Comparative Analysis of Second Order Effects by Different Structural Design Codes

João Luis Martins Soares Nogueira
joaoluismsn@tecnico.ulisboa.pt

Department of Civil Engineering, Instituto Superior Técnico, Portugal, 2017

Abstract: This work compares some methods of analysis of global and local second order effects in reinforced concrete columns, according to four structural design codes: Brazilian NBR 6118 (2014), European EN 1992-1-1, *fib* Model Code 2010 and American ACI 318M-14. To do this, a 12-storey reinforced concrete fictitious building, subjected to wind, accidental and self-weight loads with 12 column-types, was modelled using the software SAP2000. For the global analysis, while the Brazilian Standard gives the possibility of choosing between the P-Delta method and a bending moment multiplier, the EN 1992-1-1 only considers a magnification factor of horizontal forces. The two other codes do not make any reference to global second order effects. It was also done a local analysis, comparing seven different methods found in the four codes and although many similarities were found, there are some particularities in each code that make the final results differ. This comparison was done in terms of both final bending moments and reinforcement. Also, to check the accuracy of the methods, a nonlinear analysis considering both nonlinearities was performed, using SAP2000 once again. Finally, to show the immense importance of stiffness in the behaviour of a building, a case study is analysed. First, all columns were turned 90-degrees to reduce the stiffness against the wind force, and then two rigid cores were added, comparing the before and after results.

Keywords: Second order effects; Reinforced concrete building; Non-linear analysis; Comparative analysis; Structural design codes; Stiffness influence;

1. INTRODUCTION

In the past few decades, the increase of urban populational density allied with the evolution of building materials and constructive and designing techniques turned structures increasingly taller and slenderer. This evolution brought as consequence potential problems of instability – the so called second order effects – which in some cases can assume considerable magnitude and must be taken into account.

Although these effects are a worldwide problem, not all structural codes address this issue in the same way and assumptions, simplifications and approaches are different. As today's world is a global one, it was felt necessary to compare four codes and see their differences and similarities. The codes analysed herein are the Brazilian NBR 6118 (2014), European EN 1992-1-1, *fib*'s Model Code 2010 and American ACI 318M-14.

This thesis was done as the Final Project of Double Degree between IST-Lisbon (Portugal) and UFRJ (Brazil), and gives continuity to the Final Project of Bárbara Gomes [1].

2. FUNDAMENTAL CONCEPTS

2.1. SECOND ORDER EFFECTS

It is a well-known fact that an element deflects when subjected to axial force and bending moments. This interaction between the deformation and the axial force will cause an increase of the bending moment at a given section that, depending on the slenderness of the element, may take an important role in the structural design. This additional effect is what is called second order effect.

In general, structural design codes assume that second order effects can be ignored if they represent less than 10% of the first order moment. However, as this rule is not practical, regulations have developed simplified ways to verify if these effects are in fact a problem.

It is important to note that there are two distinct kinds of second order effects and both will be addressed in this work:

- Global effects (or P-Δ effects) – affect the entire structure;
- Local effects (or P-δ effects) – affect isolated elements, independently from what happens with the structure.

Although local geometric imperfections are going to be considered when analysing local second order effects, when dealing with global second order effects, global geometric imperfections will not be addressed.

2.2. METHODS OF ANALYSIS

The most common form of structural analysis is the first order linear elastic analysis, where stresses and forces are calculated through the equilibrium of the elements in their non-deformed configuration. Although this analysis is simple and accurate for the most common cases, second order effects require a more exact analysis: nonlinear analysis.

Nonlinear analyses consider both geometric and physical nonlinearity. While the first deals with the equilibrium in the deformed state, the latter considers the effects of cracking, creep and other nonlinear behaviour of the RC structure. Although accurate, it is very complex and requires the use of an appropriate computer software and the knowledge of the section reinforcement, which is not always previously known. Due to these major inconveniences, other methods for evaluating second order effects have been developed:

- P-Delta analysis: Approximated iterative method that allows calculating global second order effects by creating fictitious horizontal forces that cause an effect equivalent to the second order moments. Here, the nonlinear analysis is replaced by as many linear analyses as needed for convergence [2].
- Computational nonlinear analysis: The software shall actualize the stiffness of the RC section in each iteration, as a function of the adjusted forces and the corresponding moment-curvature diagrams, so that the stiffness of the various RC sections will depend on the actual internal forces.

3. BUILDING ANALYSIS

The fictitious RC building herein analysed is 36 meters high, 30 meters wide and 16 meters deep and was modelled using the software SAP2000 [3], defining bar elements for beams and columns, and shell elements for the slabs. The computational model and formwork drawing can be seen in Figure 3-1:

