

# STUDY OF HISTERETIC STEEL DAMPERS

## Modeling a Variable Section Cantilever

Ricardo Carvalheira

### ABSTRACT:

In this paper the study of a specific type of passive vibration control systems, the hysteretic dampers, was discussed. These dampers take advantage of the ductile capacity of metals, more precisely of steel, to dissipate the energy from the dynamic actions and are especially effective when the dynamic action applied to the structure is the seismic action.

It was dimensioned and modeled using the ABAQUS program, a type of metal damper that consisted of a cantilever beam with variable section, so that during seismic action, all sections began plastification at the same time. The effect of seismic action on the properties of steel used in the dimensioned cantilever was studied, however, although the literature suggests otherwise, these effects can be neglected since the hysteresis curves and the amount of energy dissipated by the cantilever with different constitutive properties demonstrated that the results obtained are approximately the same.

As it was demonstrated that the dynamic effects of the steel could be neglected, this type of cantilever was applied to two case studies to realize its effectiveness in mitigating the effects of the seismic action. From the results obtained, it is safe to say that these dampers, when applied to bridges, have been so much more successful in reducing longitudinal displacements the longer the structure period.

**Keywords:** Passive Control Systems, Hysteresis Steel Dampers, Hardening, Cantilever with Variable Section

### INTRODUCTION

Passive control systems are the systems that in recent decades have been the target of the greatest number of studies and developments. These systems consist of one or more devices independent of external energy sources, which incorporate in the structure, absorb or consume part of the energy transmitted to it by dynamic loading (Constantinou, Soong, & Dargush, 1998).

These systems can be divided into three groups, depending on their behavior in the structure, basic isolation systems, dynamic vibration absorbers and energy dissipation systems. The cantilever that will be the object of study in this article, belongs to one of the classes of energy dissipation systems, hysteretic dampers. These dampers are characterized by taking advantage of the ductile capacity of the metals when they are in plastic mode to dissipate the energy from the dynamic actions and are particularly efficient in relation to other vibration control systems when the dynamic action is the earthquake (Kelly, Skinner, & Heine, 1972).

Since the 1970s different hysteretic devices have been developed, most of them using steel as a material (Curadelli, 2003). The geometry of these devices depends essentially on their mode of operation, that is, depending on the stresses to which the devices are subject, their geometric configuration is dimensioned, to maximize the amount of steel that is in a plastic regime and the force-displacement curves are as stable as possible.

The purpose of this article is to measure the height that the cross-section of a consular beam must have along its length, so that for a given force, the beam enters in a plastic regime, in all the cross sections, to perceive the influence that the dynamic action, in particular the seismic action, has on the constitutive model of the adopted steel and to test the effectiveness of this cantilever when applied to two case studies. For this, two structural analysis programs, ABAQUS, were used to size the consoles using the infinitesimal elements method and the SAP2000, to analyze the effectiveness of the consoles when applied to the two case studies.

This article is organized in six chapters. In the first chapter the constitutive model used to model the effects of

dynamic actions on steel is explained. In the second chapter the pre-sizing of the cantilevers with variable section is done. In the third and fourth chapter is applied a monotonic and cyclic displacement respectively to the cantilevers admitting that their behavior is elastic perfectly plastic and their performance is analyzed. In the fifth chapter we compare the force-displacement damper model and energy dissipated by the cantilevers when we use a constitutive model that takes into account the hardening of the steel. Finally, in the last chapter, the cantilevers are applied to two case studies and their performance is analyzed.

## 1. CONSTITUTIVE MODEL

In a simplified way, when analyzing a structure in Civil Engineering, it is assumed that the steel has a linear elastic behavior up to the yield stress and then has a perfectly plastic behavior. However, if a steel specimen is subjected to a cyclic loading test, we can observe that there are differences in the behavior of the steel. The first one is that it is no longer evident that the tension is broken when the steel passes from the elastic regime to the plastic regime, passing this transition to be subtle and the second is that in plastic regime, the increase in deformation is always accompanied by an increase in tension, that is, the steel is hardened.

These differences are because deformations in steel are actually dependent on several factors that do not translate when the applied load is quasi-static. To consider these factors, the combined Chaboche model was used to characterize the mechanical properties of the steel. This model is basically the linear hardening model proposed by Ziegler with the additional part that considers the stabilization of the hysteresis curve and an isotropic model based on Voce's law (Souto, 2011). This model is given by equation (1).

$$\alpha = \frac{C_{kin}}{\gamma} \cdot (1 - e^{-\gamma \cdot \varepsilon_i^p}) + \alpha_i \cdot e^{-\gamma \cdot \varepsilon_i^p} \quad (1)$$

