Optimization Algorithms in Forest Planning Models

Extended Abstract

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ABSTRACT

Industrial development is one of the major factors contributing to wildlife habitat destruction in forests, especially due to resource exploration. The process of harvesting wood, construction or any alteration that changes the forest should follow a plan of good practices in order to ensure its consistency. The destruction of wildlife and its habitat can be minimized with the creation of a connected forest reserve. When scheduling a harvest it is advisable to take into account the arrangement of stands for the construction of a reserve for local species.

This work proposes a formulation that combines both harvest scheduling and the creation of connected and compacted reserves for the protection of species. The development of this multi-objective formulation combines, the URM of Murray [8] for the harvest scheduling and a model for connected and compacted reserves known as RCC-nR. The multi-objective formulation is presented with two models: the model CRC characterized by leaving free stands in the forest and the model CRC-T which utilize these stands to add quality to the species habitat. In both cases, the number of variables is linear for the number of stands and it is possible to verify that they calculate the Pareto front in which the CRC-T model presents a lower number of solutions. In addition, single-objective formulations were developed for both the harvest and the reserves.

KEYWORDS

satisfiability modulo theories, forest planning models, integer linear programming, harvest scheduling.

1 INTRODUCTION

Since ancient times human evolution has been combined with the destruction of wildlife and its habitat. Industrial development has led to excessive exploitation of forests in search of raw materials. The creation of new means of communication that go through forests, the diversion of rivers and other factors have been destroying wildlife's habitat and reducing its capacity of proliferation. In order to minimize the destruction of wildlife and its habitat, elaborated plans that take into account the species protection should be made before any activity that negatively impacts the forest. To protect some species and their habitat, it is customary to select connected forest areas to ensure the creation of natural reserves.

Considering a set of requirements associated with forest planning, calculating the best solution is a question of optimization. The requirements evaluated in the calculation of the best plan vary according to the problem to be treated. For instance, harvesting wood in a way that maximize the profits [7, 8], creating natural forest reserves for the protection of animal species at a reduced financial cost [6], planning the best landscape configuration of a reserve [10], build roads in such way that avoids that species have to cross it, guaranteeing low cost in the construction of the road, and other requirements.

Maximizing profits from a harvest is usually associated with a long-term plan known as harvest scheduling. When harvest scheduling, it is common for the protection of wildlife and its habitat to ensure a connected natural reserve whose stand has a defined minimum time before being harvested. There are two different approaches for harvest scheduling, namely the Unit Restriction Model (URM) approach, which creates a schedule restricting that adjacent stands are not harvested over the same period, and the Area Restriction Model (ARM) approach, in which the adjacency constraint is controlled by the maximum harvest opening sizes. URM and ARM were proposed by Murray [8] which presented a Integer Linear Programming (ILP) formulation for URM and considered heuristics and dynamic programming to solve ARM. McDill *et al.* [7] presented Mixed-Integer Linear Programming (MILP) solutions for ARM which I have to highlight the Path Algorithm and the Generalized Management Unit (GMU) model which works with the set G_i composed by all the combinations of contiguous stands in which i belongs and that meets the maximum harvest opening size.

Regarding the species protection, Onal and Briers [9] proposed the ILP formulation which is concerned with the movement of species between stands without leaving the reserve. However, this formulation has been improved by Billionnet [1] to the level of reducing the execution time by approximately 4, 5 times.

Connectivity is a factor present in the elaboration of forest plans, in which, the purpose of these plans may depreciate or appreciate its importance. An ILP formulation capable of imposing connectivity for different environmental constraints has been presented by Carvajal *et al.* [3]. It promotes the creation of old reserves with a long and narrow landscape structure. This landscape structure motivated in this work the creation of compact and connected reserves without the long and narrow shape. In related works, Onal *et al.* [10] was concerned with the optimal configuration of the reserves with the connectivity to be applied in the structural or functional form and with that purpose an ILP formulation was used.

The main objective of this work is the following:

- Propose optimization formulations to elaborate forest plans for harvest scheduling. These formulations must meet the following requirements:
  - Protection of wildlife and its habitat.
  - Creation of natural reserves connected and with wide corridors that facilitate the movement of species.

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– Maximization of harvest profit.

In order to achieve this objective, it is expected to combine the problem of creating harvest scheduling and the problem of creating reserves for habitat protection.

2 METHODS

First we present single-objective formulation for harvest scheduling and the problem of creating reserves for wildlife habitat protection. Next, these formulations are combined into a multi-objective formulation.

2.1 Harvest Scheduling

The URM model is considered as a particular case of ARM and in turn it uses less complex constraints which results in relatively shorter execution time.

The hypothesis of using the resources of URM model was evaluated in order to compute solutions of ARM model. For this was used the ILP formulation from the URM model proposed by Murray [8] and presented a variation of this same model. The following is the ILP formulation from the URM model.

