Comparative Analysis of Second Order Effects by Different Structural Design Codes

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ABSTRACT

This work compares some methods of analysis of global and local second order effects in reinforced concrete columns, according to four structural design codes: Brazilian NBR 6118 (2014), European EN 1992-1-1, *fib* Model Code 2010 and American ACI 318M-14. To do this, a 12-storey reinforced concrete fictitious building, subjected to wind, accidental and self-weight loads with 12 column-types, was modelled using the software SAP2000.

For the global analysis, while the Brazilian Standard gives the possibility of choosing between the P-Delta method and a bending moment multiplier, the EN 1992-1-1 only considers a magnification factor of horizontal forces. The two other codes do not make any reference to global second order effects.

It was also done a local analysis, comparing seven different methods found in the four codes and although many similarities were found, there are some particularities in each code that make the final results differ. This comparison was done in terms of both final bending moments and reinforcement. Also, to check the accuracy of the methods, a nonlinear analysis considering both nonlinearities was performed, using SAP2000 once again.

Finally, to show the immense importance of stiffness in the behaviour of a building, a case study is analysed. First, all columns were turned 90-degrees to reduce the stiffness against the wind force, and then two rigid cores were added, comparing the before and after results.

Keywords:

Second order effects;

Reinforced concrete building;

Nonlinear analysis;

Comparative analysis;

Structural design codes;

Stiffness influence;

RESUMO

Neste trabalho fez-se uma comparação entre alguns métodos de análise dos efeitos de segunda ordem, tanto globais como locais, em pilares de betão armado, segundo quatro normas de dimensionamento estrutural: Norma Brasileira NBR 6118 (2014), Norma Europeia EN 1992-1-1, Model Code 2010 da *fib* e Norma Americana ACI318M-14. Para tal, modelou-se um edifício fictício de betão armado de 12 andares e 12 pilares-tipo no programa SAP2000, sujeito a carregamentos de vento, carga acidental e peso próprio.

No caso da análise global, enquanto a Norma Brasileira dá a possibilidade de escolha entre o método P-Delta e um fator multiplicador de momentos fletores, a Norma Europeia apenas considera um fator multiplicador de forças horizontais. As outras duas normas não fazem qualquer referência a estes efeitos.

Também foi feita uma análise local, comparando sete métodos diferentes e apesar das muitas semelhanças, há considerações particulares de cada norma que fazem com que os resultados sejam diferentes. Esta comparação foi feita tanto em termos de momentos flectores finais como em termos de armadura. Para verificar a precisão destes métodos, fez-se também uma análise não linear, considerando ambas as não linearidades, através do mesmo programa.

Finalmente, para enfatizar a extrema importância da rigidez no comportamento de um edifício, analisou-se um caso de estudo. Primeiro, todas as colunas sofreram uma rotação de 90 graus, de forma a reduzir a rigidez contra a acção do vento, sendo adicionados de seguida dois núcleos rígidos a esse modelo, comparando os resultados antes e depois.

Palavras-chave:

Efeitos de segunda ordem;

Edifício de betão armado;

Análise não-linear;

Análise comparativa;

Normas de dimensionamento estrutural;

Influência da rigidez;

SUMMARY

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Abbreviations

eff	Effective
LoA	Level of approximation
SLS	Serviceability limit state
ULS	Ultimate limit state

Latin upper-case letters

A	Cross sectional area; Coefficient
A_c	Cross sectional area of concrete
A_S	Cross sectional area of reinforcement
B	Coefficient
C	Coefficient
E_c	Tangent modulus of elasticity of concrete
E_{cd}	Design value of modulus of elasticity of concrete
E_{ci}	Initial tangent modulus of elasticity
E_{cs}	Secant modulus of elasticity
$E_{cs}{}^*$	Secant modulus of elasticity for global stability analysis
E_{cm}	Secant modulus of elasticity of concrete
E_s	Design value of modulus of elasticity of reinforcing steel
$E_{cd}I_c$	Stiffness of the bracing elements
EI	Bending stiffness
F	Action; Force
$F_{d,SLS}$	Design value of an action for the SLS combination
$F_{d,ULS}$	Design value of an action for the ULS combination
F_G	Direct permanent actions
$F_{H,0Ed}$	First order horizontal resulting force
$F_{magnification}$	Magnification factor for applied horizontal forces
F_Q	Direct variable actions, in which F_{Q1} is the main action
$F_{V,Ed}$	Total design vertical load
$F_{V,B}$	Nominal buckling load
$F_{V,BB}$	Nominal buckling load disregarding shear deformations
H	Height of the structure
H_i	Force acting at a pavement
$H_i{}^*$	Fictitious force acting at a pavement
H_{tot}	Height of the structure
I	Second moment of area of concrete section
$I_{bracing}$	Second moment of area of the bracing elements
I_g	Moment of inertia of gross concrete section
K_c	Factor for effects of cracking, creep and similar effects
K_s	Factor for contribution of reinforcement
K_r	Correction factor depending on the axial load
K_φ	Factor taking account creep
L	Length
M	Bending moment
M_0	Final first order moment
M_{01}	Highest value of first order end moment

M_{02}	Lowest value of first order end moment
M_{0Ed}	First order moment
M_1	Final first order moment
$M_{1d,min}$	Minimum moment due to geometric imperfections
$M_{1,tot,d}$	First order toppling moment
M_2	Second order moment
$M_{d,tot}$	Total bending moment
M_A	Highest value of first order end moment
M_B	Lowest value of first order end moment
M_{Ed}	Design value of the applied internal bending moment
M_i	Moment due to the local geometric imperfections
M_{ins}	Non-sway moment
M_{is}	Sway moment
M_{Rd}	Resistant moment
$\Delta M_{tot,d}$	Increase of bending moments due to vertical actions
M_X	Bending moment in the X direction
M_Y	Bending moment in the Y direction
N	Axial force
N_B	Nominal buckling load
N_{cr}	Critical buckling load
N_{Ed}	Design value of the applied axial force
P_{cr}	Critical buckling load
P_c	Critical buckling load
P_u	Factored axial force
Q	Coefficient
V	Shear force
V_0	Basic wind velocity
V_{Ed}	Design value of the applied shear force
V_k	Characteristic velocity
$W_{pavement}$	Weight of pavement

Latin lower-case letters

$1/r$	Curvature of a section of an element
a_i	Displacements in each pavement
Δa_i	relative displacements in each storey
b	Overall width of a cross-section
c	Concrete cover
c_0	Integration factor
d	Effective depth of a cross-section
d'	Concrete cover
e	Eccentricity
e_{0d}	Eccentricity due to local geometric imperfections
e_1	First order eccentricity
e_{1d}	First order eccentricity
e_2	Second order eccentricity; Deflection
e_a	Eccentricity due to local geometric imperfections
e_i	Eccentricity due to local geometric imperfections
$e_{2d,I}$	Total eccentricity for the Level I of Approximation
$e_{2d,II}$	Total eccentricity for the Level II of Approximation

f_{cd}	Design value of concrete compressive strength
f_{ck}	Characteristic compressive cylinder strength of concrete at 28 days
f_c'	Specified compressive strength of concrete
f_{yd}	Design yield strength of reinforcement
f_{yk}	Characteristic yield strength of reinforcement
f_u	Minimum tensile stress
g	Permanent load due to self-weight, masonry and cladding
h	Height of a column; Height of a storey; Depth of a cross-section
h_{beams}	Height of the beams
$h_{columns}$	Height of the columns
h_{slabs}	Height of the slabs
i	Radius of gyration
k	Coefficient; Factor
k_i	Relative flexibility of rotational restraints
k_d	Maximum design curvature
l	Length
l_0	Effective length
l_c	Length of a storey or column
l_e	Effective length
l_{free}	Free length of a column
l_u	Unsupported length of a column
m	Number of elements
n	Relative normal force; Number of storeys
Δp	Effective pressure
q	Variable load; Dynamic pressure; Distributed wind force
q_1	Variable accidental load
q_w	Variable wind load
r	Radius of gyration
r_m	Ratio of first order end moments
w_c	Weight of concrete

Greek letters

α	Coefficient of thermal dilatation; instability parameter
α_1	Reference value for instability parameter
α_b	Coefficient that varies according to the moments in the boundary
α_{EC2}	Rüsch coefficient according to EN 1992-1-1
α_{ACI}	Rüsch coefficient according to ACI 318M-14
α_h	Reduction value for height
α_i	Coefficient
α_m	Reduction value for number of members
β	Factor
β_{dns}	Factor
γ	Specific weight of RC columns and beams
γ_{cE}	Partial factor for the elasticity modulus
$\gamma_{concrete,EC2}$	Partial factor for concrete according to EN 1992-1-1
$\gamma_{concrete,ACI}$	Partial factor for concrete according to ACI 318M-14

γ_g	Ponderation coefficient for the direct permanent actions
γ_{n1}	Additional coefficient for very slender columns
γ_{slabs}	Specific weight of RC slabs
γ_{steel}	Specific weight of reinforcement steel
γ_q	Ponderation coefficient for the direct variable actions
γ_z	Global stability parameter
δ	Maximum storey displacement
δ_i	Displacement at a given storey
δ_s	Moment amplification factor
δ_{top}	Displacement at the top of the building
ε	Strain
θ_0	Basic value of the inclination
θ_i	Inclination
κ	Dimensionless stiffness
λ	Slenderness ratio
λ_{lim}	Limit slenderness
λ_1	Limit slenderness
υ	Poisson’s coefficient; Normal dimensionless force
ξ	Coefficient
ρ	Reinforcement ratio for longitudinal reinforcement
σ	Stress
$\varphi_{(\infty,t_0)}$	Final creep coefficient
φ_{ef}	Effective creep ratio
ψ_0	Reduction factor for the combination value of a variable action for ULS
ψ_1	Reduction factor for the frequent value of a variable action for SLS
ψ_2	Reduction factor for the quasi-permanent value of a variable action for SLS
ψ_i	Ratio of stiffness at a column end
ψ_m	Average ratio of stiffness of a column
ω	Mechanical reinforcement ratio
Δ_0	Relative first order lateral deflection

1. INTRODUCTION

1.1. CONTEXT AND MOTIVATION

In the past few decades, due to the increase of urban populational density, allied with the evolution of building materials and the constructive and designing techniques, the structures of buildings have become taller and taller, and consequently more and more slender. As a result, this evolution brought a potential amplification of the bending moments, the so called second order effects.

Although these effects may be dismissed in the most common structures, in some cases they can assume a considerable magnitude, damaging the structure, which may cause both material and human losses. For this reason, the consideration of these effects may take a very important role in the design of the structures and structural codes usually have chapters dedicated to this problem.

However, even though second order effects are a worldwide problem, not all structural design codes deal with this issue in the same way and assumptions, simplifications and approaches are different, as each country has their own particular way of dealing with engineering problems.

As today's world is a global one, it was felt necessary to compare some codes and see their differences and similarities. The ones analysed herein are:

1. NBR 6118 (2014) – Brazilian Standard;
2. EN 1992-1-1 – European Standard (Eurocode 2);
3. *fib* Model Code 2010 – International Federation for Structural Concrete;
4. ACI 318M-14 – American Standard, in the metric system.

Since this thesis was done as the Final Project of Double Degree between IST-Lisbon (Portugal) and UFRJ (Brazil), the four codes were considered to be the most important to be analysed for both countries.

From the past few years, UFRJ has been making significant theoretical research on this topic. This thesis in particular gives continuity to the Final Project of Gomes (2017).

1.2. OBJECTIVES AND METHODOLOGY

The objective of this work is to analyse and to compare the already mentioned design codes regarding to:

1. Checking the requirements for consideration of second order effects;

2. Analysing and evaluating the influence of these effects in reinforced concrete (RC) structures;
3. Comparing the different analyses and approaches, showing its similarities and differences.

To fulfil this objective, the following methodology was followed:

1. Understand the fundamental concepts behind second order effects;
2. Understand the different procedures of analysis for these effects, and evaluate the sensibilities of each method;
3. Analyse the structural model of a building previously created according to each code and compare their results.

Following this procedure, this work aims to contribute for a better understanding of second order effects in RC structures and its practical applications in different countries.

1.3. OUTLINE OF THE DOCUMENT

The present work is divided in eleven chapters and five appendixes. Besides this introducing chapter, the remaining chapters are described below.

The second chapter presents the fundamental concepts regarding structural instability, starting with the definition of second order effects and how to analyse them. This will be followed by other critical definitions on the subject, such as buckling, slenderness, geometrical imperfections and creep, and the chapter ends with the basic general approach on second order effects according to different codes.

The third chapter will define the structure in analysis and the structural model used, fully detailing its material and sections' properties, applied loads and considered combinations of actions. This model will serve as base for use in the remaining chapters.

Chapters four to seven will analyse the model previously described, each one by a different code, following the order already shown in 1.1. These four chapters start with the normative definitions, followed by global and local analyses of the structure, concluding with the final bending moment and reinforcement comparison.

Chapter eight will present a computational nonlinear analysis, using SAP2000 once again.

The ninth chapter presents a case study that has the objective of analysing the influence of stiffness in the building's structure. First, all columns will be turned 90-degrees and then two rigid cores are going to be added to the structure, comparing the before and after results of a global analysis.

Finally, conclusions will be presented in chapter ten, also presenting proposals for continuing this work.

All the references that guided this work will be shown in chapter eleven.

Appendix A demonstrates the relationship between the instability parameter α found in the Brazilian code, and a column's nominal buckling load.

Appendixes B to E show the step-by-step design for one column according to the different methods of each structural design code.

2. FUNDAMENTAL CONCEPTS

2.1. SECOND ORDER EFFECTS

It is a well-known fact that when an element is subjected to axial force and bending moments, it deflects. The interaction between this deformation and the axial force will cause an increase of the bending moment at a given section that, depending on the slenderness of the element, may take a very important role in the structural design. This additional effect is what is called second order effect.

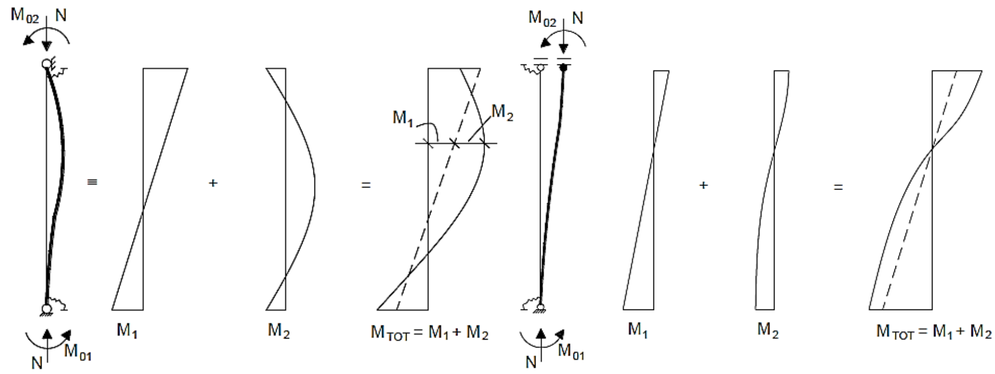

Figure 2-1– Resulting bending moments in braced and unbraced elements (Câmara, 2015).

In general, structural design codes assume that second order effects can be ignored if they represent less than 10% of the first order moment. However, this rule is not practical for the structural designer, as it requires their previous calculation to know if they are dismissible. For this reason, regulations have developed simplified ways to verify if these effects are in fact a problem.

If the case is that second order effects cannot be dismissed, a nonlinear analysis must be made. This type of analysis must consider both geometrical and physical nonlinearities, and they will be detailed in the following sub-chapters.

It is important to note that there are two distinct kinds of second order effects:

- Global effects (or P-Δ effects) affect the entire structure and occur in structures that present significant horizontal displacements when subjected to vertical and horizontal loads. This kind of structures is called sway structures.
- Local effects (or P-δ effects) affect isolated elements that suffer significant displacement when subjected to axial loads, independently from what happens with the structure. Here, the bending moment diagram is decurrent from a nonlinear behaviour, unlike to what happens to the first order moment.

Both global and local second order effects will be addressed in this work.

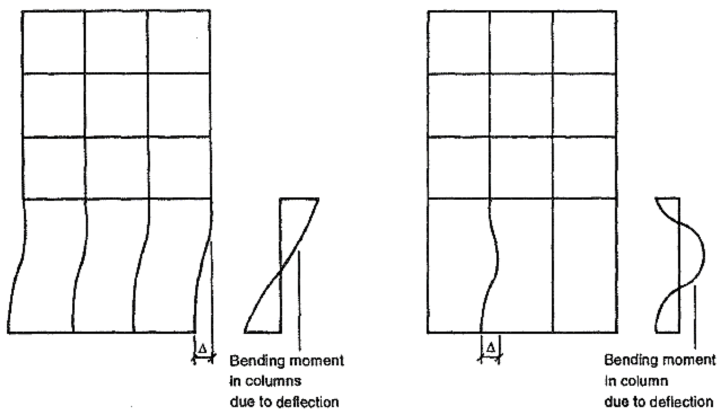

Figure 2-2– Modes of deflection of columns in a structure: (a) P-Δ effect (whole structure); (b) P-δ effect (single column) (Narayanan and Beeby, 2005).

2.2. METHODS OF ANALYSIS

First order linear elastic analysis, where stresses and forces are calculated through the equilibrium of the elements in their non-deformed configuration, is the most usual form of structural analysis, as it is simple and for most typical cases, accurate. However, in some cases nonlinear analysis is required, since a first order linear analysis does not provide accurate results. Second order effects fit this category, and nonlinearity shall be considered in their analyses.

Nonlinear analyses shall consider both geometric and physical nonlinearity. Geometric nonlinearity deals with the equilibrium in the deformed state, whereas physical nonlinearity (nonlinearity associated with the materials) considers, beside the inherent nonlinear behaviour of the materials, the effects of cracking, creep and other material behaviour of the RC structure.

Although this kind of analysis can return better results, it is very complex and requires the use of an appropriate computer software. Another problem is that a nonlinear analysis needs the knowledge of the reinforcement to be used, which is not always previously known.

Due to these major inconveniences, other methods for evaluating second order effects have been developed. The following sub-items will detail two of those methods.

2.2.1. P-DELTA ANALYSIS

The P-Delta method is an approximated iterative method that allows for calculating global second order effects by creating fictitious horizontal forces (binaries) that cause an effect equivalent to the second order moments. Here, the nonlinear analysis is replaced by as many linear analyses as needed for convergence, considering a pre-defined tolerance.

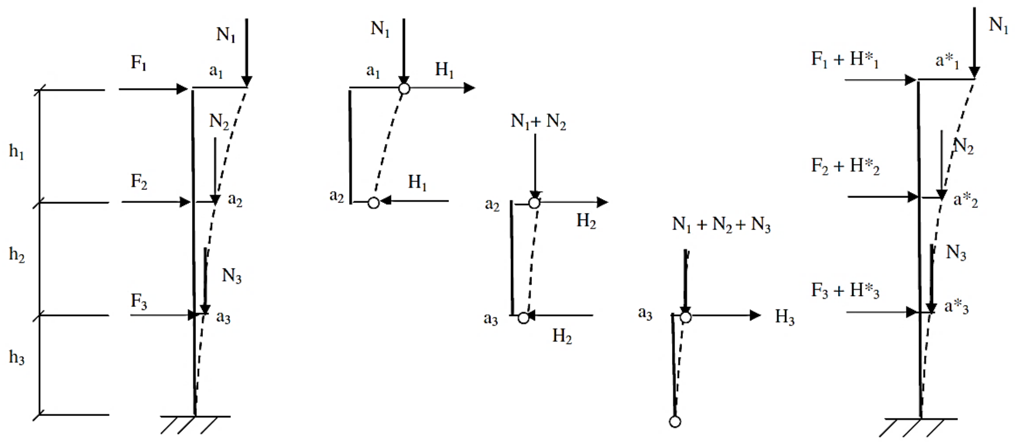

Figure 2-3 – P-Delta method for single columns (Longo, 2017).

The method will be first explained for a single column then generalized for framed structures. This sequence is followed:

1. Obtain the displacements in each pavement, a_i, by a first order linear analysis;
2. Calculate the relative displacements in each storey: $\Delta a_i = a_i - a_{i+1}$
3. Calculate the forces acting in each pavement: $H_i = N_i \frac{\Delta a_i}{h_i}$
4. In each section, fictitious horizontal forces (binaries) are defined as $H_i{}^* = H_i - H_{i-1}$. These forces shall be added to the original horizontal loads, F_i, making $F^* = F_i + H_i{}^*$.
5. Recalculate the displacements in each pavement, until the method converges.

For framed structures, the procedure is very similar and the horizontal forces will be:

$$H_i = \sum N_{j,i} \frac{\Delta a_i}{h_i} \Rightarrow H_i{}^* = H_i - H_{i-1} = \sum N_{j,i} \frac{\Delta a_i}{h_i} - \sum N_{j,i} - 1 \frac{\Delta a_{i-1}}{h_{i-1}} \qquad (1)$$

Where $\sum N_{j,i}$ is the summation of the vertical loads on level i and above.

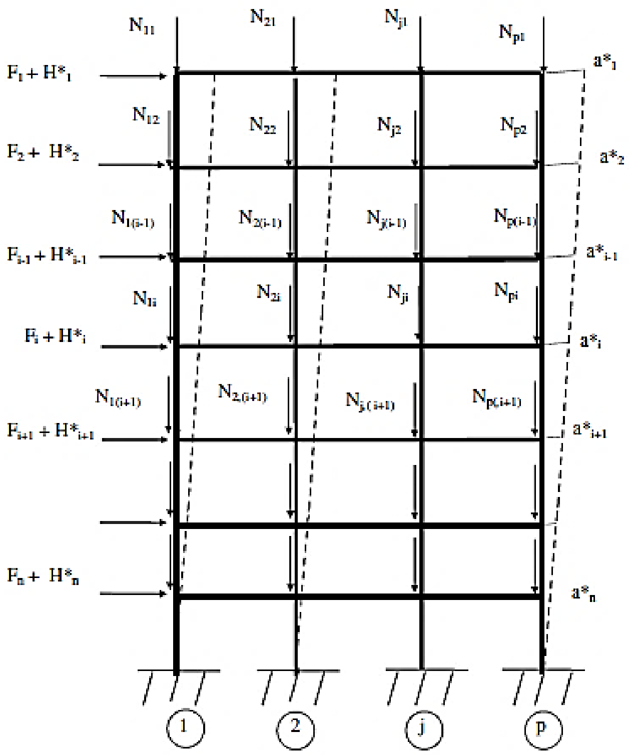

Figure 2-4 – P-Delta method for framed structures (Longo, 2017).

2.2.2. COMPUTATIONAL NONLINEAR ANALYSIS – PHYSICAL AND GEOMETRICAL

RC has a physically nonlinear behaviour since the relationship between loads and material deformations are not linear. This can be easily observed in the stress-strain design diagrams of concrete and reinforcing steel:

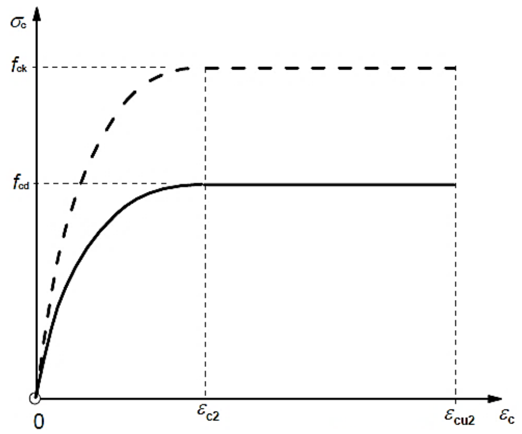

Figure 2-5 – Stress-strain diagrams for and concrete (Comité Européen de Normalisation, 2004).

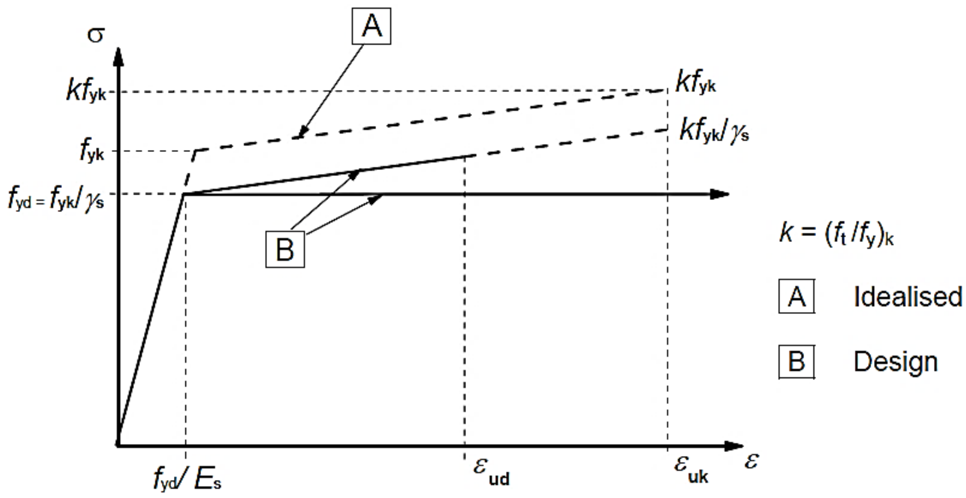

Figure 2-6 – Stress-strain diagrams for reinforcing steel
(Comité Européen de Normalisation, 2004).

As already said, to properly perform a nonlinear analysis, it is required to use an appropriated software and to have a previous knowledge of the RC section details. To take the physical nonlinearity into consideration, this software shall actualize the stiffness of the RC section in each iteration, as a function of the adjusted forces and the corresponding moment-curvature diagrams. So, the stiffness of the various RC sections will depend on the actual internal forces.

Because this consideration is very difficult to apply, some codes allow the reduction of the stiffness of the elements through approximate formulas.

2.3. BUCKLING, SLENDERNESS AND EFFECTIVE LENGTH

As already said, an element deflects when subjected to axial forces and bending moments, causing an increase of the bending moments due to the interaction between the lateral deformations and the axial forces.

The effect of pure buckling was first studied by Euler (1707-1783), whom showed that an ideal double pinned column of length l subjected to a concentrated axial force at its top has a critical buckling load equal to:

$$P_{cr} = \frac{\pi^2 EI}{l^2} \tag{2}$$

It is important to notice that the critical buckling load depends on the flexural stiffness of the column, instead of its material strength. So, to increase the buckling resistance it is needed to increase its flexural stiffness (moment of inertia).

Another important consideration is that pure buckling will not occur in real structures, since there are imperfections, eccentricities and transverse loads to be considered.

One final remark is that when $\lim_{l \to 0} P_{cr} = \infty$. This means that long columns have smaller critical buckling loads and the opposite happens to short columns. So, the notion of slenderness is necessary for understanding the concept of instability and second order effects.