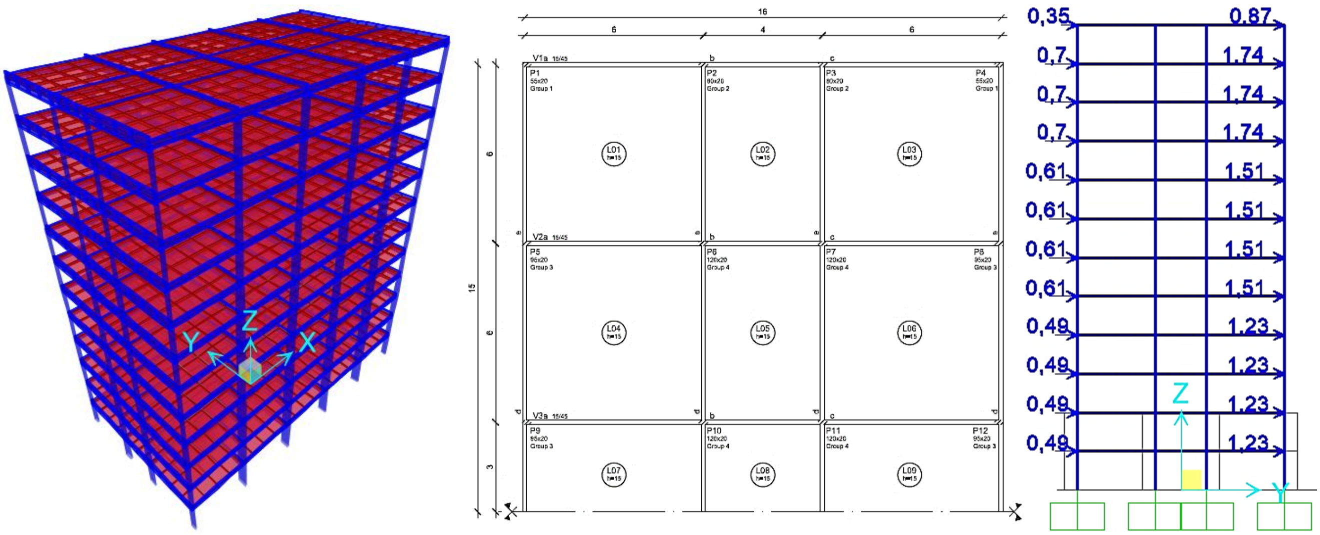

Figure 3-1 – Structural model, formwork drawing and distributed forces due to the wind, acting in the building

This structure is classified as a sway structure and although may not seem reasonable for an European designer, as it does not contain bracing elements, this structural design is quite common in Brazil and will help to accomplish the objective of this work, which is to test the approach on second order effects according to different design codes.

The acting loads and combinations were defined according to the Brazilian standard. While the permanent load was considered automatically through SAP2000, the accidental loading was defined as $q_{accidental} = 2.00\ kN/m^2$ and the wind load was applied in the model as distributed forces acting in each pavement in the Y direction (see Figure 3-1), totalling $q_{wind} = 1\ 005\ kN$. The combination of actions for the ULS was defined as:

$$F_{d,ULS} = 1.4 \cdot g + 1.4 \cdot (q_{wind} + 0.7 \cdot q_{accidental}) \quad (1)$$

The beams' cross sections were designed as 15cm x 45 cm for longitudinal beams and 15 cm x 80 cm for transversal beams, while the columns were categorized in four groups with similar levels of stress, in 3 levels, resulting in 12 different types of columns. Their height was set as $h_{columns} = 20\ cm$ and the widths were defined according to the estimated area of influence (see Table 3-1). All slabs are 15 cm thick and their weight was majored to consider so the mass of the masonry and cladding. Note that all columns have the same orientation, in which the "weak" axis of inertia is given by the X direction.

The RC used in the model presents the following properties:

$$f_{ck} = 35\ MPa; \quad \gamma = 25\ kN/m^3; \quad \gamma_{slabs} = 38.33\ kN/m^3; \quad f_{yk} = 500\ MPa;$$
$$E_{NBR} = 32\ 343\ MPa; \quad E_{EC2;fib} = 28\ 400\ MPa; \quad E_{ACI} = 31\ 800\ MPa; \quad E_s = 200\ 000\ MPa;$$

Table 3-1 – Division of the columns by groups.

Group 1	Group 2	Group 3	Group 4
P1 / P4 P21 / P24	P2 / P3 / P22 / P23	P5 / P8 / P9 / P12 / P13 / P16 / P17 / P20	P6 / P7 / P10 / P11 / P14 / P15 / P18 / P19
Corner columns	Lateral Columns (16m side)	Lateral columns (30m side)	Central columns
Low axial force	Low axial force	Low axial force	High axial force
$M_X \cong M_Y$	$M_X > M_Y$	$M_Y > M_X$	$M_Y > M_X$
G1-00: b=0.55 m G1-12: b=0.40 m G1-24: b=0.30 m	G2-00: b=0.80 m G2-12: b=0.50 m G2-24: b=0.30 m	G3-00: b=0.95 m G3-12: b=0.80 m G3-24: b=0.55 m	G4-00: b=1.20 m G4-12: b=0.80 m G4-24: b=0.40 m

4. ANALYSIS ACCORDING TO NBR 6118:2014

4.1. GLOBAL SECOND ORDER EFFECTS

To classify the structure as sway or non-sway, NBR 6118 (2014) [4] uses the coefficient γ_z, determined from a first order linear analysis, reducing the elements' stiffness to take the physical nonlinearity into account: Slabs: $(EI)_{sec} = 0.3\,E_c I_c$; Beams: $(EI)_{sec} = 0.4\,E_c I_c$; Columns: $(EI)_{sec} = 0.8\,E_c I_c$. The coefficient γ_z is given by:

$$\gamma_z = \frac{1}{1 - \frac{\Delta M_{tot,d}}{M_{1,tot,d}}} = \frac{1}{1 - \frac{673.4}{14\,430}} = 1.008 < 1.1 \tag{2}$$

Although the structure is classified as non-sway if $\gamma_z \leq 1.1$, if $1.1 \leq \gamma_z \leq 1.3$, the code allows for an approximate consideration of the global second order effects, by multiplying the horizontal loads by the factor $0.95\gamma_z$.

For this case, it is seen that the structure is non-sway and there is no need to check for global second order effects:

4.2. LOCAL SECOND ORDER EFFECTS

For framed structures, the effects of local geometric imperfections can be considered through a minimum first order moment acting in the columns:

$$M_{1d,min} = N_d \cdot e_a = N_d \cdot (0.015 + 0.03h) \tag{3}$$

It is important to notice that this moment is not going to be added to the first order moments.