Consider the notation, L represents the set of stands and T the set of the harvest periods. Having x_{it}, a binary variable of control from the stands, with $i \in L$ and $t \in T$. When $x_{it} = 1$, it indicates that stand i is harvested in period t, otherwise, $x_{it} = 0$. The constant ℓ_{it} denotes the profit associated with stand i in the period t and β_{it} is the volume of wood. I_t and S_t respectively define the lower and upper bounds of produced volume in the t period. Lastly, N_i represents the set of adjacent stands to i.

$$\text{Maximize} \sum_{t \in T} \sum_{i \in L} \ell_{it} x_{it} \tag{1}$$

Subject to:

$$\sum_{t \in T} x_{it} \leq 1 \quad \forall\, i \in L \tag{2}$$

$$I_t \leq \sum_{i \in L} \beta_{it} x_{it} \leq S_t \quad \forall\, t \in T \tag{3}$$

$$x_{it} + x_{jt} \leq 1 \quad \forall\, i > j,\ i \in L,\ j \in N_i,\ t \in T \tag{4}$$

$$x_{it} \in \{0, 1\} \quad \forall\, t \in T,\ i \in L \tag{5}$$

The objective function intends to maximize the profit of the harvest (constraint (1)). Constraint (2) imposes that a stand be harvested at most in one period and the constraint (3) ensures that a volume of wood between I_t and S_t is obtained in each period of harvest. Constraint (4) imposes the adjacency constraint that identifies the URM model, with $i > j$ so that no constraints are repeated. Finally, the domain of the variable x_{it} is bounded to binary in constraint (5).

2.1.1 URM-S Model.

In order to compute solutions of the ARM model in the URM model, it is necessary to convert the original instances into smaller instances, resulting from the stand groupings within the maximum harvest opening size. These grouped stands are considered as a stand. This makes it easier to compute solutions (decreases instance size and resolves using the URM model).

The difficulty that arises in this approach is related to the exponential number of instances converted from the original instance which varies according to the maximum area allowed. Here the idea is not to generate all the instances but generating in a stochastic way in a finite time. As it generates, it computes the solution URM and stores the best solution among the executed instances. This approach will be called Stochastic Unit Restriction Model (URM-S). The Algorithm 1 mirrors the strategy behind the stochastic generation of instances.

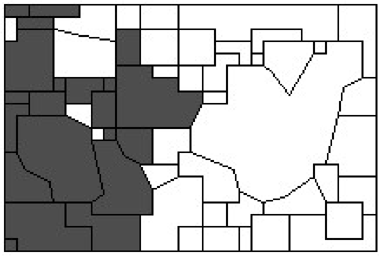
begin
  $A \longleftarrow getArea()$
  $L \longleftarrow getInstance()$
  $instance \longleftarrow \{\}$
  while $len(L) > 0$ **do**
    $i \longleftarrow$ *choose element in* L
    $L \longleftarrow L \setminus i$
    while $choose(0, 1) = 1$ **and** $len(L) > 0$ **and** $A < \alpha_i$ **do**
      $j \longleftarrow$ *choose element in* $N_i \subset L$
      if $A > \alpha_i + \alpha_j$ **then**
        $L \longleftarrow L \setminus \{j\}$
        $i \longleftarrow i \cup \{j\}$
      end
    end
    $instance \longleftarrow instance \cup \{i\}$
  end
  return *instance*
end

Algorithm 1: Strategy for instance generation.

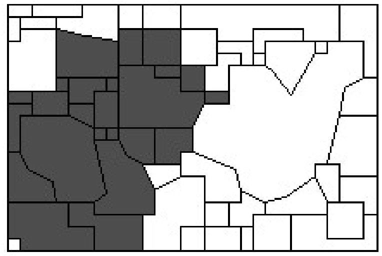
Relatively to the Algorithm 1, the letter A represents the maximum harvest opening size, whereas α_i symbolizes the area of i (where i is a set of stands). L is the checklist of the missing items to choose. For each iteration, choose a stand randomly. Then, as long as you have stands in L and the combined area among the chosen stands is within the requirements, randomly decides whether to continue choosing stands for that combination. The algorithm executes as long as there are not chosen elements in L.

2.2 Connected and Compact Reserves

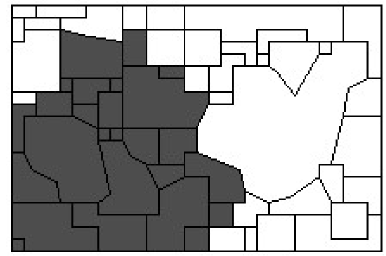
The compact reserves calculation was based in the formulation presented by Onal *et al.* [10] because it provides a solution extremely close to the one desired. It was considered that the stand in the center of the reserve should be identified a priori, according to the conditions presented by the habitat of species under study. As a result, constraints and variables were excluded and so, it was possible to create the desired formulation.



