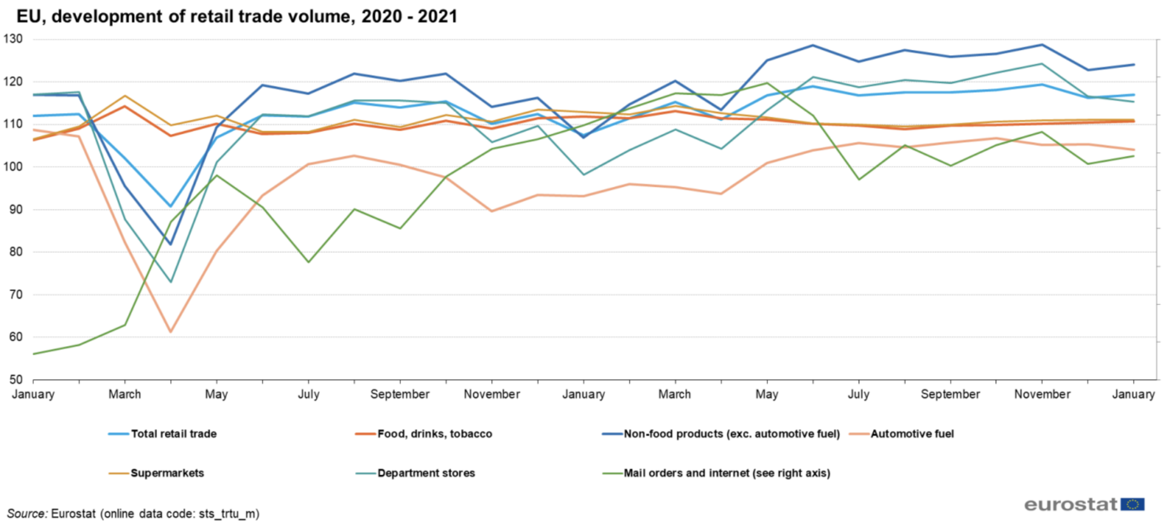


Figure 2.2: EU, development of retail trade volume, 2020-2022. Source: Eurostat, 2022

2.1.3 Future challenges

After two years living with the challenges of a pandemic, retailers have redefined themselves and adapted to the changes that have emerged. Companies have been forced to reexamine their legacy systems and to develop new strategies to address future challenges. [12]

The world saw the outbreak of war in Europe in February 2022, two years after Covid-19. The effects of Russia-Ukraine war are being amplified by the long-lasting impact of the pandemic. Although the conflict zone is small relatively to the world size, the consequences are alarming, especially in Europe. According to a recent assessment of the United Nations Conference on Trade and Development, the major concern is on food and fuels, two essential commodity markets. The Russian Federation and Ukraine are global players in agrifood markets, representing together ”53 percent of the share of global trade in sunflower oil and seeds, and 27 percent of the share of global trade in wheat". [13] Such changes will have tremendous impacts on retailers all over the world, specially the ones related to food and oil industries.

Retail headlines for 2022 suggest a change of paradigm, with new retailing trends emerging. Organizations are feeling pressured to remain agile and adaptive to new priorities. With the pandemic, companies have perceived the importance of being resilient. Organizations are becoming more customer-centric, focusing on the path of the consumer throughout the all omnichannel journey. Retailers are already investing massively in the digitalization of operations. All these trends will be discussed in detail in the following section (2.2).

2.2 Consumer and industry trends

Retailers have faced huge challenges throughout the covid-19 pandemic, whose effects are likely to be long-lasting. Having this in consideration, organizations must be anticipating the future, analysing the opportunities of the next era in retail. [12] This section will cover the main trends in the retail industry that consumers are paying attention to.

2.2.1 E-commerce

According to Babenko et al. [14], e-commerce includes the financial transactions carried out through the internet or private communication networks in which purchases and sales of goods and services are made. E-commerce is a business interaction between actors using internet technologies.

Traditional brick-and-mortar retailers have realized the importance of e-commerce, and are trying to expand their online options to compete with online players. At the same time, online retailers are starting to realize the importance of physical stores, and are opening their first store locations. This is the case of the e-commerce giant Amazon, for example.

E-commerce use is growing across Europe, as the number and share of e-shoppers increases every year. According to Eurostat [15], the greatest rise of e-shoppers occurred in 2020, with a difference of 5 pp in relation 2019 (figure 2.3), due to the outbreak of the pandemic that pushed people to shop online to avoid risky physical contact. This growth is also reflected in the numbers, all the European countries under study (37 countries [15]) saw their e-commerce gross domestic product (E-GDP) increased (figure 2.4).

In Portugal, the e-commerce trend is also growing, but at lower rates. In terms of the share of e-shoppers, there is still a difference from the european average of 15 pp in 2020, which is slightly higher than the forecast for 2021 (figure 2.3). As for the percentage of e-commerce in the GDP, the tendency is to approach the european average. While in 2017 Portugal had a difference of 0.48 pp, it was estimated that in 2021 this difference would only be 0.17 pp (figure 2.4).

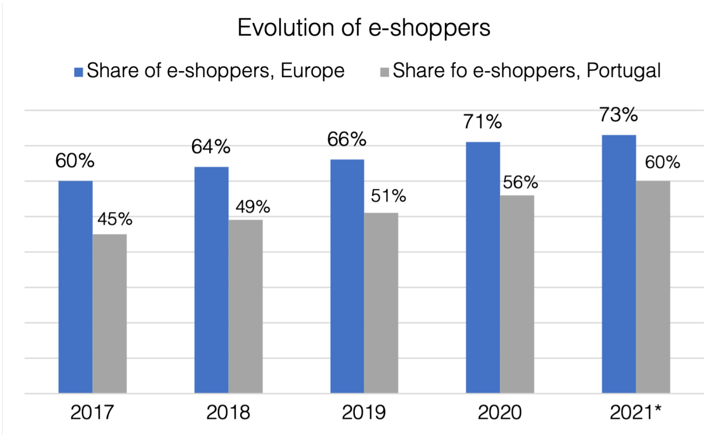


Figure 2.3: Evolution of the share of E-commerce in Europe and in Portugal. Source: Eurostat; National e-commerce associations. [15]

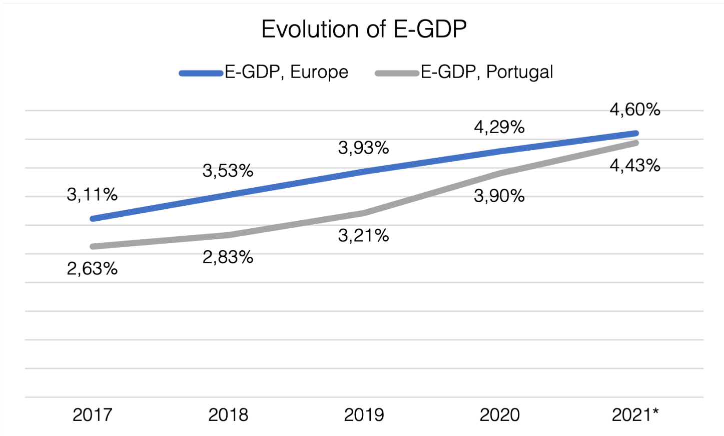


Figure 2.4: Evolution of the e-commerce share on gross domestic product (E-GDP) in Europe and in Portugal. Source: Ecommerce Europe; EuroCommerce. [15]

2.2.2 Customer-centric

According to Mitraneau [16], customer centricity is defined as the capacity to understand and respond to the customer's needs. Being able to understand what customers want is crucial for organizations to remain competitive in the market. Retailers need to be agile and forward-looking in adopting a customer-centric approach across upstream and downstream activities in the retail value chain. [17] There is a clear need for a transition from a product-centric approach to a customer-centric state.

Customers are demanding sustainable products, especially younger generations. [18] They want a seamless shopping experience, with the online stores aligned with the physical ones. Shopping must be fast, efficient, easy and intuitive. Companies should cater to consumer's needs and live up to their social and environmental responsibility claims. [18] Building an omnichannel experience can bring huge value for retailers. McKinsey's research on how to build an omnichannel distribution network has found that customers that shop online tend to buy more, and customers that pick up online orders in store often make additional in store purchases. [19] The ominichannel trend will be discussed ahead in section 2.2.5. To allow and improve e-commerce, companies have to invest in the digitalization of operations.

2.2.3 Sustainability

As referred before in section 2.2.2 and according to the National Retail Federation in association with IBM report on consumer trends, ”50 percent of consumers claim they’re willing to pay a premium for sustainability”. [18] The pandemic has changed the consumers’ vision about sustainability, making them value more companies that align themselves with these values.

However, there is still a gap between the intention of buying sustainable products and effectively converting that intention into an actual purchase. The main reasons for this gap are the higher prices, the lack of availability, the differences in quality and the shortage of information. This should be seen as an opportunity for retailers to make it easier and affordable for consumers to make sustainable choices. [18]

On the industry side, for years retailers tried to have the higher margins without sacrificing quality. In a global value chain – and especially in retail industry – the majority of the items are normally produced in low-cost developing countries. The sustainable supply chain management in developing countries differs from developed countries, because of the lack of political support, knowledge, infrastructure and the economic costs. [20]

One way to account for sustainability in the supply chain is to change the supplier selection. Diversifying the range of suppliers geographically increases the resilience of the supply chain, a trend that will be addressed in section 2.2.4. In terms of sustainability, suppliers should be chosen in an informed way and preferably local to reduce transport emissions.

As already referred, another way for retailers to be more sustainable is through transportation. Sustainable freight transportation helps to reduce energy consumption and greenhouse gas emission through the use of low-carbon fuel, electric mobility and adoption of environmental standards and green practices resulting in the substantial savings in operational costs and carbon footprint regulations. [21]

2.2.4 Resilience

The pandemic has triggered widespread concern about organizational resilience. Resilience has been defined as a dynamic process of maintaining positive adaptation and effective coping strategies in the face of adversity. [22] As already referred in section 2.1.2, companies are used to eliminate redundancy to reduce costs instead of increasing flexibility to cope with unexpected situations.

Resilient companies outperform their peers in three ways: the immediate impact of an external shock on their performance is smaller, the speed of their recovery is faster, and the extent of their recovery is wider. [3]

A recent study on ”Suppliers’ coalition strategy for green-Resilient supply chain network design” shows that the amount of released $\mathrm{CO_2}$, cost and lost demand in a resilient network model are meaningfully less than on a non-resilient model. [23] This proves that resilience and sustainability can be combined, making organizations more competitive.

2.2.5 Omnichannel

Omnichannel is defined as the synergetic management of the various channels available and customer contact points, in order to optimize the customer experience and the performance of the chain along all channels. [4]

Companies that have already vertically integrated both online and offline channels, are trying to create a connection between the two. This synergy is possible adding e-commerce applications in physical stores. Examples of this omnichannel presence is the ability to place orders online in-store, collect the online purchase in-store, see the availability of a certain product in all stores via an app, among others. [19]

The emergence of omnichannel has completely revolutionized the traditional e-commerce. Omnichannel has changed consumer's expectations by making all customer touch-points into an integrated holistic experience. [5] As a consequence, many companies fail to achieve omnichannel success given the huge challenges in terms of speed, complexity, and efficiency. [19]

2.2.6 Digital and data-driven experiences

Having operations properly digitalized is a huge advantage for companies. When done right, digitalization can bring major benefits. According a BCG report on "How-To Guide to Digital Operations", the benefits can translate into a reduction of 10% to 20% in production and supply chain costs, a 15% to 30% cut in working capital, and an incremental revenue growth of up to 6% through enhanced productivity. [24]

Technology and data are the key to build seamless omnichannel shopping experiences. By integrating consumer data across all touch-points, retailers can provide customized experiences. The data can be provided from a variety of sources, such as: [25]

- **Transactional Data** Information about pre-orders, returns, previous sales.
- **Contextual Data** Information about the region weather, cultural events, economic outlook.
- **Customer Data** Information about demography, customer's behaviour.
- **Retailer Data** Information about forecasts, shipping, operations.

However, many organizations do not see the desired outcomes. It is necessary to invest massively in this transition process, since it is needed to acquire new technical skills, get cross-functional collaboration, change the management paradigm and atract new talent. [24]

2.3 Retail Sectors

This section will address the retail sectors that are most relevant for the present work. It will emphasise the latest adopted ways to deliver products to stores and the greatest fulfillment challenges that retailers are facing in the different sectors.

To begin with, it is important to analyse the classification of the various retail sectors according to the type of product traded. Eurostat classifies the sectors according to the SCL classification - Statistical Classification of Economic Activities in the European Community (NACE Rev. 2). Table 2.1 presents the different existing sectors based on the type of product and point of sale.

From the sectors presented, three were chosen to be analysed in depth based on the following criteria: volume of retail in January 2020 (pre-pandemic), according to Eurostat.

The sectors chosen were: Retail sale of information and communication equipment; other household equipment (except textiles) , cultural and recreation goods, etc. in specialised stores that will now be referred as **Consumer Electronics Retail**; Retail sale of textiles, clothing, footwear and leather goods in specialised stores, that will be referred as **Apparel Retail**; and Retail sale of food, beverages and tobacco, that will be referred **Food Retail**.

As already mentioned in section 2.1.2, in March 2020, sales of food products soared in contrast to non-food products. From February to March 2020, the sales volume of food products rose 1.93% while the volume of apparel and electronics products fell 65.7% and 32.2%, respectively. In the second month of the pandemic (April 2020) and after the panic buying of the first month, the food retail volume fell 2.89% compared with the previous month, apparel retail fell again 27.9% and electronics retail continued to rise. At the end of 2021, the sector values returned to pre-pandemic normality as shown in figure 2.5.

Subsection 2.3.1 will present the state of food retailers. Next, in subsection 2.3.2 the state of apparel retailers will be analysed. Lastly, in subsection 2.3.3 the state of consumer electronics retailing will be discussed. In the literature review chapter (chapter 3) the sectors to be studied will be chosen based on the following criteria: the sectors with the most challenging situation, and the fewest studied solutions.

Table 2.1: Retail sectors according to SCL - Statistical Classification of Economic Activities in the European Community (NACE Rev. 2)

Code	**Retail sector**
G47	Retail trade, except of motor vehicles and motorcycles
G47_X_G473	Retail trade, except of motor vehicles, motorcyles and fuel
G47_FOOD	Retail sale of food, beverages and tobacco
G47_NFOOD	Retail sale of non-food products (including fuel)
G47_NFOOD_X_G473	Retail sale of non-food products (except fuel)
G47_NF_CLTH	Retail sale of textiles, clothing, footware and leather goods in specialised stores
G47_NF_HLTH	Dispensing chemist; retail sale of medical and orthopaedic goods, cosmetic and toilet articles in specialised stores
G47_NF_OTH	Retail sale of information and communication equipment; other household equipment (except textiles); cultural and recreation goods, etc. in specialised stores
G47_NF_OTH1	Retail sale of computers, peripheral units and software; telecommunications equipment, etc. in specialised stores
G47_NF_OTH2	Retail sale of audio and video equipment; hardware, paints and glass; electrical household appliances, etc. in specialised stores
G471	Retail sale in non-specialised stores
G4711	Retail sale in non-specialised stores with food, beverages or tobacco predominating

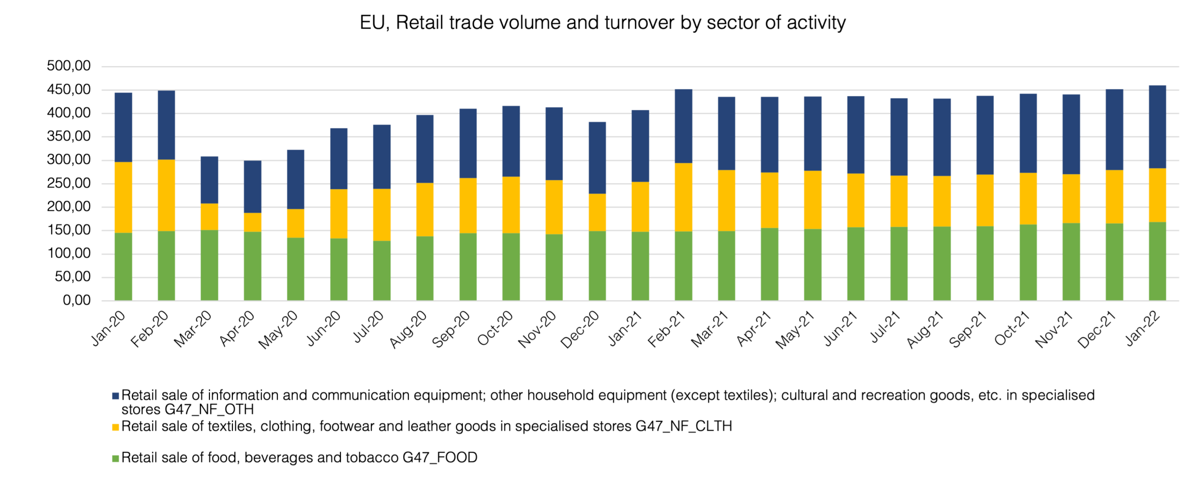


Figure 2.5: Retail trade volume and turnover by sector of activity in Europe from January 2020 to December 2021. Monthly data, seasonally and calendar adjusted (2015=100), Source: Eurostat

2.3.1 The state of food retail

As already referred to in section 2.1.2, the covid-19 pandemic increased food retail sales. Grocery retailers were one of the few retail sectors that grew consistently in 2020, while restaurants saw their doors closed. Having this in consideration, this section will mainly focus on the grocery retailing.

In addition to the increase in revenue, costs in this sector also grew due to the unprecedented demand, the spread of the virus in suppliers' factories, and all the safety measures that needed to be introduced. [26]

The imposed government restrictions and the change in consumer behaviour with a greater health concern changed the consumption patter in grocery stores. European shoppers started to reduce the frequency of physical shopping by approximately 5%, at the same time the shopping basket increased by around 16%, according to a report of McKinsey and EuroCommerce in 2021. [26]

The online channel, for instance, experienced a growth of around 55% across Europe, while traditional brick-and-mortar grocery stores only grew by 3%. In the US in 2019, only 19% of customers had bought groceries online. This number increased to 79% by the fall of 2020, according to Forbes. [27]

This behavioural change leads retailers to adapt to online grocery shopping and its challenges. To do so, it is important to understand the preferences of the consumer nowadays. It has been noticed that order pick up is winning over delivery. Unlike the pandemic, consumers have now gotten back to work, and cannot wait for a delivery at any time interval. In this way, it is more convenient to stop by a pick up point and get the groceries. Moreover, a Manifest survey showed that 52% of consumers were using online ordering to save time, instead of for health safety. [28]

There is also an increasing number of shoppers in grocery stores that are simply shopping for other customers. This is leading retailers to use regular stores that operate for online order fulfillment, or use dark stores. [29] Dark stores are, for definition, retail outlets or distribution centres that cater exclusively to the fulfillment of online orders [30], normally in urban centers close to the public but operated only by retail collaborators. This trend has started in the UK with Tesco, a british multinational grocery and general merchandise retailer, and is growing in popularity around the world. [29]

Instant delivery or quick commerce is becoming the most prominent online market segment. McKinsey and EuroCommerce report on the state of grocery retail-2022 [31] describes it as the fastest and most convenient delivery of a reduced assortment at a higher price per item. Europe's top 15 players opened more than 800 dark stores in 2021. Many traditional grocers are forming partnerships with instant delivery companies. McKinsey and EuroCommerce research suggests that, only in 2021, the instant-delivery market in Europe reached between €3 billion and €6 billion. This represents less than 1 percent of the total market, however, the the percentage of annual growth contains three digits.

