Instituto Superior Técnico / University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signals and Systems in Bioengineering

1st Semester de 2017/2018

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Epoca Especial

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Name : Number:

The duration of the exam is 3h. The score of each item is 1 when right and -0.25 if wrong. Only one option can be selected in each question.

Part 1

1. Consider the signal $y(n) = x(n - n_0)$ where $x(n) = [1; 2; 1; 0; -1; -2; -1; 0]$. What should be the value of n_0 for which the DFT of $y(n)$, $Y(k)$, is pure imaginary,
   - ■ a) 1
   - □ b) 0
   - □ c) -1
   - □ d) None of the above
2. Consider the signal $x(n) = [0; 1; 0; 1]$. What is its DFT?,
   - □ a) $e^{-k\pi/2} - e^{-3k\pi/2}$
   - ■ b) $2(-1)^k cos(k\pi/2)$
   - □ c) $cos(k\pi/2)$
   - □ d) None of the above
3. Consider the complex finite length sequence

$$x(n) = [1; 1 - j; 0; -1 + j; 1; -2 + j; -2j; -1]$$

and $y(n) = x_8(1 - n) + x_6(n + 2)$. What is the 8 length DFT value for $k = 8$, $X_8(8)$?

- ☐ a) 0.
- ☐ b) 1.
- ☐ c) j.
- ☐ d) None

4. What is the period of the signal $x(n) = e^{j0.2\pi n}$?

   - ☐ a) 0.
   - ☐ b) 5.
   - ■ c) 10.
   - ☐ d) None of the above

5. What is the impulse response of the filter $H(z) = [1 + az^{-1}]^{-1}$?

   - ☐ a) $[1, a]$
   - ■ b) $(-a)^n u(n)$
   - ☐ c) $a^n u(n)$
   - ☐ d) None

6. Consider a vector in the plane, $\mathbf{u} = [u_x, u_y]^T \in R^2$ and the following norms: $\|\mathbf{u}\|_1 = |u_x| + |u_y|$ and $\|\mathbf{u}\|_2 = \sqrt{u_x^2 + u_y^2}$. Which condition is true?

   - ☐ a) $\|\mathbf{u}\|_2 < \|\mathbf{u}\|_1$.
   - ☐ b) $\|\mathbf{u}\|_2 = \|\mathbf{u}\|_1$.
   - ☐ c) $\|\mathbf{u}\|_2 \geq \|\mathbf{u}\|_1$.
   - ☐ d) None

7. Let us consider an infinite signal, to be filtered by FIR filter with impulse response length 10. To implement the filtering process by blocks with a 2048 length FFT algorithm, what should be the length of the input blocks to not have overlap of these blocks?

   - ■ a) 2030.
   - ☐ b) 2040.
   - ☐ c) 2050.
   - ☐ d) None

8. Consider the *Linear Time Invariant* (LTI) system described by the following transfer function

$$H(z) = \frac{1}{1 + (3/2)z^{-1} + (9/16)z^{-2}} \tag{1}$$

What type of filter is this system?

- ■ a) High-pass filter .
- □ b) Band-pass filter.
- □ c) Low-pass filter.
- □ d) None

Problem (2)

Let $x(n)$ be a N length strictly positive sequence and consider the following non linear auto-regressive (AR) model

$$x(n) = c_1 x(n-1) + c_2 \log(x(n-2)) \tag{2}$$

.

Formulate the estimation problem of the vector of coefficients $\theta = [c_1, c_2]^T$ using matrix notation by minimizing the energy function

$$E(\theta) = \sum [c_1 x(n-1) + c_2 \log(x(n-2)) - x(n)]^2 \tag{3}$$

with respect to θ,

$$\theta^* = \arg\min_{\theta} E(\theta) \tag{4}$$

Part 2

1. Consider the LTI system described the following difference equations

$$y(n) = x(n) - 0.5y(n-1) \tag{5}$$

What is the mean value of the output signal if the input is $x(n) = \eta(n) + 4$ where $\eta \sim \mathcal{N}(2, 2^2)$ is white Gaussian noise?