Where  $C_{kin}$  and  $\gamma$  are parameters calibrated from the stabilized hysteresis cycle. The ratio  $C_{kin}/\gamma$  is the initial kinematic hardening modulus and also the maximum translational value of the yield surface and the parameter  $\gamma$  is the rate of change of the kinematic hardening modulus (Nip, Gardner, Davies, & Elghazouli, 2010) The value of the plastic deformation and backstress is obtained using expressions (2) and (3)

$$\varepsilon_i^p = \varepsilon_i - \sigma_i / E - \varepsilon_0^p \quad (2)$$

$$\alpha_i = \sigma_i - \sigma^s \quad (3)$$

Where  $\varepsilon_0^p$  is the amplitude of each hysteresis cycle and  $\sigma^s$  is the mean value of the first and last hysteresis cycle point,  $\sigma_1$  e  $\sigma_n$ , respectively, as shown in Figure 1.

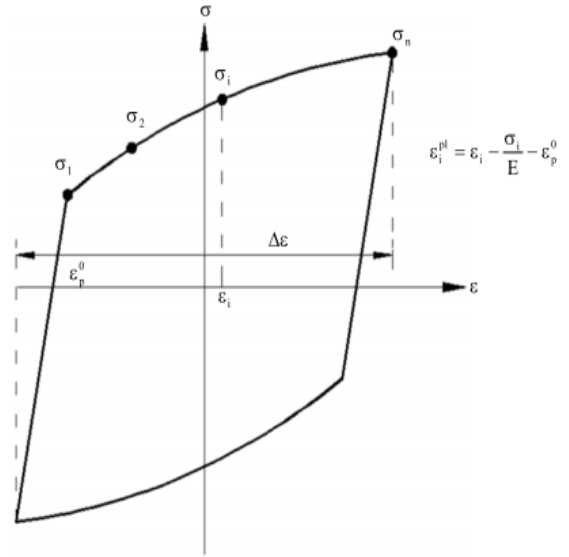


Figure 1: Calibration of kinematic hardening model. Adapted from (Nip et al., 2010)

The isotropic hardening model based on Voce's law is given by the following expression:

$$\sigma_Y = \sigma_0 + Q_{\infty} \cdot (1 - e^{-b_{iso} \cdot \varepsilon_i^p}) \quad (4)$$

Where  $\sigma_0$  is the yield stress,  $Q_{\infty}$  is the maximum yield surface increase and  $b_{iso}$  is the rate of growth of the yield surface as the plastic deformation increases (Nip et al., 2010). The size of the yielding and plastic deformation area corresponds, in each cycle, can be obtained by the following expressions:

$$\sigma_i^0 = \frac{\sigma_i^t - \sigma_i^c}{2} \quad (5)$$

$$\varepsilon_i^p = 1/2 \cdot (4i - 3) \Delta \varepsilon_p \quad (6)$$

$$\Delta \varepsilon_p = \Delta \varepsilon - 2\sigma_i^t / E \quad (7)$$

Where  $\sigma_i^t$  e  $\sigma_i^c$  are, respectively, the maximum tensile and compressive stress, as shown in Figure 2.

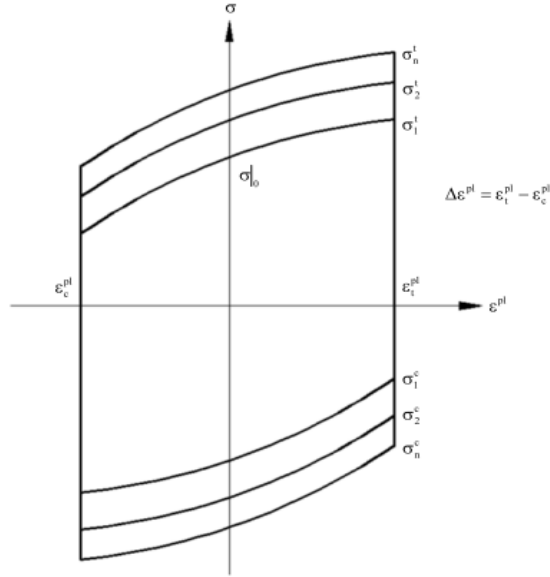


Figure 2: Calibration of isotropic hardening model.  
Adapted from (Nip et al., 2010)

## 2. DESIGN OF A HYSTERETIC DAMPER

The development of the height of the cross-section of the cantilever was made so that the moment applied to the cantilever, due to the force applied in the free end of the cantilever, was between the yield moment and the total plastification of the cross-sections, along the whole length. The applied moment was obtained by neglecting the weight of the console itself and the variation of the stiffness of the section. If we solve in function of the height, we can estimate the heights for which the sections enter in plastic regime and plasticize completely, along the length of the cantilever.

$$h_{el} = \sqrt{6 \cdot \frac{M(x)}{bf_y}} \quad (8)$$

$$h_{pl} = \sqrt{4 \cdot \frac{M(x)}{bf_y}} \quad (9)$$

The width of the console was considered constant in this study with a value of 0.15 m, the admitted length was 2.0 m. For this study, it was considered a situation in which the maximum force applied to the cantilever would be 1000 kN. In this study it is not intended to develop a damper for a specific structure, but rather to present a methodology of design, as a function of the height of the cross sections of the cantilever, that adapts to any force in the heatsink. The

steel used in this study was the steel S355J2H that has a yield stress of 443 MPa, this value was admitted based on the study made by L. Gardner, for hot rolled steels (Nip et al., 2010).