Both methods analysed herein can be only used for columns with $\lambda \leq 90$, rectangular constant section and symmetrically constant reinforcing.

- Method of the standard-column with approximated stiffness:

$$A{M_{d,tot}}^2 + BM_{d,tot} + C = 0, \text{where} \begin{cases} A = 5h \\ B = N_d h^2 - \dfrac{N_d l_e^2}{320} - 5h\alpha_b M_{1\text{d,A}} \\ C = -N_d h^2 \alpha_b M_{1\text{d,A}} \end{cases} \tag{4}$$

- Method of the standard-column with approximated curvature:

$$M_{d,tot} = \alpha_b M_{1\text{d,A}} + N_d \frac{{l_e}^2}{10}\frac{1}{r} = \alpha_b M_{1\text{d,A}} + N_d \frac{{l_e}^2}{10}\frac{0.005}{h(\nu + 0.5)} \geq M_{1\text{d,A}} \tag{5}$$

The values of the final bending moments for both methods and the reinforcement for the most penalizing case (approximated curvature method) is shown in Table 4-1.

The first two deviation columns refer to the relative deviation between the initial P-Delta analysis and the two approximated methods, only in the Y direction. The average deviation is +56% for the Stiffness method and +71% for the Curvature method. When comparing the two approximated methods, it is possible to see that the curvature is the most penalising, producing bending moments 12% to 30% higher, in average, than the ones obtained by the stiffness based method.

The relative deviations are only shown for the Y direction because the bending moments in the X direction do not represent the analysis, as the wind was only applied in the Y direction. If those deviations were to be analysed, they would be huge, particularly in the central columns (Groups 3 and 4), because they present very low values of bending moments in the initial P-Delta analysis, allied with the fact that the geometric imperfections were not taken into account in the structural model, which means that the second order effects would not have significant expression.

Note that when there is no deviation between the two methods it means that the conditioning bending moment is the one due to the wind forces, which is the same for both cases, as it was not amplified. Also, the Brazilian standard only considers the effects of biaxial bending for the bending moments due to applied loads, disregarding these effects for the minimum moments due to geometric imperfections.

Table 4-1 – Bending moments in kN·m and area of reinforcement in cm^2 for the NBR 6118 (2014).

COLUMN	P-Delta		Stiffness		Curvature		Deviation (%)				Reinforcement	
							$P\Delta$- S	$P\Delta$- C	S-C			
	M_X	M_Y	M_X	M_Y	M_X	M_Y	M_Y	M_Y	M_X	M_Y	Rebar	ρ
G1-00	23.3	44.1	44.0	46.9	55.6	53.7	6.33	21.57	20.99	12.54	4Φ12.5	0.45%
G1-12	27.9	50.4	23.9	26.7	35.7	31.7	0.00	0.00	32.95	15.74	4Φ12.5	0.61%
G1-24	26.3	36.9	11.3	11.9	16.9	14.6	0.00	0.00	32.95	18.18	6Φ12.5	1.23%
G2-00	39.3	76.1	60.3	91.1	87.4	99.8	19.69	31.15	30.99	8.74	10Φ12.5	0.77%
G2-12	44.0	24.9	38.5	46.3	55.5	52.9	85.65	111.9	30.66	12.40	8Φ12.5	0.98%
G2-24	37.1	7.5	18.5	19.4	27.6	23.8	161	219.0	32.95	18.18	6Φ12.5	1.23%
G3-00	4.5	152.1	81.0	135.2	113.9	144.4	0.00	0.00	28.87	6.37	12Φ16	1.27%
G3-12	5.2	130.6	53.8	81.3	80.1	89.7	0.00	0.00	32.86	9.40	4Φ16	0.50%
G3-24	5.2	117.9	26.4	33.1	39.4	37.9	0.00	0.00	32.95	12.69	4Φ16	0.73%
G4-00	6.0	221.1	119.0	230.4	161.1	240.2	4.24	8.64	26.15	4.05	14Φ25	2.86%
G4-12	6.1	39.9	76.9	116.2	105.0	125.4	191	214.1	26.71	7.33	6Φ25	1.84%
G4-24	5.1	13.6	37.2	41.5	51.2	47.4	204.7	247.5	27.33	12.32	4Φ25	2.45%

5. ANALYSIS ACCORDING TO THE EN 1992-1-1, EUROCODE 2

5.1. GLOBAL SECOND ORDER EFFECTS

EN 1992-1-1 [5] states that global second order effects can be dismissed if the condition below is met:

$$F_{V,Ed} \leq k_1 \frac{n_s}{n_s+1.6} \cdot \frac{\sum E_{cd} I_c}{L^2} \quad \text{where } \sum E_{cd} I_c \text{ is the stiffness of the bracing elements} \tag{6}$$

If the condition is not met and global second order effects cannot be ignored, the structure must be designed to withstand a magnified horizontal load, $F_{H,Ed}$. Comparing this to γ_z, a multiplication factor $F_{magnification}$ will be considered:

$$F_{H,Ed} = \frac{F_{H.0Ed}}{1 - \frac{F_{V,Ed}}{F_{V,B}}} \Leftrightarrow F_{magnification} = \frac{1}{1 - \frac{F_{V,Ed}}{F_{V,B}}} \tag{7}$$

However, as this building is not like the most commonly found in Europe and there are no bracing elements, two different values of stiffness are going to be calculated and compared:

1. The summation of the stiffness of every column: $(E_{cd} I_c)_1 = 4\ 069\ 002\ kN \cdot m^2$
2. Making an analogy to a cantilever beam subjected to the wind loads: $(E_{cd} I_c)_2 = \ 387\ 105\ 211\ kN \cdot m^2$

