Instituto Superior Técnico / Tecnical University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signal and Systems in Bioengineering

1st Semester of 2013/2014

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Exame 2

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Name : Number:

The duration of the test is 3h. The score of each item is 1 when right and −0.25 if wrong. Only one option can be selected in each question.

[□]Part 1

1. Let $x(n) = [1, 0, 1, 2, 3, 2]^T$. Select the signal which has a pure real DFT coefficients
   - □ a) $x((n+1)_6)$
   - □ b) $x((n-1)_6)$
   - ■ c) $x((n-2)_6)$
   - □ d) None
2. Let $p = [p_1, p_2, ..., p_N]$ a complete, non orthogonal basis of a vector space S. In this case the Graminian is
   - ■ a) Non-diagonal
   - □ b) Non-invertible
   - □ c) Non-square
   - □ d) None
3. What is the impulse response of the filter $H(z) = [1 + az^{-1}]^{-1}$?
   - □ a) $[1, a]$
   - ■ b) $(-a)^n u(n)$
   - □ c) $a^n u(n)$
   - □ d) None

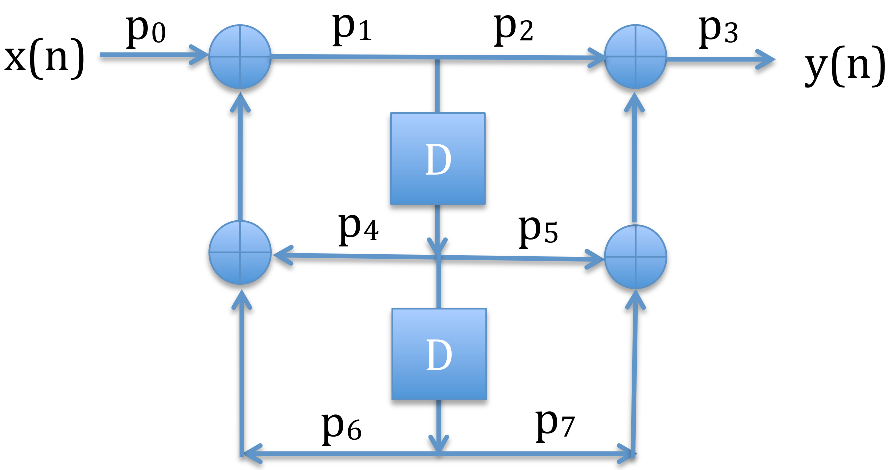

Figura 1: Direct Form II IIR filter

4. Consider the following IIR filter:

$$H(z) = \frac{2 + z^{-1} + 0.5z^{-2}}{1 - 1.25z^{-1} + 0.3z^{-2}} \quad (1)$$

What is the vector of coefficients, $\mathbf{p} = [p_0, p_1, p_2, p_2, p_3, p_4, p_5, p_6, p_7]$, at Fig.1 that implements this filter?

- ■ a) $\mathbf{p} = [1, 2, 1, 1, 1.25, 1, -0.3, 0.5]$
- □ b) $\mathbf{p} = [1, 1, 1, 1, -1.25, 1, 0.3, 0.5]$
- □ c) $\mathbf{p} = [0, 0, 0, 0, 1.25, 1, -0.3, 0.5]$
- □ d) None

5. What is the period of the signal $\sin(\pi n/2) + 0.25\cos(\pi n/3)$?

- □ a) 6
- ■ b) 12
- □ c) 24
- □ d) None

6. Let us consider an infinite signal, to be filtered by FIR filter with impulse response length 10. To implement the filtering process by blocks with a 2048 length FFT algorithm, what should be the length of the input blocks to not have overlap of these blocks?

   - ■ a) 2030.
   - □ b) 2040.
   - □ c) 2050.
   - □ d) None

7. Consider a continuous signal, $x(t)$, sampled at a sampling frequency of $f_s = 1000\text{Hz}$. What should be length N of the FFT used for spectral analysis in order to obtain a spectral separation of at most $f_1 \leq 1\text{Hz}$?

   - □ a) $N = 1000$
   - □ b) $N = 1012$
   - ■ c) $N = 1024$
   - □ d) None

8. Consider the *Linear Time Invariant* (LTI) system described by the following transfer function

$$H(z) = \frac{1}{1 + (3/2)z^{-1} + (9/16)z^{-2}} \qquad (2)$$

   What type of filter is this system?

   - ■ a) High-pass filter .
   - □ b) Band-pass filter.
   - □ c) Low-pass filter.
   - □ d) None

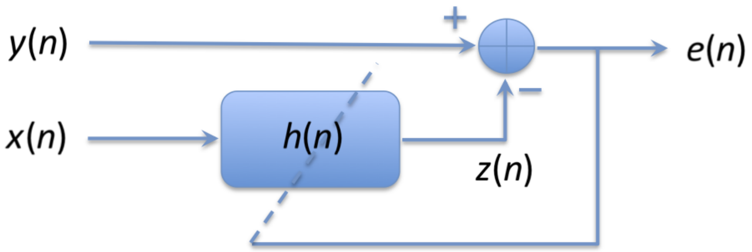

Figura 2: Adaptive filter

Problem

Consider the FIR filter $h(n)$ in Fig.2,

$$z(n) = \sum_{k=0}^{p} h(k)x(n-k) \tag{3}$$

where p is the order of the filter and $\mathbf{h} = \{h(0), h(1), ...h(p)\}^T$ are the $p+1$ coefficients of the filter to be estimated at each n^{th} sample, according the following criterion

$$\mathbf{h}(n) = \arg\min_{\mathbf{h}} J(\mathbf{h}) \tag{4}$$

where

$$J(\mathbf{h}) = \sum_{i=0}^{N-1} (y(n-i) - z(n-i))^2 \tag{5}$$

with N is the length of a window.