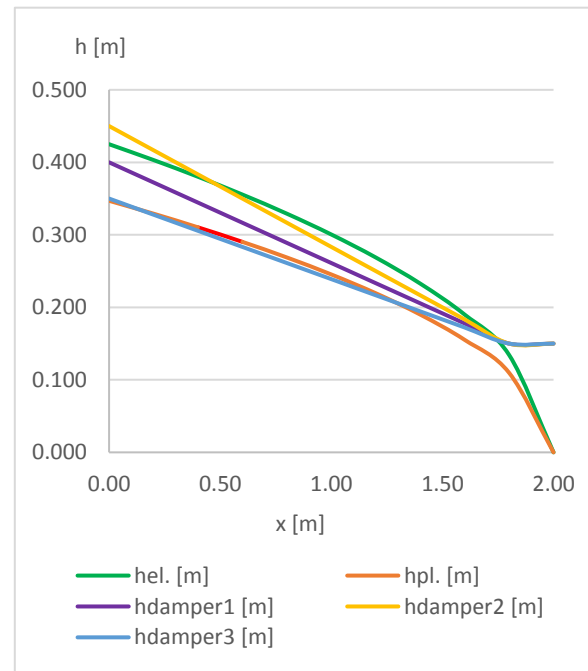
Table 1 shows a summary of the admitted values. The choice of this type of steel was because it was the only steel for which we had hardening values, but in the case of metal damper it would be more appropriate to choose a steel with a lower yielding tension.

Table 1: Dimensional constants of the cantilever

<b>b [m]</b>	0.15
<b>f<sub>y</sub> [kN/m<sup>2</sup>]</b>	443000
<b>F [kN]</b>	1000
<b>L [m]</b>	2.00

Given the variation that the height of the cantilever must have, three different configurations were tested. Despite the variation of  $h_{el}$  and  $h_{pl}$  is not linear, it was assumed that the cantilevers would have a linear variation, since from the constructive point of view it is simpler and since the slope only increases at the end of the cantilevers, in that zone, it is not important that the cross-sections enter plastic regime. **Erro! A origem da referência não foi encontrada.** shows the heights that were used for the three dampers.

Graphic 1: Heights of the three cantilevers along the length.

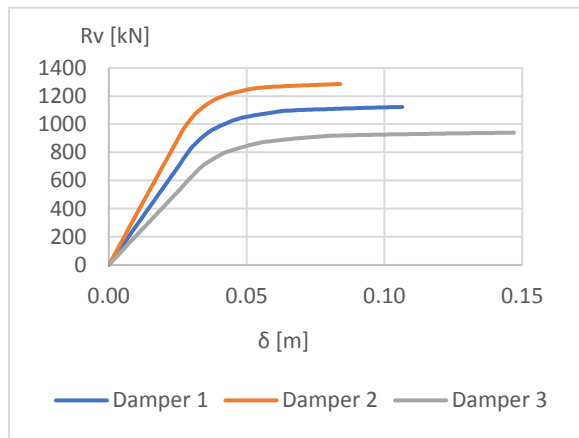


### 3. MONOTONIC BEHAVIOR

Given the geometry that the cantilevers should have, they were modeled using the finite element method, using the commercial program Abaqus 6.12 (Dassault Systèmes Simulia, 2012). In the modeling the cantilevers, were considered as a homogeneous 3D solid element with a mesh of hexahedral elements about 0.02 m in length in the transverse direction, 0.1 m in length in the longitudinal direction and width identical to the width of the cantilever. In this experimental phase, the cantilevers were modeled with an elastic-perfectly plastic behavior. According to the tests carried out by L. Gardner, for hot rolled steels the yield stress is 443 MPa and the modulus of elasticity is 212.31 GPa (Nip et al., 2010).

In **Erro! A origem da referência não foi encontrada.** it is possible to verify the reaction of the support for each one of the cantilevers as a function of the vertical displacement applied in the free end in each one of cantilevers until the

Graphic 2: Reaction in the abutment as a function of the monotonic displacement applied at the free end



first plastic hinge gets formed.

From the analysis of **Erro! A origem da referência não foi encontrada.** we can see that the higher the height of the cross-section in the abutment is, the greater the reaction will be, however, the maximum stresses will be concentrated closer to the free end and as a consequence the formation of the first plastic hinge will occur earlier.