(a) Configuration of a reserve with island stands.



(b) Configuration of a compact reserve, the non-reserved stands are not connected.



(c) Configuration of a compact reserve and non-reserved stands as well.

Figure 1: Connected reserves, the dark coloured stands represent the reserve and the white areas illustrate the non-reserved stands.

Consider the notation, L represents the set of stands of the study forest and $x_{i\gamma}$ the binary variable that represents the stand, with $i \in L$ and γ the stand center identified a priori. When $x_{i\gamma} = 1$, indicates that the stand i is part of the reserve, otherwise $x_{i\gamma} = 0$. The distance between stands γ and i is given by $D_{i\gamma}$, while the quality of the stands for the habitat is represented by Q_i. The constant Q^*, refers to the minimum quality that the reserve must have. Finally, the set of all neighbours of i is represented by N_i.

$$\text{Minimize} \sum_{i \in L} D_{i\gamma} x_{i\gamma} \tag{6}$$

Subject to:

$$\sum_{i \in L} Q_i x_{i\gamma} \geq Q^* \tag{7}$$

$$x_{i\gamma} \leq \sum_{\substack{j \in N_i \\ D_{j\gamma} < D_{i\gamma}}} x_{j\gamma} \quad \forall\ i \in L \ and \ i \notin N_\gamma \tag{8}$$

$$x_{i\gamma} \in \{0, 1\} \quad \forall\ i \in L \tag{9}$$

The objective function (6) minimizes the sum of the distance of the stands that belong to the reserve relatively to the center. Constraint (7) ensures that the created reserve has the minimum quality demanded by Q^*. The adjacency constraint is imposed by constraint (8), where, if a stand $x_{i\gamma}$ is reserved, then, at least one of its neighbours will also be reserved, as long as it is close to the center γ. Constraint (9) bounds the variable $x_{i\gamma}$ values making it binary.

The formulation is able to calculate a reserve, although, it allows the inside of the reserve to have non-reserved stands (island stands). This aspect is verified mainly when the instances have stands of varied characteristics, such as geometric form, the quality of the habitat and the number of neighbours. In Figure 1(a) the configuration of the calculated reserve is illustrated, the dark coloured stands represent the reserved stands and the existence of island stands is represented by the white colour.

This result motivated the development of two models that promote the creation of the compact reserve.

2.2.1 RCC Model. For this model, the motivation was around constraint (8), which is satisfied as long as at least one neighboring stand of $x_{i\gamma}$ closest to the center γ is reserved. Hence it was hypothesized to reserve all neighboring stands of $x_{i\gamma}$ provided that the distance is less in relation to the center γ.

For this, it is necessary to define the constant $C_{i\gamma}$ which indicates the number of neighbours of the stand i whose distance with the stand center γ is smaller. This constant can be calculated by expression (10).

$$C_{i\gamma} = \sum_{\substack{j \in N_i \\ D_{j\gamma} < D_{i\gamma}}} 1 \quad \forall\ i \in L \ and \ i \notin N_\gamma \tag{10}$$

Replacing the constraint (8) by the constraint (11) so that it ensures the reserve of all stands that are in the interior of the reserve. Making it a compact reserve.

$$C_{i\gamma} x_{i\gamma} \leq \sum_{\substack{j \in N_i \\ D_{j\gamma} < D_{i\gamma}}} x_{j\gamma} \quad \forall\ i \in L \ and \ i \notin N_\gamma \tag{11}$$

With these improvements it has been verified that a compact reserve is calculated, however in some cases unreserved stands are not connected, making them inaccessible without having to go through the reserve or go around the forest. Figure 1(b) illustrates the calculated reserve. The dark coloured stands indicate the reserve and there is a stand at the edge of the forest that was not reserved and yet is inaccessible without going through the reserve or going around the forest.

This formulation is known as Connected and Compact Reserves (RCC).

2.2.2 RCC-nR Model. This new model has as hypothesis the definition of two stand centers, one for reserve and another for non-reserved stands. α is indicated for the stand center of the non-reserved stands and γ for the reserved stand center (with $\alpha \neq \gamma$).

Adding constraint (12), will impose that each stand i becomes part of the reserve or non-reserve

$$x_{i\alpha} \neq x_{i\gamma} \quad \forall\ i \in L \tag{12}$$

Replacing constraint (8) by constraint (13) will ensure the connection of the reserve and the non-reserved stands. Resulting in the possibility of the existence of a compacted reserve and the non-reserved stand with total accessibility.

$$x_{jk} \leq \sum_{\substack{i \in N_j \\ D_{ik} < D_{jk}}} x_{ik} \quad \forall\, k \in \{\gamma, \alpha\},\ j \in L \text{ and } j \notin N_k \tag{13}$$

With this new model it is possible to compute connected and compact reserves and ensure that non-reserved stands are connected to the access without disturbing the reserve's habitat. Figure 1(c) illustrates the landscape configuration of the computed solution. The dark coloured stands represent the reserve, and the configuration of a connected and compact reserve is verified at the same time as the non-reserved stands are connected. This model is named Connected and Compact Reserves (RCC-nR).