The slenderness coefficient is given by the ratio of the effective length l_0 and the radius of gyration, i:

$$\lambda = \frac{l_0}{i} \tag{3}$$

$$i = \sqrt{\frac{I}{A}} \xrightarrow{\text{rectangular cross section}} i = \sqrt{\frac{\frac{bh^3}{12}}{bh}} = \frac{h}{\sqrt{12}} \tag{4}$$

Where I is the moment of inertia in the axis perpendicular to the buckling plan.

Effective length (or buckling length) is defined as the distance between the inflection points of the curvature of the deformed element, in an unstable condition. It depends on the support conditions of the column. As it will be seen, the Brazilian, European and American give different approximations of the effective length for framed structures.

The concept of the effective length of a column is very important, as it can generalize Euler's expression for different support conditions:

$$P_{cr} = \frac{\pi^2 EI}{{l_0}^2} \tag{5}$$

Note that the effective length may be different in the two directions, X and Y.

2.4. GEOMETRIC IMPERFECTIONS

Even though structures are ideally modelled as perfectly straight frames, when they are executed it is impossible to make them exactly as designed, as it is inevitable to have some deviation of its geometry and in the position of the acting loads.

For the ULS design, these imperfections must be considered, as they will lead to additional actions, affecting the stresses on the elements. The methods of calculation of these actions vary from code to code.

Geometric imperfections can be divided in global (entire structure) and local (isolated elements).

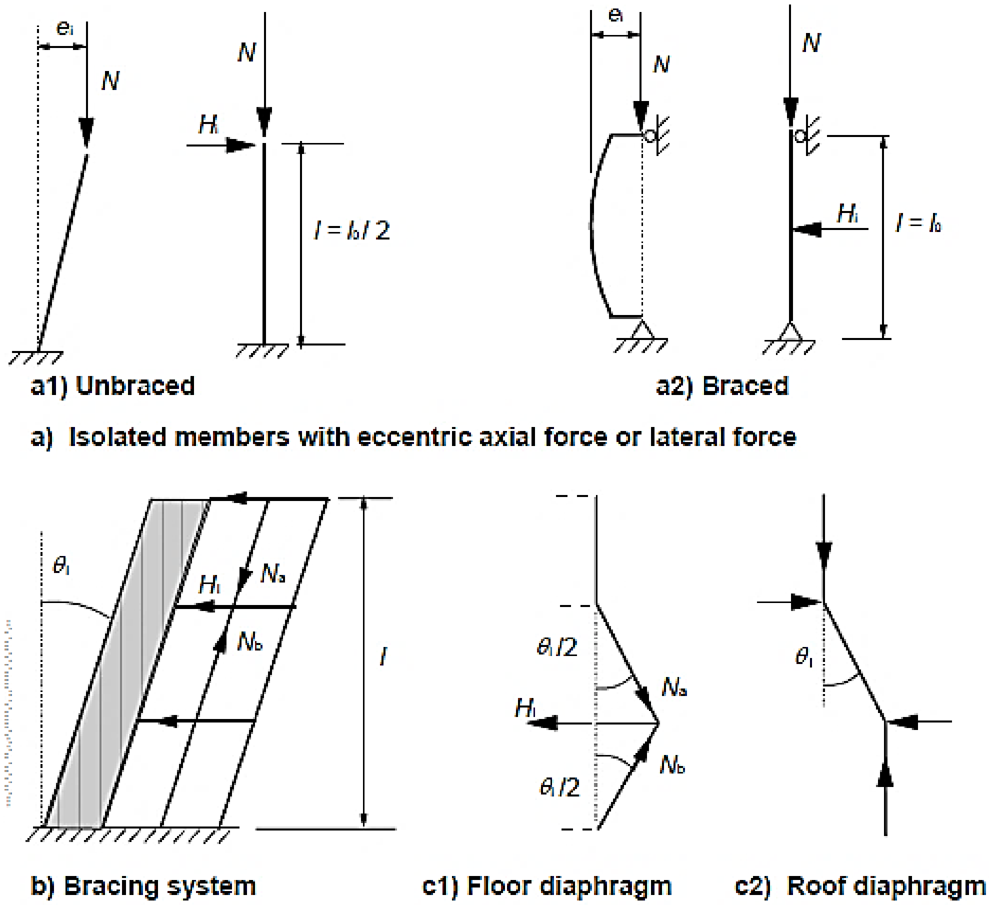

Figure 2-7 – Types of geometric imperfections (Comité Européen de Normalisation, 2004).

These effects will be detailed in the following chapters, according to each code. However, the effects of global imperfections are considered as not decisive for the design and therefore will not be analysed.

2.5. CREEP

When a RC element is subjected to a long-lasting constant load (permanent or quasi-permanent loads), its initial deformation is progressively increased in time, because of the creep effect. This increase of deformation will cause an increase of stresses and, consequently, a reduction of the element's capacity. Because creep is a consequence of the nonlinear behaviour of RC, its consideration is fundamental when evaluating second order effects.

Since the evaluation of the effects of creep is dependent on many factors, such as the composition of the cement, air humidity and age of concrete, structural regulations have developed simplified ways of calculating its influence.

2.6. GENERAL CODE APPROACH ON SECOND ORDER EFFECTS

In general, structural design codes follow a similar approach dealing with global and local second order effects (Narayanan and Beeby, 2005):

1. Classify the structure according to its stiffness against lateral deformations, as braced if it contains bracing elements (very rigid elements that can be assumed to resist to all the horizontal forces) or unbraced, if otherwise. For a local analysis, the boundary conditions of each element must be defined, to make possible to correctly evaluate its buckling length;
2. Classify the structure as sway or non-sway, according to its deflection mode. This step is only valid for global effects.
    - Sway structure – the columns are supposed to have significant deflections (sidesway of the whole structure) and global second order effects assume an important role;
    - Non-sway structure – horizontal displacements are small (there is no significative sidesway, only occurring the deflection of a single or some columns) and, generally, global second order effects can be neglected.
3. Check if the slenderness effects may be disregarded.
4. Design the structural elements, taking the slenderness effects into account if necessary.

3. BUILDING ANALYSIS

This work will use the structural model of a 12-storey fictitious RC building previously created by Gomes (2017), modelled using the software SAP2000 (Computers and Structures, 2016), defining bar elements for the beams and columns, and shell elements for the slabs.

The building in question is 36.0 meters high, 30.0 meters wide and 16.0 meters deep. The computational model and formwork drawing can be seen in Figure 3-1 and Figure 3-2 respectively.

To make the analysed structure as close as possible to a real building, the model was divided and analysed in three vertical regions with four pavements each, separated at 12, 24 and 36 meters height.

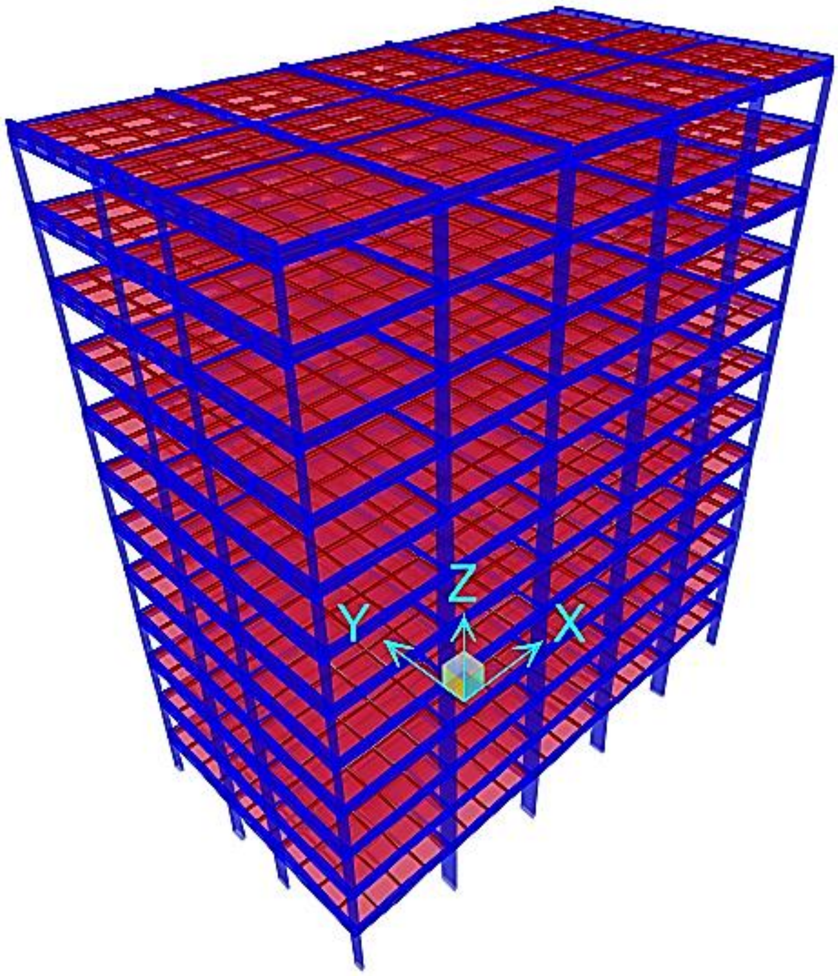

Figure 3-1 – Structural model of the building.

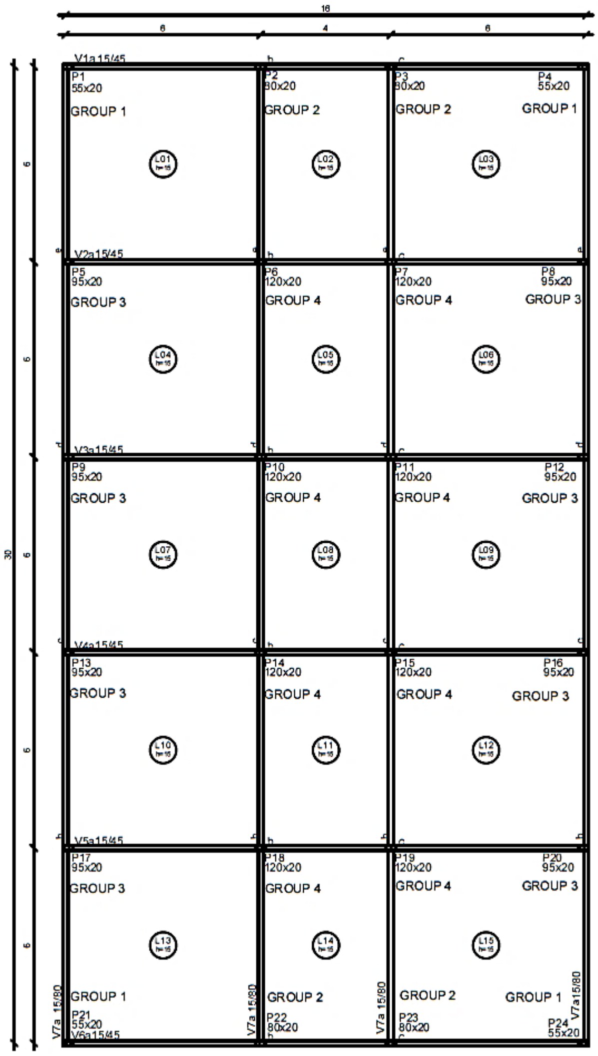

Figure 3-2 – Formwork drawing of the building, showing the different groups of columns.

A very important remark must be made about the structural design of the building. Although this structure may not seem reasonable for a European designer, as it does not contain rigid cores or walls (i.e., bracing elements), this structural design is quite common in Brazil, and will help to accomplish the objective of this work, which is to test the approach on second order effects according to different international codes.

Following the guidelines previously described in second chapter, the first thing to do is classifying the structure. As this building does not have bracing elements, the structure is classified as sway and unbraced, which means that its design should begin in considering the sidesway of the entire structure (global effects), and only after that, in the design of the individual columns for resisting to the local effects.

3.1. MATERIAL DEFINITION AND ASSIGNMENT

The RC used in the model above has the properties defined in Table 3-1. These properties are defined according to Brazilian Standard NBR 6118 (2014).

Table 3-1 – Properties of the RC used.

Concrete	Compressive strength	$f_{ck} = 35\ MPa$
	Initial tangent modulus of elasticity	$E_{ci} = 1.0 \times 5600 \times \sqrt{35} = 33\ 130\ MPa$
	Coefficient α_i	$\alpha_i = \left(0.8 + 0.2 \times \frac{35}{80}\right) = 0.888$
	Secant modulus of elasticity	$E_{cs} = 0.888 \times 33130 = 29\ 403\ MPa$
	Secant modulus of elasticity for global stability analysis	$E_{cs}{}^{*} = 29403 \times 1.10 = 32\ 343\ MPa$
	Coefficient of thermal dilatation	$\alpha = 1 \times 10^{-5}{}^{\circ}\mathrm{C}^{-1}$
	Poisson's ratio	$\upsilon = 0.2$
	Specific weight of columns and beams	$\gamma = 25\ kN/m^3$
	Specific weight of slabs (with a correction for floor finishing)	$\gamma_{slabs} = 38.33\ kN/m^3$
Reinforcement Steel	Minimum yield stress	$f_y = 500\ MPa$
	Minimum tensile stress	$f_u = 550\ MPa$
	Secant modulus of elasticity	$E_s = 200\ 000\ MPa$
	Specific weight	$\gamma_{steel} = 76.97\ kN/m^3$

Note that the specific weight of the concrete of the slabs was majored by $2\ kN/m^2$ so the mass of the masonry and cladding can be considered ($1\ kN/m^2$each):

$$\gamma_{slabs} = \frac{\gamma \times h_{slab} + 1 + 1}{h_{slab}} = \frac{25 \times 0.15 + 2}{0.15} = 38.33\ kN/m^3 \qquad (6)$$

3.2. SECTION PROPERTIES – DEFINITION AND ASSIGNMENT

The design of the structural elements followed the considerations below.

- The beams widths were set as $b_{beams} = 15\ cm$, changing its height according to their span;
- The height of the columns was set as $h_{columns} = 20\ cm$ and their widths were defined according to the estimated area of influence. To simplify the design, the columns were categorized in four groups in each reference level, grouping the ones

that present similar levels of stress, resulting in 12 types of columns. Each group is represented in Figure 3-2, while Table 3-2 explains this grouping. The reinforcement of the columns is distributed equally along the longer faces, assuming $d' = 4.0\ cm$;

- The slabs were defined with the height of $h_{slabs} = 15\ cm$, applying the same loading per area with a majored specific weight, as shown above.

Table 3-2 – Division of columns by groups.

Group 1	**Group 2**	**Group 3**	**Group 4**
P1 / P4 P21 / P24	P2 / P3 / P22 / P23	P5 / P8 / P9 / P12 / P13 / P16 / P17 / P20	P6 / P7 / P10 / P11 / P14 / P15 / P18 / P19
Corner columns	Lateral Columns (16m side)	Lateral columns (30m side)	Central columns
Low axial force	Low axial force	Low axial force	High axial force
$M_X \cong M_Y$	$M_X > M_Y$	$M_Y > M_X$	$M_Y > M_X$

All these calculations can be found in the work of Gomes (2017), and the dimensions of the cross sections are detailed in Table 3-3. The number below the group indicates the number of columns of that group.

Table 3-3 – Dimensions of the cross sections of various structural elements.

Element	**Reference Level**	**Width, b (m)**	**Height, h (m)**
Columns Group 1 (4)	0.0	0.55	0.20
	12.0	0.40	0.20
	24.0	0.30	0.20
Columns Group 2 (4)	0.0	0.80	0.20
	12.0	0.50	0.20
	24.0	0.30	0.20
Columns Group 3 (8)	0.0	0.95	0.20
	12.0	0.80	0.20
	24.0	0.55	0.20
Columns Group 4 (8)	0.0	1.20	0.20
	12.0	0.80	0.20
	24.0	0.40	0.20
Longitudinal Beams		0.15	0.45
Transversal Beams		0.15	0.80

It is important to notice that although the building in analysis will be the same one for every code, the definition of some properties such as the elasticity modulus will differ from code to code. So, the model will suffer slight changes in each analysis, to take these different properties into account.

According to SAP2000, the orientation of the section's local axis is given by:

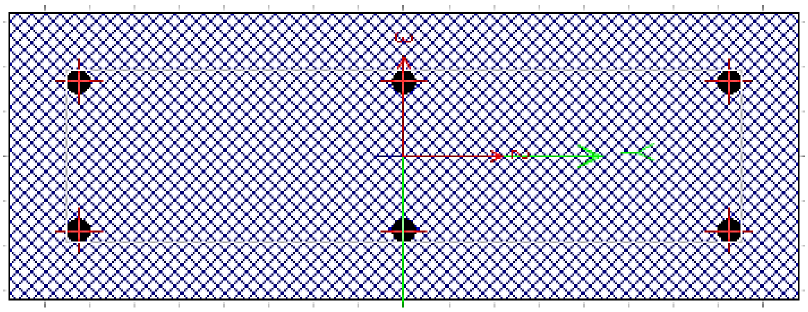

Figure 3-3 – General orientation of a column's cross section.

However, for this work, the bending moments in the direction 2 (horizontal axis in the figure above) will be called M_X and direction 3 (vertical axis in the figure above) will become M_Y. It is easy to see that the "weak" axis of inertia is given by the X direction.

3.3. DEFINITION OF THE ACTING LOADS

Although the definition of the wind action and partial safety coefficients used in the load combinations defined below vary according to the code in use (Brazilian, European or American standards), the acting forces must be the same to make a comparison possible.

The loads acting on the building are:

- Permanent loading, referring to the self-weight of the structure, masonry and cladding, that are considered directly through the weight of the concrete, as defined before;
- Accidental loading according to the NBR 6120 (1980): $q = 2.00\ kN/m^2$;
- Wind action, as previously calculated by Gomes (2017), according to NBR 6123 (1990), considering the wind in the city of Rio de Janeiro, with a basic velocity of $V_0 = 35\ m/s$. This action is resumed in Table 3-4:

Table 3-4 – Wind pressure and force acting in the structure.

Pavement	Velocity $[m/s]$	Dynamic Pressure $[N/m^2]$	Effective Pressure $[N/m^2]$	
			Windward	Leeward
1 – 4	$V_{k.12} = 25.90$	$q_{12} = 411.21$	$\Delta p_{A.12} = 411.21$	$\Delta p_{B.12} = -164.48$
5 – 8	$V_{k.24} = 28.70$	$q_{24} = 504.92$	$\Delta p_{A.24} = 504.92$	$\Delta p_{B.24} = -201.97$
9 – 12	$V_{k.36} = 30.80$	$q_{36} = 581.52$	$\Delta p_{A.36} = 581.52$	$\Delta p_{B.36} = -232.61$

Pavement	Distributed force $[N/m]$	
	Windward	Leeward
Lobby	$q_{A,L} = 616.82$	$q_{B,L} = -246.72$
1 – 4	$q_{A.12} = 1\,233.63$	$q_{B.12} = -493.44$
5 – 8	$q_{A.24} = 1\,514.76$	$q_{B.24} = -605.91$
9 – 11	$q_{A.36} = 1\,744.56$	$q_{B.36} = -697.83$
12	$q_{A.36} = 872.28$	$q_{B.36} = -348.92$

The forces above are shown in Figure 3-4, as applied in the model, in kN/m.

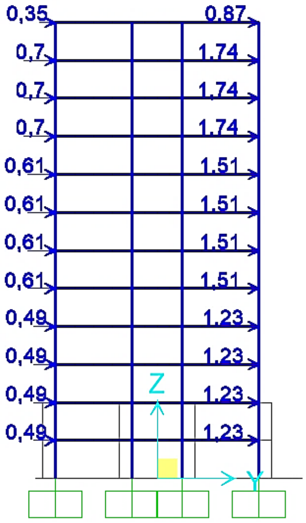

Figure 3-4 – Distributed forces due to the wind acting in the structure, in kN/m.

As this is an academic work, the wind forces are only going to be considered in the Y direction, as shown above. If this was a real project, the force in the other direction should be verified.

3.4. DEFINITION OF THE LOAD COMBINATIONS

Eurocode 0 (Comité Européen de Normalisation, 2002) states that a structure must be designed to have adequate structural resistance, serviceability and durability. This means that the structure must fulfil the requirements of both ultimate and serviceability limit states.

It also stablishes that "for each critical load case, the design values of the effects of actions shall be determined by combining the values of actions that are considered to occur simultaneously, and each combination of actions should include a leading variable action and an accidental action". The acting combinations of actions for both limit states will be the ones defined in the Brazilian Standard NBR 6118 (2014) disregarding the effects of indirect actions such as temperature and shrinking.

As the main scope of this work is to analyse the effects of the wind forces in the structure, the combination in which the wind is the main action will be the only one to be analysed, dismissing the combination that defines accidental load as the main load. However, in a real building design, all the relevant load combinations should be accordingly considered.

3.4.1. COMBINATION FOR THE ULTIMATE LIMIT STATE

As said, the ultimate limit state (ULS) is the one that concerns to the safety of people, and/or the safety of the structure. For RC buildings, this means the exhaustion of the resistance of the structural elements.

The usual combination of actions for the ULS is given by the following expression of NBR 6118 (2014):

$$F_{d,ULS} = \sum_{i=1}^{m} \gamma_{gi} \cdot F_{Gi,k} + \gamma_q \cdot \left[F_{Q1,k} + \sum_{j=2}^{n} \psi_{0j} \cdot F_{Qj,k} \right] \tag{7}$$

Considering the respective partial factors for the actions:

$$F_{d,ULS} = 1.4 \cdot g + 1.4 \cdot (q_w + 0.7 \cdot q_1) \tag{8}$$

Where:

- g – permanent load due to self-weight, masonry and cladding;
- q_w – variable wind load;
- q_1 – variable accidental load (considering the building occupation as office).

3.4.2. COMBINATION FOR THE SERVICEABILITY LIMIT STATE

Serviceability limit state (SLS) is the one that concerns the behaviour of the structure and individual structural members under normal use, the comfort of people and the appearance of the construction. This means, deformations, damages or even vibrations that are likely to adversely affect the appearance, durability, comfort of users or the functional effectiveness of the structure shall be avoided.

For the effects of wind as predominant variable load, the combination in analysis must be the frequent combination, corresponding to actions that are repeated several times during the lifespan of the structure. The frequent combination of actions for the SLS are given by the NBR 6118 (2014) as follows:

$$F_{d,SLS} = \sum_{i=1}^{m} F_{Gi,k} + \psi_1 \cdot F_{Q1,k} + \sum_{j=2}^{n} \psi_{2j} \cdot F_{Qj,k} \qquad (9)$$

Again, adopting the respective partial factors for actions:

$$F_{d,SLS} = g + 0.3 \cdot q_w + 0.3 \cdot q_1 \qquad (10)$$

Where the nomenclature is the same defined in 3.4.1.

4. ANALYSIS ACCORDING TO NBR 6118:2014

4.1. INITIAL CONSIDERATIONS

In this chapter, the previously defined structural model will be analysed according to the Brazilian standard for concrete structures, NBR 6118 (2014) (Associação Brasileira de Normas Técnicas, 2014). The analysis encompasses the global second order effects and also the local ones.

Following the methodology previously described, the first step is evaluating whether the global second order effects can or cannot be dismissed, which is done through the item 15.5 of the code mentioned above.

4.2. GLOBAL SECOND ORDER EFFECTS

4.2.1. VERIFICATION OF THE ULS – ANALYSIS THROUGH THE COEFFICIENT γ_z

A normative criterion of NBR 6118 uses the coefficient γ_z for classifying the structure as sway or non-sway, evaluating in this way the global stability of the building.

The coefficient γ_z is determined from a first order linear analysis, as defined in item 15.5 of NBR 6118. Approximate stiffness reduction factors are to be considered according to item 15.7.3 of the same standard, for taking into account the physical nonlinearity:

- Slabs: $(EI)_{sec} = 0.3\, E_c I_c$;
- Beams: $(EI)_{sec} = 0.4\, E_c I_c$ (in the general case where $A'_s \neq A_s$);
- Columns: $(EI)_{sec} = 0.8\, E_c I_c$.

Also, the elasticity modulus used in this analysis is increased in 10%, as it is shown in the material definition in Table 3-1 of this work: $E_{cs}{}^{*} = 32\,343\ MPa$

To take these reduction factors into consideration another model has been created, with the same properties as described in the previous item, but reducing the stiffness of the structural elements and increasing the elasticity modulus, as defined above.

The coefficient γ_z is calculated from the following expression:

$$\gamma_z = \frac{1}{1 - \frac{\Delta M_{tot,d}}{M_{1,tot,d}}} \tag{11}$$

Where:

- $M_{1,tot,d}$ is the first order toppling moment, the sum of the products of the horizontal forces applied to a storey and the height of that storey, with respect to the base of the structure;
- $\Delta M_{tot,d}$ is the increase of moments with respect to the first order analysis, given by the sum of the products of all the vertical design forces acting on the structure by the horizontal displacements of their respective points of application.

The structure is classified as non-sway if $\gamma_z \leq 1.1$. However, if $1.1 \leq \gamma_z \leq 1.3$, the code allows for an approximate consideration of the global second order effects, through the multiplication of the horizontal loads by the factor $0.95\gamma_z$.

Running a SAP2000 analysis, it is possible to obtain the forces and displacements in the model, for the ULS combination. The main results are shown in the following tables.

Table 4-1 – Model's structure output: Base reaction forces, in kN.

OutputCase	Case Type	Global FX	Global FY	Global FZ
ULS	Combination	0.00	-1 005	70 010
P-DELTA	NonStatic	0.00	-1 005	70 010

Table 4-2 – Model's structure output: Base reaction bending moments, in kN·m.

Output Case	Case Type	Global MX	Global MY	Global MZ
ULS	Combination	20 202	0.00	0.00
P-DELTA	NonStatic	21 128	0.00	0.00

It is important to notice that the P-Delta effect does not change the resultant of forces, only moments.

Analysing the displacements in a random column, it is possible to see that the second order effects cause an average displacement amplification of approximately 1.05:

Table 4-3 – Joint displacements in the Y direction, in various pavements, in meters.