The consumption pattern is changing, and e-grocery needs efficient fulfillment solutions. E-grocery demands intense labor due to the small unit picks, the large portfolio of SKU (stock keeping units), the big amount of different orders, and the need to deliver fast. These problems lead to automation, which can be one solution. Warehouse automation reduces costs for retailers at the same time that increases flexibility and scalability.

The transportation side can also be automated. In addition to increased truck-fill rates, automation can optimize trucking routes in real time based on traffic conditions and disruptions. This can lead to more on-time deliveries, fresher products, less $\mathrm{CO_2}$ emissions, quality issues' detection, optimized last-mile vehicle deployment, among others.

The biggest e-grocery players are already using modern technology in supply chains creating microfulfillment centers (MFC) (also called dark stores), highly automated warehouses that server e-commerce as well as local store pick-ups. Another new way of fulfillment are physical stores with entirely automated checkouts. In Portugal the idea was first implemented by Sonae MC group, and in the US and UK Amazon Go opened some partially automated grocery stores. [26]

2.3.2 The state of apparel retail

The apparel retail industry has encountered critical development and profited from innovative creations lately. To find success, enormous worldwide apparel retailers have embraced innovative advancements and have transformed themselves into more grounded organizations through mergers and acquisitions. [32]

Customer preferences are changing fast in the whole retail industry, specially in the fashion industry. Retailers have to understand customer trends and respond quickly, to enhance the customer experience.

Brick and mortar stores have been converted into interactive experiences for customers. This is done through fashion shows, samples offering, virtual fashion consultation, among others. The creation of a seamless shopping experience is the ultimate goal for the apparel industry. [32]

As already referred in the previous sections, covid-19 pandemic had a tremendous impact in retailing. Apparel industry was no exception, 2020 and 2021 were challenging years due to the closing of stores and the shift in customers' priorities.

According to a McKinsey's report on the state of fashion in 2022 [33], the only players who managed to get through this period without major losses were those who invested in the trends of the moment:

- Comfort and outdoor activities - The obligation to be at home during the pandemic caused people to buy more comfortable clothing. At the same time, people started to exercise at home, and home gym equipment sales skyrocketed

- Online shopping - Once the physical stores closed, it was necessary to invest in online shopping. Attractive and user-friendly online stores gained prominence in this phase.

McKinsey's Fashion Scenarios [33] suggest that in 2022 global fashion sales will reach 103 to 108 percent of 2019 sales. Economic growth will be top of the plan for most brands in 2022. However, changes in the market are bringing new challenges to adress. According to a McKinsey survey on the biggest challenges apparel retailing is facing, 30% of respondents mentioned supply chain issues related to logistics and inventory management, followed by 14% mentioning sustainability and 10% the pandemic recovery (figure 2.6).

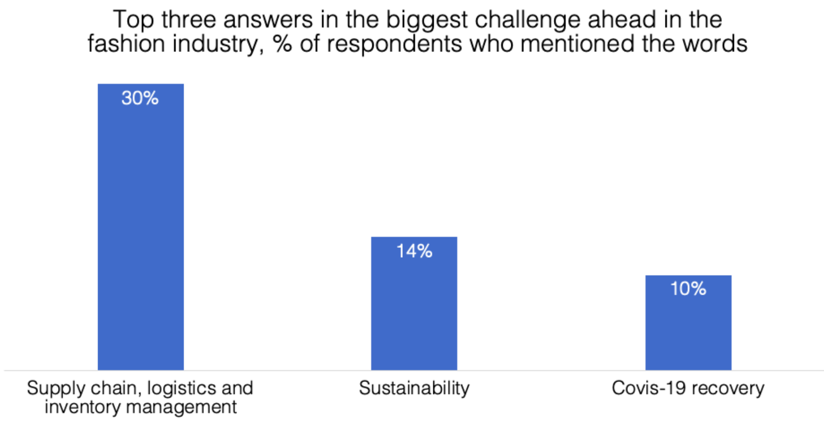


Figure 2.6: Top three answers in the biggest challenge ahead in the fashion industry, % of respondents who mentioned the words. Source: BoF–McKinsey state of fashion 2022 survey [33].

According to a survey on supply chain disruption of Oxford Economics [34], around half of global businesses suffered supply chain disruptions in 2021, with one in eight severely affected. The fashion industry is dependent on global supply chains that are experiencing unprecedented levels of pressure and disruption. The increased pressure is due to a combination of factors such as material shortage, unavailability of staff, increase in shipping costs, and transportation bottlenecks. [33]

In addition to these challenges, customers are demanding higher service levels. With new ways of delivering emerging, customers want their products delivered super-fast, in a specific time and place of their preference, and with detailed access to product tracking.

Retailers have to rethink procurement strategies while implementing cutting-edge supply chain management. There is a huge demand for flexibility in order to keep products flowing rapidly with customer demand in the coming year.

2.3.3 The state of consumer electronics retail

According to IBISWorld report [35], consumer electronics stores retail a range of new appliances, electrical goods and home entertainment products, such as dishwashers, TVs and computers, for personal use. In the past decade this sector has seen incredible changes. With the development of internet technologies and the network economy, the demand for consumer electronics has been growing year after year. This rise is due to the emergence of online channels of social and communication media. [36]

Unlike most apparel products, whose life cycle is short, consumer electronics are designed to be durable. This way, the consumers tend to make a more conscious purchase, seeking for the product in the market that best suits them. As a consequence, there is an intense competition in this industry. The technological barriers of the production decrease as the technological convergence becomes consistent and products are easily imitated. The competition among consumer electronics has become a comprehensive competition in the supply chain. [36]

There are two recent trends impacting particularly the online and offline retail channels of consumer electronics retailing, which are showrooming and webrooming. Showrooming refers to the practice of analysing products in traditional retail stores and then purchase the products online. Webrooming is the opposite trend where consumers visit retail websites to compare prices, attributes, opinions, and brands, but purchase the final product offline, at a physical store. As a result, the internet is becoming the main sales channel for most retailers, whereas their physical stores are becoming a source of information for many consumers. [37]

With the increase of sales via internet, product distribution can be a problem. Smaller items are easy to ship, however, most products (such as refrigerators, TVs, washing machines, among others) are large and fragile. This type of products are difficult to ship, and mail carrier providers do not want to assure the risk. The solution to this situation relies on the technological improvement of operations and inventory, namely on automation. [35]

2.4 Chapter conclusions

This chapter introduced the main concepts and challenges in the retail industry, covering the evolution of retail along the years, the main consumer trends and the most important sectors. Covid-19 pandemic brought huge retail implications, as demand patterns changed dramatically and only resilient supply chains could cope with the uncertainty. Two years after the pandemic, economic prosperity is expected, however the consumer has changed. Retailers have to adapt to the online channel not forgetting the physical one, building an omnichannel experience. These trends bring challenges at the supply chain level, together with the growing concern for operations sustainability.

New distribution options are emerging, and with this comes the need to understand the advantages and disadvantages of each one. It is important to study the necessity of each new form of distribution, understanding its usefulness for each industry and type of business. Some of the new ways of distribution have been covered throughout this chapter, however, the literature review chapter will provide a detailed review of all the existing options of distribution found.

This problem is especially relevant since a model to assist decision-makers in their distribution options is needed. To build the model, it is important to review the state-of-the-art literature on this topic, providing the groundwork to address the problem. This will be done in the following chapter.

Chapter 3

Literature Review

In this chapter, the state-of-the-art practices and studies applied to order fulfilment management and last-mile distribution in retailing will be presented. Section 3.1 will present an overview of the literature regarding distribution network design, from a theoretical and an operational approach. The differences between multi-channel and omnichannel concepts will also be addressed. Section 3.2 covers the delivery planning topic, which is organized into two subsections: a more theoretical subsection covering the main advantages and challenges of delivery modes (3.2.1), and section 3.2.2 addressing the delivery planning from the operational side by reviewing the literature related to the delivering routing problem. Section 3.3 covers the order fulfilment topic. Section 3.4 revises models for both the network design and order fulfilment models. Section 3.5 describes the key challenges retailers need to address. Finally, section 3.6 summarizes the literature reviewed and identifies a research gap for this study.

3.1 Retail distribution network design

Nowadays the majority of e-commerce orders are concentrated in urban areas. These areas have limited accessibility and scarce logistics infrastructure, which result in decreased efficiency and quality of retailing operations. At the same time, the customer is demanding higher service levels and new delivery options with shorter lead times. As a result, retailers have to adapt their networks in order to respond to these changes. [38]

In the literature, this problem has been mostly addressed in a multi-channel point of view (MC), and, more recently in a omnichannel approach (OC). The concepts differ in aspects such as in channel integration, management integration, and data sharing. MC retail is related to business where channels are not integrated and do not share information. MC businesses operate physical and online channels separately. OC retail refers to businesses where channels are integrated and share information and objectives. [39] The differences are summarized in table 3.1.

The network design concept has been discussed in the literature through exploratory research articles. From a MC point of view, in 2013, Lang, G. and Bressolles, G. [40] identified the differences in economic performance and customer expectations between four e-fulfillment systems. The systems

Table 3.1: Overview of MC and OC management concepts. Source: Adapted from Mirsch (2016) [39]

	Integration	Management	Data
Multi-channel retailing (MC)	Separate channels with no overlap	Separate channels with no overlap	Not integrated nor shared across channels
Omnichannel retailing (OC)	Integrated channels to provide seamless experiences	Across all channels, with cross-channel objectives, integrated and shared	Integrated and shared across all channels

differed according to the local of order preparation (distribution center or in-store) and the means of delivery to the customer (home delivery or pick-up by the customer). The work helps understanding the most suitable system for a business. From an OC approach, in 2016, Hubner et al. [41] presented an overview of the advantages and challenges of different network configurations in non-food distribution. The options differed in terms of the integration and centralization of distribution centres (DCs). Hubner et al. [42] also studied the advantages and challenges of design parameters in OC grocery retailers.

The distribution network design problem has also been studied in the literature from an operational perspective, both in MC and OC approaches.

In 2008, Aksen & Altinkemer [43] studied the transition from a brick-and-mortar to an hybrid click-and-mortar business model in a MC approach, from the distribution logistics point of view. Aksen & Altinkemer implemented a static location-routing based problem for the strategy transition, addressing the decisions related to store location, customer allocation, vehicle routing and closure of stores.

More recently, the problem has been studied from an OC point of view. In 2019, Janjevic et al. [44] proposed a method for integrating collection delivery points (CDPs) in last-mile distribution network design. While previous approaches only consider location decisions for logistics facilities, this model also considers location decisions for CDPs, accounting for changes in demand patterns. Janjevic et al. suggest that the integration of CDPs in e-commerce can bring significant cost reductions in the last-mile. In 2021, Janjevic et al. [45] proposed a multi-echelon location-routing problem (ME-LRP) that captured several relevant features of the operating environment of contemporary e-commerce last-mile distribution systems. The model addresses decisions related to the optimal number, type, and location of distribution facilities, the shape of facility-specific service areas, the modal choice, and the route configurations along which each individual delivery vehicle serves the underlying customer base. Customer demand is differentiated according to multiple time-differentiated delivery services and multiple product-exchange alternatives.

In 2020, Validi et al. [46] proposed a three-echelon bi-objective sustainable model with the objective of minimising the levels of $\mathrm{CO_2}$ emissions from transportation and the distribution networks costs. The main decisions are to open or close facilities throughout the network.

In 2022, Goedhart et al. [47] proposed the rationing of retailer inventory among the channels. The rationing is defined by the trade-off of serving in-store or online customers, since selling a product in-store is more profitable than through the online channel but the inventory cost of the online channel is lower. The model contributes to the literature in the integration of both the replenishment and rationing decision.

Finnaly, in 2022, Millstein et al. [48] studies the performance of four channel designs to fulfil online and store demand in an omnichannel distribution. Millstein et al. [48] develops a profit maximizing mixed integer linear programming model (MILP) to decide the optimal locations and capacities of omnichannel warehouses, and the product flows from suppliers to warehouses, warehouses to the stores, and warehouses and stores to online markets. Depending on product categories, some designs can be more profitable than others.

3.2 Delivery planning

This section addresses the delivery planning topic, which is organized into two subsections. First, subsection 3.2.1 presents a theoretical approach about the most important last-mile delivery modes, identifying the key advantages and challenges for each mode. Section 3.2.2 addresses the delivery planning problem from the operational point of view by reviewing the literature related to the delivering routing problem.

3.2.1 Last-mile delivery modes

With the rise of e-commerce, new delivery modes emerged to make products available to consumers. Traditional brick-and-mortar retailers entered the online business, and the traditional last-mile bulk delivery to stores could not respond to online demand. Swaminathan and Tayur [49] and Agatz et al. [50] are the first authors in the literature to present basic design options for distribution issues in e-business. They identify a closer interaction between fulfillment for e-commerce and stores as a future area of research.

OC concept is particularly relevant in the delivery modes study, since it requires the integration of the physical with the online channel. [41]

Specific analyses of the operational implications for distribution systems of retailing across channels remain scarce. [40] In 2014, Gallino and Moreno [51] also identify the need to study the integration of online and offline channels, since it was a recent practice.

According to Hubner [41], the optimization of the last-mile is the central driver for improvements in OC distribution. OC delivery modes need to fulfill customer requirements across all channels. Deciding about what delivery modes to make available is not an easy decision and it depends on aspects such as the geographic situation, the population density and demography, and the logistics configuration of the retailer. From an online customer point of view, delivery is the only moment of personal contact between the customer and the retailer. In this way, the delivery mode choice impacts customer relationship management and channel selection.

Nowadays there are many ways to deliver products to customers: (1) by delivering directly to the customer's house (3.2.1 home delivery), (2) by delivering to stores and the customer shops traditionally (3.2.1 store buying), (3) by delivering to the store a previous order from a customer (3.2.1 store pick-up),

(3) by delivering to a collection delivery point, which can be either a locker point or a service point (3.2.1 collection-and-delivery point), (4) and by having a dark store operating a certain region of customers (3.2.1 dark store).

Home delivery

Home delivery is the classic form of forward distribution for distance retailing [41]. An OC retailer that offers the home delivery mode has to deal with the picking costs of the online order and with the last-mile additional delivery cost. From the customer perspective, home delivery is a convenient service, however shipping costs are higher than pick-up options. Home delivery is subject to the possibility of the client not being at home (unattended home delivery). This typically happens when the retailer does not offer a time-slot choice to the customer. When the customer is not at the point of reception to accept a delivery, there are a couple solutions: the order can be placed in front of the customer's home to be collected after, the order goes to the delivery or reception box, or to shared reception boxes. From the logistical perspective, this is an hybrid form between home delivery and click and collect, in the way that the customer has to go to the reception box but the delivery model is quite the same for the retailer. [42]

Store Buying

Store buying is the traditional way of fulfilling customer demand through physical stores. This delivery mode enables direct contact between the customer and the retailer and it enables higher efficiency in terms of picking and transportation. From a customer perspective, by having direct contact with the products, the risk of returns is reduced. Despite being the oldest delivery mode, it has some challenges as well. From the operational point of view, the retailer incurs in high fixed costs for the brick-and-mortar stores, and the assortment and shelf space have to be managed carefully. For the customer, it might not be very convenient to visit the store depending on its location. [41]

Store pick-up

Buying online, picking-up in store allows OC retailers to save the last-mile of delivery. In this delivery mode orders are shipped in customer-ready picked parcels from a retailer or supplier distribution centre to the stores. [41] Orders can also be prepared in the store, allowing the integration of channels and a decrease in the need for online channel investments. However, picking and order preparation in a store have low efficiency and labor costs per order are high. [52] Stores are not designed to have product-picking operations. [40] It is more efficient to pick from a central distribution center than from a store, especially when handling a high volume of orders. [40]

In-store pick-ups has the advantage of increasing the probability to cross-sell products and of saving the delivery costs of the last-mile, from the retailer side. From the customer side, this mode allows virtual shelf extension, by buying online the customer has access to more products, and increased convenience. [42] The disadvantages for the retailer are the difficulty to integrate the information of both channels, by establishing real-time data access to in-store inventories, and the additional in-store handling effort. [41]

Collection-and-delivery point (CDP)

According to Weltevreden [53], collection-and-delivery points (CDPs) can be divided into two categories: locker points (unattended) and service points (attended). A locker point, or a shared reception box, is a collection of lockers where parcels can be delivered and where customers can pay, collect and, if necessary, return their parcel. They normally use PIN codes to control the delivery by the carrier and the collection of the parcel by the customer. [53] A service point is an attended point, which can be a store, a petrol station, a post office, where customers can pay, collect and return their parcel. [53] Both options have their strengths and weaknesses. Locker points do not have opening hours constraints, the collection time can happen 24/7. On the downside, lockers are not user friendly to all customers, being less attractive to customers aged over 65 years old. [53] There are also constraints related to the limited payment options and the parcel size. On the other side, service points can offer more payment options than locker points such as cash payment, and is easier to use since store personnel manage the collection procedure. Service points are also more flexible to store parcels of different shapes and sizes. On the downside, service points offer less customer autonomy due to opening hours restrictions. [53]

Dark store

Another fulfilment option for the online channel is through dark stores. The literature about dark stores is quite scarce, since it is a recent phenomenon. According to Bryson (2021) [6], a dark store is a small logistic hub located in high-density urban centre that only serves online customers in a short period of time. Dark stores can also appear in the literature as hubs that fulfill exclusively online orders.

The Covid-19 pandemic boosted the emergence of these stores, since many retailers decided to shut down their physical stores to the public, and transform them into dark stores to continue fulfilling online demand. [6]

The main advantages of dark stores include the delivery speeds improvement and the reduction of the capacity problems experienced by traditional fulfilment centres. The main challenges are the high fixed costs, since dark stores are typically operated in the urban centers.

Dark stores can be either attached to physical stores or can be separate fulfilment centers. In the first case, dark stores (or warerooms) are an integrated area of a physical store allocated only to fulfill online demand.

Table 3.2 summarizes the advantages and challenges of each delivery mode discussed.

Table 3.2: Overview of delivery modes

	Home delivery	Store buying	Store pick-up	Collection-and-delivery point (CDP)		Dark store
				Locker point	Service point	
Advantages	Increased customer convenience[42]	Direct customer contact; higher efficiency in picking and transportation; reduced risk for product returns[41]	Direct customer contact; additional store visiting; virtual shelf extension; increased customer convenience[42]	No time constraints to pick the order; increased customer autonomy[53]	Many payment options; different storage possibilities; easy use of the service; additional service point visiting[53]	Improved delivery speed; improved sku management; contact-free [6]
Challenges	Picking costs; last-mile additional cost; lead time; potential shipping fees; handling of bulky items.[42]	Fixed costs for bricks-and-mortar presence; limited assortment and shelf space[41]	Cross-channel IT requirements; increased in-store handling effort[41]	Parcel size and storage constraints; difficult use of the service; few payment options[53]	Service point opening hours constraint; less customer autonomy[53]	High fixed costs[6]

3.2.2 Delivery routing problem

Online shopping increases the need for deliveries to residential and congested areas, resulting in rising routing complexity and costs for the retailer. [43] In this sense, OC retailing is finding out new ways of getting the products to the consumer, as discussed in section 3.2.1. However, the design and planning of an urban last-mile distribution network, does not involve only the strategic decisions of network architecture and facility locations, but also the short-term operational decisions of the fleet routing. [45]

Min et al. [54] presents the location-routing problem (LRP) as a concept that recognizes the interdependence between the location of facilities, the allocation of suppliers and customers to the facilities, and the vehicle route structure. It is a logistic problem that coordinates both strategic (facility location) and operational (vehicle routing) decisions. Multiple authors find that solving location and routing problems independently results in sub-optimal solutions. [45] By integrating both decisions, the problem class of integrated LRPs rises, together with their multi-echelon counterparts, ME-LRPs. The vast majority of recent contributions in the field of strategic last-mile multi-echelon network design adopt these approach. [45]

In 2008, Aksen and Altinkemer [43] addressed the static conversion from brick-and-mortar retailing to the hybrid click-and-mortar business model by solving a static location-routing based problem. The model decides about brick-and-mortar and click-and-mortar stores locations, customer allocation and the vehicle routing structure. Aksen and Altinkemer solve the LRP through a mixed integer programming (MIP) formulation, with a Lagrangian-based solution method.