- ☐ a) 8.
- ■ b) 4.
- ☐ c) 2.
- ☐d) None of the above

2. What is the value of the following integral?

$$\int_{-\infty}^{\infty} e^{-\frac{(x-1)^2}{4}} dx \tag{6}$$

- ■ a) $2\sqrt{\pi}$.
- ☐ b) 1.
- ☐ c) ∞.
- ☐ d) None of the above

3. Consider the following decimation operation $y(n) = T_{\downarrow 2}[h(n) * x(n)]$ where $x(n) = cos(\frac{3\pi}{4}n)$ and $h(n)$ is an ideal anti-aliasing filter. What is the output signal?

- ■ a) $y(n) = 0$.
- ☐ b) $y(n) = cos(\frac{3\pi}{4}n)$.
- ☐ c) $y(n) = cos(\frac{3\pi}{8}n)$.
- ☐ d) None of the above

4. Let x and y two zero mean correlated random variables with variances σ_x^2 and σ_y^2 respectively. What is the variance of the $z = x + y$?

- ☐ a) $\sigma_x^2 + \sigma_y^2$.
- ■ b) $\sigma_x^2 + \sigma_y^2 + 2E[xy]$.
- ☐ c) $(\sigma_x + \sigma_y)^2$.
- ☐ d) None of the above

5. Consider an unitary negative feedback output topology where $G(s) = \frac{1}{s+1}$ and $C(s) = K(s+2)$ are the plant and controller transfer functions respectively. How many branches will have the root-locus of the closed loop system?

- ☐ a) 0.
- ■ b) 1.
- ☐ b) 2.
- ☐ d) None of the above

6. Using the previous example, how many branches go to ∞?

- ■ a) 0.
- ☐ b) 1.
- ☐ b) 2.
- ☐ d) None of the above

7. Consider a system with the following open-loop transfer function

$$G(s) = \frac{s+1}{s(s-1)} \tag{7}$$

and an unitary negative feedback output topology where the controller is just a gain, $C(s) = K$. What is the value of K that makes the closed loop system stable?

- ☐ a) $K = 0$
- ■ b) $K = 1$
- ☐ c) $K = \infty$
- ☐ d) None of the above

8. Consider the following open loop transfer function $G(s) = \frac{1}{(s+1)^2}$. The corresponding closed loop system with $C(s) = K(s-1)$ is... (complete the sentence)

- ☐ a) Stable for every K.
- ☐ b) Unstable for every K.
- ☐ b) Stable for $K > 0$.
- ■ d) None of the above

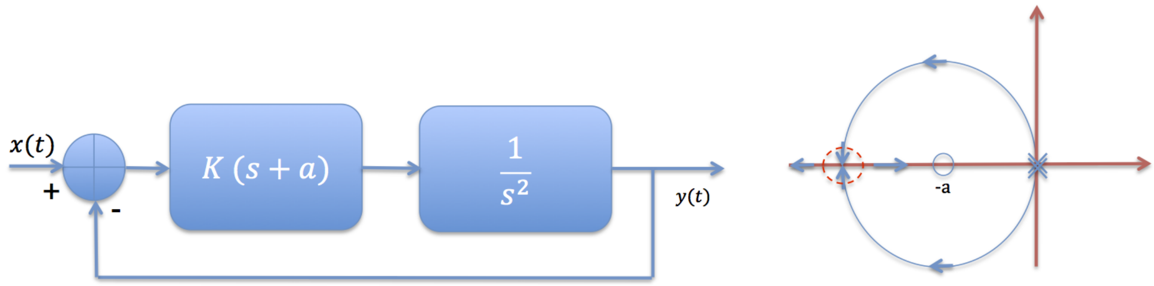

Figura 1: Unit feedback control system.

Problem (2)
Consider the feedback system represented in Fig.1.

For $a = 0$

1. Derive the corresponding root-locus for $K > 0$ and $K < 0$ without using the root-locus rules. Do it based on the analytic expression of the closed loop poles.

2. What is the stability interval for K (the interval of K for which the closed loop system is stable).

   For $a > 0$

3. The root-locus for $a > 0$ and $K > 0$ is displayed in the figure above (right).
   Derive analytically the interval of values of K that lead to closed loop complex poles.

4. What is location of the poles inside the dashed circle (in the right side of the figure) and the corresponding value of K.

5. Compute and draw the root-locus for $K < 0$. Is the system stable in this case?