Pavement	**Output Case**	**Case Type**	**Step Type**	δ
1	P-Delta	NonStatic	Max	0.001107
	ULS	Combination		0.001066
2	P-Delta	NonStatic	Max	0.003204
	ULS	Combination		0.003068
3	P-Delta	NonStatic	Max	0.005602
	ULS	Combination		0.005349
4	P-Delta	NonStatic	Max	0.007978
	ULS	Combination		0.007606
5	P-Delta	NonStatic	Max	0.010663
	ULS	Combination		0.010153
6	P-Delta	NonStatic	Max	0.013074
	ULS	Combination		0.012446
7	P-Delta	NonStatic	Max	0.015204
	ULS	Combination		0.014480
8	P-Delta	NonStatic	Max	0.017063
	ULS	Combination		0.016258
9	P-Delta	NonStatic	Max	0.019640
	ULS	Combination		0.018706
10	P-Delta	NonStatic	Max	0.021636
	ULS	Combination		0.020611
11	P-Delta	NonStatic	Max	0.022908
	ULS	Combination		0.021837
12	P-Delta	NonStatic	Max	0.023521
	ULS	Combination		0.022430

It is now possible to calculate γ_z:

$$M_{1,tot,d} = \sum_{i=1}^{12} F_{pavement_i} \times h_{pavement_i} = 14\,430\ kN \cdot m \tag{12}$$

$$\Delta M_{tot,d} = \sum_{i=1}^{12} \delta_{pavement_i} \times W_{pavement_i} = 673.4\ kN \cdot m \tag{13}$$

The weight of each floor can be found by dividing the total base reaction by the number of storeys, and dividing again by 1.4 to consider the safety factor.

$$\gamma_z = \frac{1}{1 - \frac{673.4}{14\,430}} = 1.049 \tag{14}$$

As $\gamma_z = 1.049 < 1.1$, the structure is non-sway and there is no need to check global second order effects.

4.2.2. VERIFICATION OF THE ULS – ANALYSIS THROUGH THE PARAMETER α

NBR 6118 (2014) gives another parameter to evaluate the necessity to consider global second order effects. The instability parameter α classifies the structure as non-sway if a parameter α is smaller than a reference value α_1:

$$\alpha = H_{tot}\sqrt{\frac{\sum N_k}{\sum E_c I_c}} \leq \alpha_1 = \begin{cases} 0.2 + 0.1n & if\ n \leq 3 \\ 0.60 & if\ n \geq 4 \end{cases} \tag{15}$$

Where:

- n is the number of storeys;
- H_{tot} is the total height of the structure;
- $\sum N_k$ is the characteristic value of the vertical acting loads;
- $\sum E_c I_c$ is the stiffness of a column representing all the horizontal stiffness of the building.

Although this parameter will not be used herein, it is important to register that this verification is associated with a limitation of the total vertical load in the building with respect to a fraction of its nominal buckling load (see Appendix A).

4.2.3. VERIFICATION OF THE SLS – COMPARATIVE DISPLACEMENT ANALYSIS

To verify the SLS according to the Brazilian standard, it is necessary to check the displacements of the building, considering only first order effects, using its full stiffness and with adding 10% to the modulus of elasticity.

The checking is done considering the two sets of requirements defined by the standard:

$$\frac{\delta}{H} \leq \frac{1}{1700} \cong 0.000588 \quad (16)$$

$$\frac{\delta_i - \delta_{i-1}}{h} \leq \frac{1}{850} \cong 0.0118 \quad (17)$$

Where:

- δ is the maximum storey displacement;
- δ_i is the displacement of a given storey i;
- H is the building height, 36 meters;
- h is the difference of heights between consecutive storeys, 3 meters.

Since the model considers the pavements as rigid diaphragms, the slabs have the same displacements in the XY plan, in relation to the Z axis. Thus, it is possible to choose any column to perform this analysis. The analysis of the horizontal displacements in the Y direction (axis 2) is shown in Table 4-4:

Table 4-4 – Displacements of the various pavements, both absolute and relative, in meters.

Pavement	**OutputCase**	**CaseType**	δ	$\frac{\delta_i - \delta_{i-1}}{h}$
1	SLS	Combination	0.000142	0.00005
2			0.000373	0.00008
3			0.000626	0.00008
4			0.000865	0.00008
5			0.001168	0.00010
6			0.001438	0.00009
7			0.001679	0.00008
8			0.001888	0.00007
9			0.00223	0.00011
10			0.002497	0.00009
11			0.002675	0.00006
12			0.002761	0.00003

As it is shown in Table 4-4, the requirements for the SLS according to Equation 16 and Equation 17 are fulfilled.

4.3. LOCAL SECOND ORDER EFFECTS

As long as the global second order effects were considered, it is still necessary to verify the local effects, for all the columns. All the steps of the analysis will be exposed, but explicit calculations will be shown only for one column, presented in Appendix B.

The Brazilian standard has some restrictions about the slenderness of the elements. It states that columns have a limit slenderness of 200, except in some cases where the normal force is very low.

Depending on the slenderness of the elements it is possible to use some simplified methods, as shown in the following. However, when the slenderness is higher than 140, a general ("exact") method is required.

NBR 6118 (2014) allows for a reduction in the length of the column regarding the theoretic centre-to-centre of slabs value. The effective length of the columns l_e is defined as a function of the free length l_{free}, distance between faces of beams ($l_{free} = l_{column} - h_{beam}$), height of beams h_{beam} and height of the column itself h_{column}, considering these dimensions in the same plane in which the second order displacements take place:

$$l_e = \min\{l_{free} + h_{column}; l_{free} + h_{beam}\} \quad (18)$$

For all the local analysis, the forces in the columns were obtained from SAP2000 considering the combination of actions already detailed, but also considering the effects of geometric nonlinearity, i.e., considering the global P-Delta effects.

It is important to notice that the approximated methods given in any of the four codes herein analysed were calibrated for isolated elements and do not consider the fact that in framed structures, the eccentricities of adjacent elements must be the same.

4.3.1. LOCAL GEOMETRIC IMPERFECTIONS

In framed structures, the effects from local geometric imperfections can be considered through a minimum first order moment acting in the columns:

$$\begin{cases} M_{1d,min,X} = N_d \cdot e_{a,X} = N_d \cdot (0.015 + 0.03h) \\ M_{1d,min,Y} = N_d \cdot e_{a,Y} = N_d \cdot (0.015 + 0.03b) \end{cases} \quad (19)$$

4.3.2. SLENDERNESS CRITERION FOR ISOLATED COMPRESSED MEMBERS

According to the NBR 6118 (2014), local second order effects can be neglected if the following condition is met:

$$\lambda = \frac{l_e}{i} \leq \lambda_1 = \frac{25 + 12.5 \cdot \frac{e_1}{h}}{\alpha_b} \tag{20}$$

Where $35 \leq \lambda_1 \leq 90$ and:

- λ is the element slenderness, previously described;
- e_1 is the first order eccentricity, $e_1 = \left|\frac{M_A}{N}\right|$;
- α_b is a coefficient that varies according to the moments in the boundary;

$$0.40 \leq \alpha_b = 0.60 + 0.40\frac{M_B}{M_A} \leq 1 \tag{21}$$

The values of M_A, M_B are taken from SAP2000 analysis, being $\lfloor M_A \rfloor \geq \lfloor M_B \rfloor$.

4.3.3. EFFECTS OF CREEP

According to the Brazilian standard, it is only required to consider the effects of creep for local second order effects when $\lambda > 90$. In this way, these effects can be neglected for the columns analysed in this work.

4.3.4. METHOD OF THE STANDARD-COLUMN WITH APPROXIMATED STIFFNESS

This method can be only used for columns with $\lambda \leq 90$, rectangular constant section and symmetrically constant reinforcing. The geometric nonlinearity is considered by assuming a sinusoidal deformation (standard-column approximation) and the physical nonlinearity is taken into account by considering approximate values for the effective stiffness of the columns.

The design moment is calculated through a multiplier of the first order moment:

$$M_{d,tot} = \frac{\alpha_b \cdot M_{1\mathrm{d,A}}}{1 - \frac{\lambda^2}{120\frac{\kappa}{\nu}}} \geq M_{1\mathrm{d,A}} \tag{22}$$

Where:

- ν is the normal dimensionless force, $\nu = \frac{|N_d|}{A_c \cdot f_{cd}}$;
- κ is the dimensionless stiffness, $\kappa_{approx} = 32 \cdot \nu \cdot \left(1 + 5\frac{M_{Sd,tot}}{h \cdot N_d}\right)$.

It is possible to obtain directly the total design moment by finding out the root of the following second-degree equation:

$$AM_{d,tot}{}^2 + BM_{d,tot} + C = 0, \text{where} \begin{cases} A = 5 \cdot h \\ B = N_d \cdot h^2 - \dfrac{N_d \cdot l_e^2}{320} - 5 \cdot h \cdot \alpha_b \cdot M_{1\mathrm{d,A}} \\ C = -N_d \cdot h^2 \cdot \alpha_b \cdot M_{1\mathrm{d,A}} \end{cases} \quad (23)$$

4.3.5. METHOD OF THE STANDARD-COLUMN WITH APPROXIMATED CURVATURE

Like in the previous method, this method can be only used for columns with $\lambda \leq 90$, rectangular constant section and symmetrically constant reinforcement. The geometric nonlinearity is also considered by assuming a sinusoidal deformation and the physical nonlinearity is taken into account by an approximated value assumed for the curvature in the critical section. All the necessary parameters have been defined already:

$$M_{d,tot} = \alpha_b \cdot M_{1\mathrm{d,A}} + N_d \cdot \frac{l_e{}^2}{10}\frac{1}{r} \geq M_{1\mathrm{d,A}} \quad (24)$$

$$\frac{1}{r} = \frac{0.005}{h \cdot (\nu + 0.5)} \leq \frac{0.005}{h} \quad (25)$$

4.3.6. METHOD OF THE STANDARD-COLUMN ASSOCIATED TO THE M, N AND CURVATURE DIAGRAMS

This method is an improvement of the two previous methods. It can be used in columns up to $\lambda \leq 140$, using for the curvature of the critical section the values obtained in the M, N and 1/r diagrams, for each case.

4.3.7. GENERAL METHOD

This is the “exact” method of analysis, mandatory for $\lambda > 140$. It considers the RC’s physical and geometric nonlinear behaviour by considering the actual moment-curvature relations in the sections of a discretized column, in the way that has been previously described in item 2.2.2.

If the case is that $\lambda > 140$, in addition to considering the effects of creep, it is also required to take into consideration an additional multiplier for the forces:

$$\gamma_{n1} = 1 + \left[\frac{0.01 \cdot (\lambda - 140)}{1.4}\right] \quad (26)$$

4.3.8. DISCUSSION OF RESULTS

This work will compare the bending moments for 12 columns: one column from each group, in each reference level. For this, three tables will be shown: the first two comparing the initial P-Delta analysis with the two approximated methods, while the third compares the two approximated methods with one another. The explicit calculations for column G1-00 are found in Appendix B, and the process is repeated for the other columns.

Note that the results from each method is given by the maximum value between the bending moment due to the wind forces and the amplified minimum bending moment due to geometric imperfections, since second order effects are not to be considered for the actual moments acting in the columns, as the eccentricity due to the local geometric imperfections is not added to the first order eccentricity (see also Appendix B).

The deviation in the following tables are given in percentage, according to the expression in the top of the column.

Table 4-5 – Comparison between the P-Delta analysis and the method of standard-column with approximated stiffness. Moments given in kN·m.

Column	P-Delta		SC – Stiffness		$\frac{SC_S - P\Delta}{P\Delta}$ (%)
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	23.3	44.1	44.0	46.9	6.33%
G1-12	27.9	50.4	27.9	50.4	0.00%
G1-24	26.3	36.9	26.3	36.9	0.00%
G2-00	39.3	76.1	60.3	91.1	19.69%
G2-12	44.0	24.9	44.0	46.3	85.65%
G2-24	37.1	7.5	37.1	19.4	161.01%
G3-00	4.5	152.1	81.0	152.1	0.00%
G3-12	5.2	130.6	53.8	130.6	0.00%
G3-24	5.2	117.9	26.4	117.9	0.00%
G4-00	6.0	221.1	119.0	230.4	4.24%
G4-12	6.1	39.9	76.9	116.2	191.10%
G4-24	5.1	13.6	37.2	41.5	204.66%
Average					56.06%

Table 4-6 – Comparison between the P-Delta analysis and the method of standard-column with approximated curvature. Moments given in kN·m.

Column	P-Delta		SC – Curvature		$\frac{SC_C - P\Delta}{P\Delta}$ (%)
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	23.3	44.1	55.6	53.7	21.57%
G1-12	27.9	50.4	35.7	50.4	0.00%
G1-24	26.3	36.9	26.3	36.9	0.00%
G2-00	39.3	76.1	87.4	99.8	31.15%
G2-12	44.0	24.9	55.5	52.9	111.93%
G2-24	37.1	7.5	37.1	23.8	219.01%
G3-00	4.5	152.1	113.9	152.1	0.00%
G3-12	5.2	130.6	80.1	130.6	0.00%
G3-24	5.2	117.9	39.4	117.9	0.00%
G4-00	6.0	221.1	161.1	240.2	8.64%
G4-12	6.1	39.9	105.0	125.4	214.13%
G4-24	5.1	13.6	51.2	47.4	247.48%
Average					71.16%

Table 4-7 – Comparison of the Brazilian methods. Moments given in kN·m.

Column	SC – Stiffness		SC – Curvature		$\frac{SC_C - SC_S}{SC_C}$ (%)	
	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	44.0	46.9	55.6	53.7	20.99%	12.54%
G1-12	27.9	50.4	35.7	50.4	21.71%	0.00%
G1-24	26.3	36.9	26.3	36.9	0.00%	0.00%
G2-00	60.3	91.1	87.4	99.8	30.99%	8.74%
G2-12	44.0	46.3	55.5	52.9	20.64%	12.40%
G2-24	37.1	19.4	37.1	23.8	0.00%	18.18%
G3-00	81.0	152.1	113.9	152.1	28.87%	0.00%
G3-12	53.8	130.6	80.1	130.6	32.86%	0.00%
G3-24	26.4	117.9	39.4	117.9	32.95%	0.00%
G4-00	119.0	230.4	161.1	240.2	26.15%	4.05%
G4-12	76.9	116.2	105.0	125.4	26.71%	7.33%
G4-24	37.2	41.5	51.2	47.4	27.33%	12.32%
Average					22.43%	6.30%

Although the bending moments were calculated for both directions, the relative deviations were not shown for the X direction, as those bending moments were not conditioning for the column design and therefore did not represent the analysis. If the bending moments in the X direction were to be analysed, the wind would have to be applied in the perpendicular direction.

If this relative deviation were analysed, it would be huge, particularly in the central columns (columns from Groups 3 and 4), because they present very low values of bending moments in the initial P-Delta analysis, allied with the fact that the geometric imperfections were not taken into account in the structural model, which means that the second order effects would not have significant expression. On the other hand, this effect is less significative in the extremity columns (columns from Groups 1 and 2) as the eccentricity due to gravitational loads will trigger a bending moment, that is amplified by the second order effects, which will lead to smaller differences when compared to the approximated methods.

Also, it is possible to state that for this case the method of the standard-column with approximate curvature is the most penalising method, producing bending moments 6% to 22% higher than the ones obtained by the stiffness based method.

Note that when there is no deviation between the two methods it means that the conditioning bending moment is the one due to the wind forces, which is the same for both cases, as it was not amplified.

With the final bending moments defined it is possible to design the column reinforcement. This will be done by an Excel spreadsheet (Santos, 2016), that automatically gives the interaction diagrams of the RC section. The design will be done for the most unfavourable case – Method of the standard-column with approximated curvature. The chosen reinforcement for each column-type is given below.

It is important to notice that the Brazilian standard only considers the effects of biaxial bending for the bending moments due to applied loads, disregarding these effects for the minimum moments due to geometric imperfections.

Table 4-8 – Chosen reinforcement for the Nominal Curvature method from NBR 6118. Bending moments in kN·m and area of reinforcement in cm^2.

COLUMN	NBR 6118 (2014)							
	N	M_X	M_Y	Rebar	A_S	ρ	ω	BB
G1-00	1401	55.6	53.7	4Φ12.5	4.91	0.45%	0.08	0.49
G1-12	894	35.7	50.4	4Φ12.5	4.91	0.61%	0.11	0.91
G1-24	423	26.3	36.9	6Φ12.5	7.36	1.23%	0.23	1.00
G2-00	2254	87.4	99.8	10Φ12.5	12.27	0.77%	0.14	0.51
G2-12	1437	55.5	52.9	8Φ12.5	9.82	0.98%	0.18	0.81
G2-24	691	37.1	23.8	6Φ12.5	7.36	1.23%	0.23	0.89
G3-00	3029	113.9	152.1	12Φ16	24.13	1.27%	0.24	0.21
G3-12	2011	80.1	130.6	4Φ16	8.04	0.50%	0.09	0.27
G3-24	987	39.4	117.9	4Φ16	8.04	0.73%	0.14	0.49
G4-00	4446	161.1	240.2	14Φ25	68.72	2.86%	0.53	0.23
G4-12	2875	105.0	125.4	6Φ25	29.45	1.84%	0.34	0.09
G4-24	1390	51.2	47.4	4Φ25	19.63	2.45%	0.46	0.10

From the table above, it can be seen that for most cases the biaxial bending was not conditioning for the reinforcement design, as the values of BB are not close to 1. This happens because the biaxial bending only needs to be verified for the bending moments due to the applied loads, which are not the conditioning moments, for this case.

5. ANALYSIS ACCORDING TO THE EN 1992-1-1, EUROCODE 2

5.1. INITIAL CONSIDERATIONS

As it was said before, since Eurocode 2 defines some properties differently from the NBR 6118 (2014), the structural model described before must suffer some changes to properly represent the Eurocode's approach on second order effects. These changes will be detailed in this subchapter.

5.1.1. MATERIAL PROPERTIES

According to the Eurocode 2, the modulus of elasticity is given by:

$$E_{c,EC2} = 22\left(\frac{f_{ck}+8}{10}\right)^{0.30} = 22 \times \left(\frac{35+8}{10}\right)^{0.30} \cong 34.08\ GPa \qquad (27)$$

However, for this kind of analysis, Eurocode 2 states that the design modulus of elasticity should be used:

$$E_{cd} = \frac{E_{cm}}{\gamma_{ce}} = \frac{34.08}{1.2} = 28.4\ GPa \qquad (28)$$

As far as the structural design is concerned, two other factors will be different:

- The partial safety coefficient of the concrete is $\gamma_{concrete,EC2} = 1.50$, instead of 1.40 of the NBR 6118 (2014);
- The Rüsch effect coefficient will take the value of $\alpha_{EC2} = 1.00$, instead of the 0.85 from the NBR 6118 (2014).

However, neither of these two changes will affect the computational model, only the reinforcement design. So, the structural model for this analysis will have the same properties as described before, apart from the elasticity modulus, that will assume the value above.

5.1.2. STRUCTURAL PROPERTIES

As stated, the building in analysis does not have bracing members such as walls or cores. For this reason, the calculation of the bracing members stiffness, $\sum E_{cd} I_c$ can be done by two different ways, that will be compared later:

- By the summation of the stiffness of every column. The inertia will be given by the summation of the inertia of the individual columns, considering an average section width, multiplied by the number of columns with those properties:

$$I_{bracing} = \sum n_{group} \frac{0.20 \times {b_{average}}^3}{12} = 0.1433\ m^4 \Rightarrow$$
$$\Rightarrow \sum E_{cd} I_c = 28\ 400\ 000 \times 0.1433 = 4\ 069\ 002\ kN \cdot m^2 \tag{29}$$

- Through an analogy to a cantilever beam subjected to a uniformly distributed loading of $q = \frac{1005}{36}$ and displacement at the top of $\delta_{top} = 0.0151\ m$ (displacement at the top).

$$\delta = \frac{\frac{q}{L} \cdot L^4}{8 \cdot EI} \Rightarrow E_{cd} I_c = \frac{F \cdot L^3}{8 \cdot \delta} = \frac{1005 \times 36^3}{8 \times 0.0151} = 387\ 105\ 211\ kN \cdot m^2 \tag{30}$$

5.2. GLOBAL SECOND ORDER EFFECTS

In the section 5.8.3.3 of EN 1992-1-1 (Comité Européen de Normalisation, 2004) it is possible to find information on how to deal with global second order effects, as well as an expression that allows the designer to ignore those effects, only valid for structures that fulfil the five conditions below:

1. Structure reasonably symmetrical;
2. Global shear deformations are negligible;
3. The rotations of the basis can be dismissed;
4. The stiffness of the bracing members is reasonably constant along the height;
5. The total vertical load increases by approximately the same amount per storey.

It is important to notice that although this building is not braced, for the sake of this work, this formula will be used comparing the two different values of stiffness calculated above.

$$F_{V,Ed} \leq k_1 \cdot \frac{n_s}{n_s + 1.6} \cdot \frac{\sum E_{cd} I_c}{L^2} \tag{31}$$

Where:

- $F_{V,Ed}$ is the total vertical load, $70\ 010\ kN$, given by the total vertical base reaction;
- k_1 is a coefficient that takes the value 0.31;
- n_s is the number of storeys, 12;

- $\sum E_{cd} I_c$ is the stiffness of the “bracing elements”;
- L is the building height, 36 meters.

So, using the first value of stiffness, $\sum E_{cd} I_c = 4\ 069\ 002\ kN \cdot m^2$:

$$0.31 \times \frac{12}{12 + 1.6} \times \frac{4\ 069\ 002}{36^2} = 858.8\ kN < F_{V,Ed} \tag{32}$$

As the condition is not met, global second order effects cannot be ignored. Using the expressions found in the Appendix H of the same standard, it is possible to calculate a horizontal load for which the structure should be designed.

$$F_{H,Ed} = \frac{F_{H.0Ed}}{1 - \frac{F_{V,Ed}}{F_{V,B}}} \tag{33}$$

Where:

- $F_{H.0Ed}$ is the first order horizontal resulting force;
- $F_{V,B}$ is the nominal buckling load, that (disregarding shear deformations), is equal to $F_{V,BB}$, nominal buckling load.

It is important to notice the similarity between this factor and the γ_z from the NBR 6118 (2014) previously analysed. So, the expression above can be seen as a magnification factor for the horizontal forces, at each floor, that will be named $F_{magnification}$:

$$F_{magnification} = \frac{1}{1 - \frac{F_{V,Ed}}{F_{V,B}}} \tag{34}$$

Thus,

$$F_{V,B} \cong F_{V,BB} = \xi \frac{\sum EI}{L^2} = 7.8 \cdot \frac{n_s}{n_s + 1.6} \cdot \frac{1}{1 + 0.7k} \cdot \frac{0.40 \times \sum E_{cd} I_c}{L^2} =$$
$$= 7.8 \cdot \frac{12}{12 + 1.6} \cdot \frac{1}{1 + 0.7 \times 0} \cdot \frac{0.40 \times 4\ 069\ 002}{36^2} \Leftrightarrow F_{V,B} = 8\ 643\ kN \tag{35}$$

$$F_{magnification} = \frac{1}{1 - \frac{70\ 010}{8\ 643}} = -0.141 \tag{36}$$

As the factor is negative, it is plausible to say that the result is not valid. This occurs since the actual stiffness of the building, corresponding to the frames composed by the beams and columns is much higher than the one corresponding only to the columns. So, the same steps are repeated, but now using the other previously calculated value of stiffness, $E_{cd}I_c = 387\,105\,211\ kN \cdot m^2$:

$$0.31 \times \frac{12}{12+1.6} \times \frac{387\,105\,211}{36^2} = 81\,701\ kN > F_{V,Ed} \tag{37}$$

Unlike above, global second order effects can be ignored. However, for effects of comparison, the magnification factor is going to be calculated anyway:

$$F_{V,BB} = 7.8 \cdot \frac{12}{12+1.6} \cdot \frac{1}{1+0.7\times 0} \cdot \frac{0.40 \times 387\,105\,211}{36^2} = 822\,282\ kN \tag{38}$$

$$F_{magnification} = \frac{1}{1-\frac{70\,010}{822\,282}} = 1.093 \tag{39}$$

As it was expected, the value of the magnification factor to be used is approximately the unity, as global second order effects can be disregarded.

This analysis showed that stiffness is one of the most important parameters and its correct evaluation is fundamental for an accurate analysis.

5.3. LOCAL SECOND ORDER EFFECTS

Just like in 4.3, this item will deal with local second order effects, but this time following EN 1992-1-1, aided by Westerberg (2004). Similar to the Brazilian standard, Eurocode 2 gives the designer three methods of analysis to choose from, which will be detailed in the following items. However, the complete calculations will be shown only in Appendix C.

Unlike the NBR 6118 (2014), Eurocode 2 does not put any slenderness restriction in the method of analysis, and creep must be considered in all methods, unless some conditions are met.

Another factor that changes with this standard is the definition of the effective length. For isolated members, Eurocode 2 defines:

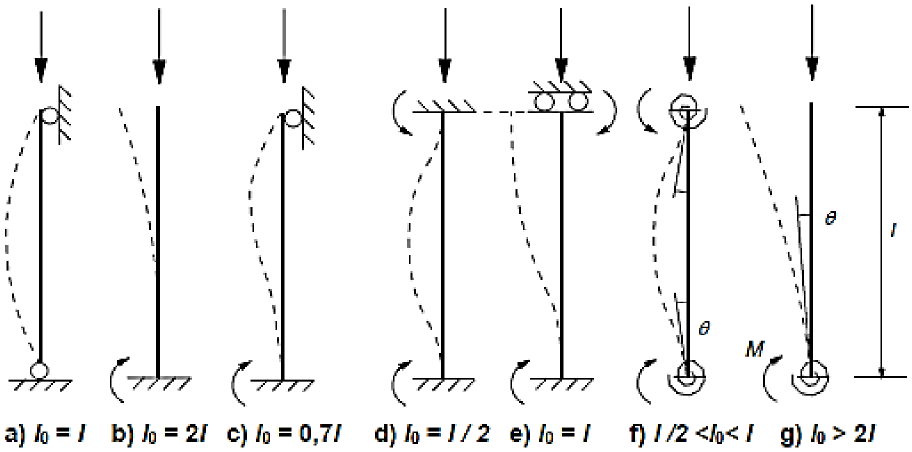

Figure 5-1 – Effective lengths for isolated members (Comité Européen de Normalisation, 2004).

When considering the structure, the effective length for unbraced columns is given by:

$$l_0 = l.\max\left\{\sqrt{1 + 10\frac{k_1 k_2}{k_1 + k_2}}; \left(1 + \frac{k_1}{1 + k_1}\right)\left(1 + \frac{k_2}{1 + k_2}\right)\right\} \tag{40}$$

Where k_i is the relative flexibility of rotational restraints at end i, $k_i = \frac{\left(\Sigma\frac{EI}{L}\right)_{columns}}{\left(4\,\Sigma\frac{EI}{L}\right)_{beams}}$

5.3.1. LOCAL GEOMETRIC IMPERFECTIONS

For isolated unbraced members, the effect of geometric imperfections may be considered as a transverse force, H_i, or as an additional eccentricity, e_i. To be consistent to the Brazilian approach, only the latter will be used:

$$e_i = \frac{\theta_i l_0}{2}, \text{ where } \theta_i = \theta_0 \alpha_h \alpha_m \tag{41}$$

- θ_0 is the basic value, recommended to be $\theta_0 = \frac{1}{200}$;
- α_h is the reduction value for height, $\alpha_h = \frac{2}{\sqrt{l}}$;
- α_m is the reduction value for number of members, $\alpha_m = \sqrt{0.5\left(1 + \frac{1}{m}\right)}$;

Considering $l = 3$ and $m = 1$ (isolated element);

$$\theta_i = \theta_0 \alpha_h \alpha_m = \frac{1}{200} \cdot \frac{2}{\sqrt{3}} \cdot \sqrt{0.5 \cdot \left(1 + \frac{1}{1}\right)} \Rightarrow e_i = \frac{l_0}{200\sqrt{3}} \tag{42}$$

Just like in item 4.2, global geometric imperfections are not to be considered.