2.3 Multi-Objective Formulation

Obtaining from the single-objectives, the multi-objective was formulated by the combination of two models. Firstly, for the harvest scheduling was defined the URM model of Murray [8]. Secondly, for the connected and compact reserve was defined the RCC-nR model.

As for the URM-S model from the harvest scheduling, its integration would transform the multi-objective solution with the same stochastic characteristic, and this is not desirable. For the compact reserve, it was verified that the RCC-nR model is what ensures the connectivity for both the reserve and the non-reserved stands and makes the harvest possible without crossing the reserve. The following is the multi-objective formulation:

$$\text{Maximize} \sum_{t \in T} \sum_{i \in L} \ell_{it} x_{it} \tag{14}$$

$$\text{Minimize} \sum_{i \in L} D_{i\gamma} x_{i\gamma} \tag{15}$$

Subject to:

$$\sum_{i \in L} Q_i x_{i\gamma} \geq Q^* \tag{16}$$

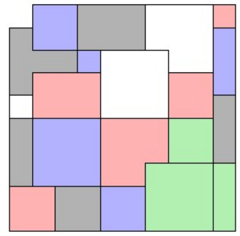
$$x_{jk} \leq \sum_{\substack{i \in N_j \\ D_{ik} < D_{jk}}} x_{ik} \quad \forall\, k \in \{\gamma, \alpha\},\ j \in L \text{ and } j \notin N_k \tag{17}$$

$$x_{i\alpha} \neq x_{i\gamma} \quad \forall\, i \in L \tag{18}$$

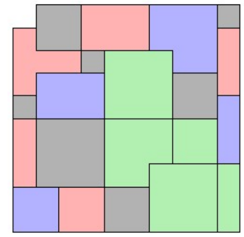
$$I_t \leq \sum_{i \in L} \beta_{it} x_{it} \leq S_t \quad \forall\, t \in T \tag{19}$$

$$x_{it} + x_{jt} \leq 1 \qquad \forall\, i > j,\ i \in L,\ j \in N_i,\ t \in T \tag{20}$$

$$x_{ik} \in \{0, 1\} \quad \forall\, i \in L,\ k \in T \cup \{\gamma, \alpha\} \tag{21}$$



(a) Harvest and reserve using constraint (22)



(b) Harvest and reserve using constraint (23)

Figure 2: Harvest scheduling with three periods and the creation of a compact reserve. The green colour indicates the reserve, the white free stands, and the other colours represent harvest periods.

The objective-functions (14) and (15), maximizes the harvest profit and minimizes the sum of the distances of the stands of reserve respectively. Constraint (16) ensures its minimum reserve quality, while the constraints (17) and (18) grant the connectivity of the reserve and non-reserved stands. For the harvest scheduling the constraint (19) imposes the upper and lower bounds of wood volume to be harvested in each period, and the constraint (20) ensures the constraints of adjacency of URM.

The first constraints that relates the (URM and RCC-nR), begins from the constraint (21) which restricts the x_{ik} variable to binary, as this variable represents both harvest and reserve.

Lastly and to complete the formulation, it is necessary to constrain each x_{it} stand so that it is not harvested in more than one period and that there is not overlap of the stand (to be harvested and reserved). For this case was used the constraint (2) from the URM model to originate the constraint (22), so that it ensures that if the stand is harvested, the same stand can not be reserved again and vice-versa.

$$x_{i\gamma} + \sum_{t \in T} x_{it} \leq 1 \quad \forall\, i \in L \tag{22}$$

After evaluations of the model, it was noticed through landscape structure that when the instance has higher quality than the required by the limit wood volume and minimum reserve quality, there will be non-reserved and not harvested stands (see Figure 2(a), these lots are called free stands).

The free stands continue to have free access without having to cross the reserve, however it would be better if they were used to increase the reserve's quality. This evaluation originated the replacement of constraint (22) with constraint (23) so that it avoids free stands.The result is illustrated in Figure 2(b).

$$x_{i\gamma} \neq \sum_{t \in T} x_{it} \leq 1 \quad \forall\, i \in L \tag{23}$$

The model with the constraint (22) will be named as Harvest Scheduling and Compact Reserve (CRC), while the constraint (23) will be called Harvest Scheduling and Total Compact Reserve (CRC-T).