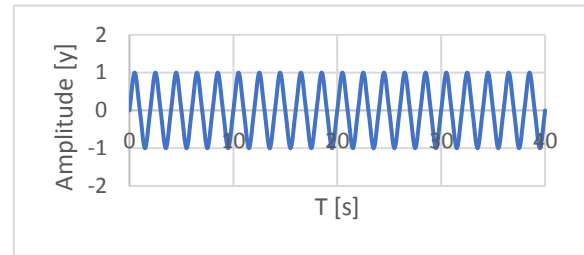
### 4. CYCLIC BEHAVIOUR

After the monotonic analysis of the consoles, the next step is to apply a constant strain controlled cycling to the cantilevers. The objective is to understand the differences between the monotonic and cyclic behavior and the differences in its behavior if we consider the steel as elastic-perfectly plastic or with hardening.

The value of the displacement applied in each of the consoles will be a value close to the displacement that

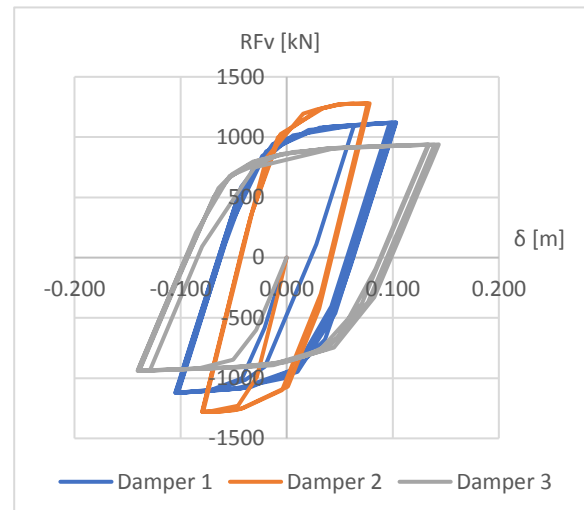
causes the first plastic hinge, determined in the previous chapter and with the frequency shown in Graphic 3.

Graphic 4: Amplitude of the cyclic displacement

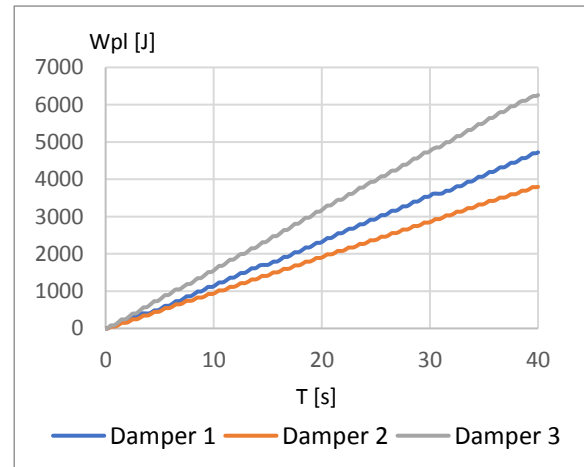


Graphic 4 shows the hysteresis curves of the three dampers and in Graphic 5 the energy dissipated over time.

Graphic 3: Hysteresis curves for the three cantilevers when the steel has a elastic-perfectly plastic behaviour



Graphic 5: Energy Dissipated over time for the three cantilevers when steel has a elastic-perfectly plastic behaviour



As would be expected the damper 2 is the one that presents a greater dissipation force, but as it is the damper that first forms a plastic hinge is the damper with a smaller hysteresis area, the damper 3 as it has a greater displacement is the console that dissipates more energy. This is not to say that console 3 is the most suitable, since for a displacement of less than 0.08 m is damper 2 that will dissipate more energy.

The behavior of the dampers when subjected to a cyclic load is now known, the difference in the hysteresis curves of the cantilevers will now be seen if we consider a constitutive model of steel that considers hardening. To perceive this influence, the same cyclic load referred to above was applied in the consoles, but for different kinematic and isotropic hardening parameters. These parameters are represented in Table 2 and were obtained based on the studies carried out by L: Gardner for steel specimens subjected to cyclic loading and different percentages of plastic deformation (Nip et al., 2010).