5.3.2. SLENDERNESS CRITERION FOR ISOLATED COMPRESSED MEMBERS

The local second order effects can be dismissed if the following condition is met:

$$\lambda = \frac{l_0}{i} \leq \lambda_{lim} = \frac{20 \cdot A \cdot B \cdot C}{\sqrt{n}} \tag{43}$$

Where:

- i is the radius of gyration of the uncracked concrete section;
- $A = \frac{1}{1+0.2\varphi_{ef}}$ but if φ_{ef} is not known $\Rightarrow A = 0.7$;
- $B = \sqrt{1+2\omega}$ but if ω is not known $\Rightarrow B = 1.1$;
- $C = 1.7 - r_m$ but if r_m is not known $\Rightarrow C = 0.7$;
- n is the relative normal force, $n = \frac{N_{Ed}}{A_c f_{cd}}$;
- φ_{ef} is the effective creep ratio;
- ω is the mechanical reinforcement ratio, $\omega = \frac{A_s f_{yd}}{A_c f_{cd}} = \rho \frac{f_{yd}}{f_{cd}}$;
- A_s is the total area of longitudinal reinforcement;
- r_m is the ratio of first order end moments, $r_m = \frac{M_{01}}{M_{02}}$, and $|M_{02}| \geq |M_{01}|$;

Using the default values of A, B and C, the limit slenderness is:

$$\lambda_{lim} = \frac{20 \times 0.7 \times 1.1 \times 0.7}{\sqrt{n}} = \frac{10.78}{\sqrt{n}} \tag{44}$$

5.3.3. EFFECTS OF CREEP

Eurocode 2 allows to ignore the effects of creep if three conditions are met:

1. $\varphi_{(\infty,t_0)} \leq 2$
2. $\lambda \leq 75$
3. $\frac{M_{0Ed}}{N_{Ed}} \geq h$

To make a better comparison between the codes, the effects of creep will be neglected for this analysis, making $\varphi_{ef} = 0$, even though the conditions above are not going to be verified.

5.3.4. METHOD BASED ON THE NOMINAL STIFFNESS

This first method considers the effects of cracking, material nonlinearity and creep to modify the RC's flexural stiffness and magnify the first order moment, calculating a final moment for which the element must be designed. The nominal flexural stiffness is given by:

$$EI = K_c \cdot E_{cd} \cdot I_c + K_s \cdot E_s \cdot I_s \tag{45}$$

Considering $\rho = \frac{A_s}{A_c} > 0.002$:

- K_c is a factor for effects of cracking, creep and similar effects, $K_c = k_1 \cdot k_2$;
- k_1 is a factor which depends on concrete strength class, $k_1 = \sqrt{\frac{f_{ck}}{20}}$
- k_2 is a factor which depends on axial force and slenderness, $k_2 = 0.30 \cdot n \leq 0.20$;
- I_c is the moment of inertia of concrete cross section;
- K_s is a factor for contribution of reinforcement, $K_s = 1$;
- E_s is the design value of the modulus of elasticity of reinforcement, $E_s = 200\ GPa$;
- I_s is the moment of inertia of reinforcement, about the centre of area of the concrete.

The moment of inertia of reinforcement is calculated assuming $\rho = 2\%$, 1% on each side, and a concrete cover of 4 cm.

With the nominal stiffness calculated, it is now possible to estimate the final moment for which the column should be designed:

$$M_{Ed} = M_{0Ed}\left[1 + \frac{\beta}{\left({N_B}/{N_{Ed}}\right) - 1}\right] \tag{46}$$

Where,

- M_{0Ed} is the maximum first order moment at one of the ends of the column;
- β is a factor that depends on the distribution of the moments, $\beta = \frac{\pi^2}{c_0}$;
- N_B is the buckling load, based on the nominal stiffness, $N_B = \frac{\pi^2}{{l_0}^2} EI$
- N_{Ed} is the design value of the axial load;

Adopting $\beta = 1$, the expression above can be simplified to:

$$M_{Ed} = \frac{M_{0Ed}}{1 - \left({N_{Ed}}/{N_B}\right)} \tag{47}$$

It is important to remark that EN 1992-1-1 states that because end moments differ, it shall be considered that $M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02}, |M_{02}| \geq |M_{01}|$. However, this consideration is only applied to non-sway structures, where the maximum bending moment is found on the middle, and not on the extremes (Câmara, 2015).

5.3.5. METHOD BASED ON THE NOMINAL CURVATURE

Instead of magnifying the first order moment, this second method adds to the first order moment, already known, a second order moment considering the column deflection, based on the effective length and an estimated maximum curvature:

$$M_{Ed} = M_{0Ed} + M_2 \tag{48}$$

- M_{0Ed} is the first order moment, the same as in 5.3.4;
- M_2 is the nominal second order moment, $M_2 = N_{Ed} \cdot e_2$;
- e_2 is the deflection, $e_2 = \left(\frac{1}{r}\right) \cdot \frac{{l_0}^2}{c}$, $c = \pi^2 \cong 10$ for constant sections;
- $\frac{1}{r}$ is the curvature, $\frac{1}{r} = K_r \cdot K_\varphi \cdot \left(\frac{1}{r_0}\right)$, where $\frac{1}{r_0} = \frac{\varepsilon_{yd}}{0.45d} = \frac{\frac{f_{yd}}{E_s}}{0.45d}$;
- K_r is a correction factor depending on the axial load, $K_r = \frac{n_u - n}{n_u - n_{bal}} \leq 1$, where $n_{bal} = 0.4$ and $n_u = 1 + \omega \cong 1.37$;
- K_φ is a factor taking account creep, $K_\varphi = 1 + \beta\varphi_{ef} \geq 1 \xrightarrow{\varphi_{ef}=0} K_\varphi = 1$.

The value of $\omega \cong 0.37$ is obtained taking $\rho = 2\%$. For a more precise result, n_u should be calculated by an iterative process.

5.3.6. GENERAL METHOD

Just like is defined in NBR 6118 (2014), this method is the “exact” one, where a computational nonlinear analysis shall be performed.

5.3.7. DISCUSSION OF RESULTS

As it was done in 4.3.8, the two approximated methods will be compared for 12 different columns, showing the step-by-step calculations for column G1-00 in Appendix C.

This time there is no minimum bending moments, and the geometric imperfections will be considered through an eccentricity that will be added to the applied moment.

Table 5-1 – Comparison between the P-Delta analysis and the method based on the nominal stiffness. Moments given in kN·m.

Column	**P-Delta**		**Nominal Stiffness**		$\frac{NS-P\Delta}{P\Delta}$ **(%)**
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	15.0	37.1	46.3	54.3	31.76%
G1-12	19.3	41.6	59.8	56.9	26.83%
G1-24	18.0	30.4	33.3	37.6	19.18%
G2-00	28.1	64.1	91.7	91.1	29.61%
G2-12	32.7	24.4	149.1	43.3	43.74%
G2-24	26.7	9.7	61.6	17.8	45.18%
G3-00	4.1	126.0	62.3	175.2	28.06%
G3-12	4.1	131.3	64.5	198.7	33.91%
G3-24	3.3	120.8	24.1	146.8	17.71%
G4-00	6.1	173.9	106.5	240.0	27.55%
G4-12	5.3	46.5	133.3	104.9	55.65%
G4-24	3.3	20.8	36.6	39.7	47.60%
Average					33.90%

Table 5-2 – Comparison between the P-Delta analysis and the method based on the nominal curvature. Moments given in kN·m.

Column	P-Delta		Nominal Curvature		$\frac{NC-P\Delta}{P\Delta}$ (%)
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	15.0	37.1	79.5	69.7	46.82%
G1-12	19.3	41.6	81.3	73.1	43.06%
G1-24	18.0	30.4	45.5	45.0	32.47%
G2-00	28.1	64.1	136.7	108.0	40.64%
G2-12	32.7	24.4	132.4	61.5	60.42%
G2-24	26.7	9.7	68.6	28.7	66.11%
G3-00	4.1	126.0	123.5	201.0	37.28%
G3-12	4.1	131.3	136.2	270.2	51.39%
G3-24	3.3	120.8	67.6	172.1	29.80%
G4-00	6.1	173.9	168.8	263.7	34.05%
G4-12	5.3	46.5	154.4	147.2	68.38%
G4-24	3.3	20.8	60.4	54.9	62.08%
Average					47.71%

Table 5-3 – Comparison of the methods from Eurocode 2. Moments given in kN·m.

Column	Nominal Stiffness		Nominal Curvature		$\frac{NC-NS}{NC}$ (%)	
	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	46.3	54.3	79.5	69.7	71.98%	22.06%
G1-12	59.8	56.9	81.3	73.1	35.93%	22.17%
G1-24	33.3	37.6	45.5	45.0	36.42%	16.44%
G2-00	91.7	91.1	136.7	108.0	49.15%	15.67%
G2-12	149.1	43.3	132.4	61.5	-11.17%	29.66%
G2-24	61.6	17.8	68.6	28.7	11.41%	38.19%
G3-00	62.3	175.2	123.5	201.0	98.27%	12.82%
G3-12	64.5	198.7	136.2	270.2	110.95%	26.45%
G3-24	24.1	146.8	67.6	172.1	180.79%	14.68%
G4-00	106.5	240.0	168.8	263.7	58.46%	8.98%
G4-12	133.3	104.9	154.4	147.2	15.77%	28.71%
G4-24	36.6	39.7	60.4	54.9	65.32%	27.63%
Average					60.27%	21.96%

Once again, the same comments from 4.3.8 can be made, as in the contrary of the structural model, both approximated methods consider the geometric eccentricity when calculating bending moments, which will increase the bending moment in the central columns from almost null to a “normal” value.

Apart from the column G2-12, the Nominal Curvature method is the most penalising method, just like in the analysis according to the Brazilian standard. However, here the bending moments are more than 41% higher, in average, than the ones obtained by the Nominal Stiffness method.

This difference between the two approximated methods may be explained because there was no iteration for the value of n_u in the method of the Nominal Curvature. If other iterations were used, perhaps the results would have been better.

Once more, the Excel spreadsheet will be used to define the column reinforcement, but this time considering $\gamma_{concrete,EC2} = 1.50$ and $\alpha_{EC2} = 1.00$. So, designing for the Nominal Curvature method, the chosen reinforcement is shown as follows:

Table 5-4 – Chosen reinforcement for the Nominal Curvature method from EN 1992-1-1. Bending moments in kN·m and area of reinforcement in cm^2.

COLUMN	EN 1992-1-1							
	N	M_X	M_Y	Rebar	A_S	ρ	ω	BB
G1-00	-1375	79.5	69.7	4Φ25	19.63	1.78%	0,33	0.89
G1-12	-884	81.3	73.1	6Φ25	29.45	3.68%	0,69	0.98
G1-24	-423	45.5	45.0	4Φ25	19.63	3.27%	0,61	0.85
G2-00	-2185	136.7	108.0	8Φ25	39.27	2.45%	0,46	0.88
G2-12	-1392	132.4	61.5	10Φ25	49.09	4.91%	0,92	0.92
G2-24	-674	68.6	28.7	6Φ25	29.45	4.91%	0,92	0.80
G3-00	-3114	123.5	201.0	6Φ25	29.45	1.55%	0,29	0.97
G3-12	-2075	136.2	270.2	10Φ25	49.09	3.07%	0,57	0.95
G3-24	-1017	67.6	172.1	4Φ25	19.63	1.78%	0,33	0.99
G4-00	-4425	168.8	263.7	10Φ25	49.09	2.05%	0,38	0.97
G4-12	-2834	154.4	147.2	12Φ25	58.90	3.68%	0,69	0.90
G4-24	-1367	60.4	54.9	4Φ25	19.63	2.45%	0,46	0.98

Note that the value of $\omega = 0.37$ was assumed for the reinforcement design. However, from the table above it is possible to see that this value is different in all 12 cases, which means that more iterations should have been done to improve the design according to this method.

Also, it is easy to see that the determinant factor in the design was the biaxial bending, as the values of BB are close to 1, and that if this was a real project, some sections would have to be redesigned, as $\rho > 4\%$.

6. ANALYSIS ACCORDING TO *fib* MODEL CODE 2010

Although *fib* Model Code does not define any criteria for global second order effects, it deals with the local effects using an interesting approach, showing only one method and giving the designer the possibility of choosing between four levels of approximation, depending on the wanted accuracy. This work will only show the results for the first two levels of approximation. For more details on this approach see Muttoni (2012).

There is a lot of similarities between *fib* and EN 1992-1-1. The effective length is calculated the same way, and *fib* uses the same coefficients found in the method based on the nominal curvature from Eurocode 2.

6.1. LOCAL SECOND ORDER EFFECTS

2010 Model Code states that the dimensioning value of the bending moment is given by:

$$M_d = -N_d \cdot e_d \tag{49}$$

Where:

- e_d is the maximum eccentricity, $e_d = e_{0d} + e_{1d} + e_{2d}$;
- e_{0d} is the eccentricity due to imperfections, $e_{0d} = \max\left\{\frac{\alpha_i l_0}{2}; \frac{h}{30}\right\}$;
- e_{1d} is the first order eccentricity, $e_{1d} = \frac{M_{1d}}{-N_d}$;
- e_{2d} is the eccentricity due to the deformation of the compression member, $e_{2d} = k_d \frac{l_0^2}{c_0}$;
- α_i is a coefficient that can be estimated as $\frac{1}{200} \geq \alpha_i = \frac{0.01}{\sqrt{l_0}} \geq \frac{1}{300}$;
- k_d is the maximum design curvature, $k_d = \frac{\varepsilon_{sd} - \varepsilon_{sd}'}{h - 2c}$;
- c_0 is an integration factor accounting for the curvature distribution along the member;
- h is the height of the column section;
- c is the concrete cover, assumed to be herein 2.50 cm.

Because the different approximations change only the eccentricity e_{2d}, the remaining calculations are the same for every level of accuracy.

6.1.1. LEVEL I OF APPROXIMATION

This first approximation assumes that:

$$\begin{cases} c_0 = \pi^2 \cong 10 \\ \varepsilon_{sd} = \dfrac{f_{yd}}{E_s} \\ \varepsilon'_{sd} = -\dfrac{f_{yd}}{E_s} \end{cases} \Rightarrow e_{2d} \cong \frac{f_{yd}}{E_s} \cdot \frac{{l_0}^2}{5(h-2c)} \tag{50}$$

6.1.2. LEVEL II OF APPROXIMATION

Here, the value of the maximum design curvature can be improved by taking into account the normal force.

$$k_d = \left(\frac{n_u - n}{n_u - n_{bal}}\right) \cdot \frac{\varepsilon_{yd}}{0.45h}, \text{and } \frac{n_u - n}{n_u - n_{bal}} \leq 1 \tag{51}$$

$$e_{2d} = \left(\frac{(1+\omega) - n}{(1+\omega) - 0.40}\right) \cdot \frac{\frac{f_{yd}}{E_s}}{0.45h} \cdot \frac{{l_0}^2}{10} \tag{52}$$

If creep and shrinkage are to be considered, an additional curvature should be taken into account: $k_{d,\infty} = \frac{|\varepsilon_{c\infty}|}{d}$. However, as it has been done in this work, these effects will be neglected.

6.1.3. LEVEL III OF APPROXIMATION

This third approximation combines uses the k_d from the Level II, at the same time refining the value of the integration factor as a summation of various integration factors as a function of the load type and boundary conditions:

$$c_0 = \pi^2 \frac{N}{N_{cr}} + \frac{\sum_{i=1}^{n} M_i}{\sum_{i=1}^{n} \frac{M_i}{c_i}} \left(1 - \frac{N}{N_{cr}}\right) \tag{53}$$

The values of c_i can be found in Figure 6-1:

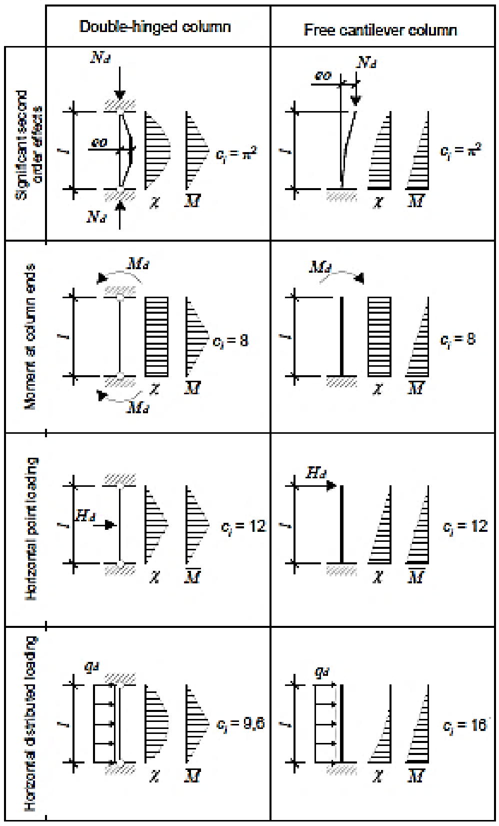

Figure 6-1 – Values of integration factors, c_i, as a function of the load type and the boundary conditions (Fédération Internationale du Béton, 2010).

6.1.4. LEVEL IV OF APPROXIMATION

This final level of approximation will be the same as the general methods described in NBR 6118 (2014) and EN 1992-1-1.

6.1.5. DISCUSSION OF RESULTS

Same as before, the comparison of the bending moments from the first two approximated methods is shown in the table below.

As it was done in 4.3.8, the two approximated methods will be compared for 12 different columns, showing the step-by-step calculations for column G1-00 in Appendix C.

Table 6-1 – Comparison between the P-Delta analysis and the Level I method found in the Model Code 2010. Moments given in kN·m.

Column	P-Delta		Level I		$\frac{LoA\ I-P\Delta}{P\Delta}$ (%)
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	15.0	37.1	97.2	81.7	54.63%
G1-12	19.3	41.6	94.2	75.8	45.10%
G1-24	18.0	30.4	49.2	45.5	33.19%
G2-00	28.1	64.1	173.7	144.0	55.46%
G2-12	32.7	24.4	168.6	72.7	66.52%
G2-24	26.7	9.7	77.7	31.1	68.65%
G3-00	4.1	126.0	183.2	264.7	52.39%
G3-12	4.1	131.3	176.4	284.0	53.76%
G3-24	3.3	120.8	76.2	172.5	29.98%
G4-00	6.1	173.9	277.1	392.7	55.71%
G4-12	5.3	46.5	248.2	193.9	76.00%
G4-24	3.3	20.8	92.6	67.7	69.23%
Average					55.05%

Table 6-2 – Comparison between the P-Delta analysis and the Level II method found in the Model Code 2010. Moments given in kN·m.

Column	P-Delta		Level II		$\frac{LoA\ II-P\Delta}{P\Delta}$ (%)
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	15.0	37.1	67.6	78.1	52.57%
G1-12	19.3	41.6	69.0	71.8	42.04%
G1-24	18.0	30.4	40.2	43.5	30.09%
G2-00	28.1	64.1	116.6	139.8	54.15%
G2-12	32.7	24.4	112.1	66.3	63.27%
G2-24	26.7	9.7	60.6	27.4	64.41%
G3-00	4.1	126.0	101.7	252.7	50.13%
G3-12	4.1	131.3	110.2	268.4	51.07%
G3-24	3.3	120.8	55.4	170.9	29.29%
G4-00	6.1	173.9	139.3	376.8	53.85%
G4-12	5.3	46.5	124.8	167.3	72.19%
G4-24	3.3	20.8	50.0	55.8	62.66%
Average					52.14%

Table 6-3 – Comparison of the methods from MC 2010. Moments in kN·m.

Column	Level I		Level II		$\frac{LoA\,I - LoA\,II}{LoA\,I}$ (%)	
	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	97.2	81.7	67.6	78.1	-43.76%	-4.56%
G1-12	94.2	75.8	69.0	71.8	-36.44%	-5.57%
G1-24	49.2	45.5	40.2	43.5	-22.27%	-4.63%
G2-00	173.7	144.0	116.6	139.8	-48.97%	-2.95%
G2-12	168.6	72.7	112.1	66.3	-50.37%	-9.72%
G2-24	77.7	31.1	60.6	27.4	-28.33%	-13.52%
G3-00	183.2	264.7	101.7	252.7	-80.16%	-4.75%
G3-12	176.4	284.0	110.2	268.4	-60.12%	-5.82%
G3-24	76.2	172.5	55.4	170.9	-37.68%	-0.98%
G4-00	277.1	392.7	139.3	376.8	-98.99%	-4.21%
G4-12	248.2	193.9	124.8	167.3	-98.83%	-15.88%
G4-24	92.6	67.7	50.0	55.8	-85.37%	-21.33%
Average					-57.61%	-7.83%

One more time, the same comments from 5.3.7 can be made.

As expected, the Level II method is less conservative than the Level I, returning values 32% smaller, in average. It is also plausible to assume that a Level III approximation would return even lower values for bending moments, closer to the real values, but still in the safe side.

The same spreadsheet from 5.3.7 will be used to define the column reinforcement. The chosen reinforcement for the Level II of approximation is given by Table 6-4:

Table 6-4 – Chosen reinforcement for the Level II of Approximation from *fib* MC 2010. Bending moments in kN·m and area of reinforcement in cm^2.

COLUMN	***fib* Model Code 2010**							
	N	M_X	M_Y	**Rebar**	A_S	ρ	ω	**BB**
G1-00	-1375	67.6	78.1	4Φ20	12.57	1.14%	0.21	0.94
G1-12	-884	69.0	71.8	6Φ20	18.85	2.36%	0.44	0.95
G1-24	-423	40.2	43.5	6Φ20	18.85	3.14%	0.59	0.87
G2-00	-2185	116.6	139.8	8Φ20	25.13	1.57%	0.29	1.00
G2-12	-1392	112.1	66.3	12Φ20	37.70	3.77%	0.70	0.97
G2-24	-674	60.6	27.4	6Φ20	18.85	3.14%	0.59	0.97
G3-00	-3114	101.7	252.7	8Φ20	25.13	1.32%	0.25	0.93
G3-12	-2075	110.2	268.4	12Φ20	37.70	2.36%	0.44	0.95
G3-24	-1017	55.4	170.9	6Φ20	18.85	1.71%	0.32	1.00
G4-00	-4425	139.3	376.8	14Φ20	43.98	1.83%	0.34	0.93
G4-12	-2834	124.8	167.3	14Φ20	43.98	2.75%	0.51	0.98
G4-24	-1367	50.0	55.8	6Φ20	18.85	2.36%	0.44	0.87

Once more time, the values of BB are close to 1, showing that the biaxial bending was determinant in the design, and just like in 5.3.7, the Level II method could be improved if the values of ω where iterated.

Although this method is very similar to the Nominal Curvature method, allied with the fact that they both had the same stresses as base, the Level II of approximation is less conservative, allowing lower levels of reinforcement in each section.

7. ANALYSIS ACCORDING TO ACI 318M-14

For this analysis, it was used the metric version of the American structural code, ACI318M-14 (American Concrete Institute, 2014). It is important to note that there are some differences of notation between this structural design code and the others analysed before. For instance, although f_c' is slightly different from f_{ck}, they will be used as the same value. See Alves de Souza and Bittencourt (2003) for more details.

Just like it happened in chapter 6, ACI 318M-14 does not deal with global second order effects, so only local effects will be analysed.

7.1. INITIAL CONSIDERATIONS

7.1.1. MATERIAL PROPERTIES

ACI gives two ways to calculate the modulus of elasticity. One only considering the concrete's resistance, and another taking into account both resistance and self-weight:

$$E_c = 4700\sqrt{f_c'} \cong 4700 \times \sqrt{35} = 27.8\ GPa \tag{54}$$

$$E_c = w_c^{1.5} \times 0.043\sqrt{f_c'} \cong 2500^{1.5} \times 0.043 \times \sqrt{35} = 31.8\ GPa \tag{55}$$

For this work, the elasticity modulus will take the value of $E_c = 31.8\ GPa$.

Like the Brazilian standard, for the second order analysis, the element's inertia is considered as a fraction of the gross concrete section:

- Slabs: $I_{slabs} = 0.25I_g$;
- Beams: $I_{beams} = 0.35I_g$;
- Columns: $I_{columns} = 0.70I_g$.

When calculating the critical buckling load of a column it is also necessary to consider an effective value of stiffness, according to section 6.6.4.4.4 of ACI 318M-14:

$$P_c = \frac{\pi^2(EI)_{eff}}{(kl_u)^2} \tag{56}$$

$$(EI)_{eff} = \frac{0.40E_cI_g}{1+\beta_{dns}} \tag{57}$$

Where:

$$\beta_{dns} = \frac{maximum\ factored\ axial\ sustained\ load}{maximum\ factored\ axial\ load} \leq 1 \tag{58}$$

To follow the same logic as the three standards before, the effects of creep will be neglected, making $\beta_{dns} = 0$.

7.2. LOCAL SECOND ORDER EFFECTS

According to ACI 318M-14, the design of a structure considering second order effects may be based on the moment magnifier approach, an elastic second order analysis or a nonlinear second order analysis.

To help the designer on how to deal with second order effects, the American structural design code gives a flowchart with the steps needed to fulfil the analysis (Figure R6.2.6). This flowchart will serve as base for the analysis below.

Some considerations found in Saatcioglu (2009), StructurePoint (2010) and Vaquinhas (2014) will be followed in the calculations.

7.2.1. LOCAL GEOMETRIC IMPERFECTIONS

ACI 318M-14 has a very similar approach to NBR 6118 (2014), regarding local geometric imperfections, as it also considers the effect of a minimum first order moment acting in the columns:

$$\begin{cases} M_{1d,min,X} = P_u \cdot (15 + 0.03h) \\ M_{1d,min,Y} = P_u \cdot (15 + 0.03b) \end{cases} \tag{59}$$

Where P_u is the factored vertical load in the column in analysis. The dimensions h, b are in mm.

7.2.2. SLENDERNESS CRITERION FOR COMPRESSED MEMBERS

For unbraced elements, slenderness effects may be dismissed if the slenderness ratio follows the condition below:

$$\frac{kl_u}{r} \leq 22 \tag{60}$$

Where,

- k is a factor of effective length of the column, given by Figure 7-1;
- l_u is the unsupported length of the column, i.e., the clear distance between the underside of the beam/slab and the top of the beam/slab below;
- r is the radius of gyration, i in the European convention, that for rectangular can be calculated as 30% of the dimension in which the instability is being considered.