Making the calculations for both cases, it is plausible to say that the first result is not valid, as $F_{magnification} < 0$. This probably occurs because the actual stiffness of the building, due to frames composed of beams and columns, is much higher than the one corresponding only to the columns. So, using the second value of stiffness, there is no need to consider global second order effects:

$$0.31 \times \frac{12}{12+1.6} \times \frac{387\ 105\ 211}{36^2} = 81\ 701\ kN > F_{V,Ed} \rightarrow F_{magnification} = \frac{1}{1 - \frac{70\ 010}{822\ 282}} = 1.093 \tag{8}$$

5.2. LOCAL SECOND ORDER EFFECTS

Unlike what happens in NBR 6118 (2014), the local geometric imperfections are calculated through an additional eccentricity that will be added to the applied moment:

$$e_i = \frac{l_0}{200\sqrt{3}} \Rightarrow M_i = N_{Ed} \cdot \frac{l_0}{200\sqrt{3}} \tag{9}$$

Just like in 4.2, two methods will be analysed and they are also based in the nominal stiffness and curvature:

- Method based on the nominal stiffness:

$$M_{Ed} = \frac{M_{0Ed}}{1 - \left(N_{Ed} \Big/ \frac{\pi^2}{{l_0}^2} EI \right)}, \text{where } EI = \sqrt{\frac{f_{ck}}{20}} \times 0.30n \cdot E_{cd} \cdot I_c + 1 \cdot E_s \cdot I_s \tag{10}$$

- Method based on the nominal curvature:

$$M_{Ed} = M_{0Ed} + N_{Ed} \cdot \left(\frac{(1+\omega) - n}{(1+\omega) - 0.40} \times \frac{\frac{f_{yd}}{E_s}}{0.45d} \times \frac{{l_0}^2}{10} \right) \tag{11}$$

The values of the final bending moments for both methods and the reinforcement for the most penalizing case (nominal curvature method) is shown in Table 5-1:

Table 5-1 – Bending moments in kN·m and area of reinforcement in cm^2 for the EN 1992-1-1.

COLUMN	P-Delta		Stiffness		Curvature		Deviation (%)				Reinforcement	
							$P\Delta$- S	$P\Delta$- C	S-C			
	M_X	M_Y	M_X	M_Y	M_X	M_Y	M_Y	M_Y	M_X	M_Y	Rebar	ρ
G1-00	15.0	37.1	46.3	54.3	79.5	69.7	31.76	46.82	71.98	22.06	4Φ25	1.78%
G1-12	19.3	41.6	59.8	56.9	81.3	73.1	26.83	43.06	35.93	22.17	6Φ25	3.68%
G1-24	18.0	30.4	33.3	37.6	45.5	45.0	19.18	32.47	36.42	16.44	4Φ25	3.27%
G2-00	28.1	64.1	91.7	91.1	136.7	108.0	29.61	40.64	49.15	15.67	8Φ25	2.45%
G2-12	32.7	24.4	149.1	43.3	132.4	61.5	43.74	60.42	-11.17	29.66	10Φ25	4.91%
G2-24	26.7	9.7	61.6	17.8	68.6	28.7	45.18	66.11	11.41	38.19	6Φ25	4.91%
G3-00	4.1	126.0	62.3	175.2	123.5	201.0	28.06	37.28	98.27	12.82	6Φ25	1.55%
G3-12	4.1	131.3	64.5	198.7	136.2	270.2	33.91	51.39	110.95	26.45	10Φ25	3.07%
G3-24	3.3	120.8	24.1	146.8	67.6	172.1	17.71	29.80	180.79	14.68	4Φ25	1.78%
G4-00	6.1	173.9	106.5	240.0	168.8	263.7	27.55	34.05	58.46	8.98	10Φ25	2.05%
G4-12	5.3	46.5	133.3	104.9	154.4	147.2	55.65	68.38	15.77	28.71	12Φ25	3.68%
G4-24	3.3	20.8	36.6	39.7	60.4	54.9	47.60	62.08	65.32	27.63	4Φ25	2.45%

Once more, the first two deviation columns refer to the relative deviation between the initial P-Delta analysis and the two approximated methods, only in the Y direction. The average deviation is +34% for the Stiffness method and +48% for the Curvature method. When comparing the two approximated methods, it is possible to see that the curvature is the most penalising, apart from column G2-12, producing bending moments 22% to 60% higher than the ones obtained by the stiffness based method.

This difference can probably be explained by the fact that in this work only the initial approximations were used for calculating the moment magnifying factor for the Nominal Stiffness method. If other iterations were used, perhaps the results would have been better. Other improvement that could have been made would be using iterations of ω in the Nominal Curvature method.

6. ANALYSIS ACCORDING TO *fib* MODEL CODE 2010

Unlike before, *fib* Model Code 2010 [6] does not deal with global second order effects, only local ones.

6.1. LOCAL SECOND ORDER EFFECTS

Similar to EN 1992-1-1, Model Code 2010 addresses local imperfections as an additional eccentricity:

$$e_{0d} = \max\left\{\frac{\alpha_i l_0}{2}; \frac{h}{30}\right\} \tag{12}$$

However, instead of giving different methods, Model Code 2010 gives one method that can be refined with different levels of approximation (see [7]). The dimensioning value of the bending moment is given by:

$$M_d = -N_d \cdot e_d = -N_d \cdot (e_{0d} + e_{1d} + e_{2d}) \quad (13)$$

- Level of approximation I:

$$e_{2d} \cong \frac{f_{yd}}{E_s} \cdot \frac{{l_0}^2}{5(h-2c)} \quad (14)$$

- Level of approximation II

$$e_{2d} = \left(\frac{(1+\omega)-n}{(1+\omega)-0.40}\right) \cdot \frac{\frac{f_{yd}}{E_s}}{0.45h} \cdot \frac{{l_0}^2}{10} \quad (15)$$

