Instituto Superior Técnico / Universidade de Lisboa

Departament of Bioengineering

Master on Biomedical Engineering

Signals and Systems in Bioengineering

1st Semester 2017/2018

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Exam 2

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Name : Number:

The duration of the Exam is 3h. The score of each item 1 when right and -0.25 if wrong. Only one option can be selected in each question.

Part 1

1. Let $x(n) = [1, 0, 1, 2, 3, 2]^T$. Select the signal which has a pure real DFT coefficients
   - ☐ a) $x((n-2)_6)$
   - ☐ b) $x((n-1)_6)$
   - ■ c) $x((n+1)_6)$
   - ☐ d) None

2. Consider a continuous signal, $x(t)$, sampled at a frequency of $f_s = 819.2$Hz and the corresponding discrete signal $x_d(n)$. What is the minimum spectral separation, in Hz, among the 2048-length FFT coefficients, $X_{2048}(k)$, of $x_d(n)$?
   - ■ a) 0.4 Hz
   - ☐ b) 0.5 Hz
   - ☐ c) 1.0 Hz
   - ☐ d) None

3. Consider a 10 length signal $\mathbf{x} = [0; 1; 2; 3; 4; 5; 6; 7; 8; 9]^T$. Sample the Fourier transform of $\mathbf{x}$, $X(\omega)$, at 8 evenly distributed frequencies, $X_8(k)$, and compute $y(n) = DFT_8^{-1}(X)$, for $n = [0, 1, ..., 7]$, where $DFT_8^{-1}()$ denotes a 8 length DFT inversion operator.

   What is $y(n)$?
   - ☐ a) $y(n) = [0; 1; 2; 3; 4; 5; 6; 7]^T$.

- ☐ b) $y(n) = [8; 9; 2; 3; 4; 5; 6; 7]^T$.
- ■ c) $y(n) = [8; 10; 2; 3; 4; 5; 6; 7]^T$.
- ☐ d) None

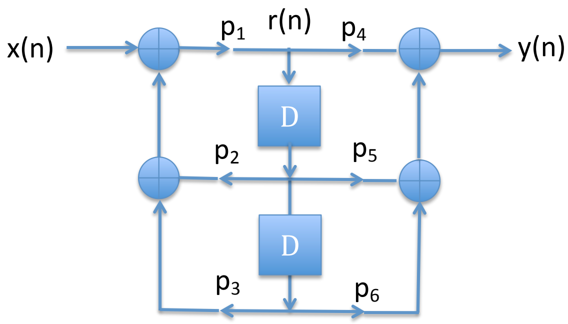

Figura 1: Direct Form II IIR filter

4. Consider the following IIR filter:

$$H(z) = \frac{1}{1 - 0.5z^{-1} + 0.25z^{-2}} \tag{1}$$

graphically represented in figure 1 in the direct form II.
What is the vector of coefficients, $\mathbf{p} = [p_1, p_2, p_3, p_4, p_5, p_6]$?

- ☐ a) $\mathbf{p} = [1, -0.5, 0.25, 1, 0, 0]$
- ■ b) $\mathbf{p} = [1, 0.5, -0.25, 1, 0, 0]$
- ☐ c) $\mathbf{p} = [1, 0, 0, 1, 0.5, -0.25]$
- ☐ d) None

5. Let us consider an infinite signal to be filtered by a 49 length impulse response FIR filter. To avoid wrong results at each processed block what should be the minimum length of the FFT_L if a 1000 length input block is used?

- ☐ a) $L = 1024$.
- ☐ b) $L = 1048$.
- ■ c) $L = 2048$.
- ☐ d) None

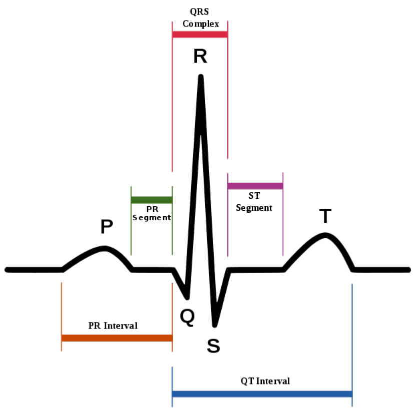

Figura 2: ECG.

6. Consider the ECG signal displayed in figure 2. In healthy subjects, with a maximum heart rate of 180 bpm, it can be assumed a minimum value for the QRS interval of 0.08 s. What is the minimum sampling frequency needed to accurately detect and capture the QRS complex and in particular the R-Peak?