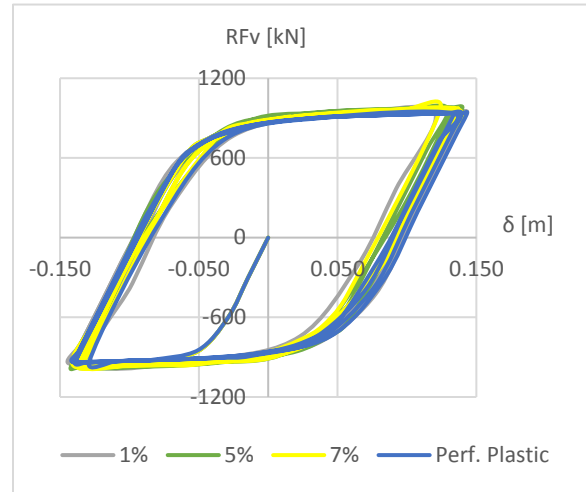
Table 2: Isotropic and kinematic hardening parameters

	40x40x4- CS-HR-1%	40x40x4- CS-HR-5%	40x40x4- CS-HR-7%
$\sigma_{ 0}$ [N/mm <sup>2</sup> ]	448	448	448
$Q_{\infty}$ [N/mm <sup>2</sup> ]	33	30	40
$b_{iso}$	0.17	6.03	3.46
$C_{kin}$ [N/mm <sup>2</sup> ]	29700	11600	16800
$\gamma$	229	68	129
$C_{kin}/\gamma$ [N/mm <sup>2</sup> ]	130	171	130

Damper 3 is the cantilever that has the greatest plastic deformation, so it will be the console that in principle will present a greater variation in the hysteresis curve, however, as can be seen in Graphic 6 and Graphic 7, the differences between the hysteresis and dissipated plastic energy are very little.

The distinction between the energy dissipated after 40 seconds in damper 3 for the different types of hardening is given in Table 3. As can be seen, the difference between the energy dissipated when the steel exhibits perfectly plastic behavior or hardening is maximum 3.2%, which means that the hardening of the steel can be neglected, considering that in the seismic action the cycles have different amplitudes and the cantilevers will not always enter inelastic behavior, as in this case where every cycle has maximum amplitude.

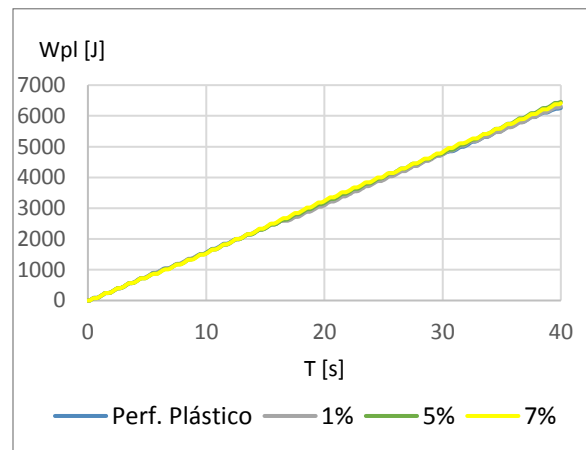
Graphic 6: Comparison between hysteresis cycles of damper 3 with different models of plasticity



Graphic 7: Comparison between dissipated plastic energy of damper 3 with different models of plasticity

Table 3: Comparison between the energy dissipated of damper 3 with different models of plasticity

	Wpl	%Wpl
Perf. Plastic	6257	100.0
40x40x4-CS-HR-1%	6308	100.8
40x40x4-CS-HR-5%	6457	103.2
40x40x4-CS-HR-7%	6411	102.5



## 5. PRACTICAL APPLICATION OF THE DAMPERS TO TWO CASE STUDIES

### 5.1. Case study 1

To understand if the application of these dampers is feasible at a practical level, they were installed on a bridge with the purpose of reducing the longitudinal seismic displacements. The model used to analyze the bridge longitudinally when it is subject to the seismic action was a model with a one degree of freedom with the characteristics expressed in Table 4.

Table 4: Bridge features for case study 1

Bridge features in case study 1	
M [Ton]	8800
K [kN/m]	86853
f [Hz]	0.50
T [s]	2.00

For the quantification of the seismic action, 10 earthquakes of type 1 and type 2 were generated, artificially generated for the municipality of Lisbon and soil type B (Carvalho, 2017). Based on these earthquakes, the average longitudinal displacement of the bridge was determined using the SAP2000 program without the dampers and with the dampers, the force generated on each damper, and the percentage reduction in longitudinal displacement when the dampers are installed. The results can be seen in Table 5

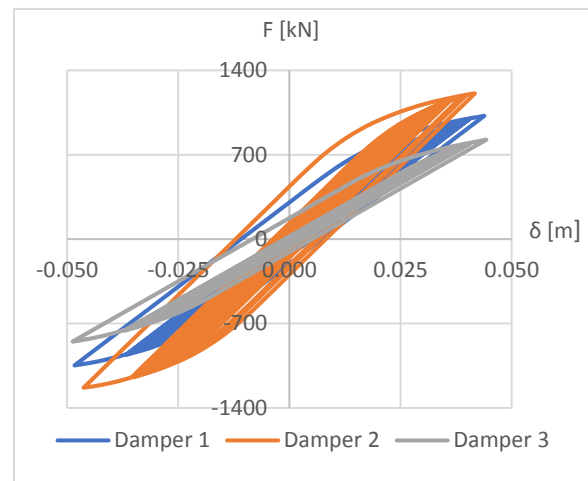
Table 5: Comparison between the displacements and forces generated in the bridge with and without dampers for case study 1