In 2011, Agatz et al. [55] addressed the attended home delivery problem of e-grocers offering narrow delivery time slots to ensure satisfactory customer service. This is a tactical, not an operational model, based on a grocery retailer in the Netherlands. The paper specifically addresses the time slot management problem (TSMP), given service requirements and average weekly demands for each zip code in the delivery region, determining the set of time slots to offer in each zip code to minimize expected delivery costs while meeting the service requirements. The results show that using service requirements has substantial savings over offering all time slots.

In 2020, Validi et al. [46] proposed a three-echelon sustainable bi-objective location-routing model. The objectives are of minimising the levels of CO_2 emissions from transportation and the distribution networks costs. The problem is to identify open and closed facilities and the optimised routing pattern throughout the network.

The year after, in 2021, Janjevic et al. [45] proposes a multi-echelon location-routing problem (ME-LRP) that captures several relevant features of the operating environment of contemporary e-commerce last-mile distribution systems. It is an integrated modelling framework for strategic last-mile design of three-tiered multi-modal networks in OC environment, with customer demand differentiated according to multiple time-differentiated delivery services and multiple product-exchange alternatives. The model decides about the optimal number, type, and location of distribution facilities, the shape of facility-specific service areas, the modal choice, and the route configurations along which each individual delivery vehicle. The main finding is the economic benefit of an integrated approach.

3.3 Order fulfilment

According to Croxton [7], order fulfillment is not only about filling customers' orders efficiently and effectively, but it is also about designing a network and a process that allows a firm to meet customer requests while minimizing the total delivered cost. Order fulfillment includes the generation, the filling, the delivery, and the service of customer orders. It is crucial to optimize the network, in order to minimize total delivered costs, including sourcing costs, while meeting service level requirements. [7]

Taylor et al. [56] defines omnichannel fulfillment strategies as processes that enable a firm to meet customer demand through the flexible sharing of fulfillment links through any combination of channels with regard to purchase origination and purchase receipt. Retailers had to adjust their logistics and supply chain management processes from fulfilling orders for each channel separately (MC approach) to integrating channels in a OC approach (omnichannel fulfillment).

In 2019, Li & Jia [57] modeled an order fulfilment planning problem from the point of view of an online retailer (e-tailer) operating its own logistics system. The network is composed of three echelons: the fulfilment centers, the delivery stations, and the final customer. The delivery mode available is only home delivery. The objective of the model is of operating cost minimization. The decisions addressed are the order fulfilment centre assignment, together with the order routing under a time window constraint. The model is developed through a Mixed Integer Programming (MIP) formulation.

In 2021, Difrancesco et al. [58] modeled the in-store fulfillment process of an omnichannel retailer using a ship-from-store strategy to fulfill online orders in a dense urban market with time-slot delivery. The paper combines a simulation-based approach with exploratory modeling to prescribe optimal fulfillment policies under various sources of uncertainty. It includes only the home delivery mode, from the brick-and-mortar stores. The paper results demonstrate the critical trade-off between operational costs and service performance in omnichannel fulfilment, with a growing arrival rate of in-store customers negatively affecting the online order service level.

Ishfaq & Bajwa [8] focused on how store-based retailers use multiple fulfillment options to fill online orders. These options include distribution centers, direct-to-customer fulfillment centers, retail stores, and vendor facilities. The objective of the paper is focused on all these fulfilment options to study how relevant operational and logistics costs affect retailers' profitability for each fulfillment option. The model evaluates fulfillment decisions over multiple periods to determine the inventory to be shipped from vendors to the different facilities in the retailer's network. The last-mile delivery modes available are home delivery and store pick-up. A non-linear mixed-integer profit maximization model of the online order fulfillment process is developed to incorporate retailer decision framework in defining the optimal order fulfillment nodes within their retail distribution networks.

In 2022, Millstein et al. [48] modeled the performance of four channel designs to fulfil online and store demand in an omnichannel distribution. The channel designs were ship-from-store, ship-from-warehouse, ship-from-store-and-warehouse, and ship-from-warehouse with back-hauling from the store inventory to the warehouse. The objective of the model is maximizing profit while deciding about optimal locations and capacities of omnichannel warehouses, and the product flows between facilities. The problem is presented through a mixed integer linear programming model (MILP). Various product categories are analysed computationally, with different cost characteristics. Results show that the ship-from-warehouse design generates greater profit than ship-from store for four of the product categories.

3.4 Modelling an omnichannel distribution model

Relevant previous studies to this research have addressed two key areas: the network design problem, and the order fulfilment problem. This section includes the model review for both problems, by summarizing the most important problem formulation aspects, the main commercial algorithms used, and the other algorithms used. Subsection 3.4.1 explains the models for the network design problem, and subsection 3.4.2 covers the models for the order fulfillment problem.

3.4.1 Models for network design

The design of distribution systems creates hard combinatorial optimization problems. Prodhon & Prins [59] evaluated research on location-routing problem. The first ideas about the combination of depot location and vehicle routing decisions arose around fifty years ago. The notion of interdependence of these decisions was recognized, however technology was not developed enough.

The location-routing problem (LRP) integrates the two kinds of decisions, (1) the facility location problem at the strategic decision level to place factories and warehouses, and (2) the vehicle routing problem at the tactical or operational levels to ensure supply to customers. Studies have shown that the integration of these two interdependent decisions account for considerable cost savings.

There are various versions of location-routing problems studied in the literature in the last years. The first models to appear considered incapacitated depots. Since 2007, the majority of authors address the LRP with capacity constraints on depots and vehicles (capacitated location-routing problem CLRP). The CLRP is NP-hard since it includes two problems known to be NP-hard: the single-source capacitated facility location problem (SS-CFLP), and the multi-depot vehicle routing problem (MDVRP).

Only very small LRP can be solved exactly by linear programming solvers, since the problem is very complex. Exact approaches begin to fail beyond 50 customers. The relaxation of integer linear models yields weak lower bounds. In this way, heuristics are required to obtain appropriate solutions in acceptable running times on the large instances that can be met in practical applications. Some constructive heuristics have been proposed for location-routing problems. Although meta-heuristics give better results, constructive heuristics are faster and simpler methods to find feasible solutions.

Multi-echelon distribution systems describe networks with multiple layers. Typically, a network can be composed of three layers: (1) factories, (2) intermediate warehouses, and (3) end-customers, with location decisions in the first or/and second layer. Most authors consider minimum cost flow network models.

In the two-echelon location-routing problems (LRP-2E) routes are added to supply the depots from several main facilities that need to be located. The LRP-2E and its variants are very hard problems to optimize. Janjevic et al. [38] formulate an optimization model that allows to integrate decisions relative to location of both satellite facilities (second echelon) and collection-and-delivery points (third echelon), and the fleet size and routing. Building on the extant literature on continuum approximation (CA) approaches for estimating near-optimal route lengths, the authors formulate an extended routing cost approximation that accounts for deliveries towards both CDPs and individual customers. They integrate these formulae in the routing component of a 2E-CLRP model. Janjevic et al. first use Gurobi Optimizer 6.5.1 as a numerical solver engine, and then propose a heuristic solution method adapted to the problem setting, yielding solutions in a reasonable computational time.

In 2008, Aksen & Altinkemer [43] solve a static location-routing based problem for companies that embrace the clicks-and-bricks strategy in their retail operations. An augmented Lagrangian relaxation method embedded in a sub-gradient optimization procedure generates lower bounds, whereas a heuristic method finds feasible solutions. Lagrangian relaxation is a powerful tool to exploit structural properties of large optimization problems and to derive bounds for the optimal objective. [60] These bounds are widely used as a core of many numerical techniques and also provide a measure for the progress of the main algorithm. A Lagrangian solution is frequently used as a starting or reference point for various heuristics and approximate techniques, as in this paper.

In 2021, Jajevic et al. [45] extended the route cost estimation formulae from the literature and incorporate them in a three-echelon capacitated location-routing problem that optimized the configuration of distribution networks in an integrated way. Routing cost estimations (RCEs) aggregate demand data in a given area to approximate near-optimal route distances and cost within the area and incorporate them into a higher-level ME-LRP. The Gurobi Optimizer is again used as a numerical solver engine.

3.4.2 Models for order fulfilment

The order fulfilment problem involves two decisions: (1) the order assignment decision, and (2) the order shipment decisions. [57]

The order assignment problem consists of assigning the orders to the fulfilment centre so that no inventory shortage is incurred and that the overall shipping cost is minimized. Most retailers subcontract third-party logistics (3PL) to fulfil online orders. The order is normally shipped directly from the fulfilment centre to the customers and the 3PL charges a shipment fee. Li & Jia [57] addressed this problem problem from the perspective of an e-tailer that operates its self-owned logistics.

The order shipment problem consists of deciding the vehicle routing along with decision of order assignment to boost fulfilment performance.

Li & Jia [57] developed a mixed integer linear program for the problem, used CPLEX 12.5 as the MIP solver, analysed its computational complexity and proposed a Benders decomposition algorithm for solving the problem. A linear mixed integer program is an optimization problem in which a nonempty subset of integer variables (unknowns) and a subset of real-valued (continuous) variables exist, the constraints are all linear equations or inequalities, and the objective is a linear function to be minimized (or maximized). Mixed integer programming has several applications, namely describing order fulfilment problems. To solve this hard class of problems, some algorithms can be used. Heuristics can also be applied to improve solution times. [61]

General Algebraic Modeling System (GAMS) is one of the most used software for solving linear and non-linear programming. There are eight categories into which optimization problems can be classified: Integer Programming (IP), Linear Programming (LP), Mixed Integer Programming (MIP), Mixed Integer Linear Programming (MILP), Mixed Integer Quadratic Programming (MIQP), Non- Linear Programming (NLP), Constraint Programming (CP), Mixed Integer Second order cone Programming (MISOCP). These problems can be solved through existing solvers, such as CPLEX, XPRESS, GuRoBi, XA, KNITRO, lp solve, GLPK, CBC, Lindo, CONOPT, among others. Subsection 3.4.3 will analyse the most used solvers and algorithms in the literature analysed.

3.4.3 Solvers analysis

CPLEX solver

CPLEX, from IBM, is one of the most advanced and accepted optimization solvers. It is a complete package for large scale problems as it offers every feature from heuristics to lazy constraints, from branch and cuts to call backs to Distributed or parallel optimization with ease of defining a complex problem into model and rest are the results shown. The main features of CPLEX are:

- Compact language for models with optimization programming language (OPL)
- Variability tester for CPLEX/CPO
- Automatic infeasibility finder

- Status window with log details

Millstein et al. solved the four profit maximizing MILP models using GAMS version 23.5 with CPLEX 12.8. [48]

GuRoBi solver

GuRoBi, from GuRoBi Optimisation, Inc., is a powerful optimizer designed from scratch to run in multi core with capability of running in parallel mode. The main features of GuRoBi are:

- Support for multiple objectives
- Automatic dualization of problems
- Automatic linearization of problems
- Support for user cuts and constraint

Aouad & Ganapathi [62] used Gurobi 9.0.1 to optimize the model mixed integer linear programs in Python.

In 2017, Anand et al. made a comparison between CPLEX and GuRoBi in terms of capabilities and problem domain. The results revealed that both CPLEX and GuRoBi provide competitive optimization solutions, however, CPLEX performs better than GuRoBi under high dimensionality problems and is able to solve non-convex mixed integer quadratic problem. [63]

KNITRO solver

KNITRO, from Ziena Opt., is a nonlinear optimization solver that combines complementary approaches to nonlinear optimization to achieve robust performance over a wide range of application requirements. It is designed for solving large-scale, smooth nonlinear programming problems, and it is also effective for the following special cases: unconstrained optimization, nonlinear systems of equations, least squares, and linear and quadratic programming. [64]

Ishfaq & Bajwa solved the MINLP model with the solver KNITRO. The script language of AMPL was used to iteratively run problem instances in an efficient manner. [8]

Benders' decomposition algorithm

Li & Jia developed an MIP formulation for the problem and showed that the problem was strongly NP-hard. In this way, the authors propose using a Benders decomposition algorithm to solve the problem. [57]

In Li & Jia model, the algorithm decomposes the order fulfilment decision into two problems: the order assignment decision at the master level, and the order routing decision at the slave level (sub-problem). [57]

For certain MIPs with a decomposable structure, CPLEX can apply a Benders' decomposition technique which has a number of levels of control [63]. The benders' decomposition method was originally proposed for MILPs with continuous sub-problems, and it has since been extended to handle a wider range of problems such as nonlinear, integer, multi-stage, and constraint programming problems. The BD method can also be a starting point to develop efficient heuristics for complex problems. [65]

Outer approximation algorithm

The complexity of the non linear mathematical model presented by Ishfaq & Bajwa [8] was handled through a solution approach based on the outer approximation technique. In 1986, Duran & Grossmann presented the outer approximation algorithm for solving mixed-integer nonlinear programming problems of a particular class. The method is based on principles of decomposition, outer-approximation and relaxation, and it effectively exploits the structure of the problems. The algorithm solves an alternating finite sequence of nonlinear programming sub-problems and relaxed versions of a mixed-integer linear master program. [66]

3.5 Retailer Challenges

Most recent literature in the omnichannel environment tries to account for the challenges retailers are facing in the last years. To achieve excellence in an OC environment, retailers must address the following key challenges: [62]

1. **Selecting the right modes of delivery**
   When designing an OC distribution network, it is crucial to choose the right modes of delivery to implement for the end customer. The mode of delivery influences not only the customer demand, but also retailer's decisions in terms of inventory and the SC configuration. This type of choice can vary according to the industry and products that the retailer wants t operate with. To overcome the challenge of providing efficient last-mile delivery, retailers need to adopt flexible operating models by adjusting transportation and facility capacities as well as demand allocations

2. **Deciding the delivery speed**
   To respond to the rising customers' expectations for faster deliveries, many retailers started to offer same-day or even same hour delivery options. These delivery options have to be carefully evaluated to asses if they compensate the huge pressure and cost to the retailer.

3. **Inventory transparency**
   Sharing reliable information about inventory availability and delivery times in all channels is key to provide a transparent and frictionless customer experience across all channels. Retailers should have access to real time inventory information. However, this capability requires a considerable capital investment on technology.

4. **Standardize cross-channel processes**
   To follow an omni-channel approach, retailers should look at the fulfilment process from a holistic point of view, considering the different touch-points between online and offline channels. [41] The different demand from the different channels should be managed as a whole. In this way, implementing standardized cross-channel processes is crucial to ensure operational readiness in an omnichannel SC. Treating each channel separately prevents retailers from exploring the potential synergies of integrating all channels.

3.6 Literature gap

To summarize all the models presented and to look for a new searching direction, all operational models analysed previously were categorized into seven parameters in table 3.3. This first column is the reference and the next four columns indicate the channel designs considered. Columns six and seven indicate whether the model considers home delivery or collection and delivery points for the last-mile. The next two columns show whether the model considers online or store demand (or both). Columns ten to fourteen present the type of decision the models incorporate. The type of objective function is also evaluated, followed by the assessment of channel approach. Finally, the last three columns describe the formulation and solving methods, indicating the type of model, in which kind of problem is the model formulated, and the solver, algorithms and heuristics used.

The review of literature presented exhibits that research on order fulfillment in an omnichannel retail setting remains scarce. Many authors have highlighted this gap in literature and identified the assessment of different order fulfillment configurations as a future area of research. [8] The aim of this study is to fill this gap (see Table 3.3) through an optimization model that:

1. Compares three different network configurations (including shipping from dark store);
2. Includes the last-mile delivery;
3. Responds to both online and store demand;
4. Accounts the following decisions: facilities' locations, capacity and closure, and order fulfilment;
5. Maximizes profit
6. Has an omnichannel approach;
7. Uses CPLEX as the solution solver.

Table 3.3: Literature review and research contributions.

Article	Network configuration				Last-mile delivery		Demand		Decision					Objective function		Approach		Formulation and solving		
	Ship from store	Ship from warehouse	Ship from store and warehouse	Ship from dark store	Home	CDP	Online	Store	Facilities' location	Facilities' capacity	Facilities' closure	Vehicle routing	Order fulfilment	Min cost	Max profit	MC	OC	Model	Problem formulation	Solution method
Aksen and Altinkemer (2008)[43]	X				X		X	X	X	X	X	X		X		X		Discrete Optimization	LRP	Lagrangian relaxation method (LR)
Janjevic et al. (2021)[45]		X			X	X	X		X		X	X		X			X	CA Optimization	LRP	Gurobi
Janjevic et al. (2019)[44]		X			X	X	X		X					X		X		CA Optimization	2E-CLRP	Gurobi; Heuristic
Li & Jia (2019)[57]		X			X		X						X	X			X	Optimization	MILP	CPLEX; Benders decomposition algorithm
Ishfaq & Bajwa (2019)[8]			X		X		X						X		X	X		Optimization	MINLP	KNITRO; Outer Approximation (AO) technique
Aouad & Ganapathi (2020)[62]		X		X	X	X	X			X	X		X	X			X	Optimization	MILP	Gurobi
Millstein et al. (2022)[48]	X	X	X		X		X	X	X	X	X		X		X		X	Optimization	MILP	CPLEX
This paper	X	X	X	X	X		X	X	X	X	X		X		X		X	Optimization	MILP	CPLEX

Chapter 4

Problem and Model formulation

This chapter focuses on the development of three different network configuration models. Section 4.1 summarizes and defines the problem under analysis. In this section, the model objective of finding the most suitable configurations of different product categories and network configurations, while maximizing profit will be detailed. Next, section 4.2 presents the developed mathematical formulation for the three different models, alongside with all assumptions, indexes, sets, parameters, constraints, and objective functions. The last section, 4.3 summarizes the main conclusions of this chapter.

4.1 Problem definition

This work seeks to study three different network configurations in the last-mile distribution so as to verify the benefits obtained from their adoption. Building up the research by Millstein [48], this work will add up the dark store as a new option in the last-mile distribution network to study whether or not it has operational efficiency and economical profitability. In order to evaluate the different configurations, we will develop the corresponding network representation models, which will integrate three to four SC echelons:

- **Suppliers** - There are a fixed number of suppliers available to the retailer at a fixed location. The model will evaluate the appropriate suppliers to provide the warehouses.
- **Warehouses** - There are a fixed number of possible warehouse locations (not in every market) with seven possible capacities. Warehouses receive the products from the suppliers and ship them to 1) online markets, 2) stores, and 3) dark stores. Warehouses are subcontracted and entail a fixed cost. The model will evaluate the most interesting warehouse location and capacity to contract.
- **Stores** - There is a store located in each market. Stores receive products from the warehouses and can sell them 1) directly to customers physically, or 2) to the online market. It is assumed that the stores are already a retailer asset. The model will decide to maintain open or to close stores in every location.