   - □ a) 6 Hz.
   - ■ b) 25 Hz.
   - □ c) 50 Hz.
   - □ d) None

7. Consider the following scalar function $g(\mathbf{x}, \mathbf{y}) = \mathbf{y}^T\mathbf{x} + \mathbf{b}$ where $\mathbf{x}, \mathbf{y}$ and $\mathbf{b} \neq \mathbf{0}$ are column vectors. $g(\mathbf{x}, \mathbf{y})$ is a metric function because is

   - □ a) linear.
   - □ b) strictly positive.
   - □ c) a symmetric function.
   - ■ d) None of the above

8. Consider the canonical adaptive filter displayed in figure 3 and two, independent, zero mean white (with autocorrelation $\phi(n) = \delta(n)$), Gaussian noise signals, $\eta(n), \epsilon(n) \sim \mathcal{N}(0, 1)$. If $d(n) = \eta(n)$ and $x(n) = \epsilon(n)$ what is the optimal impulse response of the 4-length FIR filter $h(n)$ that minimizes the norm of the error, $\|\mathbf{e}\|$?

   - ■ a) $h(n) = [0, 0, 0, 0]$.
   - □ b) $h(n) = [1, 0, 0, 0]$.
   - □ c) $h(n) = [1, 1, 1, 1]$.
   - □ d) None

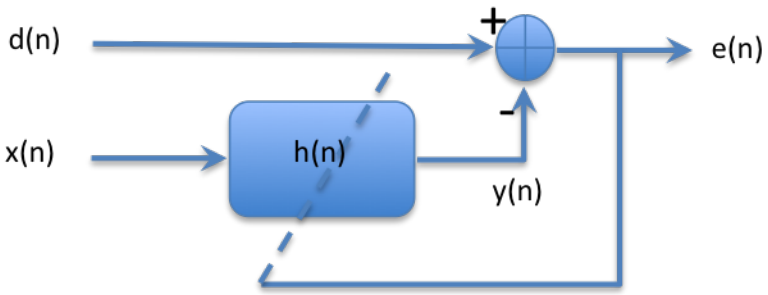

Figura 3: Adaptive filter.

Problem (2)

Let $x(n)$ be a N length strictly positive sequence and consider the following non linear auto-regressive (AR) model

$$x(n) = c_1 x(n-1) + c_2 \log(x(n-2)) \tag{2}$$

.

1. Formulate the estimation problem of the vector of coefficients $\theta = [c_1, c_2]^T$ using matrix notation by minimizing norm of error vector $\mathbf{e} = [e_1, e_2, ...e_N]^T$, where

$$e_i = c_1 x(i-1) + c_2 \log(x(i-2)) - x(i). \tag{3}$$

2. Compute the solution when $x(n) = 1$ (note that the optimal solution minimizes the norm of the error vector as well as the norm of the vector of parameters, θ).

Part 2

1. Consider the IIR LTI system described the following difference equations

$$y(n) = x(n) + 0.5y(n-1) - 0.25y(n-2). \tag{4}$$

   What is the mean value of the output signal if the input is $x(n) = \eta(n) + 3$ where η is white Gaussian noise, $\eta \sim \mathcal{N}(0, \sigma^2)$.

   - ☐ a) 8.
   - ■ b) 4.
   - ☐ c) 2.
   - ☐ d) None of the above

2. To change the sampling rate of a discrete signal $x(n)$ by a factor of $R = 1.25$ what is the appropriated sequence of operations?

   - ☐ a) $T_{\downarrow 4}[T_{\uparrow 5}[h * x(n)]]$.
   - ■ b) $T_{\downarrow 4}[h * T_{\uparrow 5}[x(n)]]$.
   - ☐ c) $T_{\downarrow 5}[h * T_{\uparrow 4}[x(n)]]$.
   - ☐ d) None

3. When changing the sampling rate f_s by a factor $R > 1$ the aliasing

   - ☐ a) always occur.
   - ☐ b) depends on the signal.
   - ☐ c) depends on f_s.
   - ■ d) None

4. Consider a first order unknown system $G(s) = A/(1 + \tau s)$ and its step $(x(t) = u(t))$ response displayed in figure 4. What is the time constant in seconds, τ, and the gain, A, (A, τ)?

   - ■ a) $(4, 4)$.
   - ☐ b) $(1, 4)$.
   - ☐ c) $(4, 0.25)$.
   - ☐ d) None

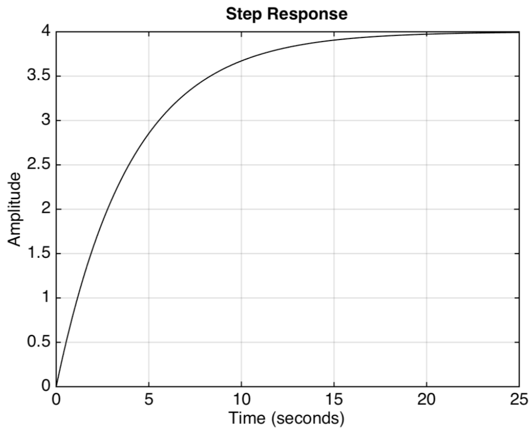

Figura 4: First order step response of an unknown system, $G(s) = A/(1+\tau s)$.