Table 6-1 – Bending moments in kN·m and area of reinforcement in cm^2 for the *fib* Model Code 2010

COLUMN	P-Delta		Level I		Level II		Deviation (%)				Reinforcement	
							$P\Delta$- I	$P\Delta$- II	LoA I-LoA II			
	M_X	M_Y	M_X	M_Y	M_X	M_Y	M_Y	M_Y	M_X	M_Y	Rebar	ρ
G1-00	15.0	37.1	97.2	81.7	67.6	78.1	54.63	52.57	-43.76	-4.56	4Φ20	1.14%
G1-12	19.3	41.6	94.2	75.8	69.0	71.8	45.10	42.04	-36.44	-5.57	6Φ20	2.36%
G1-24	18.0	30.4	49.2	45.5	40.2	43.5	33.19	30.09	-22.27	-4.63	6Φ20	3.14%
G2-00	28.1	64.1	173.7	144.0	116.6	139.8	55.46	54.15	-48.97	-2.95	8Φ20	1.57%
G2-12	32.7	24.4	168.6	72.7	112.1	66.3	66.52	63.27	-50.37	-9.72	12Φ20	3.77%
G2-24	26.7	9.7	77.7	31.1	60.6	27.4	68.65	64.41	-28.33	-13.52	6Φ20	3.14%
G3-00	4.1	126.0	183.2	264.7	101.7	252.7	52.39	50.13	-80.16	-4.75	8Φ20	1.32%
G3-12	4.1	131.3	176.4	284.0	110.2	268.4	53.76	51.07	-60.12	-5.82	12Φ20	2.36%
G3-24	3.3	120.8	76.2	172.5	55.4	170.9	29.98	29.29	-37.68	-0.98	6Φ20	1.71%
G4-00	6.1	173.9	277.1	392.7	139.3	376.8	55.71	53.85	-98.99	-4.21	14Φ20	1.83%
G4-12	5.3	46.5	248.2	193.9	124.8	167.3	76.00	72.19	-98.83	-15.88	14Φ20	2.75%
G4-24	3.3	20.8	92.6	67.7	50.0	55.8	69.23	62.66	-85.37	-21.33	6Φ20	2.36%

When comparing the initial P-Delta analysis and the two approximated methods, only in the Y direction, the average deviation is +55% for the LoA I and +52% for the LoA II. As expected, Level II is less conservative than Level I, returning values 32% smaller, in average. It is also plausible to assume that a Level III approximation would return even lower values for bending moments, closer to the real values, but still in the safe side. The same comment about ω could be made in the Level II of approximation.

7. ANALYSIS ACCORDING TO ACI 318M-14

Like *fib*, ACI 318M-14 [8] only deals with the local second order effects and similar to the Brazilian standard, the elements' inertia are considered as a fraction of the gross concrete section: Slabs: $I_{slabs} = 0.25I_g$; Beams: $I_{beams} = 0.35I_g$; Columns: $I_{columns} = 0.70I_g$. However, the column's stiffness is reduced one more time: $(EI)_{eff,column} = 0.40E_cI_g$

7.1. LOCAL SECOND ORDER EFFECTS

Like in NBR 6118, the effects of local geometric imperfections can be considered through a minimum first order moment acting in the columns, that will not be added to the first order moments and does not need to be verified against biaxial bending.

$$M_{1d,min} = P_u \cdot (15 + 0.03h) \quad (16)$$

ACI 318M-14 only provides one approximated method to deal with local second order effects – Moment magnification method. This method is used strictly for plane frames subjected to loads causing deflections in that plane. As the column in analysis is sway, the moment magnification method is applied as follows:

$$M_i = M_{ins} + \delta_s M_{is} \text{ where } \delta_s = \begin{cases} \dfrac{1}{1-1.15Q} \geq 1, if\ \delta_s \leq 1.5 \\ \dfrac{1}{1-\frac{\sum P_u}{0.75\sum P_c}} \geq 1, \text{if } \delta_s > 1.5 \end{cases} \quad (17)$$

Table 7-1 – Bending moments in kN·m and area of reinforcement in cm^2 for ACI318M-14

COLUMN	P-Delta		MMM		Deviation (%)	Reinforcement	
	M_X	M_Y	M_X	M_Y	M_Y	Rebar	ρ
G1-00	13.8	35.2	25.0	40.8	13.92%	4Φ12.5	0.45%
G1-12	17.2	32.7	17.2	35.1	7.03%	4Φ12.5	0.61%
G1-24	15.3	22.1	15.3	24.9	11.17%	4Φ12.5	0.82%
G2-00	24.8	49.1	44.6	90.0	45.45%	4Φ12.5	0.31%
G2-12	27.6	17.8	29.1	58.0	69.39%	4Φ12.5	0.49%
G2-24	20.9	7.6	20.9	18.4	58.63%	4Φ12.5	0.82%
G3-00	4.2	107.8	60.7	178.4	39.56%	4Φ12.5	0.26%
G3-12	5.1	125.2	40.7	119.2	-4.98%	4Φ12.5	0.31%
G3-24	4.3	103.7	20.1	94.0	-10.34%	4Φ12.5	0.45%
G4-00	5.8	136.4	93.2	229.1	40.46%	12Φ20	1.57%
G4-12	6.1	44.5	61.1	113.5	60.78%	6Φ20	1.18%
G4-24	4.0	16.2	30.4	39.0	58.60%	4Φ20	1.57%

Comparing the final bending moments with the initial P-Delta analysis it is possible to see that the MMM returns the less reinforced section and, consequently, with the closest values to the P-Delta analysis.