Table 1: Instance characteristics

Instance	Area (hectare)	Number of Stands
FLG_1	2.970	21
FLG_2	2.990	57
FLG_3	1.495	62
FLG_4	1.495	63
FLG_5	1.497	100
FLG_6	2.098	203

3 RESULTS AND DISCUSSION

The ILP formulations developed, were implemented and tested in Satisfiability Modulo Theories (SMT) using the Microsoft's Z3 solver [5].This solver was chosen because it provides useful tools for multi-objective formulations (*e.g.* Pareto front [11] and Box objective [2]).

The tests were performed on a 1400 MHz CPU with 132 GB RAM of memory. For the evaluation of the formulations were used several instances generated by the tool known as Forest Landscape Generator available on-line [1]. Table 1 illustrates the instance characteristics according to the number of stands and total areas of forest instance.

3.1 Connected and Compact Reserves

The Carvajal *et al.* formulation [3], imposes connectivity on instances with up to 1.000 stands and computes reserves whose landscape configuration has the long and narrow form, whereas RCC and RCC-nR are able to compute connected and compact reserves (see Figures 1(b) and 1(c)). However, RCC and RCC-nR are unable to compute optimal solutions in instances with more than 400 stands within the time limit of 4 hours.

In relation to the constraints, the RCC is characterized by the constraint (11), which imposes that the stands are reserved around the center, whereas the RCC-nR imposes the connectivity to the reserve and for the non-reserved stand (characterized by the constraint (13)).

The RCC-nR evaluation of stands non-reserved, implied that it had double the variables in relation to RCC. Considering that the number of variables is linear in the number of stands of instances.

The execution time, is greatly influenced by the size of instances [2] and the minimum reserve quality. It was verified that when the minimum quality alternates between 30% to 70% in relation to the total quality offered by the forest, the highest time is found in the calculation of optimal solution (see Figures 3(a), 3(b) and 4(a)), with a higher incidence ranging between 40% to 60%.

According to the tests, it has been generally verified that the RCC model is relatively faster than RCC-nR in what concerns to the optimal computation. The Figures 3(a), 3(b) and 4(a) illustrate the execution time in seconds to the minimum qualities in percentage indicated by the set {10, 20, 30, 40, 50, 60, 70, 80, 90}. In Figure 3(a), the RCC-nR had a maximum time of 14 seconds for each quality, while the RCC-nR needed only 4 seconds. For the Figure 3(b), with only 80% of minimum quality is that the RCC-nR was dominated by the RCC, and the RCC-nR solution needed 100 seconds maximum, while 40 seconds is the maximum time for RCC. Lastly, in Figure 4(a) is verified that RCC-nR needed more than 1 hour to obtain the optimal solution, while the the RCC, to 50% and 60% did not compute the optimal solution in less than 6 hours, however the RCC-nR solution took longer in the majority of qualities (see Figure 4(a), the time of RCC is not illustrated for 50% and 60%).

Relatively to the number of reserved stands, the RCC-nR reserves a lower number, and it has been verified that it obtains the quality and the distance of the stands equivalent to the RCC (see Figure 4(b)).

The fact that RCC-nR reserves less stands, obtain equivalent quality and allow that the non-reserved stands to be connected, assigns advantages over the RCC model. This is because the non-reserved stand is connected and larger, which is beneficial for other purposes (in this extended abstract it is beneficial for the harvest scheduling).

The approach applied to connectivity is only structural. If we consider forests with rivers and stands without a structural neighborhood, it has been verified that RCC-nR does not apply because it imposes connectivity also on non-reserved stands. The Onal *et al.* formulation [10], which based the construction of the models, considers structural and functional connectivity, being able to solve problems that RCC-nR does not solve. However, the solution of Onal *et al.* [10] allows the existence of stand islands within the reserve and RCC-nR does not. The RCC model, while not imposing functional connectivity, is capable of solving instances with stands without neighborhoods. This is possible because RCC imposes connectivity only on the reserve, leaving the stands without neighborhood and structurally separated.

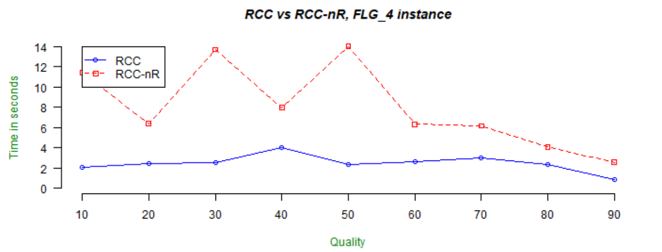
3.2 Harvest Scheduling

The MILP formulation presented by McDill *et al.* [7], namely the Path Algorithm and GMU, determines that Path Algorithm needs 5 times ou more execution time than URM to compute solutions. Since URM-S reduces the instances size before executing and has the same complexity of URM, it is verified that it needs less execution time to compute solutions. By transitivity, URM-S has a number of linear variables in the number of stands, while the two ILP formulations to ARM solved using Branch-and-Bound (B&B) by Constantino *et al.* [4] is quadratic and Path Algorithm exponential. It was also determined by McDill *et al.* [7] that the MILP GMU formulation needs more execution time and uses more variables and constraints than Path Algorithm.

