Instituto Superior Técnico / University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signals and Systems in Bioengineering

1st Semester 2015/2016

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Test 2 / Exam 1

January 18, 2016

Name : Number:

The duration of the Test is 1h30m and the Exam is 3h. The score of each item is 2 when right and -0.5 if wrong in the Test and 1 when right and -0.25 if wrong in the Exam. Only one option can be selected in each question.

1 Test 1

1. Consider a vector in the plane, $\mathbf{u} = [u_x, u_y]^T \in R^2$ and the following norms: $\|\mathbf{u}\|_1 = |u_x| + |u_y|$ and $\|\mathbf{u}\|_2 = \sqrt{u_x^2 + u_y^2}$. Which condition is true?

   - ☐ a) $\|\mathbf{u}\|_2 < \|\mathbf{u}\|_1$.
   - ☐ b) $\|\mathbf{u}\|_2 = \|\mathbf{u}\|_1$.
   - ☐ c) $\|\mathbf{u}\|_2 \geq \|\mathbf{u}\|_1$.
   - ■ d) None

2. Consider the problem $A\mathbf{x} = \mathbf{y}$ where A is a square matrix with a non-zero determinant, $|A| = \epsilon \neq 0$, where ϵ is very small. $\mathbf{y}$ are the observations (data) and $\mathbf{x}$ is the unknown to be estimated. This (inversion) problem is

   - ■ a) Well-posed.
   - ☐ b) Ill-posed.
   - ☐ c) Impossible.
   - ☐ d) None

3. Consider two finite length sequences $x_8(n)$ and $y_5(n)$ with length 8 and 5 respectively. $w(n) = x_8(n) * y_5(n)$ and $z(n) = x_8(n)(\star_9)y_5(n)$ are the linear $(*)$ and circular length 9, $(\star_9)$, convolutions respectively. Which condition is true?

- ☐ a) $w(0) = z(0)$.
- ☐ b) $w(2) = z(2)$.
- ☐ c) $w(9) = z(9)$.
- ■ d) None

4. Consider a N length sequence $x(n)$ and its corresponding DFT, $X(k)$. Let us reverse it making $Y(k) = X(-k)$. The resulting sequence $y(n) = DFT^{-1}(Y(k)) = DFT^{-1}(X(-k))$ is equal to

- ☐ a) $x(n)$.
- ■ b) $x(N - n)$.
- ☐ c) $-x(n)$.
- ☐ d) None

5. What is the frequency of the signal $x(n) = cos(\omega n)sin(\omega n)$, where $\omega = 2\pi/N$ with N integer?

- ■ a) 2ω rad/sample
- ☐ b) ω rad/sample
- ☐ c) $0.5\omega \; rad/sample$
- ☐ d) None

6. Consider the infinite length periodic sequences $x_1(n)$ and $x_2(n)$ with periods $N_1 = 8$ and $N_2 = 5$ respectively. What is the period of the sequence $y(n) = x_1(n) + x_2(n)$?

- ☐ a) $N_y = 8 + 5 = 13$ samples.
- ☐ b) $N_y = 8 - 5 = 3$ samples.
- ■ c) $N_y = 8 * 5 = 40$ samples.
- ☐ d) None

7. Consider the finite length sequences $x(n) = [0, 1, 2, 3, 4]$ and $y(n) = x((1-n)_8) + x((n+2)_6)$. What is the value of y(6)?

- ☐ a) 2.
- ■ b) 3.
- ☐ c) 4.
- ☐ d) None

8. Consider the complex finite length sequence

$$x(n) = [1; 1 - j; 0; -1 + j; 1; -2 + j; -2j; -1]$$

and $y(n) = x((1-n)_8) + x((n+2)_6)$. What is the 8 length DFT value for $k = 8$, $Y_8(8)$?

- ☐ a) 0.
- ☐ b) 1.
- ☐ c) j.
- ■ d) None

Problem (T=4/Ex=2)

Consider the following transfer function

$$H(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}}$$

1. Represent the poles and zeros of $H(z)$ in the complex plane (draw also the unit circle);
2. Draw the Bode plot (magnitude and phase characteristics) of the filter.
3. In which canonical filter representation do you classify this filter? Why?
4. Derive the difference equation describing the output of the filter.
5. Compute the expression of the impulsive response of the filter, $h(n)$.
6. Compute the impulse response of the filter $G(z) = 1/H(z)$.

2 Test 2

1. Consider the filter

$$H(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}}$$

with an input signal $x(n) = \eta(n) + 1$ where $\eta \sim \mathcal{N}(1, 1)$ is a random variable with unit mean, $\mu = 1$, and variance, $\sigma^2 = 1$. What is the mean of the output signal, $\mu_y =< y(n) >$?