Once more, the effective length is calculated differently, as ACI uses a chart similar to the ones used for steel structures, that depend on the stiffness of the adjacent members:

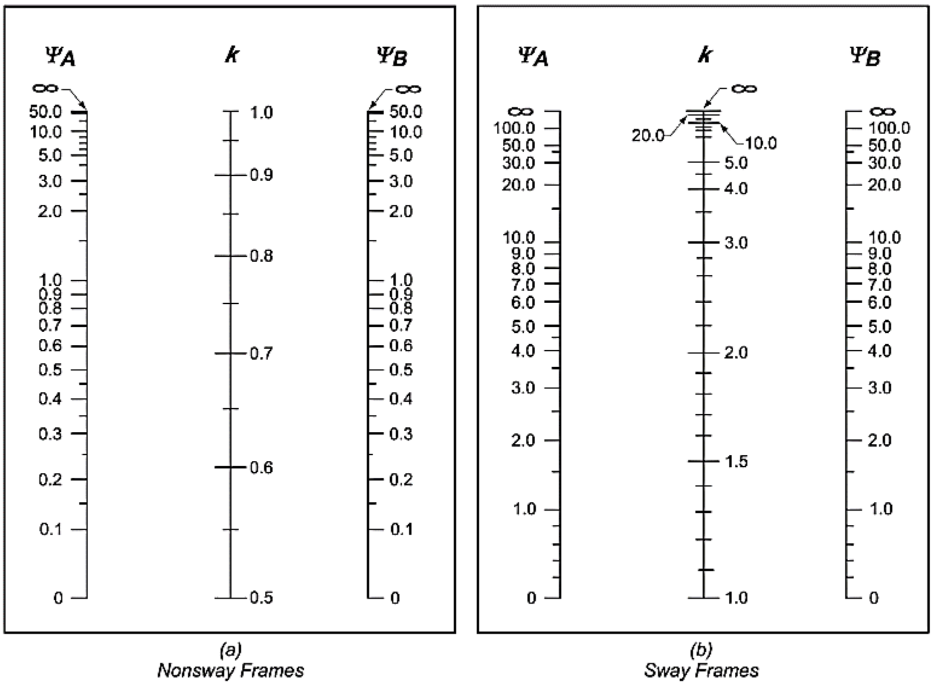

Figure 7-1 – Charts that give the effective length factor *k* (American Concrete Institute, 2014).

The ratio ψ_i is given by:

$$\psi_i = \frac{\left(\sum \frac{EI_i}{l_i}\right)_{columns}}{\left(\sum \frac{EI_i}{l_i}\right)_{beams}} \tag{61}$$

Note that if both ends of the element show no or minimal rotation, i.e., $\psi_m \cong \infty$, then $k = \infty$. On the other hand, if the rotations are completely restrained, $\psi_m \cong 0$ and $k = 1$.

Instead of using the charts above, it is possible to use analytical expressions, that for sway frames, are given by (Vaquinhas, 2014):

$$\begin{cases} k = \dfrac{20-\psi_m}{20}\sqrt{1+\psi_m} & if\ \psi_m < 2 \\ k = 0.9\sqrt{1+\psi_m} & if\ \psi_m \geq 2 \end{cases} \quad (62)$$

Where the value ψ_m is the average of the two ψ_i values.

7.2.3. MOMENT MAGNIFICATION METHOD

A first comment that must be made about this method is that it only deals with plane frames subjected to loads causing deflections in that plane. If the elements are subjected to a lateral loading that causes significant torsional displacement, a three-dimensional second order analysis should be used. If it is not, the moments given by the moment magnification method will probably be underestimated.

In the moment magnification method (MMM), the final bending moment will be given by an amplification of the first order moment, considering the member's curvature. Each value of end moment is given by the summation of a non-sway moment, M_{ins}, calculated by a first order elastic analysis, and a sway moment, M_{is}, also calculated by a first order elastic analysis, but magnified by the moment amplification factor δ_s. Note that M_{ins} is the bending moment due to loads that cause no appreciable sidesway, while M_{is} is the bending moment due to loads that cause relevant sidesway.

ACI 318 states that if $Q > 0.05$, the column or storey in analysis is considered unbraced, i.e., sway element:

$$Q = \frac{\sum P_u \Delta_0}{V_{us} l_c} \quad (63)$$

Where:

- $\sum P_u$ is the total factored vertical load in a storey or column;
- V_{us} is the horizontal storey or column shear;
- Δ_0 is the relative first order lateral deflection due to V_{us};
- l_c is the length of the storey or column, 3 m;

In addition, section 10.10.2.1 of this structural code states that the total bending moment including second order effects must not exceed $1.40M_1$, being M_1 the first order moment. This corresponds to a value of $Q \cong 0.25$. This limitation is due to the increase of the probability of instability and a need to stiff the structure laterally.

As the column in analysis is sway, the moment magnification method gives:

$$\begin{cases} M_1 = M_{1ns} + \delta_s M_{1s} \\ M_2 = M_{2ns} + \delta_s M_{2s} \end{cases} \tag{64}$$

Where:

- M_{1ns} and M_{2ns} are end moments due to factored gravity loads;
- M_{1s} and M_{2s} are the end moments due to actions that produce sidesway;
- δ_s is the amplification factor, given by one of the expressions below (Equation 65 and 66)

$$if\ \delta_s \leq 1.5 \Rightarrow \delta_s = \frac{1}{1 - 1.15Q} \geq 1 \tag{65}$$

$$if\ \delta_s > 1.5 \Rightarrow \delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1 \tag{66}$$

Note that if the applied moment due to external forces is smaller than the minimum moment due to local geometric imperfections, the column must be designed to withstand the magnified minimum moment.

Because the wind force is only being considered in the Y direction, the second order effects can only be analysed for the M_X bending moments, as there is no shear in the columns for the X direction. So, the conditioning M_X moments will be given by the minimum moment due to the local geometric eccentricities.

Just like NBR 6118, ACI 318 only considers the effects of biaxial bending for the moments due to the applied loads, not needing to verify its effects for the minimum moments. So, the reinforcements will be calculated for the most conditioning moments and the biaxial bending will be verified for the moments due to wind loads.

However, because ACI 318M-14 does not refer any approximated way of dealing with biaxial bending, the same spreadsheet will be used, considering $\gamma_{concrete,ACI} = \frac{1}{0.65} = 1.538$ and $\alpha_{ACI} = 0.85$.

The results for the moment magnification method (MMM) and its reinforcement are given in Table 7-1 and the explicit calculations for column G1-00 are given in Appendix E. Once again, for the same reasons as in the NBR 6118, the biaxial bending does not condition the reinforcement design.

Comparing the final bending moments with the initial P-Delta analysis (Table 7-2) it is possible to see that the MMM returns the less reinforced section and, consequently, with the closest values to the P-Delta analysis.

Table 7-1 – Bending moments and reinforcement for the MMM from ACI 318M-14. Bending moments in kN·m and area of reinforcement in cm^2.

COLUMN	ACI 318M-14							
	N	M_X	M_Y	Rebar	A_S	ρ	ω	BB
G1-00	-1193	25.0	40.8	4Φ12.5	4.91	0.45%	0.08	0.32
G1-12	-779	17.2	35.1	4Φ12.5	4.91	0.61%	0.11	0.56
G1-24	-380	15.3	24.9	4Φ12.5	4.91	0.82%	0.15	0.64
G2-00	-2125	44.6	90.0	4Φ12.5	4.91	0.31%	0.06	0.40
G2-12	-1388	29.1	58.0	4Φ12.5	4.91	0.49%	0.09	0.66
G2-24	-683	20.9	18.4	4Φ12.5	4.91	0.82%	0.15	0.61
G3-00	-2890	60.7	178.4	4Φ12.5	4.91	0.26%	0.05	0.30
G3-12	-1936	40.7	119.2	4Φ12.5	4.91	0.31%	0.06	0.35
G3-24	-959	20.1	94.0	4Φ12.5	4.91	0.45%	0.08	0.53
G4-00	-4438	93.2	229.1	12Φ20	37.70	1.57%	0.29	0.16
G4-12	-2911	61.1	113.5	6Φ20	18.85	1.18%	0.22	0.20
G4-24	-1446	30.4	39.0	4Φ20	12.57	1.57%	0.29	0.16

Table 7-2 – Comparison between the P-Delta analysis and the MMM. Moments given in kN·m.

Column	P-Delta		MMM		$\frac{MMM-P\Delta}{P\Delta}$ (%)
	M_X	M_Y	M_X	M_Y	M_Y
G1-00	13.8	35.2	25.0	40.8	13.92%
G1-12	17.2	32.7	17.2	35.1	7.03%
G1-24	15.3	22.1	15.3	24.9	11.17%
G2-00	24.8	49.1	44.6	90.0	45.45%
G2-12	27.6	17.8	29.1	58.0	69.39%
G2-24	20.9	7.6	20.9	18.4	58.63%
G3-00	4.2	107.8	60.7	178.4	39.56%
G3-12	5.1	125.2	40.7	119.2	-4.98%
G3-24	4.3	103.7	20.1	94.0	-10.34%
G4-00	5.8	136.4	93.2	229.1	40.46%
G4-12	6.1	44.5	61.1	113.5	60.78%
G4-24	4.0	16.2	30.4	39.0	58.60%
Average					55.05%

8. NONLINEAR ANALYSIS

As stated before, the best way to verify if the results obtained in the previous P-Delta analysis are correct is doing a nonlinear analysis considering both nonlinearities. As this process takes a lot of time and work, this analysis was only done to the EN 1992-1-1 and *fib* models previously defined in Chapter 5 and Chapter 6, using the reinforcements from Table 5-4 and Table 6-4, respectively.

In both cases, the nonlinear behaviour was considered through a model with distributed plasticity, in which the elements are subdivided and the constitutive relations are modelled in each section (Deierlein, Reinhorn and Willford, 2010). So, 5 hinges type "Fiber P-M2-M3" were defined in each column, making each hinge 60 cm long. The default stress-strain curves from SAP2000 were used, and the reinforced sections were assigned using the section design tool.

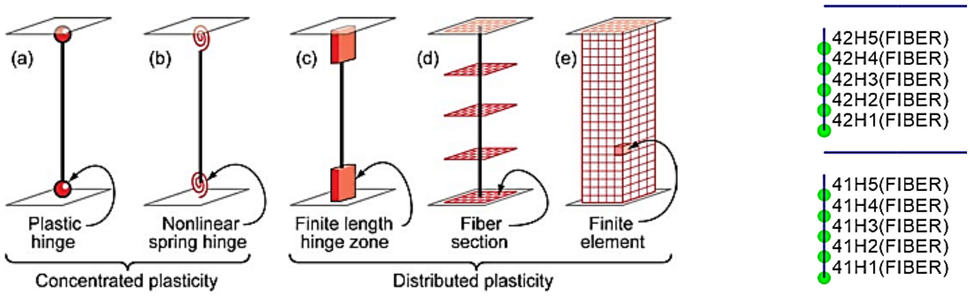

Figure 8-1 – On the left, the different models of plasticity that can be adopted (Deierlein, Reinhorn and Willford, 2010) and on the right, the division of the column elements in fiber hinges in the analysed model (Computers and Structures, 2016).

Starting from the state of full vertical loads, the wind force was applied as incremental imposed displacements, until the value of 0.1 m was reached at the control node at the top (about 2% the height of the building, as suggested by Surana (2000)). Note that this final level of displacement corresponds to a final base shear force of 6236 kN, which is way higher than the design wind load applied to the building. With the analysis completed, the step that had the base reaction closest to the full wind force was found, and the forces in the 4 columns were extracted.

8.1. DISCUSSION OF RESULTS

Only the Eurocode analysis will be shown, as the results from the *fib* model did not present any significant difference.

Also, in order to test the sensibility of this nonlinear analysis, other load histories were tested. However, one more time there was no significative change in the results for any of them.

After running the analyses, it became clear that for this case the physically nonlinear effects are not significative, as very few sections crack. So, the results from the nonlinear analysis are practically the same as the ones from the initial P-Delta analysis, that only considered the geometric nonlinearity.

Figure 8-2 compares the base force and the displacement at the top of the structure, for the nonlinear analysis and its initial tangent value. It becomes clear that the effect of the nonlinearity only becomes significative when the wind force is much higher than the considered design force.

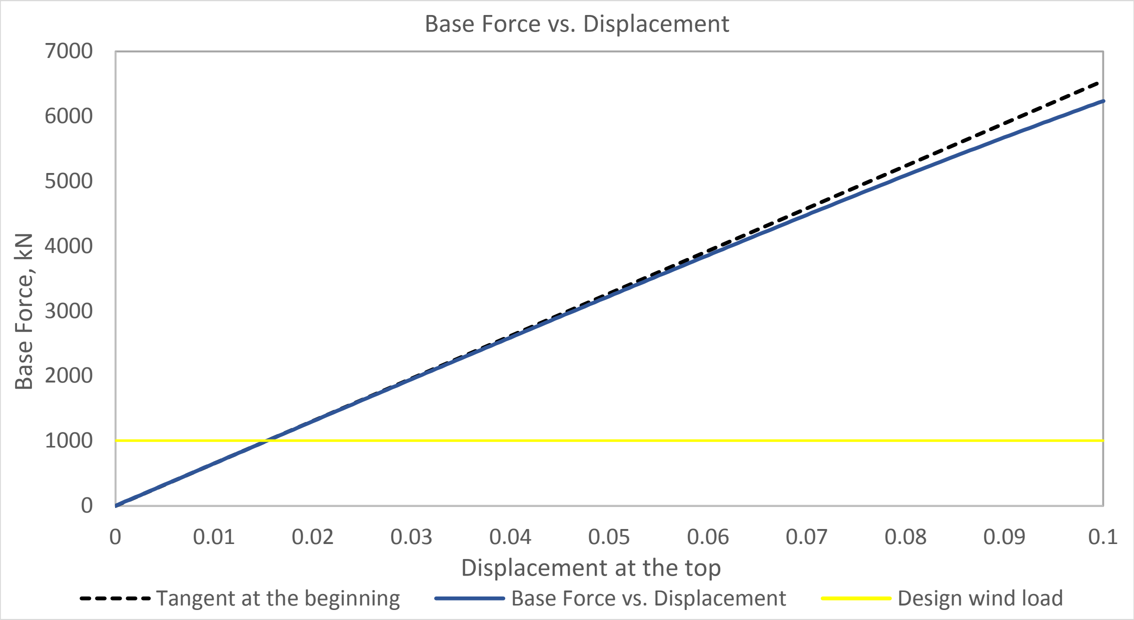

Figure 8-2 – Base force vs. Displacement at the control node at the top.

It is also possible to illustrate the difference of behaviour of an uncracked and cracked section, by analysing their Moment-Rotation diagram for both cases. Figure 8-3 shows the M-θ diagram for an uncracked column, while Figure 8-4 shows the same diagram but for a cracked column.

Note that the Moment-Curvature diagram can be obtained dividing the rotation by the length of the hinge.

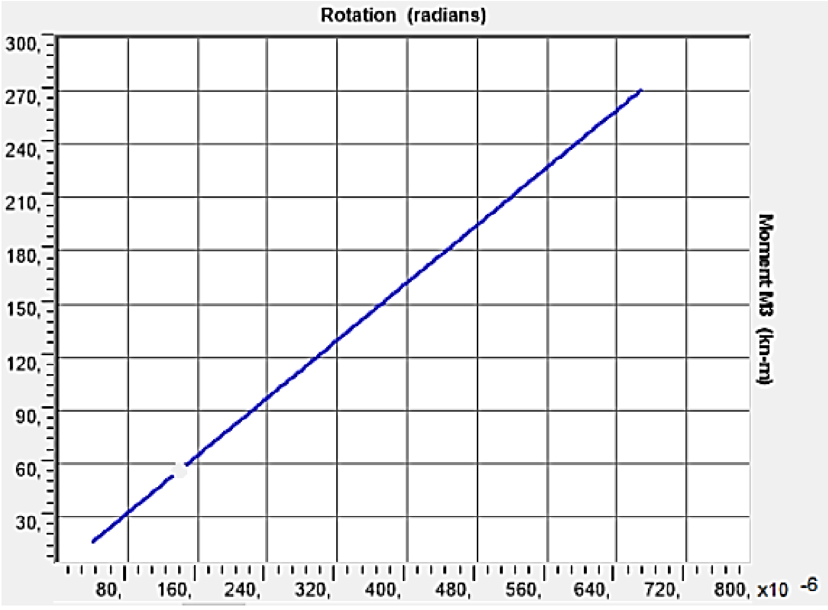

Figure 8-3 – Moment–Rotation diagram for an uncracked hinge.

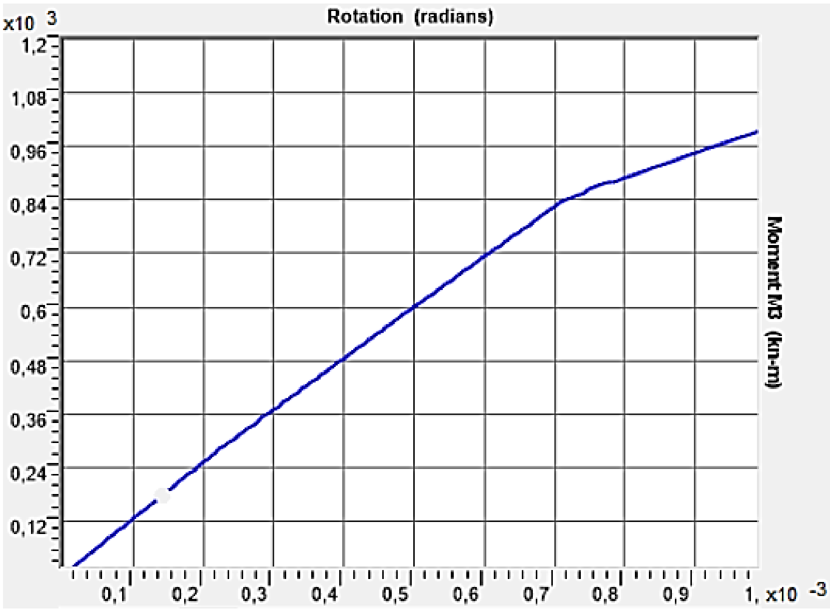

Figure 8-4 – Moment–Rotation diagram for a cracked hinge.

For both cases, the “ball” in the graphic indicates the step with the design wind force, which clearly stills presents linear behaviour, still far from cracking. So, for the considered load, the physical nonlinearity does not add significance to the P-Delta analysis.

Table 8-1 compares the bending moments obtained from the nonlinear and P-Delta analyses. The similarity of results is seen once again, as the physically nonlinearity only adds about 12% to the initial value of bending moments.

Table 8-1 – Comparison between the bending moments found in a P-Delta and nonlinear analysis, given by SAP2000 for the EN 1992-1-1. Bending moments in kN·m and deviation in %

Column	**P-Delta Analysis**		**Nonlinear**		$\frac{NLA-P\Delta}{P\Delta}$ **(%)**	
	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	15.0	37.1	18.2	49.4	21.55%	33.31%
G1-12	19.3	41.6	23.7	51.0	23.21%	22.53%
G1-24	18.0	30.4	20.7	34.7	15.26%	14.27%
G2-00	28.1	64.1	34.5	55.6	22.52%	-13.27%
G2-12	32.7	24.4	40.8	30.5	24.87%	25.42%
G2-24	26.7	9.7	34.0	11.4	27.31%	17.32%
G3-00	4.1	126.0	5.0	108.8	20.40%	-13.64%
G3-12	4.1	131.3	4.3	155.7	5.81%	18.56%
G3-24	3.3	120.8	3.4	145.1	2.59%	20.11%
G4-00	6.1	173.9	6.1	173.9	0.00%	0.00%
G4-12	5.3	46.5	5.3	46.5	0.00%	0.00%
G4-24	3.3	20.8	3.3	20.8	0.00%	0.00%
Average					13.63%	10.38%

Although it is not correct to superpose effects when performing nonlinear analyses, as the nonlinearity is almost irrelevant in this case, a new approximate comparison will be made. Here, the bending moments due to the geometric eccentricity will be added to those obtained from the nonlinear analysis already performed, and these results will be compared with the two approximated methods from EN 1992-1-1.

Even though this is a very approximated comparison, it makes clear that the bending moments due to geometric eccentricities are decisive when designing RC columns, as the relative deviations reduced drastically (see Table 8-2 and Table 8-3)

One more time, the relative deviations in the X direction are much greater than in the Y direction, for the same reasons commented in the previous chapters.

Table 8-2 – Comparison between the bending moments from the nonlinear analysis considering the geometric imperfections and the nominal stiffness method from EN 1992-1-1.

Column	Nonlinear + Eccentricity		Nominal Stiffness		$\frac{NS-NLA}{NLA}$ (%)	
	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	33.0	64.9	46.3	54.3	40%	-16.3%
G1-12	35.3	62.0	59.8	56.9	69%	-8.3%
G1-24	25.8	39.1	33.3	37.6	29%	-3.9%
G2-00	59.4	81.1	91.7	91.1	54%	12.3%
G2-12	60.4	47.3	149.1	43.3	147%	-8.6%
G2-24	42.3	17.9	61.6	17.8	46%	-0.6%
G3-00	37.8	154.4	62.3	175.2	65%	13.4%
G3-12	31.1	208.5	64.5	198.7	108%	-4.7%
G3-24	15.5	162.5	24.1	146.8	55%	-9.7%
G4-00	54.4	236.9	106.5	240.0	96%	1.3%
G4-12	42.5	99.5	133.3	104.9	214%	5.4%
G4-24	18.8	36.3	36.6	39.7	95%	9.4%
Average					85%	-0.8%

Table 8-3 – Comparison between the bending moments from the nonlinear analysis considering the geometric imperfections and the nominal curvature method from EN 1992-1-1.

Column	Nonlinear + Eccentricity		Nominal Curvature		$\frac{NC-NLA}{NLA}$ (%)	
	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	33.0	64.9	79.5	69.7	141%	7.4%
G1-12	35.3	62.0	81.3	73.1	130%	17.9%
G1-24	25.8	39.1	45.5	45.0	76%	15.0%
G2-00	59.4	81.1	136.7	108.0	130%	33.2%
G2-12	60.4	47.3	132.4	61.5	119%	30.0%
G2-24	42.3	17.9	68.6	28.7	62%	60.9%
G3-00	37.8	154.4	123.5	201.0	226%	30.1%
G3-12	31.1	208.5	136.2	270.2	338%	29.6%
G3-24	15.5	162.5	67.6	172.1	336%	5.9%
G4-00	54.4	236.9	168.8	263.7	210%	11.3%
G4-12	42.5	99.5	154.4	147.2	263%	47.9%
G4-24	18.8	36.3	60.4	54.9	222%	51.2%
Average					188%	28.4%

9. CASE STUDY – INFLUENCE OF STIFFNESS

As it became obvious in this work, the building's stiffness is a very important factor. So, to properly show the influence of this parameter, a case study will be presented.

First, all columns will rotate 90 degrees, making the building less stiff against the wind action. Then, two rigid cores will be added to this new building model, and the before and after results will be compared.

As NBR 6118 (2014) and EN 1992-1-1 are the only codes that deal with global second order effects, these will be the only codes to be compared. The same steps from 4.2 and 5.2 will be followed for both cases and every other property will stay the same as before for each model.

9.1. COLUMNS TURNED 90-DEGREES

By turning the columns 90-degrees, the stiffness of the building is going to change. Making the same considerations as in 5.1.2, the new values of stiffness are going to be:

- By the summation of the stiffness of every column:

$$I_{bracing} = \sum n_{group} \frac{b_{average} \times 0.20^3}{12} = 0.0109\ m^4 \Rightarrow$$
$$\Rightarrow \sum E_{cd} I_c = 28\ 400\ 000 \times 0.0109 = 309\ 219\ kN \cdot m^2 \quad (67)$$

- Through an analogy to the same cantilever beam, but with displacement at the top of $\delta_{top} = 0.0665\ m$

$$\delta = \frac{\frac{q}{L} \cdot L^4}{8 \cdot EI} \Rightarrow E_{cd} I_c = \frac{F \cdot L^3}{8 \cdot \delta} = \frac{1005 \times 36^3}{8 \times 0.0665} = 88\ 105\ 947\ kN \cdot m^2 \quad (68)$$

9.1.1. GLOBAL ANALYSIS ACCORDING TO NBR 6118 (2014)

Running the same analyse as in 4.2, it is possible to obtain the forces and displacements in the model, showed in the tables below:

Table 9-1 – Base reaction forces and bending moments, in kN and kN·m respectively.

OutputCase	Global FX	Global FY	Global FZ	Global MX	Global MY	Global MZ
ULS	0.00	-1 005	70 010	20 202	0.00	0.00
P-DELTA	0.00	-1 004	70 010	23 892	0.00	0.00

Once again, note that the P-Delta effect does not significantly change the resultant of forces, only global moments, while the weight of the structure stays the same.

The analysis of the displacements at each floor (Table 9-2) shows how important the column orientation is. Now, instead of 1.05, the displacements suffer an average amplification of 1.22. Clearly, the value of γ_z will be higher than the one found before:

Table 9-2 – Joint displacements in the Y direction, in various pavements, in meters.

Pavement	Output Case	Case Type	Step Type	δ
1	P-Delta	NonStatic	Max	0.00885
	ULS	Combination		0.00717
2	P-Delta	NonStatic	Max	0.01880
	ULS	Combination		0.01517
3	P-Delta	NonStatic	Max	0.02791
	ULS	Combination		0.02262
4	P-Delta	NonStatic	Max	0.03618
	ULS	Combination		0.02950
5	P-Delta	NonStatic	Max	0.04602
	ULS	Combination		0.03751
6	P-Delta	NonStatic	Max	0.05455
	ULS	Combination		0.04460
7	P-Delta	NonStatic	Max	0.06170
	ULS	Combination		0.05072
8	P-Delta	NonStatic	Max	0.06761
	ULS	Combination		0.05589
9	P-Delta	NonStatic	Max	0.07482
	ULS	Combination		0.06206
10	P-Delta	NonStatic	Max	0.07993
	ULS	Combination		0.06657
11	P-Delta	NonStatic	Max	0.08297
	ULS	Combination		0.06936
12	P-Delta	NonStatic	Max	0.08416
	ULS	Combination		0.07047

$$\gamma_z = \frac{1}{1 - \frac{2\,682}{14\,430}} = 1.228 \quad (69)$$

As $\gamma_z = 1.228 > 1.1$, the structure is classified as sway. However, since $\gamma_z = 1.228 < 1.3$, the Brazilian code allows the approximation defined previously in 4.2.1. With the results from Table 9-1 it is possible to verify that, in the analysed case, the approximation defined by the NBR 6118 (2014) is accurate:

$$20\,202 \times 0.95 \cdot \gamma_z = 23\,568 \cong 23\,892 \quad (70)$$

The deviation in this approximation is roughly 1.4%, showing that, in the analysed case, this procedure provides a good approximation.