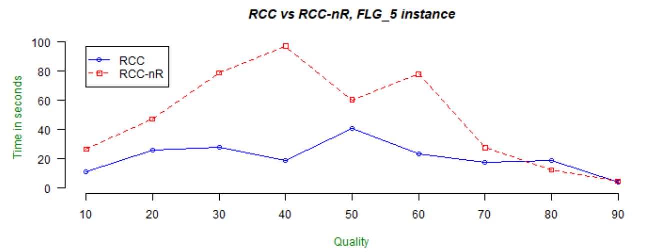
The URM-S, although better in the number of variables and execution time, is not able to prove optimality for the solutions found, due to the stochastic component used to generate instances of smaller size than the original instance.

A version of the GMU model was implemented to more accurately measure the quality of the URM-S. It was verified that GMU computes optimal solutions in less of 30 minutes in instances with up to 21 stands and limited by 10% of harvested wood volume in each period. However, for instances other than FLG_1, GMU was unable to compute optimal solutions, not even when considering 3 hours as the time limit. The same instances for URM-S presented extremely close optimal solutions. Figure 5 illustrates the optimal

[1] http://ifmlab.for.unb.ca/fmos/tools/DistributeFLG.zip
[2] Number of Stands.

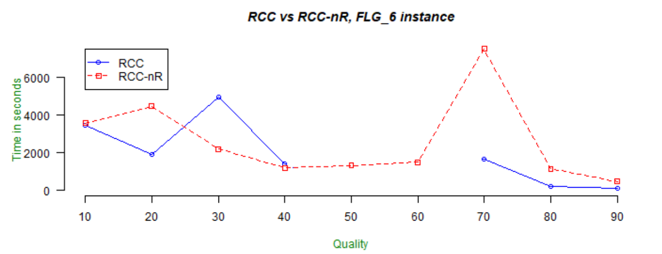


(a) FLG_4 instance, RCC model is the blue line and the RCC-nR model is the red line.

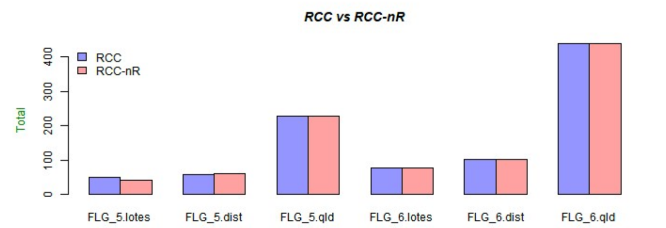


(b) FLG_5 instance, RCC model is the blue line and the RCC-nR model is the red line.

Figure 3: Execution time for connected and compact reserves, RCC *vs* RCC-nR. The vertical axis represents the time in seconds and the horizontal axis represents the percentage of minimum reserve quality to be computed.



(a) FLG_6 instance, RCC is the blue line and RCC-nR is the red line.



(b) FLG_5 and FLG_6, the number of reserved stands, the sum of the distances and reserve quality are observed.

Figure 4: Time and quality for compact and connected reserves, RCC *vs* RCC-nR.

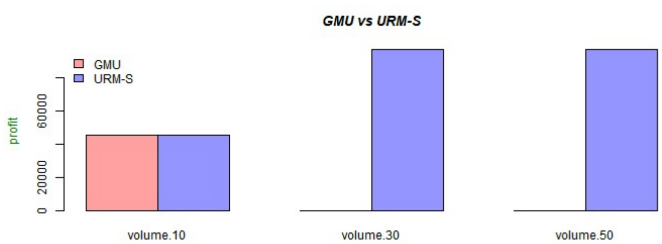


Figure 5: URM-S, with 10% of volume, was at $0,01\%$ of distance for the optimal value (which is not noticeable giving the impression that they have the same profit).

profit of GMU with the red colour and the computed by URM-S in blue.

After limiting the execution time to 30 minutes, with instances with more than 21 stands, was verified that URM-S presents higher profits than GMU which is able to compute in the same time interval. Figures 6(a) and 6(b), with the limits of the wood volumes defined by 10%, 30% and 50% of the forest total, can be verified that URM-S computes higher profits in relation to GMU.