- ☐ a) 2.
- ☐ b) 1.
- ■ c) 0.
- ☐ d) None

2. Let x and y two independent zero mean random variables with variances $\sigma_x^2 = 1$ and $\sigma_y^2 = 2$ respectively. What is the variance of the variable $z = x - 2y$

- ☐ a) 3.
- ☐ b) 5.
- ■ c) 9.
- ☐ d) None

3. Consider the graphical representation in the plane of a huge number of samples $\mathbf{x}_k = [x_1, x_2]_k^T$ that are zero mean random vectors, $\mu_{\mathbf{x}} = [0, 0]^T$ with covariance

$$C = \begin{bmatrix} 2 & 0.25 \\ 0.25 & 1 \end{bmatrix} \tag{1}$$

The shape of the resulting cloud is

- ☐ a) a single point.
- ☐ b) circularly symmetric.
- ■ c) non-horizontal ellipsoid.
- ☐ d) None

4. Let $H(z)$ be an ideal high-pass filter with cutoff frequency $\omega_c = \pi/2$ where

$$|H(\omega)| = \begin{cases} 1 & \text{if } \pi/2 \leq \omega \leq \pi \\ 0 & otherwise \end{cases} \tag{2}$$

in the interval $[0, \pi]$. Consider the input signal $x(n) = 1 + sin(\pi/3) + \eta$ where $\eta \sim \mathcal{N}(0, 1)$ is a normal zero mean random variable with unit variance.

What is the power spectrum of the output, $P_y(\omega)$ in the interval $[0, \pi]$ (where $u(\omega)$ is the step function)?

- ■ a) $P_y(\omega) = u(\omega - \pi/2)$.
- □ b) $P_y(\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - \pi/3)$.
- □ c) $P_y(\omega) = 2\pi\delta(\omega) + 1$.
- □ d) None

5. Consider a unitary feedback loop topology of the discrete system $H(z) = 1/(1 + z^{-1})$. For $K = -2$ the system is

- □ a) Stable.
- ■ b) Critically Stable.
- □ c) Unstable.
- □ d) None

6. Consider the system $G(s) = 1/s^2(s + 1)$. What is the limit of the unitary feedback loop static error, $e(t) = r(t) - y(t)$, when time goes to infinite

$$e(\infty) = lim_{t\to\infty} e(t)$$

for $K = 1$ and the input is the ramp $r(t) = tu(t)$?

- ■ a) 0
- □ b) $a > 0$
- □ c) ∞.
- □ d) None

7. Consider a unitary feedback loop of the continuous system $G(s) = 1/s^2$.

For which value of the gain, K, the output is a pure harmonic oscillation with frequency $\omega = 10$ rad/sec ?

- ☐ a) $K = 0$.
- ☐ b) $K = 10$.
- ■ c) $K = 100$.
- ☐ d) None

8. To modify the sampling rate of $x(n)$ by a rational factor of $R = 0.75 = 3/4$ the following pipeline is used:

$$x(n) \rightarrow [\uparrow 3] \rightarrow H(\omega) \rightarrow [\downarrow 4] \rightarrow y(n)$$

where $[\uparrow 3]$ and $[\downarrow 4]$ represent upsampling and downsampling operations respectively.

What should be the cutoff frequency, ω_c, of the ideal low-pass filter $H(\omega)$?

- ☐ a) $\pi/3$.
- ■ b) $\pi/4$.
- ☐ c) $\pi/(3 + 4)$.
- ☐ d) None

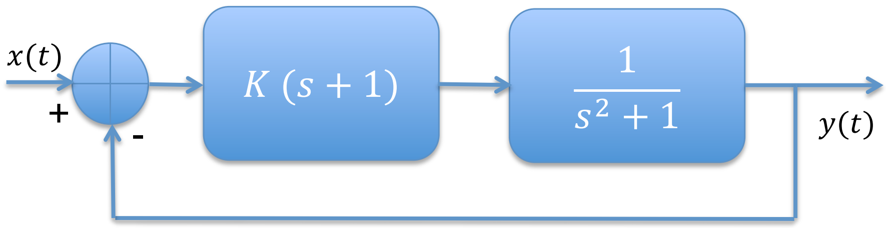

Figura 1: Unit feedback control system.

Problem (T=4/Ex=2)
Consider the feedback system represented in Fig.1.

1. **(2)** Draw the root-locus (RL) for $K > 0$ and $K < 0$. (Compute explicitly the break in and break out points at the real axis, asymptotic center and angle of the asymptotes and the sections of the real axis of RL)

2. **(0,5)** Compute the values of K for which the closed-loop poles are equal (double-poles).

3. **(0,5)** What is the minimum value of K for which the system has no overshooting?

4. **(0,5)** For which value of K the system is faster (minimum rising time, t_r)?

5. **(0,5)** Compute the discrete version of the system, $G(z)$, using the poles and zero mapping technique, needed if the design of the controller is to be performed in the discrete domain.