Comparing the results from the Brazilian approximation $ULS \times 0.95\gamma_z$ with the P-Delta analysis, for one column from each group, it can be seen that even though the criterion of NBR 6118 (2014) is not very accurate, for a simple analysis it provides a good approximation. The mean absolute deviation in all columns is 9.41%, ranging from an average 6.95% in Group 2 to 12.68% in Group 4. This is shown in Table 9-3:

Table 9-3 – Comparison of bending moments at the base of various columns.

GROUP 1 – BENDING MOMENT M2, kN·m				
PAVEMENT	**P-Delta**	**ULS**	$ULS \times 0.95\gamma_z$	$\frac{(ULS\times 0.95\gamma_z)-P\Delta}{P\Delta}$ **(%)**
1	54.7	46.6	54.4	0,52%
2	62.5	55.1	64.3	2,91%
3	57.7	51.7	60.3	4,48%
4	55.1	50.1	58.5	6,20%
5	49.3	43.9	51.2	3,91%
6	46.1	42.2	49.2	6,65%
7	42.3	39.5	46.1	8,96%
8	39.0	37.1	43.3	11,03%
9	35.5	33.1	38.6	8,77%
10	30.8	29.7	34.6	12,25%
11	26.0	25.6	29.9	14,84%
12	22.7	22.7	26.5	16,45%
GROUP 2 – BENDING MOMENT M2, kN·m				
PAVEMENT	**P-Delta**	**ULS**	$ULS \times 0.95\gamma_z$	$\frac{(ULS\times 0.95\gamma_z)-P\Delta}{P\Delta}$ **(%)**
1	67.5	55.6	64.9	3,84%
2	57.7	44.3	51.7	10,29%

3	52.0	41.0	47.8	8,14%
4	46.0	36.7	42.9	6,81%
5	38.0	30.0	35.0	8,12%
6	31.9	25.6	29.9	6,43%
7	25.7	21.1	24.6	4,00%
8	19.8	16.5	19.3	2,34%
9	16.5	13.6	15.9	3,66%
10	10.1	8.5	9.9	1,96%
11	4.0	3.3	3.9	2,89%
12	-2.2	-2.3	-2.7	24,97%

GROUP 3 – BENDING MOMENT M2, kN·m				
PAVEMENT	**P-Delta**	**ULS**	$ULS \times 0.95\gamma_z$	$\frac{(ULS\times 0.95\gamma_z)-P\Delta}{P\Delta}$ **(%)**
1	95.8	82.8	96.7	0,86%
2	111.9	101.6	118.5	5,91%
3	103.7	95.3	111.2	7,23%
4	99.9	93.2	108.8	8,90%
5	101.4	92.4	107.8	6,30%
6	95.5	89.3	104.2	9,08%
7	89.9	85.5	99.8	10,99%
8	85.8	82.9	96.7	12,75%
9	73.9	70.1	81.8	10,73%
10	68.9	67.3	78.5	13,87%
11	61.7	61.2	71.4	15,69%
12	59.7	59.7	69.6	16,60%

GROUP 4 – BENDING MOMENT M2, kN·m				
PAVEMENT	**P-Delta**	**ULS**	$ULS \times 0.95\gamma_z$	$\frac{(ULS\times 0.95\gamma_z)-P\Delta}{P\Delta}$ **(%)**
1	93.7	77.6	90.5	3,37%
2	68.3	49.8	58.1	14,84%
3	61.9	46.6	54.4	12,09%
4	53.5	40.8	47.6	11,09%
5	50.5	38.3	44.7	11,56%
6	41.2	31.7	37.0	10,30%
7	32.3	25.5	29.7	8,12%
8	23.6	18.7	21.8	7,52%
9	17.9	14.2	16.6	7,32%
10	9.6	7.5	8.8	9,16%
11	2.1	1.1	1.3	35,70%
12	-6.4	-6.6	-7.7	21,08%

As an additional comment, it can be seen that the NBR 6118 (2014) criterion gives better results in the lower storeys that in the lower ones, probably because the horizontal displacements are relatively smaller in the lower floors.

9.1.1.1. VERIFICATION OF THE SLS – COMPARATIVE DISPLACEMENT ANALYSIS

Again, it is necessary to verify the SLS according to the Brazilian standard. Using the same criteria as before (Equations 16 and 17 from 4.2.3), it is seen that the requirements are fulfilled.

It is also seen that the relative displacements are higher in the lower floors than in the upper ones, as suggested from the results from Table 9-3.

Table 9-4 – Displacements of the various pavements, both absolute and relative, in meters.

Pavement	OutputCase	CaseType	δ	$\frac{\delta_i - \delta_{i-1}}{h}$
1	SLS	Combination	0.001245	0.00042
2			0.002541	0.00043
3			0.003746	0.00040
4			0.004859	0.00037
5			0.006195	0.00045
6			0.007378	0.00039
7			0.008400	0.00034
8			0.009265	0.00029
9			0.010333	0.00036
10			0.011115	0.00026
11			0.011603	0.00016
12			0.011801	0.00007

9.1.2. GLOBAL ANALYSIS ACCORDING TO EN 1992-1-1

Following the same procedure as in 5.2, the first thing to be done is checking if global second order effects can be dismissed (Equation 32).

So, using the first value of stiffness, $\sum E_{cd} I_c = 309\ 219\ kNm^2$:

$$0.31 \times \frac{12}{12 + 1.6} \times \frac{309\ 219}{36^2} = 65.3\ kN < F_{V,Ed} \quad (71)$$

As the condition is not met, global second order effects cannot be ignored:

$$F_{V,B} \cong 7.8 \cdot \frac{12}{12+1.6} \cdot \frac{1}{1+0.7\times 0} \cdot \frac{0.40 \times 309\ 219}{36^2} \Leftrightarrow F_{V,B} = 656.8\ kN \qquad (72)$$

$$F_{magnification} = \frac{1}{1-\frac{70\ 010}{656.8}} = -0.009 \qquad (73)$$

Once again, the result is not valid. So, the same steps are repeated, but now using the other previously calculated value of stiffness, $E_{cd}I_c = 88\ 105\ 947\ kN \cdot m^2$:

$$0.31 \times \frac{12}{12+1.6} \times \frac{88\ 105\ 947}{36^2} = 18\ 595\ kN < F_{V,Ed} \qquad (74)$$

Just like in the Brazilian standard, global second order effects cannot be ignored:

$$F_{V,BB} = 7.8 \cdot \frac{12}{12+1.6} \cdot \frac{1}{1+0.7\times 0} \cdot \frac{0.40 \times 88\ 105\ 947}{36^2} = 187\ 153\ kN \qquad (75)$$

$$F_{magnification} = \frac{1}{1-\frac{70\ 010}{187\ 153}} = 1.598 \qquad (76)$$

The result is now plausible and this value means that the analysis of the building considering global second order effects must include the multiplication of horizontal forces by the factor of 1.598.

9.2. ADDING TWO RIGID CORES TO THE NEW MODEL

9.2.1. CORE MODELLING

Although modelling in shell elements returns good results, the analysis is way more complex, as the output is given as tension stress in meshing points instead of force in different sections, as it is desirable for the structural design (Ramilo, 2009 and Machado, 2010). So, for this reason, the core was modelled using frame elements.

The behaviour of the core will be simulated using vertical bars linked by horizontal rigid bars that guarantee the Bernoulli's Hypothesis of the planes sections. These rigid links will ensure the linear operation of the vertical core frames and consequently the proper model response.

The vertical bars will have the same dimensions of the core walls, and the rigid sections are placed at each floor, modelled using frame elements with the dimensions of 1 m x 1 m and elasticity modulus of 30 000 GPa, but with no weight, as this is only fictional. All the core walls are perfectly fixed to the ground.

To maximize the torsional stiffness of the building, two rigid RC cores, 25 cm thick will be added where the slabs L02 and L14 used to be (see Figure 3-2). The final structure is shown in Figure 9-1.

It is important to notice that by adding the cores some columns will no longer exist. All columns from Group 2 (G2-00, G2-12, G2-24) will disappear, while Group 4 will be reduced from eight to only four columns – the ones in the centre of the building, supporting slab L08 (see Figure 3-2).

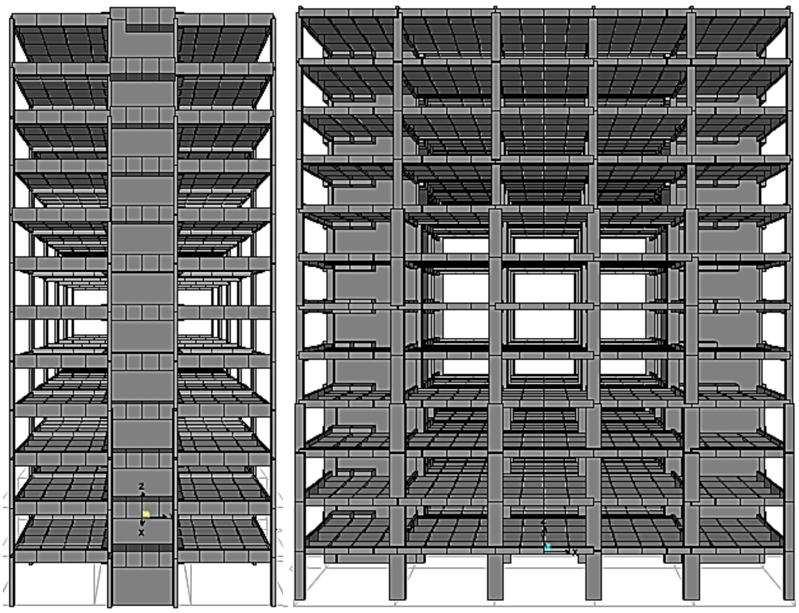

Figure 9-1 – Structural model of the building with the two cores.

9.2.2. GLOBAL ANALYSIS ACCORDING TO NBR 6118 (2014)

Just like in 4.2.1, γ_z can be calculated through the displacements and moments obtained by a SAP2000 computational analysis, with results shown below:

Table 9-5 – Model's structure output: Base reaction forces, in kN

OutputCase	Global FX	Global FY	Global FZ	Global MX	Global MY	Global MZ
ULS	0.00	-1005	75 358	20 202	0.00	0.00
P-DELTA	0.00	-1005	75 358	20 361	0.00	0.00

As expected, the weight increases from 70 010 kN to 75 358 kN, while only the global moment MX changes with the P-Delta effect.

The joint displacements are shown below and it is possible to see that the second order effects cause a displacement amplification of approximately 1.01, instead of 1.22 found previously. This means that the cores reduced the lateral displacements, as it was expected.

Table 9-6 – Joint displacements in the Y direction, in the various pavements, in meters.

Pavement	**Output Case**	**Case Type**	**Step Type**	δ
1	P-Delta	NonStatic	Max	0.00013
	ULS	Combination		0.00013
2	P-Delta	NonStatic	Max	0.00037
	ULS	Combination		0.00036
3	P-Delta	NonStatic	Max	0.00068
	ULS	Combination		0.00067
4	P-Delta	NonStatic	Max	0.00105
	ULS	Combination		0.00104
5	P-Delta	NonStatic	Max	0.00147
	ULS	Combination		0.00146
6	P-Delta	NonStatic	Max	0.00192
	ULS	Combination		0.00190
7	P-Delta	NonStatic	Max	0.00239
	ULS	Combination		0.00237
8	P-Delta	NonStatic	Max	0.00287
	ULS	Combination		0.00284
9	P-Delta	NonStatic	Max	0.00335
	ULS	Combination		0.00332
10	P-Delta	NonStatic	Max	0.00383
	ULS	Combination		0.00379
11	P-Delta	NonStatic	Max	0.00430
	ULS	Combination		0.00426
12	P-Delta	NonStatic	Max	0.00476
	ULS	Combination		0.00471

Calculating γ_z taking the same considerations as before:

$$\gamma_z = \frac{1}{1 - \frac{121.6}{14\,430}} = 1.008 \quad (77)$$

As $\gamma_z = 1.008 < 1.1$, the structure is non-sway. So, there is no need to consider global second order effects.

It can be concluded that adding two cores reduced γ_z from 1.228 to 1.008, which made disregarding global second order effects possible.

9.2.3. GLOBAL ANALYSIS ACCORDING TO EN 1992-1-1

By adding the two cores, the stiffness of the building has increased significantly. So, according to Eurocode 2 it can be now calculated as:

$$I_{bracing} = 2 \times \left(\frac{6.00 \times 4.00^3}{12} - \frac{5.50 \times 3.50^3}{12} \right) = 24.70\ m^4 \Rightarrow$$
$$\Rightarrow \sum E_{cd} I_c = 28\,400\,000 \times 24.70 = 701\,362\,098\ kN \cdot m^2 \tag{78}$$

So, making the same calculations as before, but considering the new weight and stiffness:

$$0.31 \times \frac{12}{12 + 1.6} \times \frac{701\,362\,098}{36^2} = 148\,027\ kN > F_{V,Ed} = 75\,358\ kN \tag{79}$$

Adding the two cores made the building rigid enough to handle global second order effects without any force magnification factor. However, if this multiplier were to be calculated it would be:

$$F_{V,BB} = 7.8 \cdot \frac{12}{12 + 1.6} \cdot \frac{1}{1 + 0.7 \times 0} \cdot \frac{0.40 \times 701\,362\,098}{36^2} \Leftrightarrow F_{V,BB} = 1\,489\,821\ kN \tag{80}$$

$$F_{magnification} = \frac{1}{1 - \frac{75\,358}{1\,489\,821}} = 1.053 \tag{81}$$

With the cores, the value of $F_{magnification}$ was reduced to almost equal to the unity, instead of 1.598.

10.FINAL CONSIDERATIONS

10.1. CONCLUSIONS

After this thorough comparative analysis, it is possible to conclude that the four codes herein analysed have many processes in common, and some particularities that must be considered when designing.

Regarding global second order effects, while NBR 6118 demands verification for both limit states, EN 1992-1-1 only deals with the ULS. However, both *fib* Model Code 2010 and ACI 318M-14 do not present methods for the consideration of global second order effects, only dealing with isolated elements.

For the global ULS analysis, NBR 6118 allows the designer to choose between a P-Delta computational analysis and a moment multiplying parameter γ_Z, as long as some conditions are met. However, Eurocode 2 considers the effect of a horizontal force multiplier and do not compel the designer to use the P-Delta method.

After the global analyses it was seen that in this building there was no need to consider global second order effects, for the acting wind load. The Brazilian code parameter was $\gamma_Z = 1.008$ and although there was no need to calculate the European multiplier, it had a similar value: $F_{magnification} = 1.053$. This difference may be due to different values of elasticity modulus, or to the difficulty of properly defining the stiffness of the building.

Because the building in analysis was unlike the most common buildings in Europe, as it did not have any rigid cores or walls, and all columns were oriented in the same direction, some important considerations were not clearly found in Eurocode 2 and assumptions had to be made about its stiffness. So, as it became clear that the stiffness of a building is a very important factor that must be properly defined, a case study was analysed in Chapter 0 to understand its importance.

Just by changing the orientation of the columns, global second order effects became a problem to be dealt with. The Brazilian parameter increased to $\gamma_Z = 1.228$ and turned out to be a good evaluator for global second order effects, as the consequent results of the normative approximation presented no major differences from the P-Delta analysis. On the other hand, the European multiplier increased in about 50%, as $F_{magnification} = 1.598$. Again, this difference may again be due to different values of elasticity modulus or to the difficulty of properly defining the stiffness of the building.

It was possible to see that considering only the stiffness of the columns is a wrong assumption that does not return accurate results, as the stiffness of the building due to the framing system, composed of beams and columns, is much higher than the one just due to the columns.

When the two rigid cores were added to the new model with increased flexibility, the global second order effects could be ignored again, making clear the importance of bracing elements. For the Brazilian model, the amplification of displacements was reduced from 1.22 to 1.01, while the parameter γ_Z passed from 1.228 to 1.008. For the European model, although there was no need for this calculation, it was seen that $F_{magnification}$ was reduced from 1.598 to 1.053.

As far as local second order effects are concerned, it was shown that only *fib* and Eurocode consider the geometric imperfections the same way. However, both NBR 6118 and ACI 318 evaluate the effects of these imperfections through a separate analysis, instead of adding the moments due to the imperfections to the first order moments.

Also, the effective length of the columns is a significant parameter, since it is important in the definition of other fundamental factors. Although it is considered in the same way in EN 1992-1-1 and *fib* Model Code, NBR 6118 and ACI 318M-14 define it differently. While the first only takes into consideration the dimensions of the adjacent beams, the latter resorts to a chart that considers the relative stiffness of both column ends.

While analysing the standards it was possible to notice some similarities in the way of dealing with end moments. NBR 6118 uses the criterion α_b (item 15.8.2), EN1992-1-1 uses the parameter M_{0e} (item 5.8.8.2(2)) and ACI 318M-14 uses C_m (item 6.6.4.5.3). However, while NBR 6118 uses this criterion independently from the boundary conditions of the column, both Eurocode 2 and ACI 318 only apply their approximations when the column in analysis is non-sway. The results of the local analysis presented previously in this work can be summarized below:

Table 10-1 – Comparison of the different approximated methods for each column, in the X direction and absolute value. Moments in kN·m.

Column	**NBR 6118 (2014)**		**EN 1992-1-1**		***fib* MC 2010**		**ACI 318M-14**
	Stiffness	**Curvature**	**Stiffness**	**Curvature**	**Level I**	**Level II**	**MMM**
G1-00	44.0	55.6	46.3	79.5	97.2	67.6	25.0
G1-12	23.9	35.7	59.8	81.3	94.2	69.0	17.2
G1-24	11.3	16.9	33.3	45.5	49.2	40.2	15.3
G2-00	60.3	87.4	91.7	136.7	173.7	116.6	44.6
G2-12	38.5	55.5	149.1	132.4	168.6	112.1	29.1
G2-24	18.5	27.6	61.6	68.6	77.7	60.6	20.9
G3-00	81.0	113.9	62.3	123.5	183.2	101.7	60.7
G3-12	53.8	80.1	64.5	136.2	176.4	110.2	40.7
G3-24	26.4	39.4	24.1	67.6	76.2	55.4	20.1
G4-00	119.0	161.1	106.5	168.8	277.1	139.3	93.2
G4-12	76.9	105.0	133.3	154.4	248.2	124.8	61.1
G4-24	37.2	51.2	36.6	60.4	92.6	50.0	30.4

Table 10-2 – Comparison of the different approximated methods for each column, in the Y direction and absolute value. Moments in kN·m.

Column	NBR 6118 (2014)		EN 1992-1-1		*fib* MC 2010		ACI 318M-14
	Stiffness	Curvature	Stiffness	Curvature	Level I	Level II	MMM
G1-00	46.9	53.7	54.3	69.7	81.7	78.1	40.8
G1-12	26.7	31.7	56.9	73.1	75.8	71.8	35.1
G1-24	11.9	14.6	37.6	45.0	45.5	43.5	24.9
G2-00	91.1	99.8	91.1	108.0	144.0	139.8	90.0
G2-12	46.3	52.9	43.3	61.5	72.7	66.3	58.0
G2-24	19.4	23.8	17.8	28.7	31.1	27.4	18.4
G3-00	135.2	144.4	175.2	201.0	264.7	252.7	178.4
G3-12	81.3	89.7	198.7	270.2	284.0	268.4	119.2
G3-24	33.1	37.9	146.8	172.1	172.5	170.9	94.0
G4-00	230.4	240.2	240.0	263.7	392.7	376.8	229.1
G4-12	116.2	125.4	104.9	147.2	193.9	167.3	113.5
G4-24	41.5	47.4	39.7	54.9	67.7	55.8	39.0

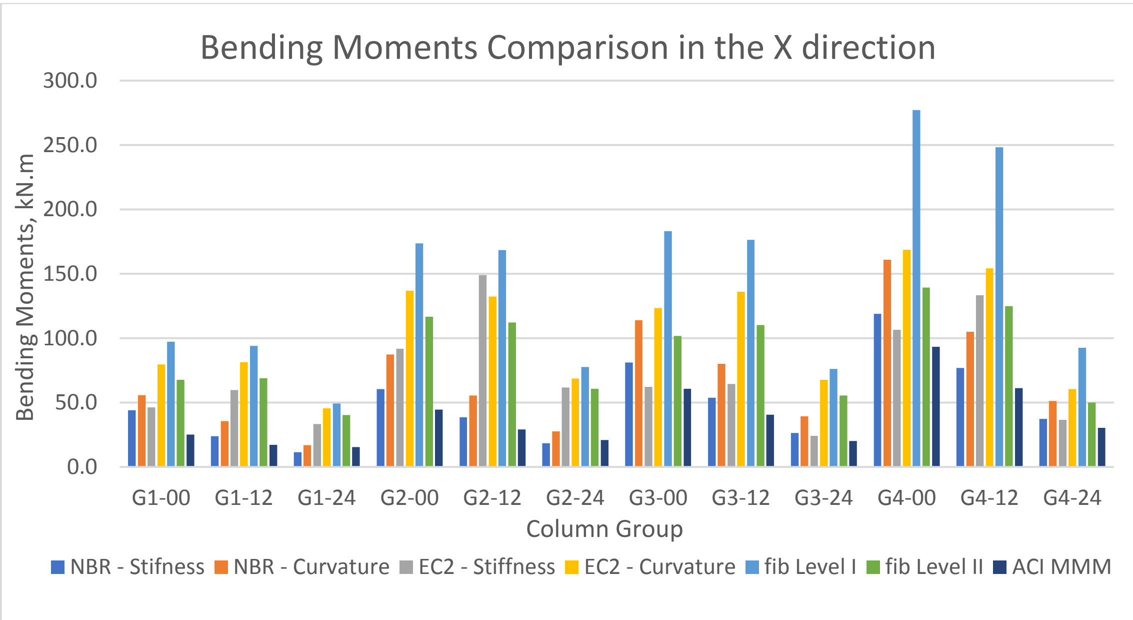

Figure 10-1 – Bending moments comparison in the X direction.

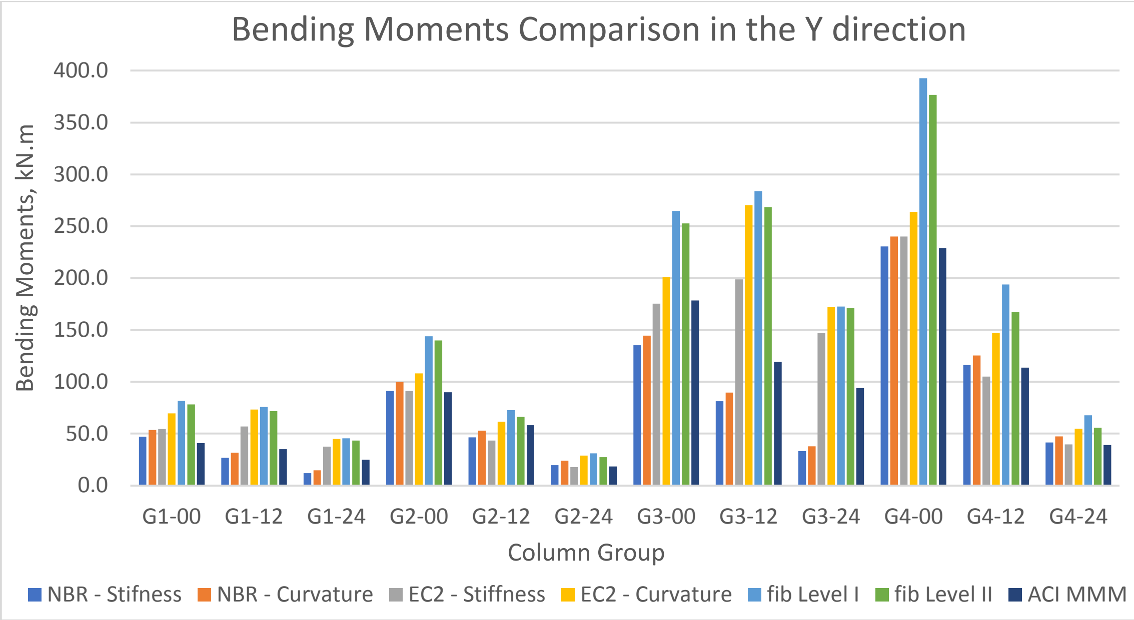

Figure 10-2 – Bending moment comparison in the Y direction.

It can be seen that for both NBR 6118 and Eurocode 2 standards, the Nominal Curvature method returns more conservative values than the Nominal Stiffness method, i.e., the Nominal Curvature methods will result in higher safety factors, when compared with more refined methods.

The discrepancy of results found between Brazilian, American and European codes occurs because the latter adds the bending moments due to geometric imperfections to the first order moments due to the wind load. Also, the three codes have different criteria for defining the column effective length.

As said in item 5.3.7, the difference between the two European methods may be due to the basic approximations made in the Nominal Stiffness methods. It can be also seen that the Level II *fib* approach is very similar to the curvature method of EN 1992-1-1, and that bending moments present lower, more economic values, as long as hypotheses are refined from Level I to Level II

Table 10-3 resumes the chosen reinforcements for the four cases, and the same results can be shown in graphic form in Figure 10-3:

Table 10-3 – Summary of the chosen reinforcements for the differente codes.
Area of reinforcement in cm^2.