8. PHYSICALLY NON-LINEAR ANALYSIS

To verify if the previous results are correct, a nonlinear analysis was done. The nonlinear behaviour was considered through a model with distributed plasticity, with 5 hinges type "Fiber P-M2-M3" in each column, using the default stress-strain curves from SAP2000. Although this analysis was only done to the EN 1992-1-1 and *fib* models previously defined, and other load histories were tested to check the sensibility of the analysis, the results were very similar and therefore only the results from the Eurocode 2 model will be shown.

After running the analyses, it became clear that for this case the physically nonlinear effects are not significative, as very few sections crack. So, the results from the nonlinear analysis are practically the same, only about 12% higher than the P-Delta analysis. When the base force is compared with the displacement at the top of the structure, for the nonlinear analysis and its initial tangent value, it becomes clear that the effect of the nonlinearity only becomes significative when the wind force is much higher than the considered design force (Figure 8-1).

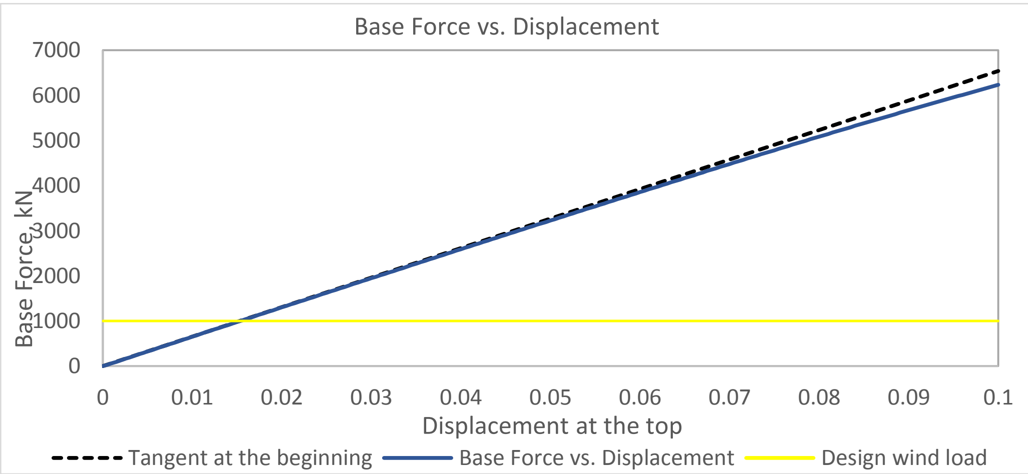

Figure 8-1 – Base force vs. Displacement at the control node at the top.

Although it is not correct to superpose effects when performing nonlinear analyses, as the nonlinearity is almost irrelevant, a new approximate comparison will be made. The bending moments due to the geometric eccentricity will be added to those obtained from the nonlinear analysis, and this will be compared with the two approximated methods from EN 1992-1-1. It becomes clear that the moments due to geometric eccentricities are decisive when designing RC columns, as the relative deviations reduced drastically (see Table 8-1Table 8-1). One more time, the relative deviations in the X direction are much greater than in the Y direction, for the same reasons commented in the previous chapters.

Table 8-1 – Comparison between the nonlinear analysis, the approximated methods and the initial linear analysis, for the EN 1992-1-1. Bending moments in kN·m and deviation in %

Column	Nonlinear + Eccentricity		Nominal Stiffness		Deviation (%)		Nominal Curvature		Deviation (%)	
	M_X	M_Y	M_X	M_Y	M_X	M_Y	M_X	M_Y	M_X	M_Y
G1-00	33.0	64.9	46.3	54.3	40%	-16.3%	79.5	69.7	141%	7.4%
G1-12	35.3	62.0	59.8	56.9	69%	-8.3%	81.3	73.1	130%	17.9%
G1-24	25.8	39.1	33.3	37.6	29%	-3.9%	45.5	45.0	76%	15.0%
G2-00	59.4	81.1	91.7	91.1	54%	12.3%	136.7	108.0	130%	33.2%
G2-12	60.4	47.3	149.1	43.3	147%	-8.6%	132.4	61.5	119%	30.0%
G2-24	42.3	17.9	61.6	17.8	46%	-0.6%	68.6	28.7	62%	60.9%
G3-00	37.8	154.4	62.3	175.2	65%	13.4%	123.5	201.0	226%	30.1%
G3-12	31.1	208.5	64.5	198.7	108%	-4.7%	136.2	270.2	338%	29.6%
G3-24	15.5	162.5	24.1	146.8	55%	-9.7%	67.6	172.1	336%	5.9%
G4-00	54.4	236.9	106.5	240.0	96%	1.3%	168.8	263.7	210%	11.3%
G4-12	42.5	99.5	133.3	104.9	214%	5.4%	154.4	147.2	263%	47.9%
G4-24	18.8	36.3	36.6	39.7	95%	9.4%	60.4	54.9	222%	51.2%

9. CASE STUDY – INFLUENCE OF STIFFNESS

To properly study the influence of the building's stiffness, a case study was done. First, all columns were rotated 90-degrees, making the building more flexible to the wind action. Then, two rigid cores were added to this new model, and the before and after results were compared, considering the analysis according to both NBR 6118 (2014) and EN 1992-1-1, verifying the same steps from 4.1 and 5.1

9.1. COLUMNS TURNED 90-DEGREES

Turning the columns 90-degrees, the stiffness of the bracing elements is going to be reduced:

- Summation of the stiffness of every column in each direction: $(E_{cd}I_c)_{1'} = 309\ 219\ kN \cdot m^2$
- Analogy to a cantilever beam subjected to the loads described above: $(E_{cd}I_c)_{2'} = 88\ 105\ 947\ kN \cdot m^2$

For the Brazilian model, since $\gamma_z = 1.228 < 1.3$, the use of the previously referred approximation, instead of P-Delta method, is allowed. So, comparing the majored global moments in the X direction given by the base reaction, with the value obtained with the P-Delta analysis, it is possible to see that this procedure provides a good approximation:

$$20\ 202\ kN \cdot m \times 0.95 \cdot \gamma_z = 23\ 573\ kN \cdot m\ \Leftrightarrow \cong 1.33\%\ \text{deviation} \quad (18)$$

Repeating this comparison, this time for one column from each group, in the 12 pavements, it is possible to see once again that this approximation provides good results, as the average deviation in each group ranges from 6.95% to 12.68%.