5. Consider an unknown system $G(s)$ and the step ($x(t) = u(t)$) response displayed in figure 5. The poles of this system are all

- ☐ a) real
- ☐ b) imaginary
- ☐ c) real but one that is complex
- ■ d) None

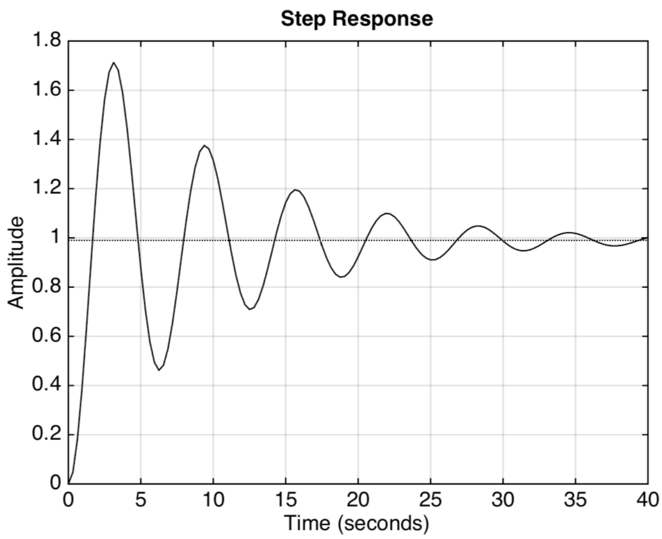

Figura 5: Step response of an unknown system.

6. Consider the following open-loop transfer function

$$G(s) = \frac{10}{s(s-1)} \tag{5}$$

and an unity negative feedback output topology where the controller is the following $C(s) = K(s+1)$. What is the value of K that makes the closed loop system critically stable?

- ☐ a) $K = 0$
- ■ b) $K = 1/10$
- ☐ c) $K = 10$
- ☐ d) None of the above

7. Consider an unity negative feedback output topology where $G(s) = \frac{1}{s(s+1)}$ and $C(s) = K(s+2)$ are the plant and controller transfer functions respectively. How many branches will have the root-locus of the closed loop system?

- ☐ a) 0.
- ☐ b) 1.
- ■ b) 2.
- ☐ d) None of the above

8. In the previous example, how many branches go to ∞?

- ☐ a) 0.
- ■ b) 1.
- ☐ b) 2.
- ☐ d) None of the above

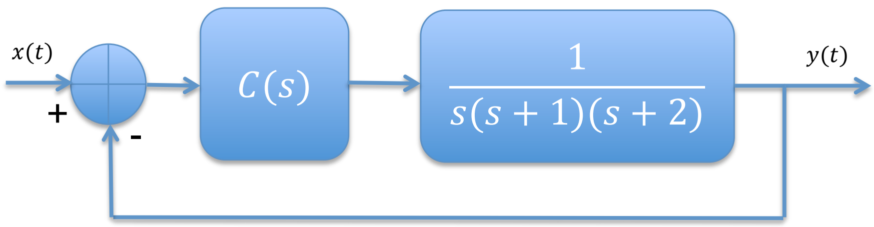

Figura 6: Unit feedback control system.

Problem (2)

Consider the feedback system represented in Fig.6 with a proportional controller $C(s) = K$.

1. **(1)** Draw the root-locus (RL) for $K > 0$ and $K < 0$. Compute explicitly the **break in** and **break out** points at the real axis, **asymptotic center and angle** of the asymptotes and the **sections of the real axis** of the RL.

2. **(0,5)** Stability interval

   - Compute the stability interval, K, $[K_{min}, K_{max}]$, where the closed loop system is stable.
   - For the limits of that interval compute the corresponding location of the three closed loop poles.
     (Notice that when the difference between the number of poles and zeros is greater or equal to 2, $N - M \geq 2$, the sum of the open loop and closed loop poles is equal, $\sum_i p_{ol}(i) = \sum_i p_{cl}(i)$).

3. **(0,5)** Consider now the following controller $C(s) = K(s + 0.5)$. Sketch the new RL and comment about the stability of the new closed loop system (compute again the asymptotic center and angles).