- **Dark stores** - Dark stores can be located in every market and their capacity depends on that location. These stores are closed-to-customer facilities that receive products from the warehouses and ship them to the online market. These are also assumed to be subcontracted and entail a fixed cost. The model will evaluate the most interesting dark store locations.

Multichannel distribution counts with warehouses and stores to fulfill online and store demand. This research adds the dark store facility to help fulfill the online demand in concentrated urban centres. To build an omnichannel distribution model, warehouses, stores and dark store should be integrated in order to respond to both online and store demand. To do so, three different channel configurations were designed to be studied:

- **Ship from store and warehouse (SFSW)** This channel design is the most traditional. With three echelons, the network starts in the suppliers that provide products to the warehouses. Here, products either follow two directions: 1) are shipped to the stores, or 2) are shipped directly to customers of any market to fulfill online demand. The products that end in the stores can be purchased by customers physically or can also be shipped to any market to fulfill online demand. This channel design was built upon the Millstein model [48].

- **Ship from dark store and warehouse (SFDSW)** This channel design adds the dark store to the previous network. It is analogous to the SFSW model until the warehouses echelon. Here, products either follow three directions: 1) are shipped to the stores, 2) are shipped to dark stores, or 3) are shipped directly to customers of any market to fulfill online demand. The products that end in the stores can only be purchased by customers physically, unlike the SFSW formulation. In the dark stores, products are shipped to the same market to fulfill online demand.

- and **Ship from store, dark store and warehouse (SFSDSW)** This channel design is the combination of the SFSW and SFDSW models. It is analogous to the SFDSW model until the moment products arrive at stores and dark stores. The difference lies in the fact that both stores and dark stores can satisfy the online demand. With the slight distinction that stores can ship products to all markets whereas dark stores ship only to the same market.

The three channel designs are represented in figure 4.1, for a better understanding of the differences.

To conclude, given a set of suppliers, warehouses, stores, and dark stores, and potential locations and capacities for each, the problem aims to identify the optimal locations of warehouses and dark stores, the closure or opening of stores, and the flows between all network nodes, so as to maximize the profit of the distribution.

4.1.1 Objectives

The main objective is to study the advantages of a network configuration with dark stores, versus a traditional configuration with stores and warehouses ensuring the last-mile distribution. To do so, the models were constructed not with the goal of minimizing all the operational costs, but with the profit maximization goal in mind. As highlighted in the previous chapters, the customer is changing and new

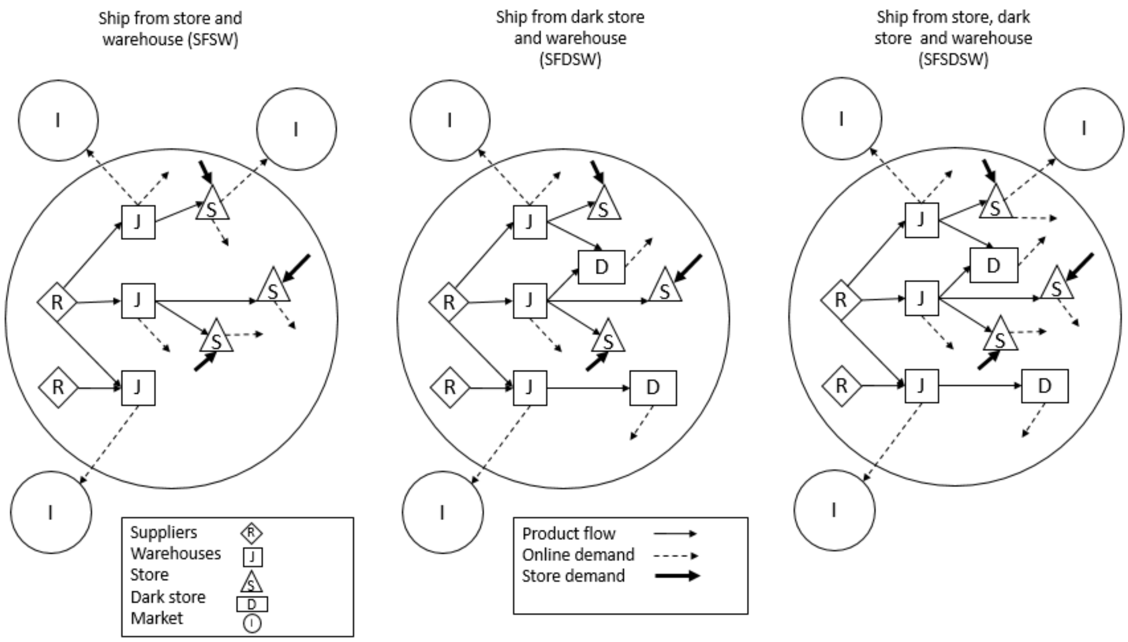


Figure 4.1: Channel designs illustration.

ways of shopping are emerging. [18] The retailers' goal should not be only to reduce costs but to expand their network to account for more efficient ways to fulfill demand, even if an initial investment is required. In this work we want to assess in which conditions is profitable or not to include dark stores into a retailers' network.

Another objective is to analyse the three channel designs for different product categories. These categories are in line with the retail sectors described in section 2.3: Electronics, Apparel and Food. Depending on the product characteristics, and for different levels of online and store demand, this research will give the best configuration with the optimal flows, locations and capacities for the omnichannel model, to maximize profit.

4.2 Mathematical formulation

In this section the mathematical formulation of the three models developed is presented. The models were constructed based on the work of Millstein from 2022 [48] with adaptations to include a dark store network configuration. Like Millstein, we formulated our problem as a profit maximizing mixed-integer programming model (MILP) incorporating costs for transportation to serve online and store demands, costs for warehouses and dark stores (inventory, handling and fixed costs), and costs for stores (inventory, handling). Assumptions were made throughout the formulation of the models, and will be explicated in subsection 4.2.1. Next, the definition of sets, parameters, functions, and decision variables will be presented in subsections 4.2.2, 4.2.3, and 4.2.4, respectively. Lastly, subsections 4.2.5, 4.2.6, and 4.2.7 depict the constraints and objective functions for the three networks representation's formulations.

4.2.1 Assumptions

Before starting formulating the model, a few assumptions were taken. In order to build a mixed integer linear programming model, some variables had to be exogenously defined as parameters to ensure the linearity of the model. In each model a single product category retailer is studied, subject to two markets with two different demands: physical store demand and online demand. The total demand for each market is assumed to be a given parameter, as well as the indication of whether each market includes retail stores, warehouses and suppliers. In addition, for each market, the market share of retailer's stores and the online percentage of retailer's demand are exogenously specified.

By studying only one product category for each retail sector, we assumed an average for all profit and costs parameters, to improve the efficiency of the model. This is not the ideal situation since the majority of retailers sell different products within the same category.

Another assumption is related to the facilities. In these models, stores are assumed to be a retailer asset. In this way, the only store cost considered is the inventory holding and handling cost. Unlike stores, we assume warehouses are subcontracted. Thus, a fixed cost is associated with this facility, in addition to the inventory holding and handling costs. Dark stores cost structure is analogous to the warehouses. Since we assume that the retailer wants to assess the feasibility of incorporating this facility in the network, we assume beforehand that the best way is to rent the dark store space.

Finally, dark stores were assumed to have only one possible capacity, unlike warehouses. This was assumed since these facilities take place in urban spaces that are quite limited to commercial space areas.

4.2.2 Sets definition

As illustrated in figure 4.1, the network is modelled through nodes representing markets. Each market accommodates online and store demand, as well as four types of entities/facilities: 1) suppliers, 2) warehouses, 3) stores, and 4) dark stores. To formulate this mathematically, let I be the set of markets (represented by a circle in figure 4.1). Each market corresponds to a specific location at the network.

The remaining sets representing the 1) suppliers, 2) warehouses, 3) stores, and 4) dark stores, are actually subsets of set I. R is the set of nodes representing the suppliers, with $R \subseteq I$. J is the set of the candidate warehouse locations, belonging also to the set of markets I, $J \subseteq I$. Let S be the set of store locations in the network, also belonging to the set of markets I ($S \subseteq I$). Analogously, D is the set of dark stores locations, with $D \subseteq I$. Finally, set K represents the possible warehouse capacities for each potential location.

The set definition was based on the Millstein model [48], except for the set of dark stores locations.

Table 4.1 summarizes all sets and subsets used in the three models.

Table 4.1: Sets for the models.

Sets	Description
I	The set of nodes representing markets
R	The set of nodes representing the suppliers, $R \subseteq I$
J	The set of potential warehouse locations, which belong to the set of markets, $J \subseteq I$
K	The set of warehouse sizes (capacities)
S	The set of store locations, which belong to the set of markets, $S \subseteq I$
D	The set of dark store locations, which belong to the set of markets, $D \subseteq I$

4.2.3 Parameters and calculation formulas

Parameters represent the input data to the optimization model. Some option parameters are modelled as function of other input parameters, as costs, profits, demands, among others. First, the input parameters for the functions will be enunciated in **Input parameters for the calculation formulas**, then the functions will be explained in **Parameters' calculation formulas**, and then the remaining parameters will be presented in **Parameters for the models**.

Input parameters for the calculation formulas

All input parameters for the functions are listed in table 4.2.

Cwl_j, Crl_s, and Cdl_d are the unitary handling cost of an online order fulfilled from a warehouse j, a store s, and a dark store d, respectively. Let Crh_s be the unitary inventory holding cost of a store located at s.

The total demand in market i is $Demand_i$. The demand is exogenous, and corresponds to the total average units. This parameter includes both the online and store demand, that is why $OLpct$ is needed to indicate the percentage of demand available to the online channel. G indicates the percentage of store demand available to the store channel.

The parameter $Distance$ represents the distance in kilometers between every pair of facilities/entities. $profit$ represents the gross profit (sell-price of purchase) for the retailer for both the online and store channel.

Let F be a demand adjustment coefficient which utility will be explained in the next subsection. Finally, WDD is the maximum distance that can be covered during a work day.

Parameters Cwl_j, Crh_s, Crl_s, $Demand_i$, F, G, $OLpct$, $profit$ were adapted from Millstein et al. (2022) model, and parameters Cdl_d, $Distance$, WDD were originally defined.

Parameters' calculation formulas

Having in consideration all parameters explained in the previous subsection, now the calculation formulas will be explained. All formulas are listed in table 4.3. The first six formulas concern the shipping

Table 4.2: Input parameters for the calculation formulas used in the models.

Parameters	Description
Cdl_d	Dark store handling cost per unit for an online order fulfilled from a dark store located at d
Cwl_j	Warehouse handling cost per unit for an online order fulfilled from a warehouse located at j
Crh_s	Inventory holding cost per unit for store located at s
Crl_s	Handling cost per unit of online orders fulfilled from a store located at s
$Demand_i$	Total demand in market i
$Distance_{i(j,s,d)}$	Kilometers to ship to every online market i from every online fulfillment facility node (warehouse, store or dark store) located at (j, s, d)
$Distance_{(s,d)j}$	Kilometers to ship to every facility node (store or dark store (s, d) from every warehouse located at j
$Distance_{jr}$	Kilometers to ship to every warehouse located at j from every supplier located at r
F	Demand adjustment coefficient
G	Percentage of store demand available to the store channel
$OLpct$	Percentage of total demand available to the online channel
$profit$	Gross profit per unit for both channels (online and store)
WDD	Distance covered during a work day

costs between two entities. The shipping cost is achieved by multiplying the distance by a cost factor depending on the route taken.

Regarding demand, Dr_i is the demand available at the retailer's stores at the physical channel in market i. Don_i is the total online demand in market i. Let $T_{i(j,s,d)}$ represent the number of delivery days needed to ship a product from a fulfillment facility to an online market i. This is achieved by dividing the distance by the maximum distance that can be covered during a work day WDD. $Donl_{i(j,s)}$ represents the total online demand for market i available to the retailer from a specific facility (warehouse or store) at location j or s. This formula reflects the influence of the delivery time in the demand. F_i is a demand adjustment exogenous coefficient for market i that reflects demand elasticity based on delivery time and the level of competition in the online market. The fraction $\frac{1}{T_{i(j,s)}*F}$ captures the benefits of fast delivery and the level of online competition. $Donl_{id}$ is analogous to $Donl_{i(j,s)}$ for online orders served from a dark store at d.

$pcow_{ij}$ represents the profit achieved for an online order in market i fulfilled from a warehouse located at j, by subtracting the warehouse handling cost and the shipping cost from the gross profit. pcr_s represents the unitary profit when a product is purchased physically at a store at s, subtracting the inventory holding cost from the gross profit. $pcos_{is}$ is the profit per unit for an online order in market i fulfilled from a store at s, obtained by subtracting the handling cost and the shipping cost from the gross profit. Finally, the last profit formula ($pcod_{id}$) is for online orders fulfilled from dark stores at d, resulting from a subtraction of both handling and shipping costs from the gross profit.

Parameters $Csow_{ij}$, $Csos_{is}$, $Csrw_{sj}$, $Csrr_{jr}$, Dr_i, Don_i, $T_{i(j,s,d)}$, $pcow_{ij}$, pcr_s, $pcos_{is}$ were adapted from the Millstein et al. (2022) model, and parameters $Cdrw_{dj}$,$Csod_{id}$, $Donl$, $pcod_{id}$ were originally defined.

Parameters for the models

The remaining parameters are listed in table 4.4. Cap_{jk}, $Caps_s$, and $Capds_d$ are capacity parameters, in number of units, for a warehouse of size k located at j, for a store located at s, and for a dark

Table 4.3: Parameters' calculation formulas.

Formulas	Description
$Cdrw_{dj} = factor * Distance_{dj}$	Shipping cost per unit to fulfil a dark store located at d from a warehouse located at j
$Csow_{ij} = factor * Distance_{ij}$	Shipping cost per unit for an online order in market i from a warehouse located at j
$Csos_{is} = factor * Distance_{is}$	Shipping cost per unit for an online order in market i from a store located at s
$Csod_{id} = factor * Distance_{id}$	Shipping cost per unit for an online order in market i from a dark store located at d
$Csrw_{sj} = factor * Distance_{sj}$	Shipping cost per unit to fulfil a store located at s from a warehouse located at j
$Csrr_{jr} = factor * Distance_{jr}$	Shipping cost per unit to fulfil a warehouse located at j from a supplier located at r
$Dr_i = Demand_i * (1 - OLpct) * G$	Retail demand for the retailer at store(s) in market i (total retail market size)
$Don_i = Demand_i * OLpct$	Total online demand in market i (online market size)
$T_{i(j,s,d)} = Distance_{i(j,s,d)}/800$	Number of delivery days to ship to every online market $i \in I$ from the fulfillment facility node (warehouse,store or dark store) located at j or s or d
$Donl_{i(j,s)} = Demand_i * OLpct * \frac{1}{T_{i(j,s)} * F}$	Total online demand for market i available to the retailer when the market is served from a facility (warehouse or store) at location j or s
$Donl_{id} = Demand_i * \frac{OLpct}{20} * \frac{1}{T_{id} * F}$	Total online demand for market i available to the retailer when the market is served from a dark store at location d
$pcow_{ij} = profit - Cwl_j - Csow_{ij}$	Profit per unit for an online order in market i fulfilled from warehouse located at j
$pcr_s = profit - Crh_s$	Profit per unit for a store order located at s
$pcos_{is} = profit - Crl_s - Csos_{is}$	Profit per unit for online order located in market i fulfilled from a store located at s
$pcod_{id} = profit - Cdl_d - Csod_{id}$	Profit per unit for online order located in market i fulfilled from a dark store located at d

store located at d (respectively). Cdo_d and Cwo_{jk} are both fixed costs for a dark stores at d and for a warehouse of capacity k at j, respectively. Cwh_{jk} is the inventory holding cost per unit for a warehouse of size k located at j. HD and HS are the minimum number of orders to keep a dark store and a store open, respectively. And finally, M is a very large number that will be useful in the constraint formulation.

Here, parameters Cap_{jk}, $Caps_s$, Cwo_{jk}, Cwh_{jk}, HS, M were based on Millstein et al. (2022) formulation [48] and $Capds_d$, Cdo_d, HD were originally defined.

Table 4.4: Parameters for the models.

Parameters	Description
Cap_{jk}	Capacity in number of units of a warehouse of size k located at j
$Caps_s$	Capacity in number of units of a store located at s
$Capds_d$	Capacity in number of units of dark store located at d
Cdo_d	Fixed cost of a dark store located at d
Cwo_{jk}	Warehouse fixed cost for warehouse of size k located at j
Cwh_{jk}	Inventory holding cost per unit for a warehouse of size k located at j
HD	Minimum number of orders to keep the dark store open
HS	Minimum number of orders to keep the store open
M	A very large number, M=M_1, M_2, M_3, M_4, M_5

4.2.4 Decision Variables

There are two types of decision variables in this problem: binary ones, which indicate if a given entity or facility is opened or not, and continuous variables. All decision variables are summarized in table 4.5.

There are three binary variables concerning the use of three facilities. Y_{jk} denotes if warehouse of size k is opened at location j, St_s indicates if a store is open at s, and Dt_d that shows if a dark store is to be opened at location d.

The remaining decision variables are non-negative and continuous. The first is X_{ij}, indicating the number of units shipped to online market i from a warehouse located at j, then RS_{jr} is the number of units shipped to a warehouse located at j from a supplier located at r, then V_{sj} the total number of units shipped to a store located at s from a warehouse located at j, and P_{dj} the number of units shipped to a dark store located at d from a warehouse located at j.

Qs_s is the quantity of units sold to store customers at a store located at s and U_{is} the quantity of the ones shipped to online market i. Finally, W_{id} indicates the number of units shipped to online market i from a dark store located at d.

Dt_d, P_{dj}, and W_{id} were originally defined for the models, the remaining decision variables were adapted from the Millstein et al. (2022) formulation [48].

Table 4.5: Decision variables for the models.

Decision variables	Description
Y_{jk}	Binary variable that indicates if a warehouse of size k is open ($Y_{jk} = 1$) or closed ($Y_{jk} = 0$) at location j
St_s	Binary variable that indicates if a store is open ($St_s = 1$) or closed ($St_s = 0$) at location s
Dt_d	Binary variable that indicates if a dark store is open ($Dt_d = 1$) or closed ($Dt_d = 0$) at location d
X_{ij}	Total number of units shipped to online market i from a warehouse located at j
RS_{jr}	Total number of units shipped to a warehouse located at j from a supplier located at r
V_{sj}	Total number of units shipped to a store located at s from a warehouse located at j
P_{dj}	Total number of units shipped to a dark store located at d from a warehouse located at j
Qs_s	Total number of units sold to store customers at a store located at s
U_{is}	Total number of units shipped to online market i from a store located at s
W_{id}	Total number of units shipped to online market i from a dark store located at d

4.2.5 Constraints for the Ship from Store and Warehouse (SFSW) model

As already explained in section 4.1, the SFSW model is the most traditional one where all online orders are shipped either from stores or warehouses, depending on what is more profitable. Stores are replenished by warehouses and, consequently, these are replenished from suppliers. Stores can also sell products physically to customers.