Once again, for the EN 1992-1-1 model, the first value of stiffness does not represent the building. However, when considering the cantilever analogy, it is required the horizontal forces to be multiplied by $F_{magnification} = 1.598$.

9.2. ADDING TWO RIGID CORES TO THE NEW MODEL

Two RC cores 25 cm thick were added where the slabs L02 and L14 used to be (see Figure 3-1). They were modelled with frame elements, using vertical bars linked by horizontal rigid bars [9],[10]. By adding the cores all columns from Group 2 will disappear and Group 4 will be reduced from eight to only four columns, and the stiffness will now be $\sum E_{cd}I_c = 701\ 362\ 098\ kN \cdot m^2$

$$\gamma_z = \frac{1}{1 - \frac{121.6}{14\,430}} = 1.008\,;\; F_{magnification} = \frac{1}{1 - \frac{75\,358}{1\,489\,821}} = 1.053 \quad (19)$$

In both cases the building is non-sway and there is no need to consider global second order effects. Also, the amplification of displacements before and after P-Delta effects were considered was reduced from 1.22 to 1.01.

10. RESULTS AND CONCLUSION

After this comparative analysis, it is possible to conclude that the four codes herein analysed have many processes in common, and some particularities that must be considered when designing.

Regarding global second order effects, while NBR 6118 demands verification for both limit states, EN 1992-1-1 only deals with the ULS. However, both *fib* Model Code 2010 and ACI 318M-14 do not present methods for the consideration of global second order effects, only dealing with isolated elements.

After both analysis it was seen that in this building there was no need to consider global second order effects for the acting wind load, and that the difference between the Brazilian parameter and European multiplier may be due to different values of elasticity modulus, or to the difficulty of properly defining the stiffness of the building.

It was also seen that by changing the orientation of the columns, global second order effects became a problem to be dealt with, as the Brazilian parameter increased to $\gamma_Z = 1.228$ and the European multiplier got about 50% higher. γ_Z turned out to be a good approximation for the global second order effects, as the consequent results of the normative approximation presented no significant differences from the P-Delta analysis. Also, when two rigid cores were added to the new model with increased flexibility, the global second order effects could be ignored again, making clear the importance of bracing elements.

For the local effects, the effective length of the columns is a very important parameter. Although it is considered in the same way in EN 1992-1-1 and *fib* Model Code, NBR 6118 and ACI 318M-14 define it differently.

The results of the local analysis can be summarized in Figure 10-1, Figure 10-2 and Figure 10-3:

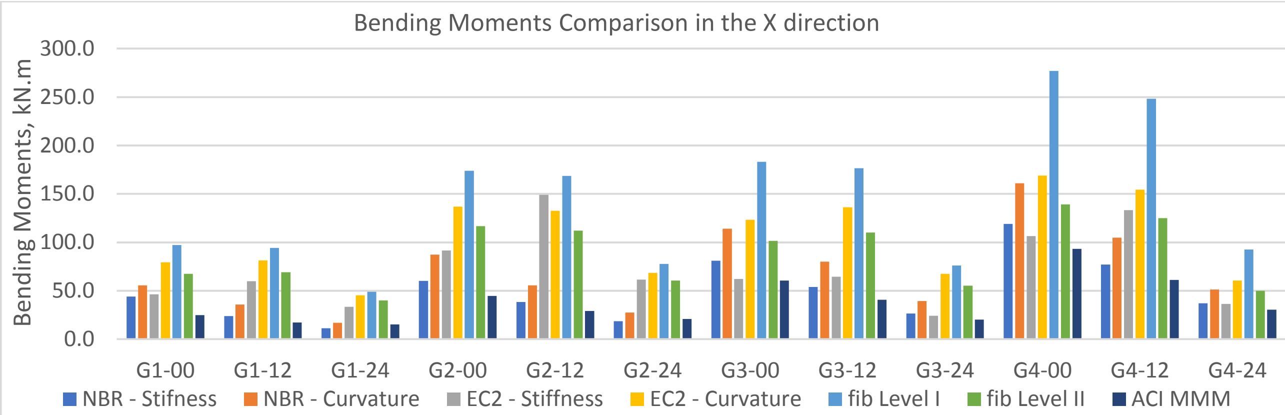

Figure 10-1 – Bending moment comparison in the X direction.

