TÉCNICO LISBOA

Instituto Superior Técnico / University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signals and Systems in Bioengineering

1st Semester de 2014/2015

João Miguel Sanches

Exame 2

January 31, 2015

Name : Number:

The duration of the exam is 3h. The score of each item is 1 when right and -0.25 if wrong. Only one option can be selected in each question.

Part 1

1. Consider the signal $y(n) = x(n - n_0)$ where $x(n) = [1; 2; 1; 0; -1; -2; -1; 0]$. What should be the value of n_0 for which the DFT of $y(n)$, $Y(k)$, is pure imaginary,
   - ■ a) 1
   - □ b) 0
   - □ c) -1
   - □ d) None of the above
2. Consider the signal $x(n) = [0; 1; 0; 1]$. What is its DFT?,
   - □ a) $e^{-k\pi/2} - e^{-3k\pi/2}$
   - ■ b) $2(-1)^k cos(k\pi/2)$
   - □ c) $cos(k\pi/2)$
   - □ d) None of the above
3. Consider the signal $x(n) = (1/2)^n u(n)$, $n \in N$. What is the period of the periodic signal $\tilde{y}(n) = x((n)_4)$?,
   - □ a) $\tilde{y}(n) = x(n)$, $n \in N$

- ■ b) $\tilde{y}(n) = \frac{2^4}{(2^4-1)} x(n)$, $0 \leq n < 4$
- □ c) $\tilde{y}(n) = x(n)$, $0 \leq n < 4$
- □ d) None of the above

4. Consider the following optimization problem,

$$\mathbf{c}^* = \arg\min_{\mathbf{c}} \|A\mathbf{c} - \mathbf{x}\|_1 \tag{1}$$

where A is a $N \times L$ matrix, $\mathbf{x}$ is a N length column vector of observations and $\mathbf{c}$ is a L length vector of coefficients to be estimated. $\|e\|_1 = \sum_{k=1}^{L} |e_k|$ is called $\mathcal{L}_1$ norm where $|a|$ is the absolute value of the scalar a. This optimization problem can solved using the *Iteratve Reweighted Least Squares* (IRLS) algorithm as follows

$$\mathbf{c}^{t+1} = \arg\min_{\mathbf{c}} \mathbf{e}^T W(\mathbf{c}^t)\mathbf{e} \tag{2}$$

where $\mathbf{e} = A\mathbf{c} - \mathbf{x}$ and $W(\mathbf{c}^t)$ is a diagonal matrix with elements

- □ a) $w_{i,i} = |e_i|$
- □ b) $w_{i,i} = 1/|e_i|^2$
- □ c) $w_{i,i} = 1/e_i^2$
- ■ d) None of the above.

5. Let $x(n)$ be a real 4 length signal with the following DFT: $X(k) = [1; 1-j; 3j; X(3)]$. What is the value of $X(3)$?

- □ a) $X(3) = 0$
- □ b) $X(3) = 1 - j$
- ■ c) $X(3) = 1 + j$
- □ d) None of the above.

6. Consider a discrete signal, $x_d(n), n \in N$, that is obtained by uniformly sampling a continuous signal, $x_c(t), t \in R$, where $x_d(n) = x_c(nT_s)$. $T_s = 1/f_s$ is the sampling period and $f_s = 1000Hz$ is the sampling frequency. What is the minimum frequency separation in the discrete domain (digital frequency axis) if a 1024 length *Fast Fourier Transform* (FFT_{1024}) is used in the analysis?

- ■ a) $\pi/512$ rad/sample.
- □ b) 1000/1024 Hz.
- □ c) 2π rad/sample.

- ☐ d) None of the above

7. Let us consider an infinite signal to be filtered by a 49 length impulse response FIR filter. To avoid wrong results at each processed block what should be the minimum length of the DFT_L if a 1000 length input block is used

   - ☐ a) $L = 512$.
   - ☐ b) $L = 1024$.
   - ■ c) $L = 1048$.
   - ☐ d) None

8. What is the period of the the following signal: $x(n) = sinc((\pi/5)n)$?

   - ☐ a) $N = 10$.
   - ☐ b) $N = 5$.
   - ☐ c) $N = 1$.
   - ■ d) None

Problem (2)

1. Consider the finite N length signal, $x(n)$, and let $y(n)$ be a M length sequence, obtained from $x(n)$, by sampling its Fourrier transform in $M < N$ evenly spaced frequencies, including $(X(\omega)|\omega = 0)$. Compute $y(n)$.
2. If $x = [5, 4, 3, 2, 1, 0, -1, -2, -3, -4]$ represent graphically the signal $y(n)$ for $M = 8$.