3.3 Multi-Objective Formulation

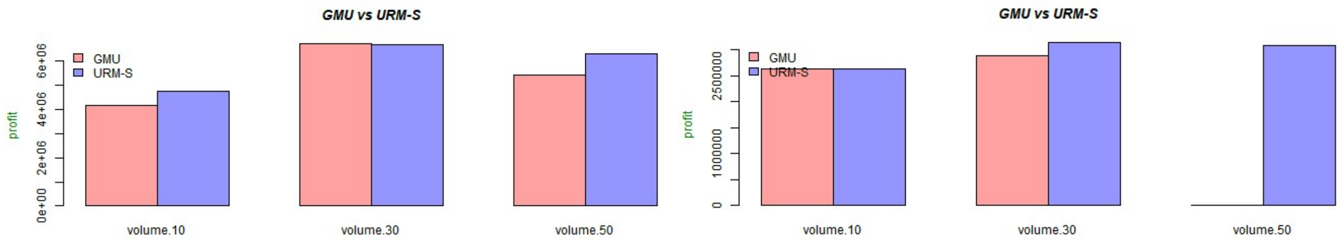
In the Section 2.3, was verified that the CRC and CRC-T models provide solutions with impact on stand utilization, when the upper limit of wood volume combined with the minimum reserve quality does not impose the use of all stands. By way of illustration, Figure 2(b) illustrates a solution with full stand utilization (provided by CRC-T) and in Figure 2(a) is found that there are free stands (provided by CRC).

The free stands are taken advantage of and added to the compact reserve by the CRC-T model. In this case, the model CRC computes solutions with compact reserves whose quality is very close to the minimum quality. While the CRC-T model reuses all free stands to add quality to the reserve.

Regarding the number of variables, the multi-objective formulation combines the variables of the URM and RCC-nR models. Both are linear in the number of stands, so the formulation is linear.

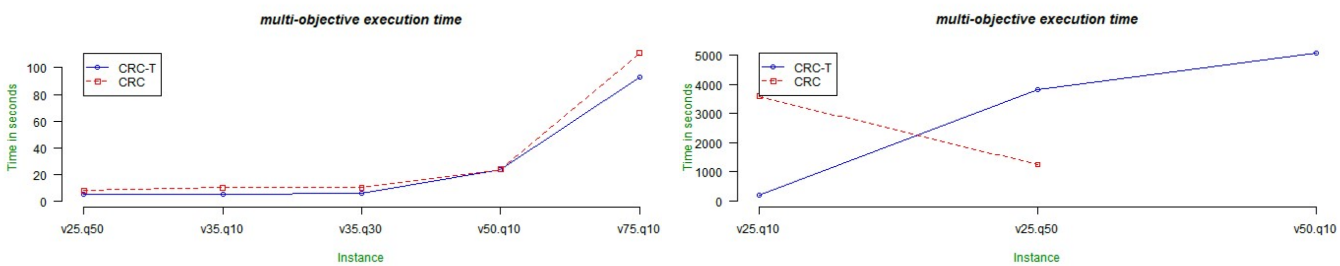
As for the execution time to compute a solution, the CRC model proved to be slower to solve in relation to CRC-T model. Figure 7(a) illustrates the execution time for both and Figure 7(b) shows a moment in which CRC-T took longer but that after the wood volume variation CRC was unable to compute a solution not even with 8 hours time limit.

Another aspect that has been verified is that CRC-T is also relatively faster in computing the Pareto front. This aspect was verified



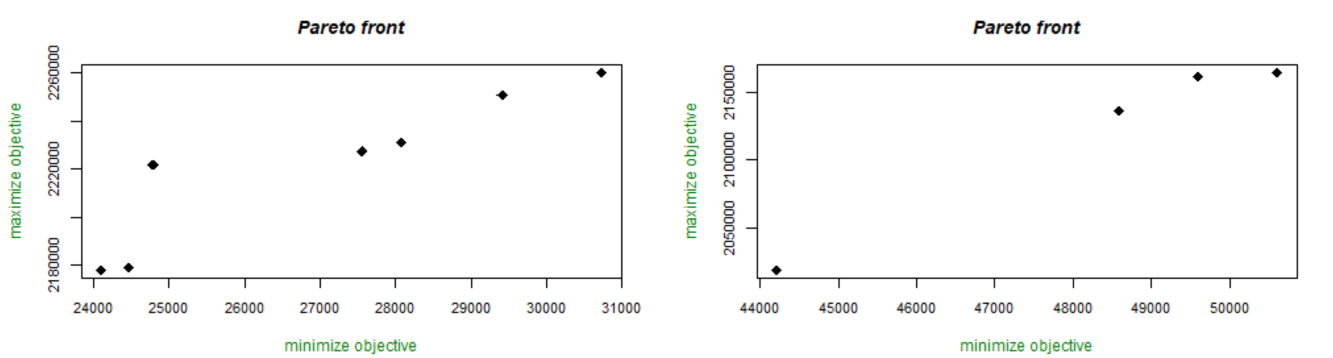
(a) FLG_2 instance, profit acquired. **(b)** FLG_3 instance, profit acquired.

Figure 6: Profit acquired with harvest, GMU represented in red and URM-S by the blue colour. For each instance, the lower and upper bounds of the wood volume were varied. Boundaries vary in percentage relatively to the total of the forest.



(a) Execution time for FLG_1 instance. **(b)** Execution time for FLG_2 instance.

