Instituto Superior Técnico / University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signals and Systems in Bioengineering

1st Semester de 2017/2018

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Test 1

November 15, 2017

Name : Number:

The duration of the test is 1h30m. The score of each item is 2 when right and -0.5 if wrong. Only one option can be selected in each question.

1. Consider the *Linear Time Invariant* (LTI) system described by the following difference equation

$$y(n) = x(n) + y(n-1) - 0.25y(n-2). \tag{1}$$

What type of filter is this system?

- ☐ a) High-pass filter .
- ☐ b) Band-pass filter.
- ■ c) Low-pass filter.
- ☐ d) None

2. Consider the chirp signal $x(t) = sin(2\pi f(t)t$ for $0 \leq t < 1$ second with $f(t) = 100 + 900t^2$. What is the band of frequencies occupied by this signal?

- ☐ a) $[0, 1000]$ Hz
- ☐ b) $[100, 1000]$ Hz
- ■ c) $[100, 2800]$ Hz
- ☐ d) None

3. Consider a discrete signal $x(n)$ obtained with a sample rate of $f_s = 100$ kHz. What is the frequency in Hz of the continuous spectral component that corresponds to the 96^{th} coefficient of a 1024 length DFT, $X_{1024}(96)$?

- ■ a) 9375.00 Hz.
- □ b) $1041,67$ Hz.
- □ c) $1024,00$ Hz.
- □ d) None

4. The following inner product

$$\left\langle e^{j\frac{2\pi}{N}kn}, e^{j\frac{2\pi}{N}rn} \right\rangle \tag{2}$$

is

- □ a) $\delta(k-r)$.
- ■ b) $N\delta(k-r)$.
- □ c) 0.
- □ d) None

5. Consider an unknown continuous scalar function $f(t)$ defined in the interval $t \in [0,1]$ and a set of M observations $F = [f_1, f_2, ..., f_M]^T$ taken at random time points, t_k, from the interval where $f(t)$ is defined. Let also consider a continuous function $g(t) = \sum_{k=0}^{N-1} c_k \phi_k(t)$, a linear combination of N known basis functions, $\phi_k(t)$, evenly distributed in the same time interval, $\phi_k(t) = \phi(t/\Delta - k), k = 0...N-1$ where $\phi(t)$ is a mother interpolation function and Δ is the distance between interpolation functions.

The optimal set of coefficients, $\mathbf{c} = [c_0, c_1, ..., c_{N-1}]^T$ are computed by minimizing the norm of the error $E(\theta) = \|F - G\|_2^2$ where $G = \{g(t_k)\}$ with t_k is the time point of the k^{th} observation.

Using the appropriated formulation (laboratory work) the solution is

$$\mathbf{c}^* = (\Theta^T\Theta)^{-1}\Theta^T F$$

where Θ is a function of the observations and interpolation functions.
Under this formulation what are the dimensions of matrix Θ?

- □ a) $N \times M$.
- ■ b) $M \times N$.
- □ c) $N \times N$.

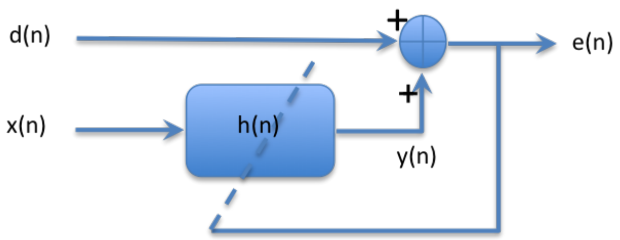

Figura 1: Adaptive filter.

- ☐ d) None

6. Consider the canonical adaptive filter displayed in Fig. 1 where $d(n) = x(n-1)$ and $h(n)$ is a 4 length FIR filter. In these conditions what is the optimal impulse response of the FIR that minimizes the norm of the error, $\|\mathbf{e}\|$?

- ■ a) $h(n) = [0, -1, 0, 0]$.
- ☐ b) $h(n) = [0, 1, 0, 0]$.
- ☐ c) $h(n) = [-1, 0, 0, 0]$.
- ☐ d) None

7. What is the period of the signal $y(n) = sin(n)$?

- ☐ a) 1 sample.
- ☐ b) 2π rad/sample.
- ☐ c) 1 second.
- ■ d) None

Part II - Problems

A (3) Let $X(k)$ and $Y(k)$ the DFTs of the N-length $x(n)$ and $y(n)$ sequences respectively where

$$Y(k) = \begin{cases} X(k) & \text{if } k \text{ is even} \\ -X(k) & \text{otherwise} \end{cases} \tag{3}$$

a) (2) What is the relation between $y(n)$ and $x(n)$

b) (1) Compute $y(n)$ when $x(n) = [1; 2; 3; 4; 5; 6; 7; 8]$.

B (3) Let (x_i, y_i, z_i) be N triplets of strictly positive observations with the underlying model

$$z_i = \alpha x_i^{\beta+\gamma y_i} \tag{4}$$

a) (1)Derive the **expression of the square norm of the error vector**, $\mathbf{e} = \{e_i\}$, where

$$e_i = \log(z_i/\alpha x_i^{\beta+\gamma y_i}). \tag{5}$$

b) (1) Derive the **closed form solution of the minimizer vector of parameters**, $\theta = \{\alpha, \beta, \gamma\}$,

$$\theta^* = \arg\min_{\theta} \|\mathbf{e}(\theta)\|_2^2 \tag{6}$$

c) (1) Propose an iterative algorithm to compute the optimum vector of parameters θ that minimizes the L_1 norm of the error vector

$$\theta^* = \arg\min_{\theta} \|\mathbf{e}(\theta)\|_1 \tag{7}$$