The majority of the formulation of the SFSW model is based on Millstein et al. (2022) [48]. The objective function for SFSW model is stated in equation 4.1. The first term of the equation is the gross profit earned from store. The next two terms are the gross profit earned from online sales (for orders fulfilled by warehouses and by stores), respectively. The fourth term is the warehouses fixed costs.

The fifth term is the warehouse inventory holding cost, which depends on the number of units shipped to fulfill online orders (X_{ij}) and the number of units shipped to stores (V_{sj}). This term is nonlinear as the binary warehouse size and location variable (Y_{jk}) is multiplied by the continuous flow variables (X_{ij} and V_{sj}). To linearize the term, Millstein et al. (2022) [48] defined the positive variable WHC_{jk} as the warehouse holding cost and replaced the nonlinear term with $\sum_{j\in J}\sum_{k\in K} WHC_{jk}$, by adding the constraint 4.3. In this constraint, the constant M is set to be higher than the sum of the maximum possible inventory holding cost of a warehouse. Note that if Y_{jk} is equal to 1, then equation 4.3 requires the warehouse holding cost in the objective to be at least as large as the sum of order flows times the holding cost per unit. Because of the profit maximization, this cost will be no higher than the minimum required. If Y_{jk} equals 0, then the warehouse holding cost will be zero to maximize the profit.

The sixth and seventh terms represent the shipping costs of the product flow between warehouses and stores, and between suppliers and warehouses, respectively.

$$Maximize\ profit = \sum_{s\in S} pcr_s * Q_s + \sum_{i\in I}\sum_{j\in J} pcow_{ij} * X_{ij} + \sum_{i\in I}\sum_{s\in S} pcos_{is} * U_{is} - \sum_{j\in J}\sum_{k\in K} Cwo_{jk} * Y_{jk} - \sum_{j\in J}(\sum_{k\in K} Cwh_{jk}Y_{jk} * (sum_{i\in I} X_{ij} + sum_{s\in S} V_{sj})) - \sum_{s\in S}\sum_{j\in J} Csrw_{sj} * V_{sj} - \sum_{j\in J}\sum_{r\in R} Csrr_{jr} * RS_{jr} \tag{4.1}$$

The objective function for the SFSW model is now reformulated for the linearization issue in equation 4.2.

$$Maximize\ profit = \sum_{s\in S} pcr_s * Q_s + \sum_{i\in I}\sum_{j\in J} pcow_{ij} * X_{ij} + \sum_{i\in I}\sum_{s\in S} pcos_{is} * U_{is} - \sum_{j\in J}\sum_{k\in K} Cwo_{jk} * Y_{jk} - \sum_{j\in J}\sum_{k\in K} WHC_{jk} - \sum_{s\in S}\sum_{j\in J} Csrw_{sj} * V_{sj} - \sum_{j\in J}\sum_{r\in R} Csrr_{jr} * RS_{jr} \tag{4.2}$$

$$Cwh_{jk} * (\sum_{i\in I} X_{ij} + \sum_{s\in S} V_{sj}) - M(1 - Y_{jk}) \leq WHC_{jk}\ for\ all\ j \in J\ and\ for\ all\ k \in K \tag{4.3}$$

Constraint 4.4 ensures that if a warehouse of capacity k located at j is closed, then no flow can exist between that warehouse and any market i. Constraint 4.5 is analogous, but for the flow between the warehouse and all suppliers at r. Constraint 4.5 was not adapted from Millstein model.

Constraints 4.6 and 4.7 limit the online demand in market i served from warehouse j, and from store s, respectively, to the maximum level assigned in table 4.3. Constraint 4.8 limits the total online demand of market i served from all the warehouses and all stores for those that can ship within one day. Constraint 4.9 is similar, however, this equation limits the online demand for warehouses and stores that ship within a particular number of delivery days ($T_{ij} = \{1, 2, 3, 4, 5, 6, 8\}$). Constraint 4.10 ensures that the number of units sold at a store at s do not exceed the available store demand for the retailer.

Constraint 4.11 ensures that the total units shipped to the online markets and to stores from warehouse j do not exceed the total units shipped from suppliers to that warehouse. Equation 4.12 is another shipment balance constraint, but for any store open at s. This constraint ensures that the quantity of units sold at a store s and the quantity shipped from that store to online markets do not surpass the quantity shipped from the warehouses to that store.

Constraint 4.13 ensures that the quantity shipped from warehouse j is limited by the actual capacity of that warehouse. Constraints 4.14, 4.15, and 4.16 ensure that the total units shipped to online markets from store s, sold at store s, and shipped from warehouses to store s, are limited to the capacity of that store, respectively. These three constraints were originally formulated in this work.

Constraint 4.17 makes sure that each warehouse j is opened at only one capacity k. Constraint 4.18 ensures that if a store at s is closed, then nothing can be sold at that store. Constraint 4.19

is analogous, if a store at s is closed, then no flow can exist between that store and online markets. Similarly, constraint 4.20 ensures that a store at s is open if the quantity sold at that store exceeds the predetermined minimum number of orders.

Finally, the last two constraints, 4.21 and 4.22 are variable domain constraints.

$$X_{ij} \leq \sum_{k \in K} Y_{jk} * M_1 \; for \; all \; i \in I \; and \; for \; all \; j \in J \tag{4.4}$$

$$RS_{jr} \leq \sum_{k \in K} Y_{jk} * M_2 \; for \; all \; j \in J \; and \; for \; all \; r \in R \tag{4.5}$$

$$X_{ij} \leq Donl_{ij} \; for \; all \; i \in I \; and \; for \; all \; j \in J \tag{4.6}$$

$$U_{is} \leq Donli_{is} \; for \; all \; i \in I \; and \; for \; all \; s \in S \tag{4.7}$$

$$\sum_{j \in J} X_{ij} + \sum_{s \in S} U_{is} \leq Demand_i * OLpct * \frac{1}{F} \; for \; all \; i \in I \tag{4.8}$$

$$\sum_{j \in J | T_{ij} = t} X_{ij} + \sum_{s \in S | T_{ij}} U_{is} \leq Demand_i * OLpct * \frac{1}{F * t} \; for \; all \; i \in I \; and \; T_{ij} = \{1, 2, 3, 4, 5, 6, 8\} \tag{4.9}$$

$$Q_s \leq Dr_s \; for \; all \; s \in S \tag{4.10}$$

$$\sum_{i \in I} X_{ij} + \sum_{s \in S} V_{sj} \leq \sum_{r} RS_{jr} \; for \; all \; j \in J \tag{4.11}$$

$$Qs_s + \sum_{i \in I} U_{is} \leq \sum_{j} V_{sj} \; for \; all \; s \in S \tag{4.12}$$

$$\sum_{i \in I} X_{ij} + \sum_{s \in S} V_{sj} \leq \sum_{k} Y_{jk} Capjk \; for \; all \; j \in J \tag{4.13}$$

$$\sum_{i \in I} U_{is} \leq Caps_s \; for \; all \; s \in S \tag{4.14}$$

$$Qs_s \leq Caps_s \; for \; all \; s \in S \tag{4.15}$$

$$\sum_{j \in J} V_{sj} \leq Caps_s \; for \; all \; s \in S \tag{4.16}$$

$$\sum_{k \in K} Y_{jk} \leq 1 \; for \; all \; j \in J \tag{4.17}$$

$$Qs_s \leq St_s * M_3 \; for \; all \; s \in S \tag{4.18}$$

$$U_{is} \leq St_s * M_4 \; for \; all \; i \in I \; and \; for \; all \; s \in S \tag{4.19}$$

$$(HS - Qs_s) \leq (1 - St_s) * M_5 \; for \; all \; s \in S \tag{4.20}$$

$$Y_{jk}, St_s \in \{0, 1\} \tag{4.21}$$

$$X_{ij}, U_{is}, V_{sj}, Qs_s, RS_{jr} \geq 0 \tag{4.22}$$

4.2.6 Constraints for the Ship from Dark Store and Warehouse (SFDSW) model

The SFDSW model is a variation of the SFSW model with a new entity: dark store. Here all online orders are shipped either from dark stores or warehouses, depending on what is more profitable, and stores only fulfill physical demand. Both stores and dark stores are replenished by warehouses and, consequently, these are replenished from suppliers. In this way, the objective function for SFDSW model is stated in equation 4.23. It is quite similar to the objective function for the SFSW model (4.2, however with slight changes to accommodate the addition of the dark stores.

The first three terms are the gross profit earned from store and online sales (for orders fulfilled by warehouses and by dark stores), respectively. The fourth and fifth terms are the warehouses and the dark stores fixed costs, respectively. The sixth term is the linearized warehouse inventory holding cost, already explained in equation 4.1. The last three terms represent the shipping costs of the product flow between: warehouses and stores, warehouses and dark stores, and suppliers and warehouses, respectively.

$$Maximize \; profit = \sum_{s \in S} pcr_s * Q_s + \sum_{i \in I} \sum_{j \in J} pcow_{ij} * X_{ij} + \sum_{i \in I} \sum_{d \in D} pcod_{id} * W_{id} - \sum_{j \in J} \sum_{k \in K} Cwo_{jk} * Y_{jk} - \sum_{d} Cdo_d * Dt_d - \sum_{j \in J} \sum_{k \in K} WHC_{jk} - \sum_{s \in S} \sum_{j \in J} Csrw_{sj} * V_{sj} - \sum_{d \in D} \sum_{j \in J} Cdrw_{dj} * P_{dj} - \sum_{j \in J} \sum_{r \in R} Csrr_{jr} * RS_{jr} \tag{4.23}$$

This objective function is subject to constraints 4.4, 4.5, 4.17, 4.10, 4.15, 4.16, 4.6, 4.18, 4.20, 4.21, 4.22 and

$$Cwh_{jk} * (\sum_{i \in I} X_{ij} + \sum_{s \in S} V_{sj} + \sum_{d \in D} P_{dj}) - M(1 - Y_{jk}) \leq WHC_{jk} \; for \; all \; j \in J \; and \; for \; all \; k \in K \tag{4.24}$$

$$W_{id} \leq Donlin_{id} \; for \; all \; i \in I \; and \; for \; all \; d \in D \tag{4.25}$$

$$\sum_{j \in J} X_{ij} + \sum_{d \in D} W_{id} \leq Demand_i * OLpct * \frac{1}{F} \; for \; all \; i \in I \tag{4.26}$$

$$\sum_{j \in J | T_{ij} = t} X_{ij} + \sum_{d \in D | T_{ij}} W_{id} \leq Demand_i * OLpct * \frac{1}{F * t} \; for \; all \; i \in I \; and \; T_{ij} = \{1, 2, 3, 4, 5, 6, 8\} \tag{4.27}$$

$$\sum_{i \in I} X_{ij} + \sum_{s \in S} V_{sj} + \sum_{d \in D} P_{dj} \leq \sum_{r} RS_{jr} \; for \; all \; j \in J \tag{4.28}$$

$$Qs_s \leq \sum_{j} V_{sj} \; for \; all \; s \in S \tag{4.29}$$

$$\sum_{i \in I} W_{id} \leq \sum_{j} P_{dj} \; for \; all \; d \in D \tag{4.30}$$

$$\sum_{i \in I} X_{ij} + \sum_{s \in S} V_{sj} + \sum_{d \in D} P_{dj} \leq \sum_{k} Y_{jk} Capjk \; for \; all \; j \in J \tag{4.31}$$

$$\sum_{i \in I} W_{id} \leq Capds_s \; for \; all \; d \in D \tag{4.32}$$

$$\sum_{j \in J} P_{dj} \leq Capds_s \; for \; all \; d \in D \tag{4.33}$$

$$W_{id} \leq Dt_d * M_1 \; for \; all \; i \in I \; and \; for \; all \; d \in D \tag{4.34}$$

$$(HD - \sum_{i} W_{id}) \leq (1 - Dt_d) * M_2 \; for \; all \; d \in D \tag{4.35}$$

$$Dt_d \in \{0, 1\} \tag{4.36}$$

$$P_{dj}, W_{id} \geq 0 \tag{4.37}$$

Constraint 4.24 is analogous to 4.3 including the flow between the warehouse and dark stores. Constraint 4.25 is analogous to 4.7 but instead of the flow being between stores and online markets, it is between dark stores and online markets. Constraints 4.26 and 4.27 are analogous to constraints 4.8 and 4.9. Constraint 4.28 is analogous to 4.11.

Constraint 4.29 is analogous to 4.12 without the flow between the store and online markets. Constraint 4.30 ensures that the total units shipped to the online markets from dark store at d do not exceed the total units shipped from warehouses to that dark store. Constraint 4.31 is analogous to 4.13. Constraint 4.32 ensures that the quantity shipped from dark store d to online markets is limited by the actual capacity of that dark store. Similarly, constraint 4.33 ensures that the quantity shipped from warehouses to dark store at d is also limited by dark store capacity. Constraint 4.34 ensures that if a dark store at d is closed, then no flow can exist between that dark store and online markets. Similarly, constraint 4.35 ensures that a dark store at d is only open if the quantity sold at that dark store exceeds the predetermined minimum number of orders. Finally, the last two constraints, 4.36 and 4.37 are variable domain constraints that complement constraints 4.21 and 4.22, respectively.

Equation 4.29 was adapted from the Millstein et al. (2022) model [48] and the remaining were originally formulated for this model.

4.2.7 Constraints for the Ship from Store, Dark Store and Warehouse (SFSDSW) model

The SFSDSW model is a combination of the two previous models, SFSW and SFDSW. In this formulation both stores and dark stores can ship from their facilities to online markets, depending on what is more profitable. Warehouses can also still ship from their facilities to online markets. Similarly to what already happened, stores and dark stores are replenished by warehouses and, consequently, these are replenished from suppliers. In this way, the objective function for SFSDSW model is stated in equation 5.1. It is very similar to the objective function for the SFDSW model (4.23), only with minor changes to accommodate the addition of the shipment to online markets from stores.

The first fourth terms are the gross profit earned from store and online sales (for orders fulfilled by warehouses, stores and dark stores), respectively. The fifth and sixth terms are the warehouses and the dark stores fixed costs, respectively. The seventh term is the linearized warehouse inventory holding cost, already explained in equation 4.1. The last three terms represent the shipping costs of the product flow between: warehouses and stores, warehouses and dark stores, and suppliers and warehouses, respectively.

$$Maximize\ profit = \sum_{s \in S} pcr_s * Q_s + \sum_{i \in I} \sum_{j \in J} pcow_{ij} * X_{ij} + \sum_{i \in I} \sum_{s \in S} pcos_{is} * U_{is} + \sum_{i \in I} \sum_{d \in D} pcod_{id} * W_{id} - \sum_{j \in J} \sum_{k \in K} Cwo_{jk} * Y_{jk} - \sum_{d} Cdo_d * Dt_d - \sum_{j \in J} \sum_{k \in K} WHC_{jk} - \sum_{s \in S} \sum_{j \in J} Csrw_{sj} * V_{sj} - \sum_{d \in D} \sum_{j \in J} Cdrw_{dj} * P_{dj} - \sum_{j \in J} \sum_{r \in R} Csrr_{jr} * RS_{jr} \tag{4.38}$$

This objective function is subject to constraints 4.4, 4.5, 4.17, 4.10, 4.14, 4.15, 4.16, 4.6, 4.18, 4.19, 4.20 4.21, 4.22, 4.24, 4.25, 4.7, 4.28, 4.12, 4.30, 4.31, 4.32, 4.33, 4.34, 4.35, 4.36, 4.37 and two additional constraints:

$$\sum_{j \in J} X_{ij} + \sum_{d \in D} W_{id} + \sum_{s \in S} U_{is} \leq Demand_i * OLpct * \frac{1}{F} \ for \ all \ i \in I \tag{4.39}$$

$$\sum_{j \in J | T_{ij}=t} X_{ij} + \sum_{d \in D | T_{ij}} W_{id} + \sum_{s \in S | T_{ij}=t} U_{is} \leq Demand_i * OLpct * \frac{1}{F * t} \ for \ all \ i \in I \ and \ T_{ij} = \{1, 2, 3, 4, 5, 6, 8\} \tag{4.40}$$

Constraints 4.39 and 4.40 are analogous to constraints 4.26 and 4.27, accommodating the flow between stores and online markets. All these constraints are original work.

4.3 Chapter conclusions

In this chapter the problem at hand was described, alongside with its assumptions, while providing context for the developed omnichannel distribution network optimization models.

The objectives of the model were presented, with an illustration of the three different channels designs. The mathematical model proposed is a Mixed Integer Linear Programming (MILP). The modeling assumptions were presented, alongside with the detailed description of the mathematical formulation and notion. In a general way, the models consider a multi-echelon SC with the following entities: suppliers, warehouses, stores and dark stores. Each formulation has its entities, the SFSW formulation is the basis of the work, considering only suppliers, warehouses and stores. The most robust model, the SFSDSW formulation, incorporates all echelons. The following chapter presents the dissertation's data collection methods used to apply the developed model, the validation of the model and the correspondent results obtained.

Chapter 5

Model implementation and results

In this chapter the main results will be presented and discussed. The chapter starts with section 5.1, the data collection process to implement the models. Here, a brief description of the experiments to be carried out is also presented. In section 5.2, the models must be proven plausible relying on the used data. Section 5.3 is an overview of the model statistics. Section 5.4 presents the main model results. Here, results are categorized into channel design performance related results 5.4.1, and economic performance related results 5.4.2. In section 5.5 a sensitivity analysis is performed on the parameters subject to higher uncertainty. Finally, there is a general discussion of results in section 5.6, followed by some limitations and recommendations for future work.

5.1 Data Collection

To implement and validate the constructed models, input data was needed to assess the feasibility of the solutions. As already stated, we used Millstein (2022) SFS+W [48] as a reference model to the developed formulations. In this sense, similarly to Millstein, Daskin (1995) [67] data set was used as a reference for our model demand. This data set is based on the US state capitols geographic locations and the distribution of the population. It includes 49 markets (48 state capitols and Washington DC.). Each state demand represents its' population size.

The distances between market nodes were calculated using reference coordinates of each state. The distance between two entities of the same state was assumed to be around 30km. The delivery time (T) between each fulfillment entity and market i is calculated based on the distance (as explained in table 4.3). The WDD (work day distance) was assumed to be of 800km. The demand adjustment coefficient F_i was assumed to be: 1) $F_i = 0.5$ to benefit short periods of deliveries; 2) $F_i = 2$ to a pessimistic scenario; and 3) $F_i = 1$ for a neutral approach. To model online demand we considered four scenarios with $OLpct = \{10\%, 30\%, 50\%, 75\%\}$. For retailers' physical store market size we considered $G_i = \{30\%, 50\%, 75\%\}$.

In terms of the set definition, we assumed that each state has a store of the retailer, that can be kept open or can be closed depending on the optimization decision. For the dark store study, we assumed that only one dark store can be opened in every state and each dark store served only the market where it is located. The warehouse distribution is only in 15 of the previously defined states, and within that options the model will tell where to open warehouses. Finally, like the warehouse distribution, suppliers are available in 6 states. The assignment of locations to warehouses and suppliers is on tables 5.1 and 5.2, respectively.

Table 5.1: Warehouse location assignment.

Warehouse locations														
Sacramento	SaltLakeCity	Phoenix	Austin	Springfield	Madison	Montgomery	Nashville	Columbus	Tallahassee	Atlanta	Richmond	Trenton	Albany	Harrisburg

Table 5.2: Suppliers location assignment.