Part 2

1. What is the value of the following integral?

$$\int_0^\infty e^{-\frac{x^2}{2}} dx \tag{3}$$

- ☐ a) $2\sqrt{\pi}$.
- ☐ b) 1.
- ■ c) $\sqrt{\pi/2}$.
- ☐ d) None of the above

2. Consider the following decimation operation $y(n) = T_{\downarrow 2}[h(n) * x(n)]$ where $x(n) = cos(\frac{3\pi}{4}n)$ and $h(n)$ is an ideal anti-aliasing filter. What is the output signal?

- ■ a) $y(n) = 0$.
- ☐ b) $y(n) = cos(\frac{3\pi}{4}n)$.
- ☐ c) $y(n) = cos(\frac{3\pi}{8}n)$.
- ☐ d) None of the above

3. Consider the signal $x(n) = \eta(n) + k$ where $\eta(n)$ is a zero mean white Gaussian noise component, $\eta \sim \mathcal{N}(0, \sigma^2)$, and k is a constant. What is the Power Spectrum of $x(n)$?

- ☐ a) $P_x(\omega) = 1 + 2\pi k$.
- ■ b) $P_x(\omega) = \sigma^2 + 2\pi\delta(\omega)k^2$.
- ☐ c) $P_x(\omega) = 2\pi\sigma^2\delta(\omega) + k$.
- ☐ d) None of the above

4. Consider the LTI system described the following difference equations

$$y(n) = x(n) + 0.5y(n-1) - 0.25y(n-2) \tag{4}$$

What is the mean value of the output signal if the input is $x(n) = \eta(n) + 2$ where $\eta \sim \mathcal{N}(\mu, \sigma^2)$ is white Gaussian noise with $\mu = 0$ and $\sigma = 1$?

- ☐ a) 0.
- ■ b) 8/3.
- ☐ c) 8.
- ☐ d) None of the above

5. Consider an unitary negative feedback output topology where $G(s) = \frac{1}{s+1}$ and $C(s) = K * (s+2)$ are the plant and controller transfer functions respectively. How many branches will have the root-locus of the closed loop system?

- ☐ a) 0.
- ■ b) 1.
- ☐ b) 2.
- ☐ d) None of the above

6. Using the previous example, how many branches go to ∞?

- ■ a) 0.
- ☐ b) 1.
- ☐ b) 2.
- ☐ d) None of the above

7. Consider a system with the following open-loop transfer function

$$G(s) = \frac{1}{s(s-1)} \tag{5}$$

and an unitary negative feedback output topology where the controller is just a gain, $C(s) = K$. What is the value of K that makes the closed loop system stable?

- ☐ a) $K = 0$
- ☐ b) $K = 1$
- ☐ c) $K = \infty$
- ■ d) None of the above

8. Consider the following open loop transfer function $G(s) = \frac{1}{(s+1)^2}$. The corresponding closed loop system with $C(s) = K(s-1)$ is... (complete the sentence)

- ☐ a) Stable for every K.
- ☐ b) Unstable for every K.
- ☐ b) Stable for $K > 0$.
- ■ d) None of the above

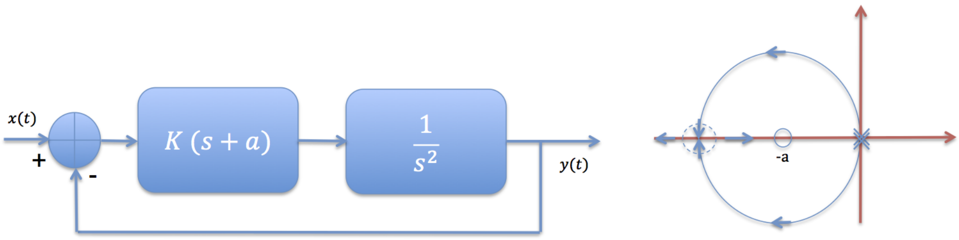

Figura 1: Unit feedback control system.

Problem (2)
Consider the feedback system represented in Fig.1.

For $\alpha = 0$

1. Derive the corresponding root-locus for $K > 0$ and $K < 0$ without using the root-locus rules. Do it based on the analytic expression of the closed loop poles.

2. What is the stability interval for K (the interval of K for which the closed loop system is stable).

   For $\alpha > 0$

3. The root-locus for $\alpha > 0$ and $K > 0$ is displayed in the figure above (right). Derive analytically the interval of values of K that lead to closed loop complex poles.

4. What is location of the poles inside the circle (in the right side of the figure) and the corresponding value of K.

5. Compute and draw the root-locus for $K < 0$. Is the system stable in this case?