	E. type 1	E. type 2
$U_{\text{withoutDampers}}$ [m]	0.0469	0.0229
$F_b$ [kN]	4076	1986
$U_{\text{Damper1}}$ [m]	0.0425	0.0219
$F_{\text{Damper1}}$ [kN]	986	622
% $U_{\text{Damper1}}$	9.48	4.13
$U_{\text{Damper2}}$ [m]	0.0413	0.0218
$F_{\text{Damper2}}$ [kN]	1175	787
% $U_{\text{Damper2}}$	11.96	4.59
$U_{\text{Damper3}}$ [m]	0.0427	0.0219
$F_{\text{Damper3}}$ [kN]	862	608
% $U_{\text{Damper3}}$	9.03	4.11

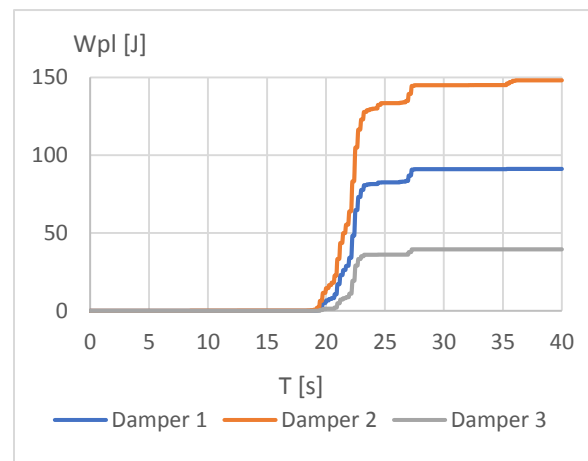
If we analyze the results of Table 5 we can see that the reduction in displacements is not significant, which is perceived since the basal force generated by the earthquake in the bridge is about four times higher than the force generated by the damper 2, which is the cantilever with the highest damping force.

To better understand the behavior of the bridge over time without and with installed dampers, we select earthquakes with the closest longitudinal displacements, quake # 08 and quake # 04, for type 1 and type 2 earthquakes, respectively. When analyzing the hysteresis curves of the type 1 # 08 earthquake, for the three dampers, we can see from Graphic 8 that the inelastic component in the dampers is small, that is they are not dissipating the desired energy, only in the area between 20 and the 25 s where the bridge exhibits the maximum displacements, the dampers are dispersing large amounts of energy from the earthquake, as can be seen in Graphic 9.

Graphic 8: Hysteretic curves of the type 1 #08 earthquake for the three dampers in case study 1.

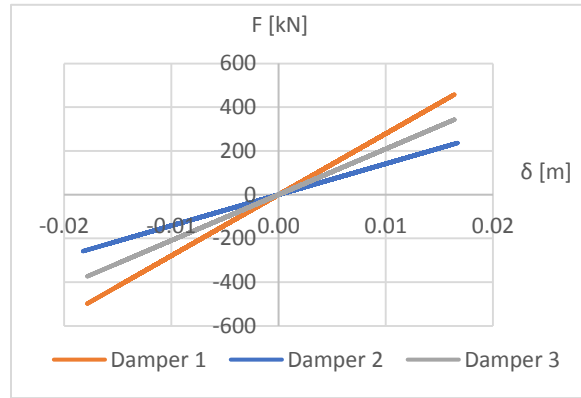


Graphic 9: Dissipated plastic energy of the type 1 #08 earthquake for the three dampers in case study 1.



For quake type 2 # 04, since the displacements are not large enough for the dampers to work in the inelastic regime, the addition of the dampers only increases the rigidity of the bridge, and the steel does not even yield, as can be verified by the hysteresis curves in Graphic 10.

Graphic 10: Hysteretic curves of the type 2 #04 earthquake for the three dampers in case study 1.



In conclusion, the dampers are not suitable for this case study, either because the frequency of the structure is very large or because the dissipation force generated by them is not sufficient. The solution in this case would be to add more than one damper so that they work in parallel or to size a damper that is better suited to this case study, however as the intended is not to size a damper for this case study, but rather to realize if these dampers are suitable, it was decided to test these dampers for a different case study.