Supplier locations					
CarsonCity	BatonRouge	Indianapolis	Frankfort	Columbia	Hartford

The three models were run for three different product categories (electronics, apparel and food) to assess the difference of the benefits of installing dark stores on different product types. These categories are a generalization of a big range of products within each category. Table 5.3 summarizes the differences between the three categories by presenting the relative profits and costs of the food and electronics categories, having the apparel category as a reference. Millstein et al. (2022) [48] studied five different product categories, of which we adapted three to our model: Perishable Food as Food, Fashion as Fashion, and Consumer electronics as electronics.

The actual values of the demand, market locations coordinates, profits and costs per product categories that were used during the computational test were adapted from Millstein (2022) [48] and can be consulted in Appendix A. The reference model presented only one value for each parameter regardless of the market, so it was assumed that the data for the various markets followed a normal distribution whose mean was the reference value and the standard deviation was 25% of the mean value.

Table 5.3: Product categories relative profits and costs.

Product category	Unitary profit	Inventory holding cost	Inventory handling cost	Warehouse fixed cost	Dark store fixed cost	Transportation cost
Apparel	1	1	1	1	1	1
Eletronics	1.5	1.5	3	1.5	1	3
Food	0.75	0.75	1	2	2	2

5.2 Model Validation

Models can be validated by comparing output to independent field or experimental data sets that align with the simulated scenario. In this sense, to validate the built model, a comparison of results will be performed with the Millstein et al., 2022 [48], since this work served as a reference and starting point for this study. As already referred, Millstein established a comparison between four model configurations that included warehouses and stores. We adapted the SFW+S [48] configuration and added new ones (SFSDSW and SFDSW) to account for the dark store facility.

In this sense, to validate the implementation of the model, we will compare the results of the reference paper with ours. The reference work does not present a direct result of its SFW+S formulation, but only of comparisons between this and its other model configurations. The only result that can be comparable is the number of opened warehouses and stores in the different experiments. Figure 5.1 shows the number of warehouses and stores open in the reference model, with the number varying between 3 and 5 open warehouses and 42 and 47 open stores, for the SFW+S configuration. The average number of open warehouses and stores of our model is detailed in table 5.4. The number of warehouses of the reference model is in the same order of magnitude as our model, which can validate the adaptation of the SFW+S with ours. The number of stores is not precisely in the same order of size. Although it is not very far away, this difference can be explained by a set of factors, as the non consideration of all product categories, adaptations and assumptions in data, and different partitions of demand considered.

		Number of Open Warehouses			Number of Open Stores		
$F=1$		SFW	SFS	SFS+W	SFW	SFS	SFS+W
	OLPCT = 10%	3	3	3	42	42	42
	OLPCT = 25%	5	3	5	46	44	46
	OLPCT = 50%	5	5	5	47	47	47
	OLPCT = 75%	6	5	5	46	43	43
	OLPCT = 90%	7	5	5	45	47	47
$F=2$							
	OLPCT = 10%	3	3	3	42	42	42
	OLPCT = 25%	3	3	3	43	43	43
	OLPCT = 50%	3	3	3	44	43	43
	OLPCT = 75%	3	3	3	45	44	44
	OLPCT = 90%	3	3	3	44	44	43
$F=3$							
	OLPCT = 10%	3	3	3	43	43	43
	OLPCT = 25%	3	3	3	44	44	44
	OLPCT = 50%	3	3	3	43	44	44
	OLPCT = 75%	3	2	3	41	42	43
	OLPCT = 90%	3	1	3	42	25	40

Figure 5.1: Average number of open warehouses and stores in the Millstein (2022) model [48]

Table 5.4: Average number of open warehouses and stores in our model for F=1.

		Number of open warehouse		Number of open stores	
	Olpct	SFSDSW	SFSW (SFW+S)	SFSDSW	SFSW (SFW+S)
Electronics	10%	2	2	31	31
	30%	2	2	27	27
	50%	2	2	20	20
	75%	4	3	8	8
Fashion	10%	1	1	31	31
	30%	3	3	27	27
	50%	3	3	20	20
	75%	4	5	8	8
Food	10%	1	1	31	31
	30%	2	2	27	27
	50%	3	3	20	20
	75%	4	4	8	8

The CPU times of both formulations can also be compared to assess the veracity of the models. Millstein et al. (2022) states that all models were solved using GAMS version 23.5 with CPLEX 12.8 on an INTEL Core i5 CPU 2.70 GHz with 8 GB of RAM. His three models (SFW, SFW and SFS+W) were solved with CPU times ranging from 48 to 7678 s. Our SFSW configuration were solved using GAMS version 39.2 with CPLEX 20.1 on an Intel(R) Core(TM) i7-4500U CPU @ 1.80GHz with 8 GB of RAM, with CPU times ranging between 2s and 9522s to run. This time is not very far away from the reference model time, which supports the hypothesis that the constructed model is valid. The remaining model statistics will be presented next in section 5.3.

5.3 Model statistics

This section shows the most important model statistics. All models are solved using GAMS version 39.2 with CPLEX 20.1 on an Intel(R) Core(TM) i7-4500U CPU @ 1.80GHz with 8 GB of RAM. The SFSW, SFDSW, and SFSDSW models solved with CPU times ranging from 1.859 to 9522.953 s. All models were slower when run to the fashion category. Table 5.5 shows five statistics: resource usage (total computational time), number of iterations, number of nodes, number of single equations, number of single variables, and finally number of discrete variables.

The highest computational time happened for the SFSDSW formulation for the fashion category and the lowest for the SFSW formulation for the food category. In table 5.5 there are the minimum and maximum times, iterations and nodes for all channel designs and all product categories. The lowest number of iterations was 2520 and the highest 30573806, and the lowest number of nodes was 4 and the highest was 634410, for all runs. The number of single equations and variables, and discrete variables is a channel design characteristic, so it is the same for all runs within a channel design

Table 5.5: Model statistics.

	Dark			Store			Both		
	Electronics	Fashion	Food	Electronics	Fashion	Food	Electronics	Fashion	Food
Minimum resource usage (time)	8.375	72.954	3.594	14.578	10.969	1.859	27.453	17.531	3.36
Minimum number of iterations	31615	231027	2520	20647	32412	4486	28023	32530	3840
Minimum number of nodes	286	2560	4	672	600	47	490	547	35
Maximum resource usage (time)	2368.562	8835.234	125.266	532.75	7758.984	71.422	755.047	9522.953	133.031
Maximum number of iterations	5433014	30573806	251407	858158	27999432	112344	945975	26362518	143044
Maximum number of nodes	112250	631480	2482	16802	634410	1299	20880	594286	1469
Number of single equations	2.622	2.622	2.622	7.208	7.208	7.208	7.503	7.503	7.503
Number of singles variables	2.702	2.702	2.702	4.27	4.27	4.27	5.103	5.103	5.103
Number of discrete variables	203	203	203	154	154	154	203	203	203

5.4 Model Results

This section will present the most relevant results of the model implementation. Subsection 5.4.1 presents the operational results, related to the different channel designs. Subsection 5.4.2 details the economic related results, measuring profitability with different scenarios.

In order to present our results in a synthetic way and to reduce the number of model runs in some of the analysis, we considered as a reference:

- Electronics category
- $F = 1$

- $Olpct = 30\%$ and $G = 50\%$

This electronics category was chosen as a reference because of its higher profitability performance, the demand adjustment coefficient was set to the neutral scenario, and an intermediate demand partition was chosen.

5.4.1 Channel design performance

In this problem, operational efficiency is not quite easy to study, since we have simplified the solution to a single product type in each retail sector. However, some Key Performance Indicators (KPI) can be studied to asses the three channel design performances. Firstly, we studied the number of facilities open. Then, we measured the number of markets reached in each of the three model configurations for the three types of retail sectors, as well as the percentage of reached online demand. Finally, the unitary cost of satisfying online demand in the 3 echelons (warehouse, store and dark store) was also studied, for one product category.

Facilities openings

The number of facilities opened was studied for three types of facilities: warehouse, store and dark store, and for the three model configurations: SFSDSW, SFDSW, and SFSW. Then, we specified the openings for the three product categories under study: Electronics, Fashion and Food. In terms of demand, we only present the four different levels of online demand $Olpct$, which of them aggregating the three possible $G = 30\%, 50\%, 75\%$ as an average (percentage of store demand available to the store channel). The results are present in Table 5.6.

It is to note that the dark store column for the SFSW model is empty since this configuration does not integrate the dark store facility.

From table 5.6 we can conclude that as $Olpct$ grows (the partition of online demand available), the number of warehouses and dark stores also grow. Stores are not following this trend since they mainly serve store and not online demand. Warehouses and dark stores serve mainly online demand.

There are no significant changes between product categories in these results.

In the number of dark stores and warehouses, the SFDSW typically exceeds the SFSDSW, since it only has dark stores and warehouses fulfilling the online demand, which does not happen in the SFSDSW configuration (where stores also fulfill).

Market coverage

The market coverage was studied to understand the dimension of markets reached. Table 5.7 shows the average percentage of demand fulfilled in the markets that are reached. Table 5.8 shows the number of markets covered. This study was performed to the reference partition ($Olpct = 30\%$ and $G = 50\%$) for all model configurations and product categories.

We can see that the greatest market coverage is achieved by the SFDSW configuration for the fashion category. For the SFSDSW and SFSW the electronics category is the one with best coverage, followed

Table 5.6: Facilities openings per category per model configuration.

		Nr warehouses			Nr stores			Nr dark stores		
	Olpct	SFSDSW	SFSW	SFDSW	SFSDSW	SFSW	SDSW	SFSDSW	SFSW	SFDSW
Electronics	10%	2	2	3	31	31	31	10	0	29
	30%	2	2	4	27	27	28	32	0	34
	50%	2	2	5	20	20	20	42	0	39
	75%	4	3	5	8	8	8	43	0	43
Fashion	10%	1	1	3	31	31	31	14	0	24
	30%	3	3	4	27	27	27	33	0	33
	50%	3	3	5	20	20	20	36	0	37
	75%	4	5	5	8	8	8	38	0	40
Food	10%	1	1	2	31	31	31	12	0	24
	30%	2	2	3	27	27	27	25	0	39
	50%	3	3	3	20	20	20	38	0	44
	75%	4	4	4	8	8	8	40	0	42

by fashion and food categories. We have a service level in terms of fulfilling demand higher than 63% with an average of 75%. It is also interesting to see that the model configuration with only dark store has the highest average level of service.

In terms of number of markets reached, fashion and food categories cover 100% of the markets, in contrast with electronics.

We assume that this analysis is representative to most demand partition configurations.

Table 5.7: Market coverage per product category for the $Olpct$=30% and G=50% partition.

	$Olpct$=30% G=50%		
	SFSDSW	SFSW	SFDSW
Electronics	87%	90%	55%
Fashion	64%	73%	94%
Food	63%	67%	86%
Average	71%	77%	78%

Table 5.8: Number of markets covered per product category for the $Olpct$=30% and G=50% partition.

	$Olpct$=30% G=50%		
	SFSDSW	SFSW	SFDSW
Electronics	38	38	49
Fashion	49	49	49
Food	49	49	49

Unitary cost of fulfilling demand

To have a better understanding of the operational efficiency and the cost of fulfillment, we studied the unitary cost of fulfilling online demand in each of the fulfillment facilities. This assessment was done to the reference partition ($O = 30\%$ and $G = 50\%$) and the reference product category (Electronics), for all of the three model configurations.

To measure this KPI, we have averaged the facility's operating costs weighted by the actual number of units that fulfill online demand. As it can be seen in table 5.9. It was already expected that the warehouse had the lowest cost, since it is a facility normally located in less populated areas which corresponds to lower facility costs. The lowest warehouse fulfillment cost is in the SFSDSW and the highest in the SFSW. The highest fulfilment cost is for the store facility. This happens mainly because we are only considering the online demand and these facilities serve mostly store demand. This was also already expected due to the high fixed costs of operating an open-to-public store. In the dark store facility, we can see that it in the SFSDSW is cheaper to fulfill demand through dark stores than in the SFDSW model. Overall, we can see that the SFSDSW model, the one with higher flexibility to ship from between the three entities, is the one with lowest operating costs in terms of fulfilling online demand.

For this specific analysis results for fashion and food categories were very similar so they were not presented. The conclusions are the same as for the electronics.

Table 5.9: Unitary cost of fulfilling demand for each facility, for the $O = 30\%$ and $G = 50\%$ partition and the electronics product category.

	Olpct=30% G=50%		
Unitary fulfillment cost	SFSDSW	SFSW	SFDSW
Warehouse	2.51	3.81	3.70
Store	8.46	8.68	
Dark Store	5.23		6.99
Average	5.40	6.25	5.35

5.4.2 Economic profitability

Regarding the model's economic profitability, many analysis were conducted to see how profit and costs evolved by changing certain parameters.

In this subsection the following KPIs will be detailed:

- Evolution of profit in all product categories and partitions considered;
- Evolution of profit with F for all product categories;
- Evolution of profit with a limitation in the warehouse number for different demand partitions and the three model configurations for the electronics category;
- Evolution of profit with a limitation in the warehouse number and growing demand for the three model configurations for the electronics category;
- Evolution of profit with a limitation in the dark store number for different demand partitions and growing demand for two model configurations for the electronics category;
- Distribution of profit by the facilities.

Evolution of profit in all product categories and partitions considered

First, we saw the evolution of profit with the different partitions of the online and physical demand ($Olpct$ and G), for the three product categories (figure 5.2). In general, profit increases with the increase in both the online and physical demand fraction.

This relation is almost linear in the fashion and food categories (5.2b and 5.2c). However, in the electronics category in the SFDSW model, for a physical demand fraction of 50%, profit is the same for both a 10% and 30% $Olpct$ (5.2a). This can happen since with the demand growing, the operational costs in the scenario of O=30% decreases the overall profit, making it equivalent to the scenario with less online demand available.

In this study the most profitable product category is the Electronics, followed by Fashion and Food. The food category is mostly affected by to the requirement of special shipping or warehousing for perishable products, leading to increased fixed and transporting costs. The Fashion category has also additional costs when compared to the electronics, due to the size of the products and the assortment.

The most profitable configuration varies accordingly to the product category. In the electronics category the most profitable configuration is the SFSW, however it is only 0.2% more profitable than the SFSDSW. In the fashion category the most profitable configuration is the one that incorporates more entities (SFSDSW). In the food products it is preferable to implement the ship from store and warehouse (SFSW) design. Overall we can conclude that the SFSW design is the safest option in the short term. However, the SFSDSW configuration has only 0.09% less profit, which can be negligible and we can state that both SFSW and SFSDSW offer the same level of profit.

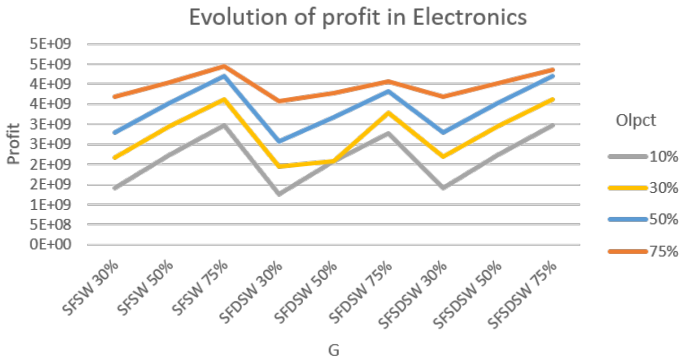
Evolution of profit with F for all product categories

Then, we studied the impact of the demand adjustment coefficient F_i in the overall profit, for all product categories. As explained in 4.2.3, F_i is a demand adjustment exogenous coefficient for market i that reflects demand elasticity based on delivery time and the level of competition in the online market. The impact was studied only for three different demand partitions: $Olpct$=10% G=50% ; $Olpct$=30% G=50% ; and $Olpct$=50% G=50%. We can conclude that with the increase in the demand the profit increases, at the same time that profit is always higher in the optimistic scenario (F_i=0.5), followed by the neutral scenario (F_i=1), and by the pessimistic scenario (F_i=2). However, the differences between scenarios are amplified with the increase in the demand available. The three graphs in figure 5.3 illustrate these conclusions.

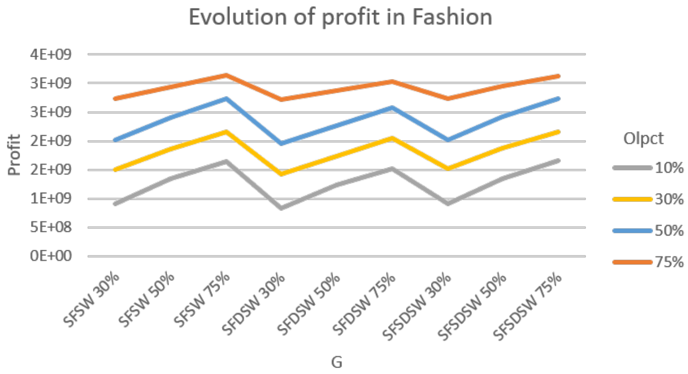
Evolution of profit with a limitation in the warehouse number for different demand partitions and the three model configurations for the electronics category

To analyse the effect of fixing the number of warehouses, we studied the evolution of profit with the fixation of the number of warehouses (w) to the following scenarios: number of warehouses= 2, 4, 6, 8, 10. This analysis was performed to the reference product category (Electronics) and for four different partitions: $Olpct$=30%, 50% and G=30%, 75%.

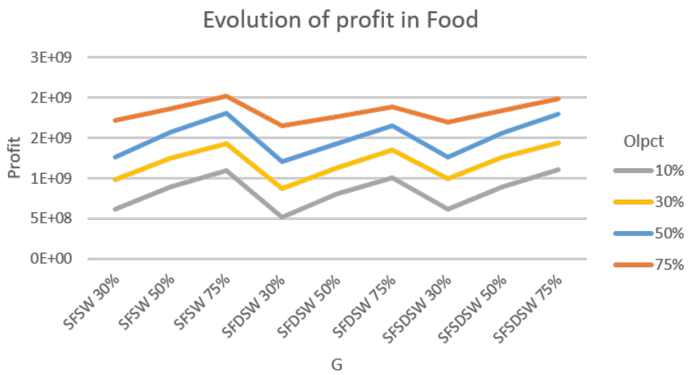
We concluded that for the 5.4a SFSDSW configuration, the fixed costs of opening a warehouse meant that, generally, the fewer warehouses opened, the more profit, except in one specific partition. With $Olpct$=50% and G=30%, the profit for the w=2 is 2.71E9, while for the w=4 is 2.75E9, (1,47% under).



(a) Evolution of profit with G and $Olpct$ in electronics.



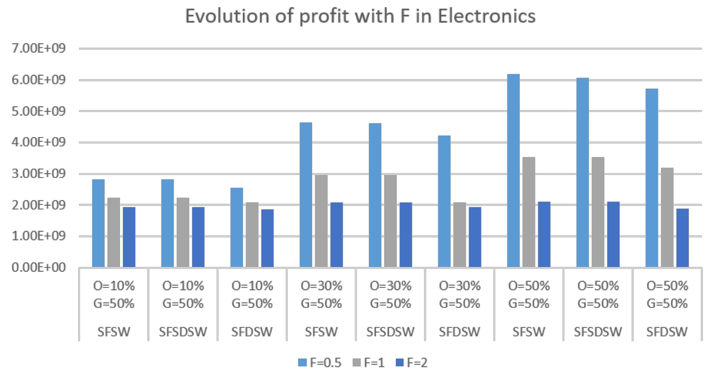
(b) Evolution of profit with G and $Olpct$ in fashion.