Figure 7: Execution time for the calculation of the first solution at the Pareto front. Red line identifies CRC and the blue line identifies CRC-T. It can be seen in Figure 7(b) that CRC does not solve for an instance when the time is limited to 8 hours. Instance variations were made with alternation of the wood volume and the quality of the reserve (*e.g.* $v50.q10$ means 50% of wood volume as a limit and 10% of minimum reserve quality).



(a) Model CRC. **(b)** Model CRC-T.

Figure 8: Pareto front computed in less than 30 minutes, instance FLG_1 and 3 periods. 50% of minimum quality for reserve and 25% of volume of wood for harvest. The vertical axis represents the maximization of the objective function of the harvest scheduling and the horizontal axis represents the minimization of the objective function of the compact reserve.

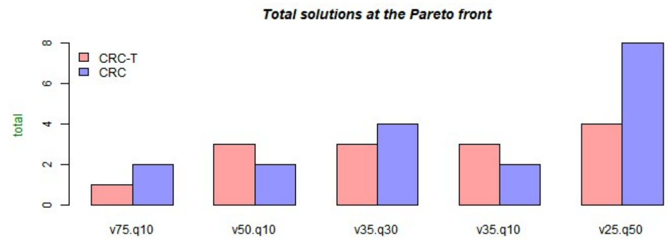


Figure 9: Total solutions at the Pareto front. The red colour represents the CRC-T model and the CRC by the blue colour. FLG_1 instance varies in wood volume and reserve's quality (*e.g.* $v75.q10$ means 75% of wood volume as a limit and 10% of minimum reserve quality).

for all the tests, even when CRC did not present any solution of the Pareto front.

The Pareto front for the CRC model is shown in Figure 8(a) and in Figure 8(b) the Pareto front for the CRC-T model is shown. It is clear that CRC-T has a lower number of solutions.

Another aspect observed in Figures 8(a) and 8(b) is that CRC obtains better solutions for the objective functions. Obtaining a higher profit and a lower distance, relatively to what CRC-T computes. For the CRC-T model, the solutions of the objective function of the reserve, is justified by the reuse of the free stands, while the solutions for the objective functions of the harvest is justified by the connectivity of non-reserved stands imposition.

Although the number of solutions at the Pareto front are not uniform for CRC-T and CRC (comparatively), it can be seen that CRC-T has a lower number of solutions in more occasions. This is shown in Figure 9.

Evaluating CRC and CRC-T models, we conclude that both have a linear number of variables inherited from the combination of the URM and RCC-nR models. Although, there is optimality in both, CRC-T is relatively faster in the calculation of the first solution and in the Pareto front. It was also verified that the model CRC-T creates reserves with better qualities, resulted from the free stands utilization of CRC.

In terms of the landscape presentation, CRC leaves free stands when the parameters (wood volume and minimum reserve quality) allow. While CRC-T takes advantage of these stands to add qualities to the reserve. In this aspect, it is not verified which one is the best, however, for the performance of the results obtained from the evaluation, it is verified that CRC-T is better in both execution time and in the lower number of solutions at the Pareto front (generalizing).

4 CONCLUSION

This work has developed three formulations related to harvesting and protection of species, namely:

- Formulation for harvest scheduling that implements constraints which ensure a balanced harvest for all periods and whose objective function maximizes harvest profit.
- Formulation for the creation of connected and compact reserves that ensure the development of a reserve with the necessary qualities for the species habitat.
- Multi-objective formulation that combines the previous formulations and guarantees the creation of a harvests scheduling preserving as much as possible the habitat of the species.

In the harvest scheduling formulation, URM-S was developed with a stochastic component that compute solutions of ARM with the complexity of URM. It is concluded that URM-S is unable to prove the optimality of the solutions, however, within a limited time of 30 minutes it computes solutions with better quality than GMU, for instances with more than 21 stands.

Relatively to the formulation for connected and compact reservations, two models have been developed: RCC, which imposes connectivity exclusively for creation of the reserve and RCC-nR that imposes connectivity for the reserve and for the non reserved stands. From the results, it is concluded that RCC is relatively faster computing solutions. However, their solutions can separate the non-reserved stands from the reserve and the total of reserved stands is higher. This formulation, irrespective of the model, has fulfilled the objective of computing solutions with acceptable landscape configurations, that is, the reserves do not have a long and narrow configuration.

URM for harvest scheduling and RCC-nR for connected and compact reserves was used for the multi-objective formulation in which has two models, namely CRC and CRC-T. The CRC allows the occurrence of free stands and presents higher harvest profit, whereas CRC-T does not allow such occurrence and includes those stands in the reserve (adds the quality of the reserve and increases the distance of the stands to with the center). The CRC-T approach reduces the harvest profit because of RCC-nR behaviour which imposes connectivity for the reserve and for the non-reserved stand.

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