COLUMN	NBR 6118		EN 1992-1-1		*fib* Model Code		ACI 318M	
	A_S	ρ	A_S	ρ	A_S	ρ	A_S	ρ
G1-00	4.91	0.45%	19.63	1.78%	12.57	1.14%	4.91	0.45%
G1-12	4.91	0.61%	29.45	3.68%	18.85	2.36%	4.91	0.61%
G1-24	7.36	1.23%	19.63	3.27%	18.85	3.14%	4.91	0.82%
G2-00	12.27	0.77%	39.27	2.45%	25.13	1.57%	4.91	0.31%
G2-12	9.82	0.98%	49.09	4.91%	37.70	3.77%	4.91	0.49%
G2-24	7.36	1.23%	29.45	4.91%	18.85	3.14%	4.91	0.82%
G3-00	24.13	1.27%	29.45	1.55%	25.13	1.32%	4.91	0.26%
G3-12	8.04	0.50%	49.09	3.07%	37.70	2.36%	4.91	0.31%
G3-24	8.04	0.73%	19.63	1.78%	18.85	1.71%	4.91	0.45%
G4-00	68.72	2.86%	49.09	2.05%	43.98	1.83%	37.70	1.57%
G4-12	29.45	1.84%	58.90	3.68%	43.98	2.75%	18.85	1.18%
G4-24	19.63	2.45%	19.63	2.45%	18.85	2.36%	12.57	1.57%

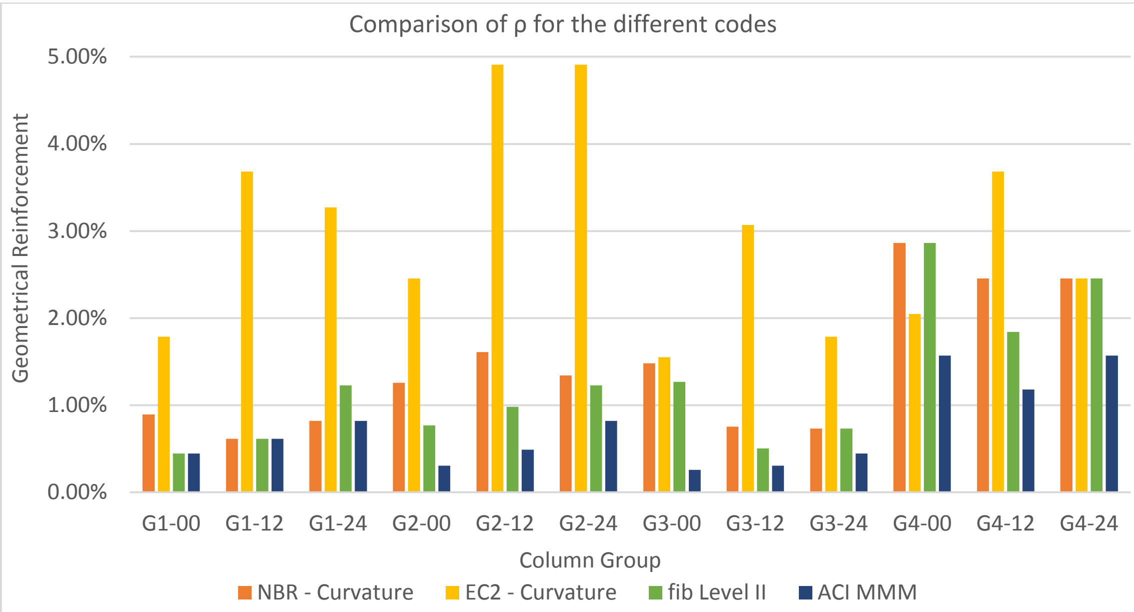

Figure 10-3 – Comparison of ρ for the different codes.

The American code presents the lowest bending moments and consequently the less reinforced sections. On the other hand, the Level II of Approximation gives the highest bending moments.

Unlike what happened for the other two codes, for both NBR 6118 and ACI 318 the biaxial bending was not decisive in the reinforcement design, as the conditioning moments were due the geometric imperfections and those bending moments do not need to be verified against biaxial bending. So, the needed reinforcement is lower.

It is important to notice that although the values of bending moments for EC 2 and MC 2010 are very close, the first presents higher moments in the X direction, which is more conditioning for the biaxial bending, as $M_{Rd,Y} \gg M_{Rd,X}$, and therefore will lead to higher reinforcing.

After performing the nonlinear analyses, it was seen that, for this case, the physically nonlinearity does not add much significance, as it returned values only 12% higher, in average, than the ones obtained from a simple P-Delta analysis. For this case, the geometric imperfections turned out to be decisive, with much more importance than the nonlinear effects of RC.

10.2. SUGGESTIONS FOR FUTURE WORKS

The trend of future structural design codes will certainly be in favour of using nonlinear design analysis. If so:

- Approximated methods like the ones analysed in this work should see their usage restricted to isolated elements, which correspond to practical situations such as bridge columns during constructive phases or columns of industrial buildings;
- New studies should be conducted to recalibrate the parameters to be adopted in a geometric and physically nonlinear analysis, or even think of new forms of safety designing;
    - It is important to refer that MC 2010 already has developed something on this subject as it presents chapters that aid to verify the robustness of the structure, or to verify the safety of the structure aided by numerical simulation or testing.
- New studies for the proposal and calibration of the effective stiffness's to be used in P-Delta analyses, as this is a considerably simpler and much more accessible method;
- To better understand the effects of geometric imperfections, it is also important to extend and complement the existing information for isolated elements, with indications for values to be considered in global analyses such as P-Delta or nonlinear considering both nonlinearities.

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APPENDIX A – RELATIONSHIP BETWEEN α AND NOMINAL BUCKLING LOAD

In the item 4.2.2, it was introduced an instability parameter α used in the Brazilian standard. During the writing of this work it was noted that there was a relationship between this parameter and the buckling load.

In a structure of a building, the vertical loads do not act like a concentrated force at the top but like an evenly distributed load. In this case, it is demonstrated (Timoshenko and Gere, 1963) that the critical value of the buckling load is given by:

$$(ql)_{cr} = \frac{7.837EI}{H^2} \Rightarrow EI = \frac{(ql)_{cr} \cdot H^2}{7.837}$$

By putting $\alpha = 0.50$ it is possible to show that

$$\alpha = H_{tot}\sqrt{\frac{N_k}{E_{cs}I_c}} \Rightarrow 0.50 = H\sqrt{\frac{N_k}{EI}} \Leftrightarrow 0.25 = H^2\frac{N_k}{EI}$$

Replacing that in the equation above:

$$0.25 = \frac{H^2N_k}{\frac{(ql)_{cr} \cdot H^2}{7.837}} \Leftrightarrow (ql)_{cr} = 31.35N_k \Leftrightarrow N_k = 0.032(ql)_{cr}$$

So, if $\alpha = 0.50$ the total vertical load will be approximately 3% of the resulting distributed buckling load.

It is very important to notice the difference between this case and the one of a cantilever column with only a concentrated top load. The critical load is three times lower when compared with the distributed load:

$$P_f = \frac{\pi^2EI}{(2H)^2} \Rightarrow 0.25 = \frac{\pi^2}{4P_f}N_k \Leftrightarrow P_f = \pi^2N_k \Leftrightarrow N_k \cong 0.10 \cdot P_f$$

APPENDIX B – EXAMPLE OF LOCAL SECOND ORDER EFFECTS ANALYSIS FOR NBR 6118 (2014)

As stated before, this and the three following appendixes will show the explicit calculations for local second order effects, for column G1-00, with $b = 0.55\,m$ and $h = 0.20\,m$, making $A_c = 0.11\,m^2$.

According to the Brazilian standard, it is initially necessary to calculate the effective length and the slenderness coefficient in each direction:

$$\begin{cases} l_{free,X} = 3 - 0.45 = 2.55\,m \\ l_{free,Y} = 3 - 0.80 = 2.20\,m \end{cases} \Rightarrow \begin{cases} l_{e,X} = \min\{2.55 + 0.45; 2.55 + 0.20\} = 2.75\,m \\ l_{e,Y} = \min\{2.20 + 0.80; 2.20 + 0.55\} = 2.75\,m \end{cases}$$

$$\begin{cases} \lambda_X = 2.75 \times \dfrac{\sqrt{12}}{0.20} = 47.63 > 35 \\ \lambda_Y = 2.75 \times \dfrac{\sqrt{12}}{0.55} = 17.32 < 35 \end{cases}$$

NBR 6118 (2014) considers 35 to be the upper limit for very short columns, so there is no need to check for second order effects due to the wind forces in the Y direction, only in the X direction.

Through a SAP2000 computational analysis it was possible to obtain the forces in column G1-00, which gives the following relations:

$$\begin{cases} N_d = -1401\,kN \\ M_{X,A} = -23.3\,kN \cdot m \\ M_{X,B} = 12.4\,kN \cdot m \\ M_{Y,A} = 44.1\,kN \cdot m \\ M_{Y,B} = -38.8\,kN \cdot m \end{cases} \xrightarrow{X\ direction} \begin{cases} e_1 = \dfrac{M_{X,A}}{N_d} = \dfrac{23.3}{1401} = 0.017\,m \\ \dfrac{M_{X,B}}{M_{X,A}} = \dfrac{12.4}{-23.3} = -0.53 \\ \alpha_b = 0.60 + 0.40\dfrac{M_B}{M_A} = 0.39 \xrightarrow{\alpha_b \geq 0.40} \alpha_b = 0.40 \end{cases}$$

With the values above, it is possible to calculate the limit slenderness in the X direction:

$$\lambda_{1,X} = \frac{25 + 12.5 \times \frac{0.017}{0.20}}{0.40} = 65.10 > \lambda_X = 47.53$$

So, as $\lambda_{1,X} > \lambda_X$, local second order effects due to the wind forces may be disregarded in both directions. However, the check for biaxial bending is necessary, as shown in the end of this Appendix.

It is necessary to evaluate the local geometric imperfections, through the minimum bending moment:

$$\begin{cases} M_{1d,min,X} = 1401 \times (0.015 + 0.03 \times 0.20) = 29.41\,kN \cdot m \\ M_{1d,min,Y} = 1401 \times (0.015 + 0.03 \times 0.55) = 44.12\,kN \cdot m \end{cases}$$

This result gives the eccentricity due to imperfections:

$$\begin{cases} e_{1,X} = \dfrac{29.41}{1401} = 0.021\ m \\ e_{1,Y} = \dfrac{44.12}{1401} = 0.032\ m \end{cases}$$

The limit slenderness for the minimum bending moments is:

$$\begin{cases} \lambda_{1,X} = \dfrac{25 + 12.5 \cdot \frac{0.021}{0.20}}{1.00} = 26.3 \Rightarrow \lambda_{1,X} = 35 \\ \lambda_{1,Y} = \dfrac{25 + 12.5 \cdot \frac{0.032}{0.55}}{1.00} = 25.71 \Rightarrow \lambda_{1,Y} = 35 \end{cases}$$

For the minimum moment, the coefficient α_b is always equal to the unity, $\alpha_b = 1$.

As the limit slenderness is very low in both directions, local second order effects due to the minimum moment must be considered in X and in Y.

The design moment according to the method of the standard-column with approximated stiffness can be obtained as the root of a second-degree equation, with the coefficients found below:

$$X\ direction: \begin{cases} A = 5 \times 0.20 = 1.00 \\ B = 1401 \times 0.20^2 - \dfrac{1401 \times 2.75^2}{320} - 5 \times 0.20 \times 1.00 \times 29.41 = -6.49 \\ C = -1436 \times 0.20^2 \times 1.00 \times 29.41 = -1648 \end{cases}$$

$$Y\ direction: \begin{cases} A = 5 \times 0.55 = 2.75 \\ B = 1401 \times 0.55^2 - \dfrac{1401 \times 2.75^2}{320} - 5 \times 0.55 \times 1.00 \times 44.12 = 269.3 \\ C = -1401 \times 0.55^2 \times 1.00 \times 44.12 = -18692 \end{cases}$$

So, the design bending moments according to this stiffness-based method are:

$$A{M_{d,tot}}^2 + BM_{d,tot} + C = 0 \Leftrightarrow \begin{cases} M_{d,tot,X} = 44.0\ kN \cdot m \\ M_{d,tot,Y} = 46.9\ kN \cdot m \end{cases}$$

Now, using the method of the standard-column with approximated curvature, the normal dimensionless force is:

$$\upsilon = \frac{1401}{0.11 \times \frac{35000}{1.40}} = 0.509$$

The curvature in the critical section will be:

$$\begin{cases} \left(\dfrac{1}{r}\right)_X = \dfrac{0.005}{0.20 \times (0.509 + 0.5)} = 0.0248 \\ \left(\dfrac{1}{r}\right)_Y = \dfrac{0.005}{0.55 \times (0.509 + 0.5)} = 0.0090 \end{cases}$$

Thus, the design moment according to this method will be:

$$\begin{cases} M_{d,tot,X} = 1.00 \times 29.41 + 1401 \times \dfrac{2.75^2}{10} \times 0.0248 = 55.6\ kN \cdot m \\ M_{d,tot,Y} = 1.00 \times 44.12 + 1401 \times \dfrac{2.75^2}{10} \times 0.0090 = 53.7\ kN \cdot m \end{cases}$$

The interaction diagrams are given below, using the Excel spreadsheet described in 4.3.8:

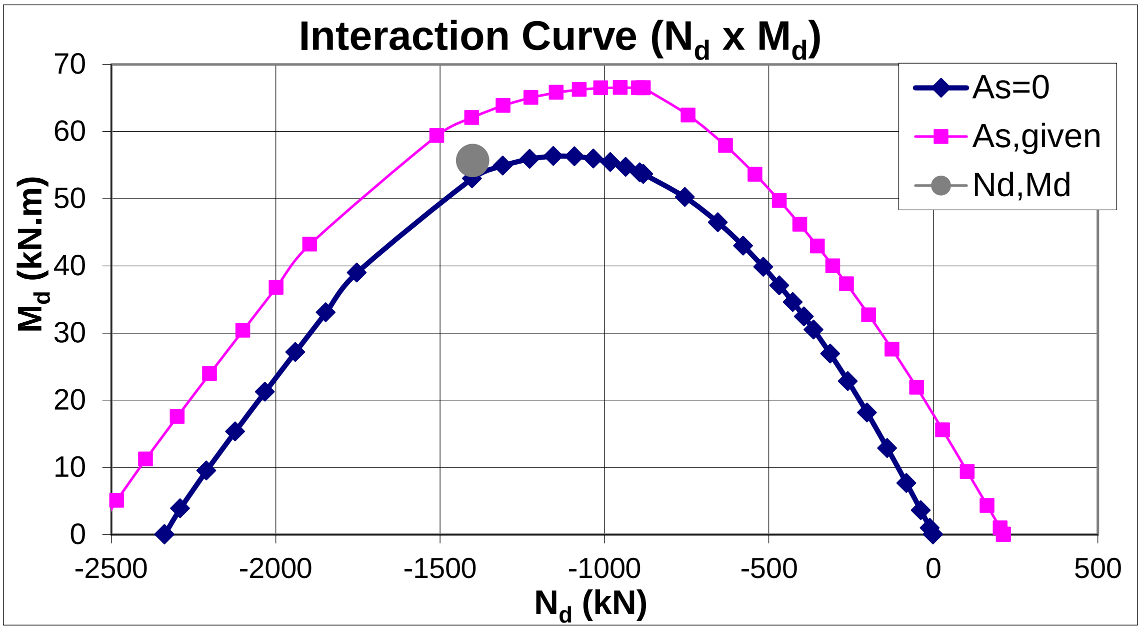

Figure B-1 – Concrete interaction diagram for X direction, NBR 6118 (2014).

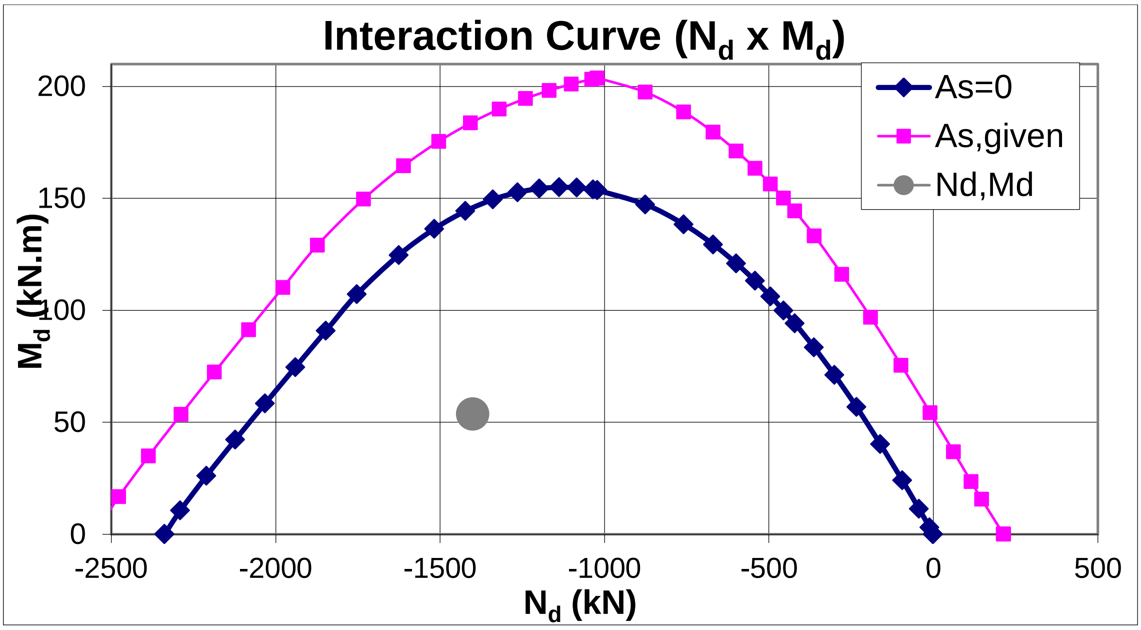

Figure B-2 – Concrete interaction diagram for Y direction, NBR 6118 (2014).

Note that although the minimum reinforcement could be sufficient for each direction separately, biaxial bending requires a bigger area of reinforcement. The chosen reinforcement was 4Φ12.5 mm ($A_S = 4.91\ cm^2$). The section detailing is shown below:

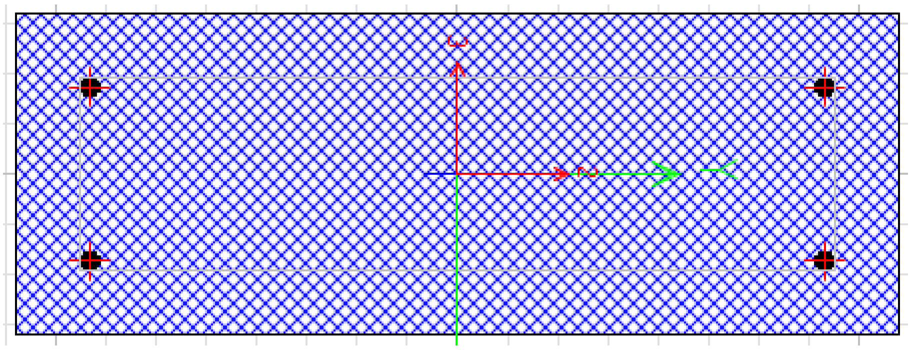

Figure B-3 – Cross section detail for the NBR 6118 (2014). (Computers and Structures, 2016).

For this reinforcement, the resistant moments will be:

$$\begin{cases} M_{Rd,X} \cong 62\ kN \cdot m \\ M_{Rd,Y} \cong 182\ kN \cdot m \end{cases}$$

According to NBR 6118, the minimum moments due to geometric imperfections do not require the verification of the biaxial bending. So, verifying the biaxial bending for the results of SAP 2000 for the applied moments:

$$\left(\frac{23.3}{62}\right)^{1.2} + \left(\frac{44.1}{182}\right)^{1.2} = 0.49 < 1 \text{ (OK)}$$

APPENDIX C – EXAMPLE OF LOCAL SECOND ORDER EFFECTS ANALYSIS FOR EN 1992-1-1

This analysis according to EN 1992-1-1 will use the same column from Appendix B. However, the buckling length definition is different in the European standard.

For the X direction:

$$k_{1,X} = 0 \rightarrow \text{ rigid rotational restraint}$$

$$k_{2,X} = \frac{\left(\sum \frac{EI}{L}\right)_{columns}}{\left(4\sum \frac{EI}{L}\right)_{beams}} = \frac{\frac{0.55\times0.20^3}{12}\frac{1}{3}\times 2}{\frac{0.15\times0.45^3}{12}\frac{4}{6}} = 0.322$$

$$l_{0,X} = l \times \max\left\{\sqrt{1+10\times 0}; \left(1+\frac{0}{1+0}\right)\left(1+\frac{0.322}{1+0.322}\right)\right\} = 3 \times 1.24 = 3.73$$

$$\lambda_X = l_{0,X}\frac{\sqrt{12}}{h} = 3.73 \times \frac{\sqrt{12}}{0.20} = 64.61$$

For the Y direction:

$$k_{1,Y} = 0 \rightarrow \text{ rigid rotational restraint}$$

$$k_{2,Y} = \frac{\left(\sum \frac{EI}{L}\right)_{columns}}{\left(4\sum \frac{EI}{L}\right)_{beams}} = \frac{\frac{0.20\times0.55^3}{12}\frac{1}{3}\times 2}{\frac{0.15\times0.80^3}{12}\frac{4}{6}} = 0.433$$

$$l_{0,Y} = l \times \max\left\{\sqrt{1+10\times 0}; \left(1+\frac{1}{1+0}\right)\left(1+\frac{0.433}{1+0.433}\right)\right\} = 3 \times 1.30 = 3.91$$

$$\lambda_Y = l_{0,Y}\frac{\sqrt{12}}{h} = 3.91 \times \frac{\sqrt{12}}{0.55} = 24.61$$

Through a SAP2000 computational analysis the forces in the column were obtained:

$$\begin{cases} N_d = -1375\ kN \\ M_{01,X} = 7.9\ kN\cdot m \\ M_{02,X} = -15.0\ kN\cdot m \\ M_{01,Y} = 36.7\ kN\cdot m \\ M_{02,Y} = 37.1\ kN\cdot m \end{cases}$$

The first order eccentricity is calculated as:

$$\begin{cases} e_{1,X} = \frac{M_{0e,X}}{N} = \frac{15.0}{1375} = 0.0109\ m \\ e_{1,Y} = \frac{M_{0e,Y}}{N} = \frac{37.1}{1375} = 0.0270\ m \end{cases}$$

The relative normal force is the same in both directions:

$$n = \frac{N_{Ed}}{A_c f_{cd}} = \frac{1375}{0.11 \times \frac{35000}{1.50}} = 0.536$$

Considering the default values of the coefficients A, B and C, the limit slenderness will be given by:

$$\lambda_{lim} = \frac{10.78}{\sqrt{n}} = \frac{10.78}{\sqrt{0.536}} = 14.7$$

So, as $\lambda_X, \lambda_Y > \lambda_{lim}$, local second order effects must be considered in both directions.

As defined before, the eccentricity due to local geometric imperfections are:

$$\begin{cases} e_{i,X} = \frac{l_{0,X}}{200\sqrt{3}} = \frac{3.73}{200\sqrt{3}} = 0.0108\, m \\ e_{i,Y} = \frac{l_{0,Y}}{200\sqrt{3}} = \frac{3.91}{200\sqrt{3}} = 0.0113\, m \end{cases}$$

So, the moment due to the local geometric imperfections will be:

$$\begin{cases} M_{i,X} = N_{Ed} \times e_{i,X} = 1375 \times 0.0108 = 14.8\, kN \cdot m \\ M_{i,Y} = N_{Ed} \times e_{i,Y} = 1375 \times 0.0113 = 15.5\, kN \cdot m \end{cases}$$

Thus, the final first order moment will be:

$$\begin{cases} M_{0,X} = M_{i,X} + M_{0e} = 15.0 + 14.8 = 29.8\, kN \cdot m \\ M_{0,Y} = M_{i,Y} + M_{0e} = 37.1 + 15.5 = 52.6\, kN \cdot m \end{cases}$$

Regarding the method based on the nominal stiffness, it is initially necessary to calculate the column flexural stiffness:

$$\begin{cases} K_C = (\min\{0.30 \times 0.536; 0.20\}) \times \sqrt{\frac{35}{20}} = 0.213 \\ K_S = 1 \end{cases}$$

$$\begin{cases} EI_X = 0.213 \times 28\,400\,000 \times 0.00037 + 1 \times 200\,000\,000 \times 0.000016 = 5\,381\, kN \cdot m^2 \\ EI_Y = 0.213 \times 28\,400\,000 \times 0.00277 + 1 \times 200\,000\,000 \times 0.000243 = 65\,337\, kN \cdot m^2 \end{cases}$$

The total moments are:

$$\begin{cases} M_{d,tot,X} = \dfrac{29.8}{1 - \left(1375 \Big/ \frac{10\times5\,381}{3.73^2}\right)} = 46.3\ kN\cdot m \\ M_{d,tot,Y} = \dfrac{52.6}{1 - \left(1375 \Big/ \frac{10\times65\,337}{3.91^2}\right)} = 54.3\ kN\cdot m \end{cases}$$

Now, considering the method based on the nominal curvature, the deflection in the centre of the column will be given by:

$$\begin{cases} e_{2,X} = \dfrac{1.37-0.536}{1.37-0.40}\times 1 \times \dfrac{\frac{435}{200000}}{0.45\times(0.20-0.04)}\times\dfrac{3.73^2}{10} = 0.0362\ m \\ e_{2,Y} = \dfrac{1.37-0.536}{1.37-0.40}\times 1 \times \dfrac{\frac{435}{200000}}{0.45\times(0.55-0.04)}\times\dfrac{3.91^2}{10} = 0.0124\ m \end{cases}$$

$$\begin{cases} M_{2,X} = N_{Ed}e_{2,X} = 1375\times 0.0362 = 49.7\ kN\cdot m \\ M_{2,Y} = N_{Ed}e_{2,Y} = 1375\times 0.0124 = 17.1\ kN\cdot m \end{cases}$$

So, the total moment will be:

$$\begin{cases} M_{d,tot,X} = 15.0+14.8+49.7 = 79.5\ kN\cdot m \\ M_{d,tot,Y} = 37.1+15.5+17.1 = 69.7\ kN\cdot m \end{cases}$$

The interaction diagrams are given below, using the Excel spreadsheet described in 5.3.7:

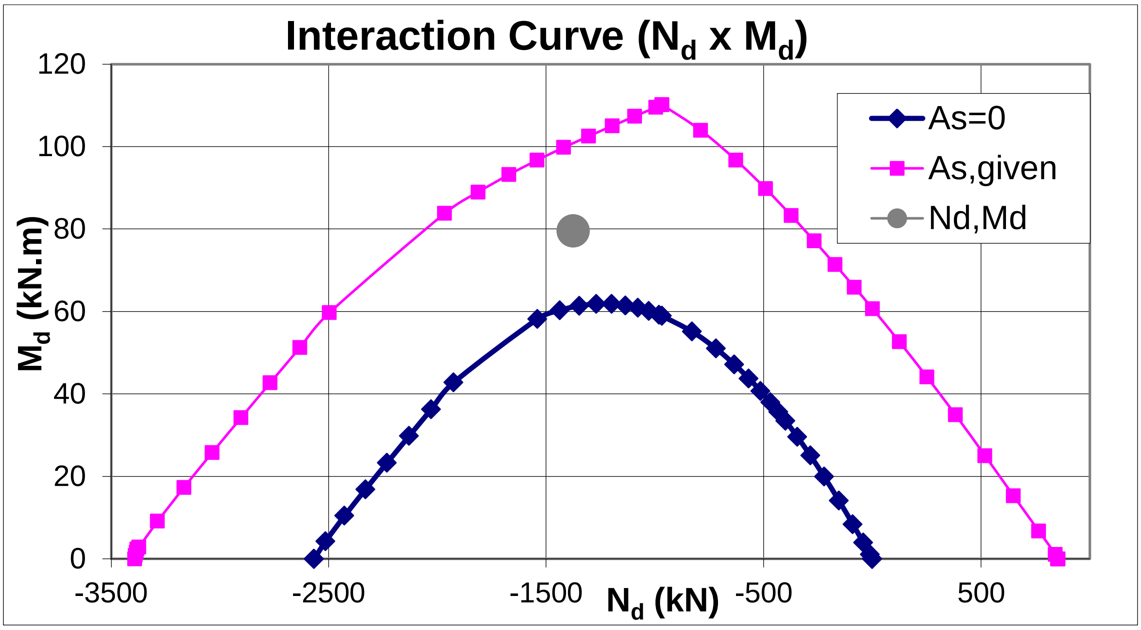

Figure C-1 – Concrete interaction diagram for X direction, for EN 1992-1-1.