## 5.2. Case study 2

In case study 2, as in case 1, the bridge structure was reduced to an oscillator with one degree of freedom and the characteristics of the bridge are expressed in Table 6

Table 6: Bridge features for case study 2

Bridge features in case study 2	
M [Ton]	8800
K [kN/m]	13896
f [Hz]	0.2
T [s]	5

The earthquakes used were the same as those used for study case 1. The displacements in case 2, with and without dampers, the basal force exerted on the bridge, the dissipation force exerted by each of the dampers and the reduction, in percent, in the maximum longitudinal

displacement on the bridge with the dampers, can be seen in Table 7

Table 7: Comparison between the displacements and forces generated in the bridge with and without dampers for case study 2

	E. type 1	E. type 2
$U_{\text{withoutDampers}}$ [m]	0.2133	0.1371
$F_b$ [kN]	2964	1905
$U_{\text{Damper1}}$ [m]	0.0993	0.0655
$F_{\text{Damper1}}$ [kN]	1117	1080
% $U_{\text{Damper1}}$	53.45	52.22
$U_{\text{Damper2}}$ [m]	0.0960	0.0597
$F_{\text{Damper2}}$ [kN]	1282	1252
% $U_{\text{Damper2}}$	54.98	56.47
$U_{\text{Damper3}}$ [m]	0.1034	0.0670
$F_{\text{Damper3}}$ [kN]	935	914
% $U_{\text{Damper3}}$	51.54	51.11

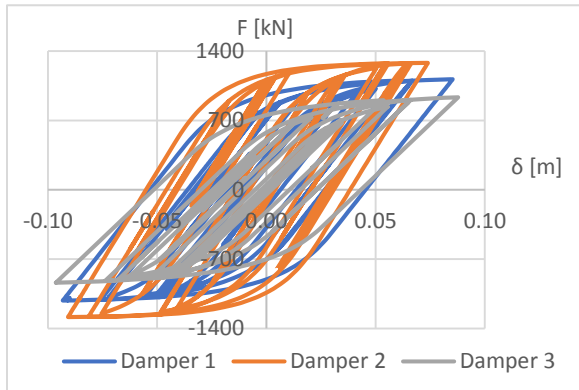
From the analysis of Table 7, we can see that the force produced by the dampers is about half of the basal force on the bridge, that is, it was still possible to optimize the dampers so that the forces were equal, but with only half the force, the displacements in the bridge are halved which demonstrates the effectiveness of this type of damper.

Compared to case 1, it can be observed that case 2 presents much higher displacements (about 4.5 larger), since stiffness is greater in case 1, this means that the more flexible the bridge the greater the efficiency of the dampers in reducing the displacements.

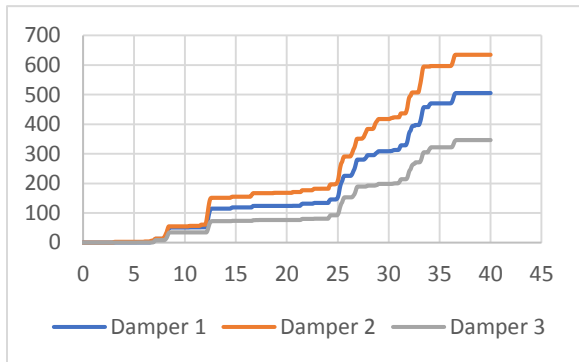
If we analyze over time the behavior of the bridge, as we did for case 1, for type 1 # 08 and type 2 # 04 earthquake, it is also possible to notice that the longitudinal displacements, in relation to case 1, although they increase in amplitude, decrease in frequency. In addition, when the bridge is equipped with the dampers unlike in case 1, longitudinal displacements take a completely different course over time.

The hysteresis curves of the three dampers when the bridge is subjected to quake # 08, can be seen in Graphic 11. From the analysis of the curves we can conclude that, since the displacements are similar for the three dampers, damper 2 as it is the cantilever that presents the greatest dissipation force will be the one that will present greater dissipation of energy. This fact can be confirmed in Graphic 12 where the energy dissipated for the three dampers is represented.

Graphic 11: Hysteretic curves of the type 1 #08 earthquake for the three dampers in case study 2.



Graphic 12: Dissipated plastic energy of the type 1 #08 earthquake for the three dampers in case study 2.



In Table 8 it is possible to observe the maximum longitudinal displacements in the bridge when the dampers are installed as well as their reduction in relation to the longitudinal displacement of the bridge without dampers, the maximum dissipation force of each of the devices and the energy dissipated after the 40 s for the quake type 1 # 08.

Table 8: Bridge features with dampers for earthquake type 1 #08 in case study 2.