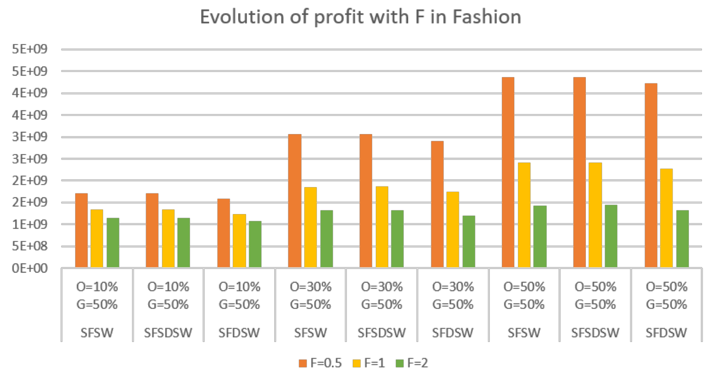
(c) Evolution of profit with G and $Olpct$ in food.

Figure 5.2: Evolution of profit in all product categories, for all the partitions considered.

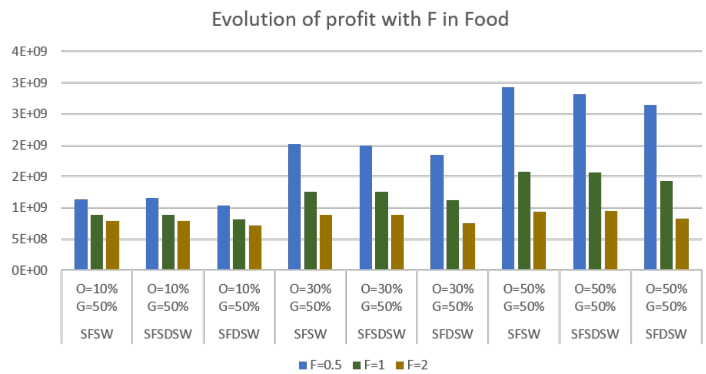
In the 5.4b SFDSW configuration, the distribution of profit with the number of warehouses is quite different. With $Olpct$=30% and G between 30% and 75%, the configuration with 4 warehouses is the most profitable one, followed by w=6, 2, 8 and 10. With $Olpct$=50% and G between 30% and 75%, the configuration with 4 warehouses is still the most profitable one, but in this case followed by w=6, 8, 10 and 2. It is interesting to see that in this configuration, having only 2 warehouses available is very detrimental to profitability, due to the lack of capacity of dark stores and the impossibility of warehouses sharing capacity with physical stores (not present in SFDSW).



(a) Evolution of profit with F_i in electronics.



(b) Evolution of profit with F_i in fashion.

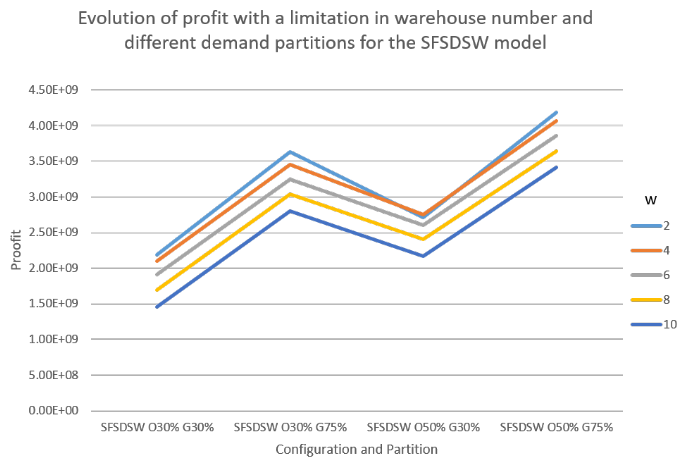


(c) Evolution of profit with F_i in food.

Figure 5.3: Evolution of profit with F_i in all product categories, for 3 different partitions.

For the 5.4c SFSW configuration, analogously to 5.4a, the fixed costs of opening a warehouse meant that, generally, the fewer warehouses opened, the more profit, except in one specific partition. With $Olpct$=30%, G=50% the profit for the w=2 is 2.66E9, while for the w=4 is 2.76E9, (3,76% under). With the increase in demand, the capacity of only 2 warehouses is not enough to account for the operational cost saving in comparison to the lost sale burden.

The results for the other product categories are expected to be similar to the electronics.



(a) SFSDSW configuration.

(b) SFDSW configuration.

(c) SFSW configuration.

Figure 5.4: Evolution of profit with a limitation in the warehouse number for different demand partitions and the three model configurations for the electronics category.

Evolution of profit with a limitation in the warehouse number and growing demand for the three model configurations for the electronics category

To continue the analysis of the effect of fixing the number of warehouses, we studied the evolution of profit with growing demand. With the previous defined demand for the model, we run the 3 configurations for a demand of half (50%) of the original one, and a second scenario with 150% of the original demand. We studied this evolution for only the reference partition: $Olpct$=30% and G=50%. The three model configurations were included in this analysis.

In the 5.5a SFSDSW configuration, the fewer warehouses opened, the more profit. This is mostly due to the reduction in the fixed costs.

In the 5.5b SFDSW configuration, the relation between the number of warehouses open and the profit is not linear with the demand. The scenario with the number of warehouses fixed in four is the one with the greatest profit regardless of the demand. In the half demand scenario, the profit is decreasingly ordered as follows: w=4, 2, 6, 8, 10. In the original demand scenario, the profit is decreasingly ordered as follows: w=4, 6, 2, 8, 10. Finally, in the increased demand scenario (150% Demand), the order is as follows: w=4, 6, 8, 2, 10. With this we can conclude that the higher the demand, the higher the need to have more capacity (translated in number of warehouses). In conclusion, the w=2 curve is descendant since with the increase in demand, the capacity needed increases, and the profit resents the lost sales.

The 5.5c is analogous to 5.5a.

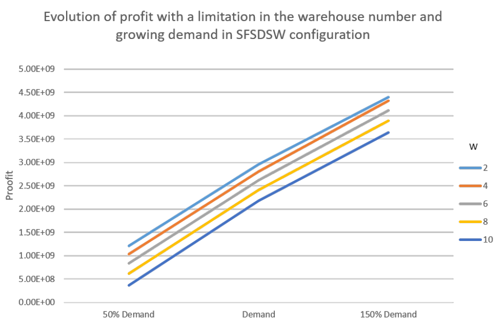
These results are assumed to be similar for the other product categories.

Evolution of profit with a limitation in the dark store number for different demand partitions and growing demand for two model configurations for the electronics category

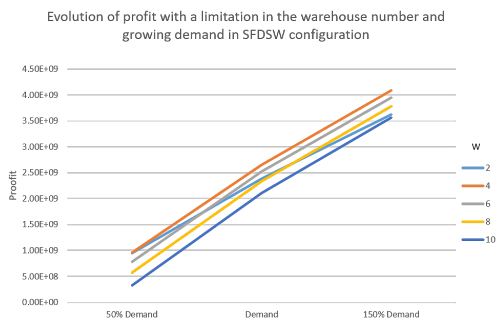
Another interesting study is for the optimal number of dark stores. Analogously to the analysis of fixing warehouse number, we now fixed the dark store number in a range between 0 and 12 dark stores (DS). The following number of considered dark stores was as follows: DS= 0, 3, 6, 9, 12. The objective is to study how the network and the results evolve by setting the number of dark stores and make it grow. This will be done for two model configurations: SFSDSW and SFDSW, the ones with dark stores involved. The partitions and demand variation considered is analogous to the previous analysis with the number of warehouses.

Figures 5.6 and 5.7 show the evolution of the dark store number with the different demand partitions and evolving demand, respectively. Here the results are as expected, the profit increases with the increase in demand and with the number of dark stores, that follow this growth.

Another interesting note is about the allocation of dark stores to the market. By limiting the number of dark stores, it is relevant to study where they are located. We observed that in the SFSDSW model, the most chosen markets to allocate dark stores were Sacramento, Austin and Olympia (chosen in more than 80% of the scenarios). For the SFDSW channel design, Austin was the market chosen in all scenarios (100%), followed by Harrisburg that was chosen 75% of the times. It is to note that this allocation in not related to the market with highest demands.



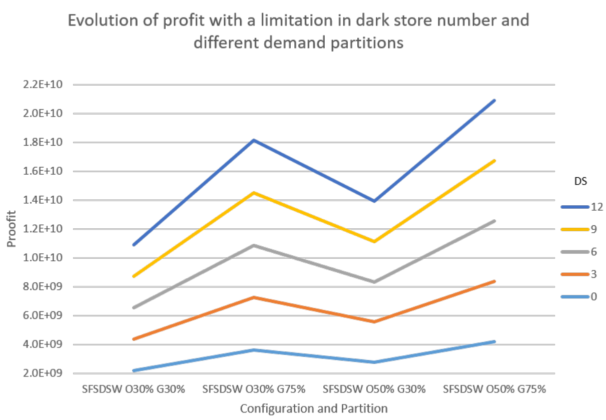
(a) SFSDSW configuration.



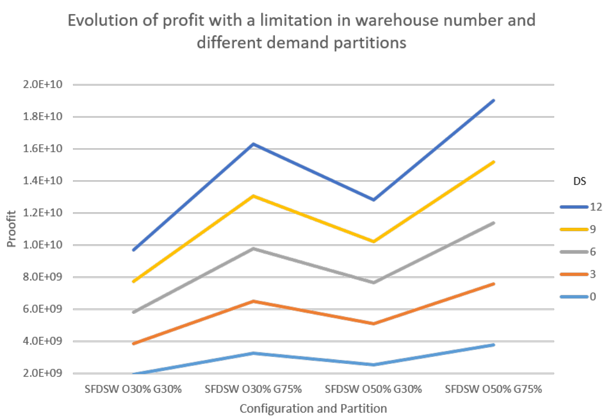
(b) SFDSW configuration.

(c) SFSW configuration.

Figure 5.5: Evolution of profit with a limitation in the warehouse number and growing demand for three model configurations for the electronics category with Olpct=30% and G=50%.



(a) SFSDSW configuration.



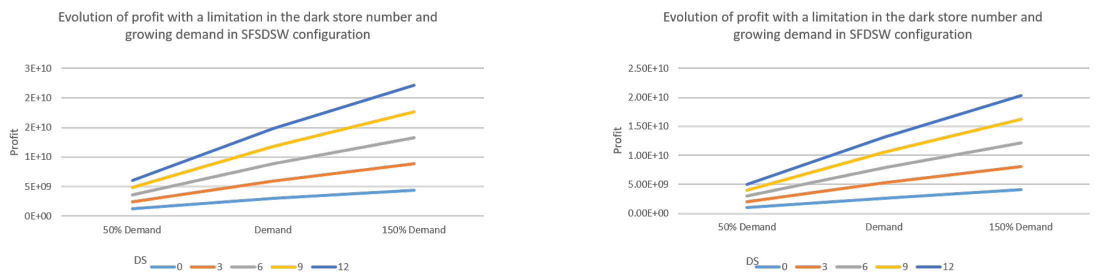
(b) SFDSW configuration.

Figure 5.6: Evolution of profit with a limitation in the dark store number and different demand partitions for two model configurations for the electronics category.

The results for the other product categories are expected to be similar to the electronics.

Distribution of profit by the facilities

In order to asses which facilities bring the greatest benefit to the model, we studied the distribution of profit through the three different entities: warehouses, stores and dark stores. In table 5.10 there are



(a) SFSDSW configuration.

(b) SFDSW configuration.

Figure 5.7: Evolution of profit with a limitation in the dark store number and growing demand for two model configurations for the electronics category with $Olpct$=30% and G=50%.

the correspondent percentages of profit divided by product category and the two model configurations with dark stores. This study was carried for the reference partition ($Olpct$=30% and G=50%).

First, we calculated the part of profit that each entity adds to the model. For example, for the store facility in the electronics SFSDSW model, the profit was the product of the gross profit of the product (sales price) and the number of units sold:

$$Storeprofit = (profit * Qs_s) + (profit * U_i, s)$$

$$Storeprofit = 3749414000$$

Then, we calculated the percentage of total profit aggregated by the facility. In the example, the percentage that the store profit has from the total. In this case it is more than 100%, which means that the store itself aggregates more than the total profit and it is the facility that leverages the model profitability.

$$Storeprofit = 3749414000$$

$$Totalprofit = 2956349700$$

$$percentageofprofit = 3749414000/2956349700$$

$$percentageofprofit = 126\%$$

Of course not every facility can aggregate such a significant part of profitability. It is clear that the dark store is the facility that brings less value to the network. This was already expected since we considered the dark store space to be subcontracted and strategical located in highly populated areas (with high fixed costs). However we can see that in a configuration with both stores and dark stores (SFSDSW), the fashion category is the one were dark stores add greater benefit. In the SFDSW configuration, it is in the food category were dark stores are most valuable.

Overall, the facility that aggregates more profit in the SFSDSW and SFSW models is the store and in the SFDSW model is the warehouse.

Table 5.10: Distribution of profit by the three facilities for the three product categories($Olpct$=30% and G=50% partition).

	Percentage of profit		
	Store	Warehouse	Dark Store
Electronics SFSDSW	126%	15%	3.04%
Fashion SFSDSW	75%	49%	3.43%
Food SFSDSW	115%	57%	2.82%
Electronics SFDSW	76%	77%	3.63%
Fashion SFDSW	64%	79%	2.78%
Food SFDSW	75%	86%	3.81%
Electronics SFSW	68%	16%	
Fashion SFSW	60%	56%	
Food SFSW	67%	60%	

Net profit of dark stores

Having the low relevance of dark stores to the overall profit in consideration, we need to assess whether or not this facility is beneficial to the retailer's financial condition. This is, if the net profit of the dark store is contributing positively or not to the overall profit.

This KPI was calculated as follows:

$$Net\ profit\ of\ dark\ stores\ = \sum_{i \in I} \sum_{d \in D} pcod_{id} * W_{id} - \sum_{d} Cdo_d * Dt_d - \sum_{d \in D} \sum_{j \in J} Cdrw_{dj} * P_{dj} - \quad (5.1)$$

Table 5.11 shows the net profit of dark stores of all product categories and channel designs for the reference partition. As we can see, the net profit is positive for all scenarios. We can assume that the reference partition is representative of the others. In this way, we can conclude that it the dark store facility does not damage the overall profitability of any of the models.

Table 5.11: Net profit of dark stores for all product categories and channel designs ($Olpct$=30% and G=50% partition).

	Net profit of dark stores
Electronics SFSDSW	7.8E+07
Fashion SFSDSW	5.8E+07
Food SFSDSW	2.9E+07
Electronics SFDSW	8.4E+07
Fashion SFDSW	5.1E+07
Food SFDSW	3.6E+07

5.5 Sensitivity analysis

The input parameters used to solve the present model and presented in chapter 4 were mainly collected from information provided by Daskin, 1995 data set [67]. However, due to the absence of data, some parameters are based on premises. Thus, some parameters are subject a significant level of uncertainty and it is important to analyse how this uncertainty affects the results achieved in the previous sections.

Accordingly, in this section a sensitivity analysis is performed on these critical parameters to test the robustness of the model results. These parameters are the ones related to the dark store cost structure, namely, the dark store fixed cost, and handling cost. Next subsections will present the sensitivity analysis for each of the two cost parameters of dark stores.

5.5.1 Dark store fixed cost

The dark store fixed cost Cdo_d covers all costs related to the space renting. Since we assumed that dark stores are not an asset for the retailer, like warehouses, there is an inherent cost of subcontracting these facilities. This parameter is particularly interesting to vary in order to understand the relevance of not being an asset for the model results. We want to understand if the dark store having a fixed cost has an impact on the profit, in the sense that it affects the models with dark stores (SFSDSW and SFDSW) in relation to the other (SFSW). Basically, we want to understand if this cost has an impact on the most profitable channel design solution for the retailer.

Table 5.12 shows the impact on profit for the SFSDSW and SFDSW channel designs with the variation of Cdo_d in comparison to the scenario with no change in Cdo_d. This analysis was performed for the reference partition ($Olpct$=30% and G=50%) and the electronics category. The dark store fixed cost suffered a decrease of 10%, 20%, 30%, 40%, 50%, and -100% in relation to the original value.

As outlined in Table 5.12, the variation of this parameter does not have a great impact on the total profit, even for the scenario with no fixed cost (-100%). The order of magnitude of these variations neither changes the decisions of which model configuration is more profitable. As shown in the third row of the table, the difference between the profit of the two models remains roughly constant, with the SFSDSW model making about 10% more profit than the SFDSW. A further remark concerns the difference between the two models, as the fixed cost of the dark store is reduced, the gap is also softened. At the same time the profit increases as the Cdo_d decreases, corroborating the model's robustness.

This low impact is probably explained due to the higher role of the other facilities (warehouse and store) on profit. As already analysed on table 5.10 the dark store is the facility that contributes the least to the overall profit of the model.

It is worth noting that this analysis was only done for the reference partition and for the electronics category. However, given the results obtained and the low levels of variation, it is safe to say that it would probably have no impact on the conclusions for the remaining partitions and categories.

Table 5.12: Profit variation with the change in dark store fixed cost (Reference partition for electronics $Olpct$=30% and G=50%).

% of DS fixed cost change	-10%	-20%	-30%	-40%	-50%	-100%
Profit SFSDSW variation	+0.002%	+0.003%	+0.005%	+0.006%	+0.008%	+0.016%
Profit SFDSW variation	+0.006%	+0.011%	+0.017%	+0.022%	+0.028%	+0.058%
SFSDSW-SFDSW (%)	+10.366%	+10.363%	+10.359%	+10.356%	+10.352%	+10.352%

5.5.2 Dark store handling cost

The dark store handling cost Cdl_d covers all costs related to the storage and movement of goods in the facility that is preparing a shipment. In this case, Cdl_d represents the costs incurred in storing and preparing orders for customer delivery. Thus, these are costs incurred during the time period from when goods leave storage to when they are about to be delivered to the shipper.

The handling cost for the preparation of an online order in a dark store was considered to be lower than in a warehouse and a store. Dark stores are facilities that are built only for the preparation of online orders. This way, they can specialise in this operation and so optimise costs. In an open-to-public store, online order preparation happens in parallel with the retailing operation, making it more costly to deliver online orders in comparison to the dark store.

We want to understand the impact of Cdl_d on the overall profit for the models with dark stores. In this sense, a similar analysis to the fixed cost approach was performed here.

Table 5.13 shows the impact of profit for the SFSDSW and SFDSW channel designs with the variation of Cdl_d in comparison to the scenario with no change in Cdl_d. This analysis was done for the reference partition ($Olpct$=30% and G=50%) and the electronics category. The dark store handling cost suffered an increase of 10%, and a decrease of 10%, 20%, and 30%, in comparison to the original value. This variations are not the same as the ones in table 5.12 since the handling cost cannot be disregarded as the fixed cost could. This way the variations were lower and a scenario with an increase in the cost was also studied (+10%).

Table 5.13 shows that the variation of this parameter has a relatively greater degree of influence on profit than Cdo_d had. However, it continues to not have a great impact on the total profit. The order of magnitude of these variations does not change the decisions of which model configuration is more profitable. As shown in the third row of the table, the difference between the profit of the two models remains roughly constant, with the SFSDSW model making about 10% more profit than the SFDSW. It is also to note that an increase in the handling cost causes a reduction in profit, and a decrease causes an increase in profit, as expected.