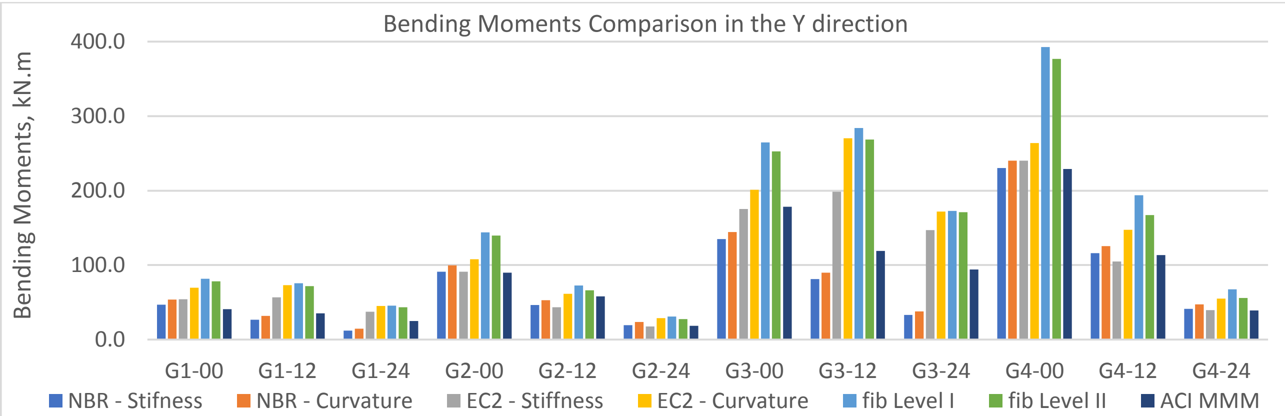

Figure 10-2 – Bending moment comparison in the Y direction.

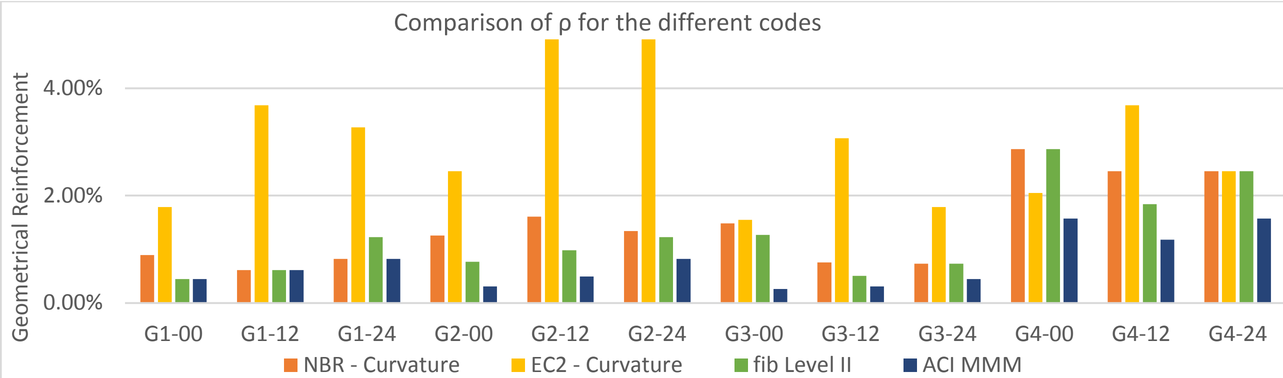

Figure 10-3 – Comparison of ρ for the different codes.

It can be seen that for both NBR 6118 and Eurocode 2 standards, the Nominal Curvature method returns more conservative values than the Nominal Stiffness method, i.e., the Nominal Curvature methods will result in higher safety factors, when compared with more refined methods.

The discrepancy of results found between Brazilian, American and European codes occurs because the latter adds the bending moments due to geometric imperfections to the first order moments due to the wind load. Also, the three codes have different criteria for defining the column effective length.

As said before, the difference between the two European methods may be due to the basic approximations made in the Nominal Stiffness methods. It can be also seen that the Level II *fib* approach is very similar to the curvature method of EN 1992-1-1, and that bending moments present lower, more economic values, as long as hypotheses are refined from Level I to Level II

Unlike what happened for the other two codes, for both NBR 6118 and ACI 318 the biaxial bending was not decisive in the reinforcement design, as the conditioning moments were due the geometric imperfections and those bending moments do not need to be verified against biaxial bending. So, the needed reinforcement is lower.

It is important to notice that although the values of bending moments for EC 2 and MC 2010 are very close, the first presents higher moments in the X direction, which is more conditioning for the biaxial bending, as $M_{Rd,Y} \gg M_{Rd,X}$, and therefore will lead to higher reinforcing.

After performing the nonlinear analyses, it was seen that, for this case, the physically nonlinearity does not add much significance, as it returned values only 12% higher, in average, than the ones obtained from a simple P-Delta analysis. For this case, the geometric imperfections turned out to be decisive, with much more importance than the nonlinear effects of RC.

11. REFERENCES

[1] B. C. Gomes, "Estudo dos fatores de redução de rigidez da NBR 6118:2014 para análise aproximada da não linearidade física," Universidade Federal do Rio de Janeiro, 2017.

[2] H. I. Longo, *Efeitos de Segunda Ordem em Estruturas de Edificações Efeitos de Segunda Ordem em Estruturas de Edificações*. Rio de Janeiro, Brazil, 2017.

[3] Computers and Structures, Inc., "SAP2000 v19." Berkeley, California, USA, 2016.

[4] Associação Brasileira de Normas Técnicas, *NBR 6118 - Projeto de estruturas de concreto - Procedimento*. Rio de Janeiro, Brazil, 2014.

[5] Comité Européen de Normalisation, *European Standard EN 1992-1-1:2004 Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*. Brussels, Belgium, 2004.

[6] Fédération Internationale du Béton, *Model Code 2010 Final Draft Volume 2*. Lausanne, Switzerland, 2010.

[7] A. Muttoni, "The levels-of-approximation approach in MC 2010," *Structural Concrete*, vol. 13, no. 1. pp. 32–41, 2012.

[8] American Concrete Institute, *Building Code Requirements for Structural Concrete (ACI 318M-14) and Commentary (ACI 318RM-14)*. 2014.

[9] N. M. C. Ramilo, "Modelação de Núcleos de Edifícios," Instituto Superior Técnico, 2009.

[10] P. O. de A. Machado, "Projecto de Estruturas de um Edifício," Instituto Superior Técnico, 2010.