	Damper 1	Damper 2	Damper 3
<b>Udamper [m]</b>	0.0932	0.0908	0.0964
<b>Fdamper [kN]</b>	1116	1282	935
<b>% Udamper</b>	68.57	69.38	67.50
<b>Wpl [J]</b>	505	635	347

Although the damper 2 is the cantilever that presents the greatest energy dissipation, for these displacements is the only cantilever that comes to form a plastic hinge. Since the reduction in displacements is similar and the difference

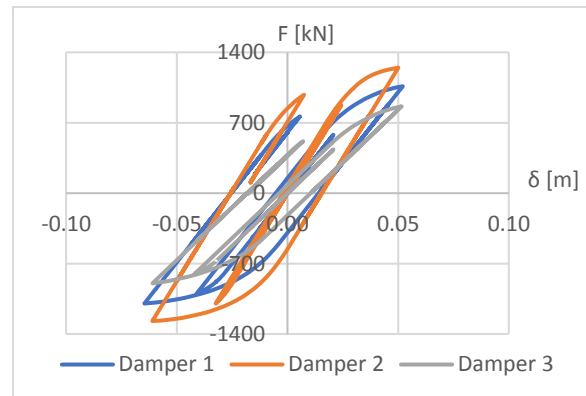
between the dissipated energy is negligible, it would be more appropriate to choose damper 1.

As for the quake type 2 #04 although there is a substantial reduction of the displacements (about 50%), as can be seen from Table 9, through the analysis of Graphic 13 we can see that the hysteresis curves for the dampers enter inelastic regime fewer times than for quake type 1 #08, fact that can be confirmed in Graphic 14, where the energy dissipated for each of the consoles is represented.

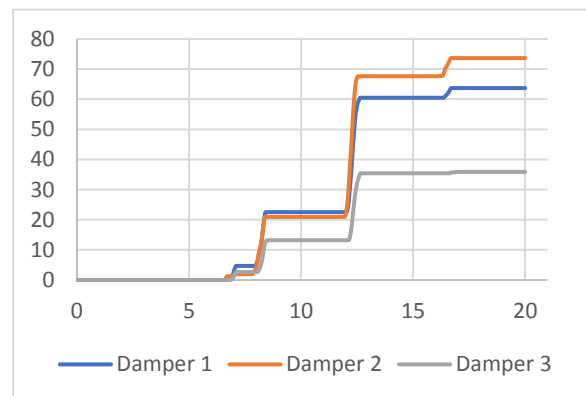
Table 9: Bridge features with dampers for earthquake type 2 #04 in case study 2.

	Damper 1	Damper 2	Damper 3
<b>Udamper [m]</b>	0.0646	0.0610	0.0609
<b>Fdamper [kN]</b>	1094	1268	919
<b>% Udamper</b>	52.16	54.85	54.89
<b>Wpl [J]</b>	64	74	36

Graphic 13: Hysteretic curves of the type 2 #04 earthquake for the three dampers in case study 2.



Graphic 14: Dissipated plastic energy of the type 2 #04 earthquake for the three dampers in case study 2.





As the dissipation force is closer to the basal force of the bridge without damper, it was expected that the hysteresis curves did not plasticize as much, however, it is observed that although the difference between the forces is smaller, the reduction of the displacements is practically the same, in the earthquake type 1 # 08 and earthquake type 2 # 04, which probably means to obtain a greater reduction in the displacements it is preferable to decrease the yielding tension of the cantilevers so that they begin inelastic behavior sooner than to increase the force of dissipation that they generate.

## 6. CONCLUSION

In this article it was possible to realize that metallic dampers are a good solution for the control of vibrations in structures, since they are versatile: The different geometric configurations that this sort of dampers can have, allow these dampers to be adapted to any type of structures provided they are properly sized.

The method used in the design of the cantilever with variable cross-section has proved to be adequate to define the height that the cantilever must have along their length, according to the dissipation force that they are expected to exert. As long as the height is between  $h_{pl}$  and  $h_{el}$ , then the cantilever will initiate the inelastic behavior to the dissipation force for which it was design. In addition, it was possible to observe that if we increase the height of the section next to the upper limit, then the cantilever will concentrate the efforts near the free end and although it presents greater amounts of steel and dissipation force than the cantilevers with a smaller height, will initiate the process earlier than other cantilevers.

As previously mentioned, the choice of the steel used was conditioned by the need to analyze the dynamic effects that the seismic action had on the steel properties. It would be more appropriate to have these mechanisms dimensioned with a steel having a lower yield stress so that the yielding occurred earlier and the amount of plasticized steel was higher. Although the literature suggests that these devices must be design considering the dynamic effects on the properties of steel, this study showed that for the apparatus in question, the influence of the dynamic properties of the steel on the damper behavior are insignificant.

The studied geometric configuration of a cantilever with variable section proved to be a good solution for bridges

with a low frequency of its own. In these bridges where the longitudinal displacements are high, their incorporation in the structure, allows to reduce the displacements a lot. However, with larger frequencies these dampers aren't very efficient.

Given the results obtained using the finite element program, the next step would be to build one of these types of dampers on scale and see if their behavior would be similar.

In the future, it would be interesting to see whether, in a building structure, the application of this type of dampers would also be appropriate, since they could, because of their length, be installed in such a way as to reduce displacements between floors when the building was subject to seismic action.

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