It is worth noting that this analysis was only done for the reference partition and for the electronics category. However, given the results obtained and the low levels of variation, it is safe to say that it would also probably have no impact on the conclusions for the remaining partitions and categories.

Table 5.13: Profit variation with the change in dark store handling costs (Reference partition for electronics $Olpct$=30% and G=50%).

% of DS handling cost change	+10%	-10%	-20%	-30%
Profit SFSDSW variation	-0.014%	+0.014%	+0.029%	+0.044%
Profit SFDSW variation	-0.029%	+0.029%	+0.059%	+0.089%
SFSDSW-SFDSW (%)	+10.384%	+10.356%	+10.343%	+10.329%

5.5.3 Sensitivity analysis remarks

With the two analysis conducted, we can conclude that the low expression of dark stores on profit is not totally related to the costs associated with this facility. The low profit variations suggest that there are other reasons for the dark stores not having the desired outcome.

To explore this issue further, it was assumed that the model favoured opening stores and warehouses over dark stores. In this line of thought, as there is more responsiveness on the part of stores and warehouses, the dark store cannot add as much profit. This way, the study of aggregated profit per facility was repeated, but now limiting the number of stores and warehouses openings.

We considered the electronics category for the SFSDSW model for the reference partition in table 5.10 as a reference. In table 5.14 both the results for the reference and the test scenario are presented. In the test scenario we limited the number of warehouses and stores to be opened to 1 each.

We can see that the store and warehouse continue to be the facilities that aggregate more profit, however, there was a significant increase in the weight of dark stores. It is not to ignore that warehouses have a storage capacity 100 times higher than the dark store, and stores have 10 times more capacity than dark stores. This scale factor certainly affects the results.

In conclusion, dark stores cannot leverage the profit of a retailer as the traditional facilities do. However, these facilities can be a great add on to the strategy of a retailer.

Table 5.14: Distribution of profit for the electronics SFSDSW for two scenarios ($Olpct$=30% and G=50% partition).

	Percentage of profit		
	Store	Warehouse	Dark Store
Electronics SFSDSW (reference)	126%	15%	3.04%
Electronics SFSDSW (test)	115%	34%	13%

5.6 General discussion, Limitations and Recommendations

This section contains a general discussion of the results, its major limitations and recommendations for future work.

5.6.1 General Discussion

The great amount of uncertainty associated with the parameters used to design the model experiment makes it challenging to draw conclusions with a high degree of confidence. Thus, it would be recommended to invest more in data collection. Once it was not possible to have a retailer who shared real data with this project, all work was based on a data set and many assumptions were made. We based most parameters in the ones used in Millstein et al. (2022) [48] work, but these were not enough for the construction of new network configurations.

In this study we reached the conclusion that the most profitable product category is the Electronics, followed by Fashion and Food. The disadvantage in the food category is mostly due to the requirement of special shipping or warehousing for perishable products, leading to increased holding and handling costs. The fashion category has also additional costs when compared to the electronics, due to the size of the products and the assortment. With the study of the impact of F_i in the results, it was possible to see that the smallest the delivery time, the highest level of competition in the market, leading to a considerable boost in profit. Customers are increasingly more sensitive to service levels and delivery time. Many shoppers will abandon the checkout process if it takes a long time to deliver. Retailers must be aware of these factors in order to improve its performance.

It was also noted that in the model, shops are the main channel for profit aggregation, bringing liquidity to the model. This was due to certain factors. The first is the defined cost structure, since stores were considered to be assets of the retailer with presence in all markets. Secondly, the warehouses did not have such a wide distribution coverage of the markets, so the transport costs dampened the potential for deliveries. Finally, dark stores, as novelties in the supply chain, played a minor role in profit addition as high fixed costs were assumed due to strategic location. A recommendation for future research would be to study the potential conversion of the stores into dark stores.

The most profitable configurations across all product categories are the more traditional and the more flexible configurations, SFSW and SFSDSW respectively. Interestingly the SFDSW configuration is the one with the highest average level of service (average demand fulfilled in reached markets) and the higher number of markets fulfilled. On the other hand, for the electronics category, SFSW is the channel designs with the highest fulfillment cost, at the same time it has the highest market coverage. This happens due to the high store presence of this model, which obviously also implies a cost. Another interesting point is the fact that the SFDSW design has the lowest unitary fulfillment cost. All these findings indicate that the investment in new facilities that respond exclusively to online demand are worth to incorporate in a traditional retailer network.

5.6.2 Limitations and Recommendations

Data

As already referred, one of the biggest limitations of this study is related to data collection, making it challenging to draw conclusions with a high degree of confidence. The difficulty in finding a Portuguese retailer willing to share its data with us, made this study dependent on an American data-set with the

Millstein et al. (2022) [48] reference. In the future it would be valuable to find a real retailer with whom we can work to develop an optimal and dynamic omnichannel network.

Market coordinates

Another limitation of the model is related to the definition of the market coordinates. In the model development, the state capitals coordinates were considered to reference each market. In shipments between the same market were considered to be all the same distance of around 30 kilometers. In a future work, it would be important to design coordinates in a more accurate way with the use of postcodes for the exact facility locations and a random distribution of postcodes inside a market for the order locations.

Deterministic demand and time

In this study demand was treated as a static, deterministic and single period parameter. To deal with this simplification, we developed various scenarios where demand was modified to simulate increases and decreases in the overall demand, in retailer market share, and in the partition of online demand available. A coefficient was also integrated in the model to make it react to small delivery times and proximity to customer. However, in the future this work can be extended to a dynamic and time bounded model, with demand changing over time. Many more scenarios can be studied in a dynamic model, for example the store transition to dark store in real time due to an unexpected increase in online demand (for example in the Covid-19 pandemic). It would be interesting to see the omnichannel distribution network evolution with sudden shifts in significant parameters like demand and costs.

Following this general discussion and limitations of the results obtained, the following chapter wraps up the developed work with the main conclusions and suggestions for improvement and future work.

Chapter 6

Conclusions

This chapter wraps up the project by presenting the most relevant conclusions that can be drawn from the developed work and presenting some future work suggestions.

6.1 Achievements

This study extends the research on channel design for omnichannel distribution by evaluating the network profitability evolution by adding a new fulfillment node: dark stores.

Retailers need to adapt their operations to streamline with the evolution of e-commerce in the last years, supported by the Covid-19 pandemic and other trends. Additionally, customers rising expectations for fast deliveries and enhanced customer experience underline the need for wider options for order fulfillment. Omnichannel is a potential solution that on one side integrates the online and offline channels, integrating inventory and operations and also reducing friction for customers, but is challenging to implement. The evolution of retail along the years brought many challenges in the industry, with new trends emerging.

Covid-19 pandemic boosted the emergence of new distribution options. Home delivering became the most convenient distribution option to the customer, enhancing shopping experience. However, this mode came with great operational challenges in terms of handling and picking costs. The traditional option of store buying is loosing relevance, being complemented with store pick-up. Dark stores are a recent phenomenon that appeared in the pandemic. These are closed-to-public facilities normally located in large urban centres, which provide a quick response to demand through a small assortment of products. The literature on dark stores is quite scarce so this work explored in-depth this delivery mode.

The main goal of this work was to evaluate the operational efficiency and profitability of dark stores in three different omnichannel configurations. Dark stores were treated as a new fulfillment node to support online demand fulfillment integrated into an online and offline retail network. We formulated the fulfillment problem into a profit maximizing Mixed Integer Linear Programming (MILP) model, considering four potential echelons in our design: suppliers, warehouses, stores, and dark stores. Suppliers replenish warehouse capacity. Warehouses and dark stores' locations can be decided through the model solution, whereas stores are given as retailer assets that can be maintained or closed. Warehouses can be installed with different capacities, whereas dark stores, if installed, only have a fixed capacity available.

Three different channel designs were studied:

- **Ship from store and warehouse (SFSW)** The network starts with the suppliers that provide products to the warehouses. Here, products either follow two directions: 1) are shipped to the stores, or 2) are shipped directly to customers of any market to fulfill online demand. The products that end in the stores can be purchased by customers physically or can also be shipped to any market to fulfill online demand.

- **Ship from dark store and warehouse (SFDSW)** This channel design adds the dark store to the previous network. Here, products either follow three directions: 1) are shipped to the stores, 2) are shipped to dark stores, or 3) are shipped directly to customers of any market to fulfill online demand. Stores fulfill only the online demand, whereas dark stores fulfill the online demand of the same market where they are located.

- **Ship from store, dark store and warehouse (SFSDSW)** This channel design is the combination of the SFSW and SFDSW networks. In this case, both stores and dark stores can satisfy the online demand: stores can ship products to all markets whereas dark stores ship to the same market only.

The three models were developed to account the interaction of different product categories, variable online demand percentages, the elasticity of online demand based on the delivery time, and the level of competition in the market. In addition, this study highlights several important considerations for omnichannel firms deciding which channel design to utilize. First, by optimizing the number, location and size of omnichannel warehouses, as online demand increases, the firm can increase profit compared to using fixed warehouse locations and sizes. Second, the unit profitability for a product category influences which channel design is optimal, resulting in different optimal channel designs for different product categories. Third, with increasing levels of competition in a market, the different levels of market share result in different solutions in different product categories.

Results show that optimizing the number, location and capacities of warehouses benefits SFSW and SFSDSW more than SFDSW for most product categories, because of the store presence in all markets where demand can be fulfilled quickly with lower inventory and handling costs, thereby increasing profitable market share. In this study the highly profitable product category is electronics, followed by fashion and food. The food product category, the least profitable one, requires special shipping and warehousing, so SFSW is the most profitable configuration in this case. The advantage of SFSW being generally closer to all store markets, and therefore having lower transportation costs becomes less important as online demand grows. The two additional omnichannel designs, SFDSW and SFSDSW have the flexibility to ship online orders from more echelons in the supply chain, which makes this channel design more profitable than SFSW alone as online demand grows.

In conclusion, customers are increasingly more sensitive to service levels and delivery time. Many shoppers will abandon the checkout process if it takes a long time to deliver. Retailers must be aware of these factors in order to improve its performance.

Next, subsection will wrap up the most relevant conclusions by answering the research questions proposed in chapter 1.

6.1.1 Answer to research questions

1. **Which channel design should be adopted in different retailing contexts in order to improve profitability?**

   The most profitable channel designs across all product categories are the more traditional and the more flexible configurations, SFSW and SFSDSW respectively. Interestingly the SFDSW channel design is the one with the highest average level of service (average demand fulfilled in reached markets) and the higher number of markets fulfilled.

   Being more specific, for the electronics category, SFSW and SFSDSW are in the same range of profit. In the fashion category the most profitable configuration is the one that incorporates more entities (SFSDSW). In the food products it is preferable to implement the ship from store and warehouse (SFSW) design.

2. **How are profits and costs distributed among the different entities?**

   The warehouse is the facility with lower cost of fulfilling demand and the store is the one with the highest. In terms of channel designs, the most traditional configuration (SFSW) is the one with higher unitary cost of fulfilling demand, followed by the model with only dark stores (SFDSW) and the most flexible channel (SFSDSW).

   In terms of profit, in the SFSDSW and SFSW configurations, it is the store the facility that aggregates more profit with a large margin of difference, followed by the warehouse and dark store. In the SFDSW model, it is the warehouse the facility that aggregates more profit, followed by the store and dark store.

   It is to note that the dark store is the facility that bring less value to the network. This could be explained by various factors as its cost structure or the higher preponderance of the other facilities to the profit structure. With the sensitivity analysis conducted, we came to the conclusion that the low contribution of the dark store to the profit is due to the higher scale of number and capacity of stores and warehouses to respond to the demand, in comparison to dark stores.

3. **Is it beneficial to include dark stores as new fulfillment options in a retailer supply chain?**

   The answer to this question is not immediate. It is true that the models with dark stores were not the most profitable in the majority of the scenarios. However, in the most profitable product category (Electronics), the SFSDSW was in the same level of profit of the most traditional channel (SFSW).

The major uncertainty related to these facilities appears in the profit distribution analysis. Here, dark stores were the facilities that aggregated less profit, with a great distance to the others.

To clear all doubts, the net profit of dark stores was calculated to see if these facilities affected negatively the economic performance. Here we came to the conclusion that although they do not add a lot of value, they do not damage profit. The net profit of dark stores is positive for all scenarios.

To sum up, dark stores do not leverage the profit of a retailer as the traditional facilities do. However, since they are a recent phenomenon, there is a great space for optimization and for growth. Our recommendation is for retailers to start investing in this type of facilities to respond to more demanding customers.

6.2 Future Work

Our results suggest some directions for further work. While we varied the online demand market share, retail demand market share available to the retailer was held constant, and vice-versa. In the future, these parameters could be dynamically incorporated into the models.

In this study demand was treated in a static and single period manner. In a future research, demand could be modeled stochastically to incorporate population changing patterns across various geographies while maintaining elasticity. In addition, a time variable could also be incorporated to give dynamism to the model, by changing online demand levels over time to study the evolution of an optimal omnichannel network. For example, this changes could be applied to simulate the demand patterns during the Covid-19 pandemic, for example, or to simulate unforeseen events.

Another suggestion for further work would be to explore the opportunity of not only closing physical stores, but to convert these retail facilities into dark stores for a greater support to omnichannel distribution. Sustainability-related drivers can also be studied, for example by exploring an objective function that aims not only to maximise profit but also, for example, to reduce CO2 emissions.

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Appendix A

Data

Table A.1: Cost Structure.

Parameter	Description		fashion	food	electronics
p	profit		20	15	30
cwl(j)	Warehouse handling cost per unit for an online order fulfilled from a warehouse located at j		1	1	3
crh(s)	Inventory holding cost per unit for store(s) located at s		3	2.25	4.5
crl(s)	Store handling cost per unit of online orders fulfilled from a store located at s		1.1	1.1	3.3
cdl(d)	Dark store handling cost per unit for an online order fulfilled from a dark store located at d		0.77	0.77	2.31
csow(i,j)	Shipping cost per unit per km for an online order in market i from a warehouse located at j		0.0006	0.00072	0.0018
csos(i,s)	Shipping cost per unit per km for an online order in market i from a store located at s		0.0008	0.00096	0.0024
Csrw(s,j)	Shipping cost per unit to store s from a warehouse located at j		0.0004	0.00048	0.0012
Csrr(j,r)	Shipping cost per unit to warehouse j from a supplier		0.0002	0.00024	0.0006
Caps(s)	Capacity of store located at s		3000000		6000000
Capds(d)	Capacity of dark store located at d		250000		500000
Cdo(d)	Dark store fixed cost located at d	k=20MM	38750	77500	38750
		k=20MM	3.3	2.475	4.95
		k=30MM	3.05	2.2875	4.575
		k=40MM	2.8	2.1	4.2
Cwh(k,j)	Warehouse inventory holding cost per unit for a warehouse of size k million located at j	k=60MM	2.3	1.725	3.45
		k=90MM	1.8625	1.396875	2.79375
		k=150MM	1.675	1.25625	2.5125
		k=240MM	1.3	0.975	1.95
		k=20MM	77500000	155000000	116250000
		k=30MM	89375000	178750000	134062500
		k=40MM	100000000	200000000	150000000
Cwo(k,j)	Warehouse fixed cost for warehouse of size k located at j	k=60MM	117500000	235000000	176250000
		k=90MM	142812500	285625000	214218750
		k=150MM	190625000	381250000	285937500
		k=240MM	260000000	520000000	390000000

Table A.2: State demands and coordinates.

City	State Demand	City Demand	Longitude	Latitude	Long*	Lat*
Sacramento	29760021	369365	121.4674	38.56685	-2.12001	0.673119
Albany	17990455	101082	73.79902	42.66575	-1.28804	0.744658
Austin	16986510	465622	97.75052	30.30588	-1.70607	0.528937
Tallahassee	12937926	124773	84.2814	30.457	-1.47099	0.531575
Harrisburg	11881643	52376	76.8845	40.27605	-1.34189	0.70295
Springfield	11430602	105227	89.64465	39.78143	-1.56459	0.694317
Columbus	10847115	632910	82.98738	39.98893	-1.4484	0.697939
Lansing	9295297	127321	84.554	42.7091	-1.47575	0.745414
Trenton	7730188	88675	74.76422	40.2234	-1.30488	0.702031
Raleigh	6628637	207951	78.65875	35.82195	-1.37285	0.625211
Atlanta	6478216	394017	84.42259	33.7629	-1.47345	0.589274
Richmond	6187358	203056	77.47458	37.53105	-1.35219	0.65504
Boston	6016425	574283	71.01789	42.33603	-1.2395	0.738903
Indianapolis	5544159	731327	86.1462	39.7764	-1.50353	0.694229
JeffersonCity	5117073	35481	92.19046	38.5719	-1.60903	0.673207
Madison	4891769	191262	89.38752	43.0798	-1.56011	0.751884
Nashville	4877185	488374	86.78483	36.17155	-1.51468	0.631313
Olympia	4866692	33840	122.8938	47.04192	-2.1449	0.821036
Annapolis	4781468	33187	76.50303	38.97165	-1.33523	0.680184
StPaul	4375099	272235	93.10369	44.94774	-1.62497	0.784486
BatonRouge	4219973	219531	91.12604	30.44897	-1.59045	0.531435
Montgomery	4040587	187106	86.28429	32.3544	-1.50594	0.564691
Frankfort	3685296	25968	84.8652	38.19077	-1.48118	0.666555
Phoenix	3665228	983403	112.0714	33.54255	-1.95601	0.585428
Columbia	3486703	98052	80.88634	34.03924	-1.41173	0.594097
Denver	3294394	467610	104.8727	39.76804	-1.83037	0.694083
Hartford	3287116	139739	72.68387	41.7657	-1.26857	0.728949
OklahomaCity	3145585	444719	97.51349	35.46705	-1.70193	0.619017
Salem	2842321	107786	123.0221	44.9245	-2.14714	0.78408
DesMoines	2776755	193187	93.61741	41.57674	-1.63393	0.725651
Jackson	2573216	196637	90.20759	32.3205	-1.57442	0.564099
Topeka	2477574	119883	95.692	39.0379	-1.67014	0.68134
LittleRock	2350725	175795	92.35408	34.7224	-1.61188	0.60602
Charleston	1793477	57287	81.63044	38.35055	-1.42472	0.669343
SaltLakeCity	1722850	159936	111.9299	40.77727	-1.95355	0.711698
Lincoln	1578385	191972	96.68817	40.8164	-1.68753	0.712381
SantaFe	1515069	55859	105.9541	35.6785	-1.84925	0.622707
Augusta	1227928	21325	69.72971	44.33065	-1.21701	0.773716
CarsonCity	1201833	40443	119.7432	39.14833	-2.08991	0.683267
Concord	1109252	36006	71.56008	43.23159	-1.24896	0.754534
BoiseCity	1006749	125738	116.2261	43.60665	-2.02853	0.76108
Providence	1003464	160728	71.41973	41.82195	-1.24651	0.729931
Helena	799065	24569	112.0204	46.59652	-1.95512	0.813263
Pierre	696004	12906	100.3225	44.37298	-1.75096	0.774455
Dover	666168	27630	75.51744	39.15869	-1.31803	0.683448
Bismarck	638800	49256	100.7673	46.80547	-1.75872	0.81691
Washington	606900	606900	77.01617	38.90505	-1.34419	0.679021
Montpelier	562758	8247	72.57185	44.26648	-1.26662	0.772596
Cheyenne	453588	50008	104.7923	41.14545	-1.82897	0.718124