1. $\mathbf{z} = [z(0), z(1), ..., z(N-1)]^T$ can be expressed as $\mathbf{z} = A\mathbf{h}$. **Define A**.
2. Express $J(\mathbf{h})$ by using matrix notation.
3. Derive a closed-form solution for the optimization problem described in (4).
4. What is the optimal filter $h(n)$ if $p = 10$ and $y(n) = x(n-2)$?

[☐] Part 2

1. Let x and y two random variables with variances σ_x^2 and σ_y^2 respectively. What is the variance of the $z = x + y$?

   - ☐ a) $\sigma_x^2 + \sigma_y^2$.
   - ■ b) $\sigma_x^2 + \sigma_y^2 + 2E[(x - \mu_x)(y - \mu_y)]$.
   - ☐ c) $\sigma_x^2 + \sigma_y^2 + E[xy]$.
   - ☐ d) None of the above

2. Consider the following LTI system

$$H(z) = \frac{0.5}{1 - 0.5z^{-1}} \tag{6}$$

   . If the input is zero mean ($\mu_x = 0$) white noise with variance $\sigma_x^2 = 1$, $x \sim N(0,1)$. What is the *power spectral density* (PSD) of the output?

   - ☐ a) $\frac{1}{1.25 - 0.5cos(\omega)}$.
   - ☐ b) $\frac{1.5}{1.0625 - 0.5cos(\omega)}$.
   - ■ c) $\frac{.25}{1.25 - cos(\omega)}$.
   - ☐ d) None of the above

3. Consider a LTI system described by the equation $y(n) = x(n) + 0.25z^{-2}$ where the autocorrelation of the input signal is $\phi_{xx}(m) = 4$. What is the mean of the output signal?

   - ■ a) 8/3.
   - ☐ b) 16/9.
   - ☐ c) 0.
   - ☐ d) None of the above

4. Consider $z = xy$ where x and y are two random variables with mean μ_x and μ_y respectively. The mean of z, μ_z, is

   - ☐ a) 0.
   - ■ b) $\mu_x \mu_y$ if x and y are independent.
   - ☐ c) $\mu_x + \mu_y$.
   - ☐ d) None of the above

Consider a unity feedback control system with a controller $C(s)$ and a system (plant) to control $G(s) = 1/(s+1)$, as shown in Fig.3.

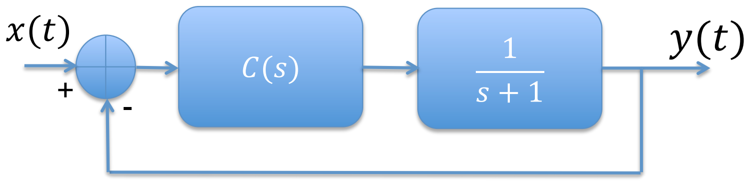

Figura 3: Unity feedback control system

5. For a proportional controller, $C(s) = K$, what is the value of K for which the static error, $e(t) = x(t) - y(t)$, is zero?

- ☐ a) $K = 0$.
- ☐ b) $K = -1$.
- ■ c) $K = \infty$.
- ☐ d) None of the above

6. Which controller leads to a finite error to a ramp input, $x(t) = tu(t)$?

- ☐ a) $C(s) = K(s + 10)$.
- ■ b) $C(s) = K(s + 10)/s$.
- ☐ c) $C(s) = K(s + 10)/(s + 1)$.
- ☐ d) None of the above

7. The overshooting of the closed loop response for $C(s) = K$:

- ☐ a) Increases with K.
- ☐ b) Decreases with K.
- ☐ c) There is no overshooting.
- ■ d) None of the above

8. The settling time of the closed loop response to a step,$x(t) = u(t)$,

- ☐ a) Increases with K.
- ■ b) Decreases with K.
- ☐ c) Is independent of K.
- ☐ d) None of the above

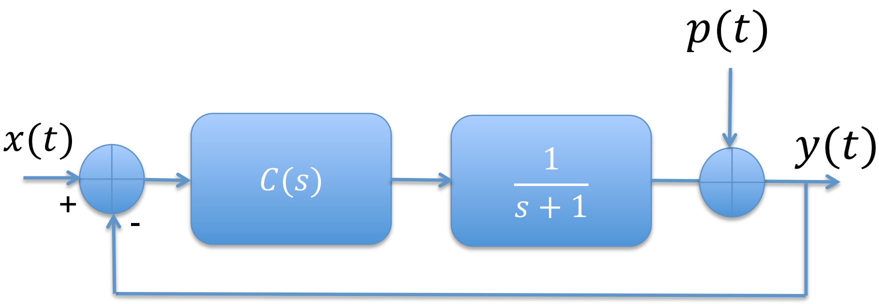

Figura 4: Unity feedback control system with disturbances

Problem

Consider the previous feedback system with disturbances,$p(t)$, at the output, as shown in Fig. 4.

1. Compute the dependence of the output, $y(t)$, with the disturbances, $p(t)$, $H(s) = Y(s)/P(s)$.
2. What is the attenuation performed by te closed-loop system to a 1 rad/s sinusoidal disturbance, $p(t) = sin(t)$?
3. What is the*Power Spectral Density* (PSD) of the output when the disturbance is white noise with unit variance $\sigma_p^2 = 1$ for $K = 9$?
4. What is the*Power Spectral Density* (PSD) of the output when the disturbance and the input are both white noise with unit variances, $\sigma_p^2 = 1$ and $\sigma_x^2 = 1$, for $K = 9$.