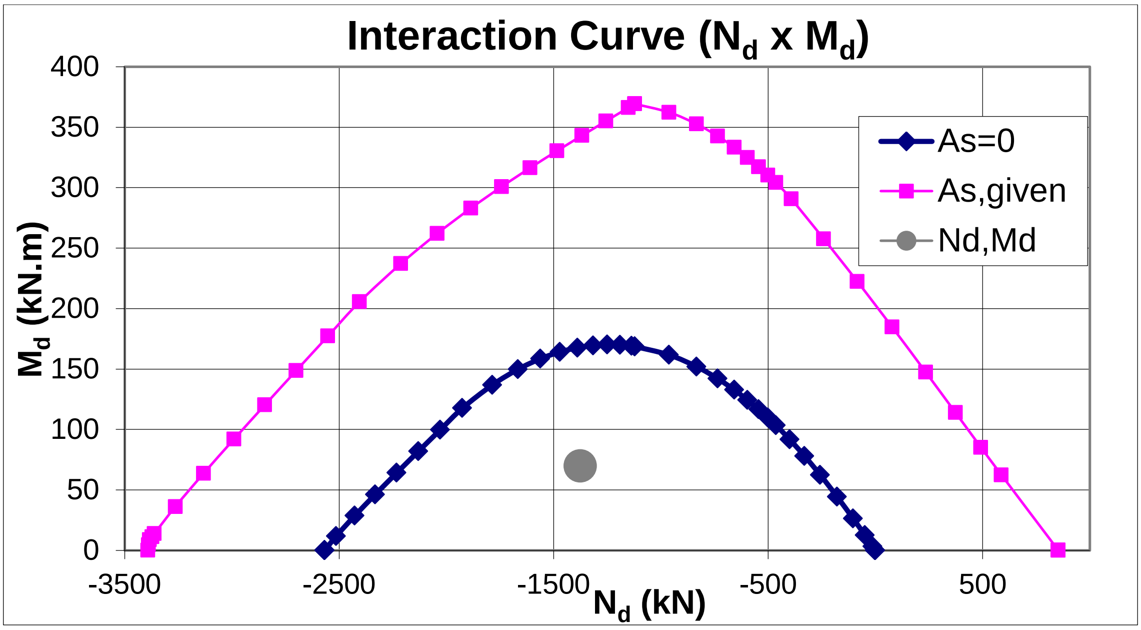

Figure C-2 – Concrete interaction diagram for Y direction, for EN 1992-1-1.

Due to the biaxial bending, the chosen reinforcement was 4Φ25 mm ($A_s = 29.45\ cm^2$), verified below. The section detailing is shown below.

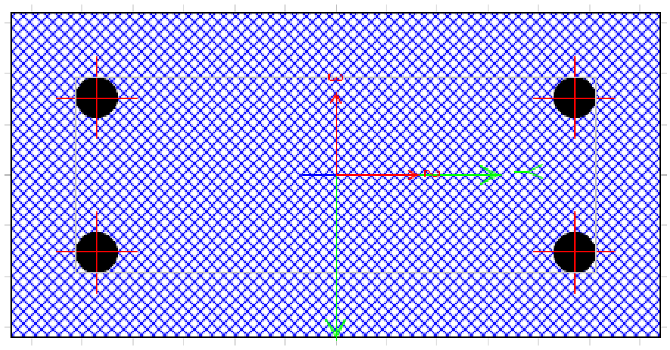

Figure C-3 – Cross section detail for the EN 1992-1-1 (Computers and Structures, 2016).

With this reinforcement, the resistant moments will be:

$$\begin{cases} M_{Rd,X} \cong 100\ kN \cdot m \\ M_{Rd,Y} \cong 340\ kN \cdot m \end{cases}$$

Verifying the biaxial bending, the design is finished.

$$\frac{N_{Ed}}{N_{Rd}} = \frac{N_{Ed}}{A_c f_{cd} + A_s f_{yd}} = \frac{1375}{0.11 \times \frac{35000}{1.50} + 0.0020 \times \frac{500000}{1.15}} = 0.401 \rightarrow a \cong 1.25$$

$$\left(\frac{79.5}{100}\right)^{1.25} + \left(\frac{69.7}{340}\right)^{1.25} = 0.89 \text{ (OK)}$$

Finally, as $\rho = \frac{0.001963}{0.11} = 1.78\% \Rightarrow \omega = \frac{\frac{500}{1.15}}{\frac{35}{1.50}} \times \frac{1.78}{100} 0.33$. This means that the choice of $\omega = 0.37$ was a good approximation.

APPENDIX D – EXAMPLE OF LOCAL SECOND ORDER EFFECTS ANNALYSIS FOR *fib* MODEL CODE 2010

This analysis according to *fib* Model Code 2010 will use the same column from Appendix B, and the same buckling length, slenderness and forces as in Appendix C.

$$\begin{cases} N_d = -1375\ kN \\ M_{01,X} = 7.9\ kN \cdot m \\ M_{02,X} = -15.0\ kN \cdot m \\ M_{01,Y} = 36.7\ kN \cdot m \\ M_{02,Y} = 37.1\ kN \cdot m \end{cases} \Rightarrow \begin{cases} e_{1,X} = \dfrac{M_{0e,X}}{N} = \dfrac{15.0}{1375} = 0.0109\ m \\ e_{1,Y} = \dfrac{M_{0e,Y}}{N} = \dfrac{37.1}{1375} = 0.0270\ m \\ n = 0.555 \end{cases}$$

For both directions, the α_i coefficient can be estimated as:

$$\begin{cases} \alpha_{i,X} = \dfrac{0.01}{\sqrt{3.73}} = 0.0052 \xrightarrow{\alpha_i \le \frac{1}{200}} \alpha_{i,X} = 0.005 \\ \alpha_{i,Y} = \dfrac{0.01}{\sqrt{3.91}} = 0.0051 \xrightarrow{\alpha_i \le \frac{1}{200}} \alpha_{i,Y} = 0.005 \end{cases}$$

The eccentricity due to imperfections are:

$$\begin{cases} e_{0d,X} = \max\left\{\dfrac{0.005 \times 3.73}{2}; \dfrac{0.20}{30}\right\} = 0.0093\ m \\ e_{0d,Y} = \max\left\{\dfrac{0.005 \times 3.91}{2}; \dfrac{0.55}{30}\right\} = 0.0183\ m \end{cases}$$

For the Level I of approximation:

$$\begin{cases} e_{2d,I,X} \cong \dfrac{435}{200000} \cdot \dfrac{3.73^2}{5 \times (0.20 - 2 \times 0.04)} = 0.0504\ m \\ e_{2d,I,Y} \cong \dfrac{435}{200000} \cdot \dfrac{3.91^2}{5 \times (0.55 - 2 \times 0.04)} = 0.0141\ m \end{cases}$$

$$\begin{cases} e_{d,I,X} = 0.0093 + 0.0109 + 0.0504 = 0.071\ m \\ e_{d,I,Y} = 0.0183 + 0.0270 + 0.0141 = 0.059\ m \end{cases}$$

$$\begin{cases} M_{d,tot,I,X} = 14375 \times 0.071 = 97.2\ kN \cdot m \\ M_{d,tot,I,Y} = 1375 \times 0.059 = 81.7\ kN \cdot m \end{cases}$$

For the Level II of approximation:

$$\begin{cases} e_{2d,II,X} = \min\left\{\left(\dfrac{1.37 - 0.536}{1.37 - 0.40}\right)\dfrac{\frac{435}{200000}}{0.45 \times 0.20}\dfrac{3.73^2}{10}; \dfrac{\frac{435}{200000}}{0.45 \times 0.20}\dfrac{3.73^2}{10}\right\} = 0.0289\ m \\ e_{2d,II,Y} = \min\left\{\left(\dfrac{1.37 - 0.536}{1.37 - 0.40}\right)\dfrac{\frac{435}{200000}}{0.45 \times 0.55}\dfrac{3.91^2}{10}; \dfrac{\frac{435}{200000}}{0.45 \times 0.55}\dfrac{3.91^2}{10}\right\} = 0.0115\ m \end{cases}$$

$$\begin{cases} e_{d,II,X} = 0.0093 + 0.0109 + 0.0289 = 0.049\ m \\ e_{d,II,Y} = 0.0183 + 0.0270 + 0.00115 = 0.057\ m \end{cases}$$

$$\begin{cases} M_{d,tot,II,X} = 1375 \times 0.049 = 67.6\ kN \cdot m \\ M_{d,tot,II,Y} = 1375 \times 0.057 = 78.1\ kN \cdot m \end{cases}$$

The Excel worksheet used on Appendix C is also used here to define the column reinforcement, considering the second level of approximation. The interaction diagrams are given below.

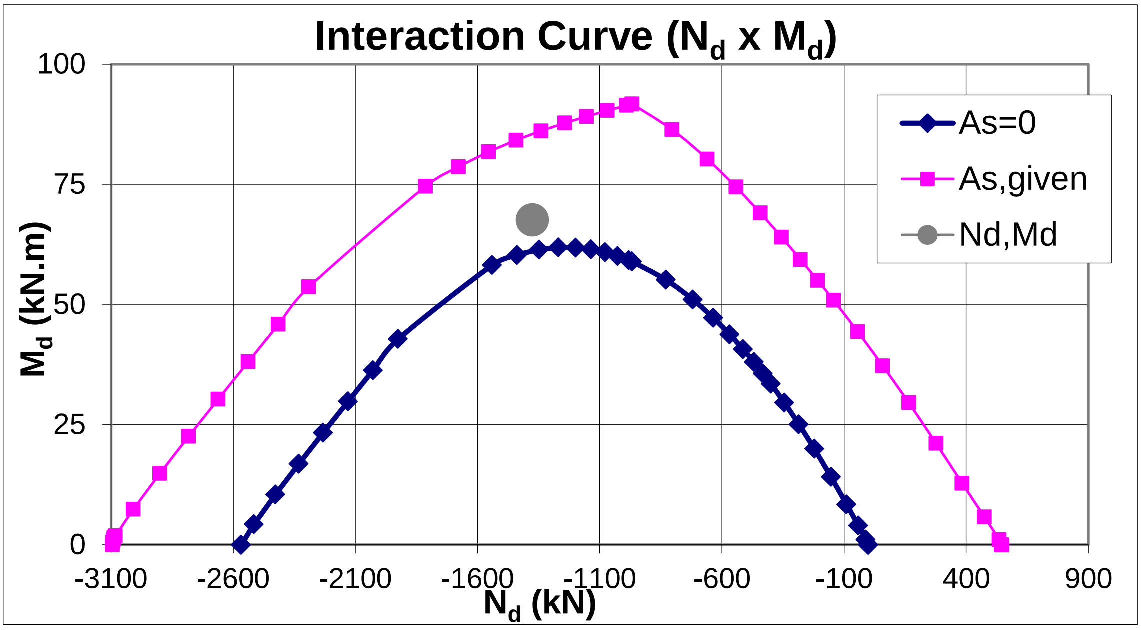

Figure D11-1– Concrete interaction diagram for X direction, for *fib* Model Code 2010.

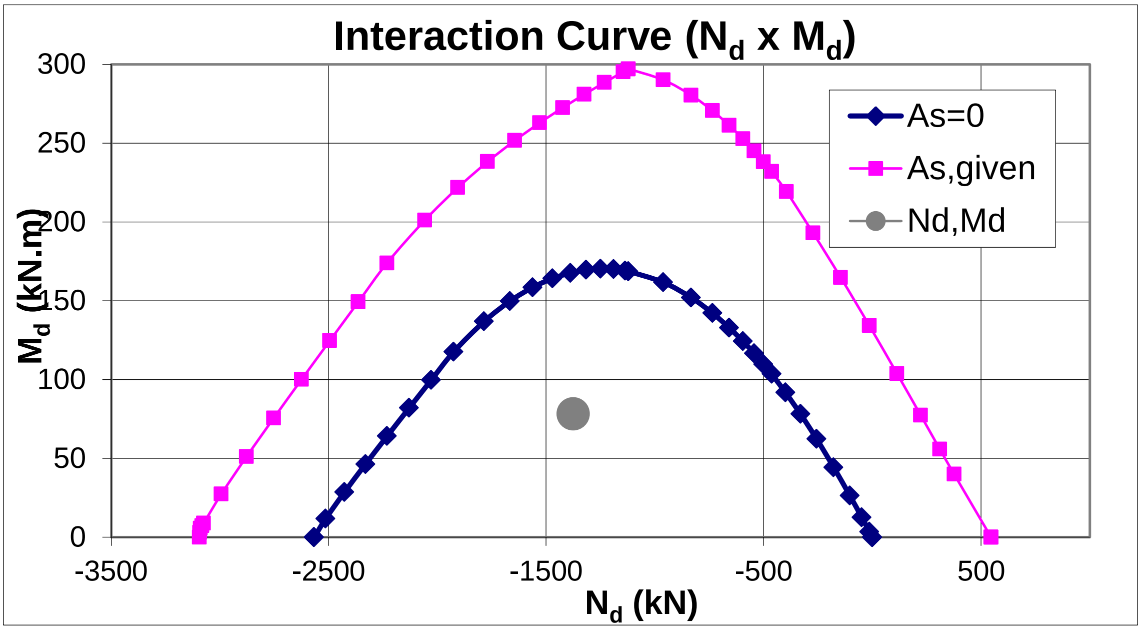

Figure D-2 – Concrete interaction diagram for Y direction, for *fib* Model Code 2010.

The chosen reinforcement is 4Φ20 mm, and the section detail is shown in Figure D-3. With this reinforcement, the resistant moments will be:

$$\begin{cases} M_{Rd,X} \cong 85 \ kN \cdot m \\ M_{Rd,Y} \cong 275 \ kN \cdot m \end{cases}$$

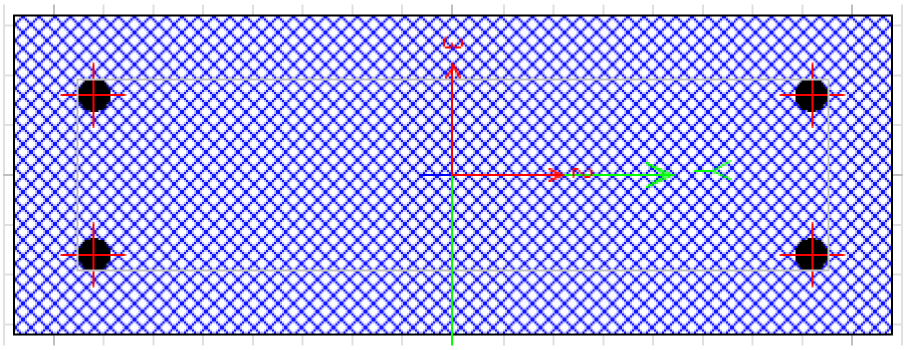

Figure D-3 – Cross section detail for *fib* Model Code 2010 (Computers and Structures, 2016).

Verifying the biaxial bending:

$$\frac{N_{Ed}}{N_{Rd}} = \frac{N_{Ed}}{A_c f_{cd} + A_s f_{yd}} = \frac{1375}{0.11 \times \frac{35000}{1.50} + 0.001257 \times \frac{500000}{1.15}} \rightarrow a \cong 1.28$$

$$\left(\frac{67.6}{85}\right)^{1.28} + \left(\frac{78.1}{275}\right)^{1.28} = 0.94 \text{ (OK)}$$

For this reinforcement, $\rho = \frac{0.001257}{0.11} = 1.14\%$, which means $\omega = \frac{\frac{500}{1.15}}{\frac{35}{1.50}} \times \frac{1.14}{100} = 0.21$. This shows that the initial assumption of $\omega = 0.37$ is not very accurate and the method could be improved with another iteration.

APPENDIX E – EXAMPLE OF LOCAL SECOND ORDER EFFECTS ANNALYSIS FOR ACI 318M-14

This analysis according to ACI 318M-14 will use the same column and considerations from Appendix B. As the effective length is defined differently in the American standard, its calculation must be the first thing to be done.

So, for the X direction:

$$\begin{cases} \psi_{1_X} = 0 \rightarrow \text{ rigid rotational restraint} \\ \psi_{2_X} = \dfrac{\frac{0.55\times0.20^3}{12}\frac{1}{3}\times 2}{\frac{0.15\times0.45^3}{12}\frac{1}{6}} = 0.229 \end{cases} \Rightarrow \psi_{m,X} = 0.115 \rightarrow k_X = 1.05$$

$$l_{u_X} = 2.20\ m$$

$$r \cong 0.3 \times 0.20 = 0.06\ m$$

$$\left(\frac{kl_u}{r}\right)_X = \frac{1.05\times2.20}{0.06} = 38.49 > 22 \rightarrow \text{Local second order effects must be considered}$$

For the Y direction:

$$\begin{cases} \psi_{1_Y} = 0 \rightarrow \text{ rigid rotational restraint} \\ \psi_{2_Y} = \dfrac{\left(\sum\frac{EI}{L}\right)_{columns}}{\left(\sum\frac{EI}{L}\right)_{beams}} = \dfrac{\frac{0.20\times0.55^3}{12}\frac{1}{3}\times 2}{\frac{0.15\times0.80^3}{12}\frac{1}{6}} = 9.74 \end{cases} \Rightarrow \psi_{m,Y} = 4.87 \rightarrow k_Y = 2.18$$

$$l_{u_Y} = 2.55\ m$$

$$r \cong 0.3 \times 0.55 = 0.165\ m$$

$$\left(\frac{kl_u}{r}\right)_Y = \frac{2.18\times2.55}{0.165} = 33.70 > 22 \rightarrow \text{Local second order effects must be considered}$$

Evaluate the local geometric imperfections through the minimum bending moment:

$$\begin{cases} M_{1d,min,X} = 1193 \times (15 + 0.03 \times 200) = 25.0\ kN \cdot m \\ M_{1d,min,Y} = 1193 \times (15 + 0.03 \times 550) = 37.6\ kN \cdot m \end{cases}$$

Through a SAP2000 computational analysis it is possible to obtain the stresses in that column. The coefficients needed for the calculation of Q are also given below:

$$\begin{cases} M_{1ns,X} = 7.9\ kN \cdot m \\ M_{2ns,X} = -13.8\ kN \cdot m \\ M_{1s,X} = 0\ kN \cdot m \\ M_{2s,X} = 0\ kN \cdot m \end{cases} ; \begin{cases} M_{1ns,Y} = 10.1\ kN \cdot m \\ M_{2ns,Y} = -20.2\ kN \cdot m \\ M_{1s,Y} = 20.3\ kN \cdot m \\ M_{2s,Y} = -14.9\ kN \cdot m \end{cases} ; \begin{cases} P_u = -1193\ kN \\ V_{us,X} = 10.1\ kN \\ \Delta_{0,X} = 0.000015\ m \\ V_{us,Y} = 7.2\ kN \\ \Delta_{0Y} = 0.001268\ m \end{cases}$$

Because the bending moments due to the acting forces are smaller than the minimum moment due to local geometric imperfections, the moment to be amplified will be the minimum.

So, as

$$Q_X = \frac{\sum P_u \Delta_{0X}}{V_{us,X} l_c} = \frac{1193 \times 0.000015}{10.1 \times 3} = 0.006 < 0.05$$

$$Q_Y = \frac{\sum P_u \Delta_{0Y}}{V_{us,Y} l_c} = \frac{1193 \times 0.001268}{7.2 \times 3} = 0.0696 > 0.05$$

It is only necessary to consider the second order effects in the Y direction.

The moment amplification factor δ_s is now calculated:

$$\delta_{s,Y} = \frac{1}{1 - 1.15 \times 0.0696} = 1.09$$

The final moments will be given by:

$$\begin{cases} M_{d,tot,X} = 25.0\ kN \cdot m \\ M_{d,tot,Y} = 1.09 \times 37.6 = 40.8\ kN \cdot m \end{cases}$$

Using the spreadsheet presented in 7:

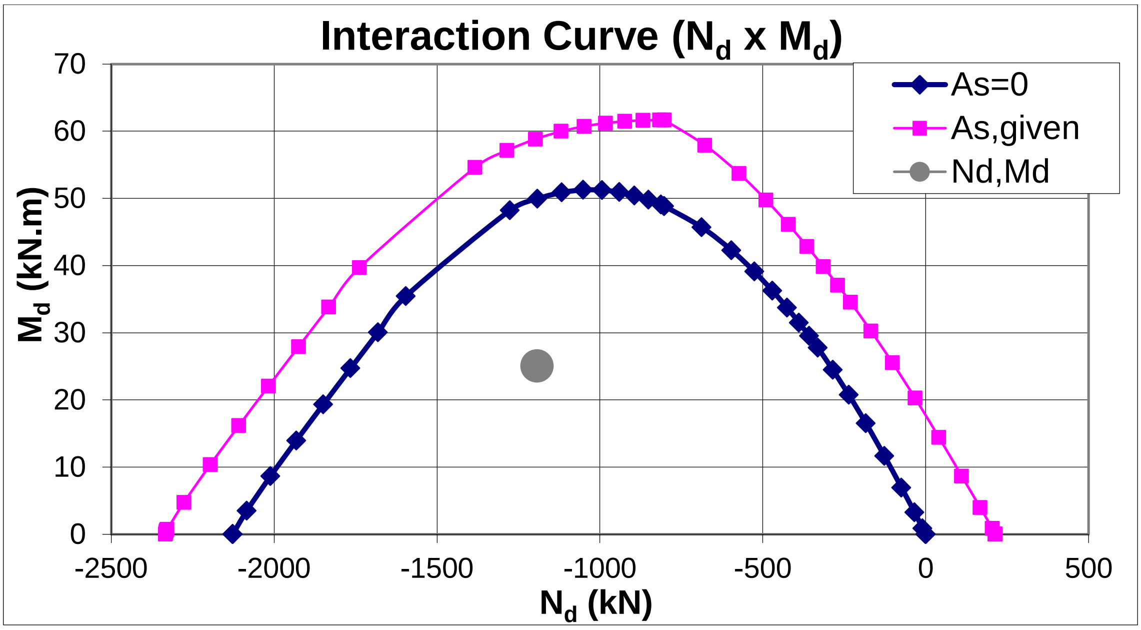

Figure E11-4 – Concrete interaction diagram for X direction, for ACI 318M-14.

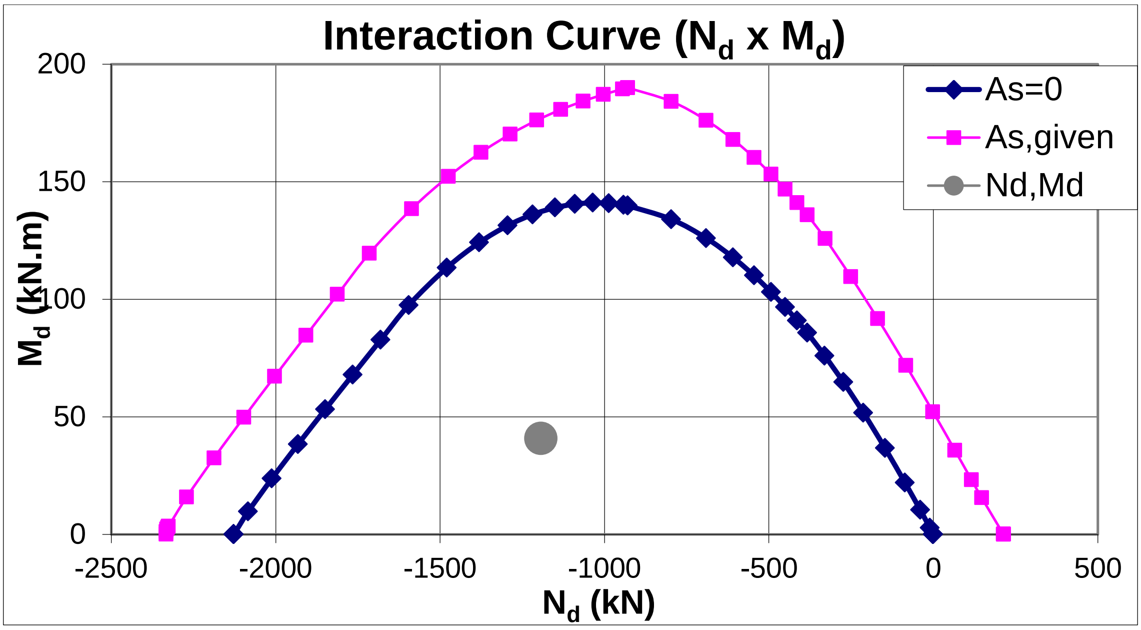

Figure E-2 – Concrete interaction diagram for Y direction, for ACI 318M-14.

The chosen reinforcement was 4Φ12.5 mm ($A_S = 4.91\ cm^2$), and the section detailing is found in the image below:

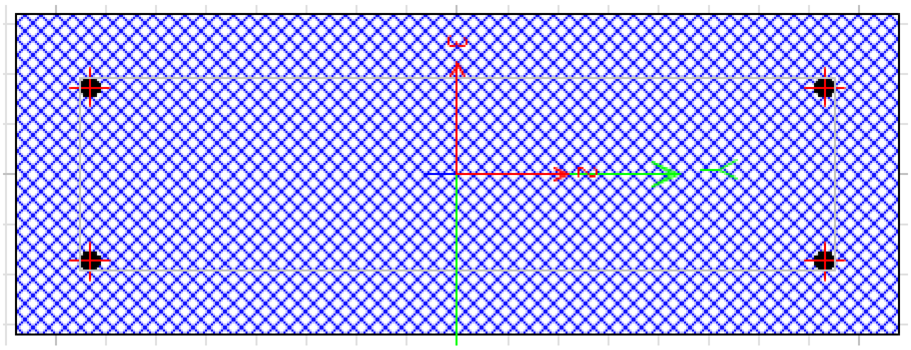

Figure E-3 – Cross section detail for the ACI 318M-14 (Computers and Structures, 2016).

With this reinforcement, the resistant moments will be approximately:

$$\begin{cases} M_{Rd,X} \cong 59\ kNm \\ M_{Rd,Y} \cong 176\ kNm \end{cases}$$

As ACI 318 does not suggest any approximated method for dealing with the biaxial bending, the same method from NBR 6118 will be used, making the same verifications:

$$\left(\frac{23.3}{59}\right)^{1.2} + \left(\frac{44.1}{176}\right)^{1.2} = 0.32 < 1 \text{ (OK)}$$