Instituto Superior Técnico / Tecnical University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signal and Systems in Bioengineering

1st Semester de 2013/2014

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Test 1

Novembro 13, 2013

Name : Number:

The duration of the test is 1h30m. The score of each item is 2 when right and -0.5 if wrong. Only one option can be selected in each question.

1. Consider the *Linear Time Invariant* (LTI) system described by the following difference equation

$$y(n) = x(n) + y(n-1) - 0.25y(n-2). \tag{1}$$

What type of filter is this system?

- ☐ a) High-pass filter .
- ☐ b) Band-pass filter.
- ☐ c) Low-pass filter.
- ☐ d) None

2. Compute the angle between signals t and t^2, defined in the interval $[-1, 1]$?

- ☐ a) 0.
- ☐ b) 1/2.
- ☐ c) 1/4.
- ☐ d) None

3. Consider a discrete signal obtained with a sample rate of 20kHz. What is the corresponding continuous spectral frequency in Hz of the FFT_{1024} coefficient $X(16)$?

- ☐ a) 312.5 Hz.
- ☐ b) 0.0982 rad/sample.
- ☐ c) 125 Hz.

- ☐ d) None

4. Consider a 10 length signal $x = [0; 1; 2; 3; 4; 5; 6; 7; 8; 9]$. Sample the Fourier transform of x, $X(\omega)$, at 8 evenly spaced frequencies, $X_8(k)$, and compute $y(n) = DFT_8^{-1}(X)$, for $n = [0, 1, ..., 7]$, where $DFT_8^{-1}()$ denotes a 8 length DFT inversion operator.

   What is $y(n)$?

   - ☐ a) $y(n) = [0; 1; 2; 3; 4; 5; 6; 7]$.
   - ☐ b) $y(n) = [8; 9; 2; 3; 4; 5; 6; 7]$.
   - ☐ c) $y(n) = [8; 10; 2; 3; 4; 5; 6; 7]$.
   - ☐ d) None

5. Consider a 8 length DFT, $X(k) = [1; X(1); 1-j; j-j; 1+j; 1-j]$, of a real signal $x(n)$. What is the value of $X(1)$?

   - ☐ a) $X(1) = 0$.
   - ☐ b) $X(1) = 1 + j$.
   - ☐ c) $X(1) = 1 - j$.
   - ☐ d) None

6. Consider a band pass filter with the following transfer function;

$$H(z) = \frac{1}{1 - (3/2)z^{-1} + (13/16)z^{-2}} \tag{2}$$

   with poles $p_{1,2} = \frac{3}{4} \pm j\frac{1}{2}$. What is central frequency of this filter?

   - ☐ a) $\omega_0 = 0$ rad/sample.
   - ☐ b) $\omega_0 = 1$ rad/sample.
   - ☐ c) $\omega_0 = arctan(2/3)$.
   - ☐ d) None

7. The inner product $< \phi_k(n), \phi_r(n) >$ with

$$\phi_\tau(n) = \frac{1}{\sqrt{N}} e^{j\frac{2\pi}{N}\tau n} \tag{3}$$

   where k, r and τ are integers and N is the total length of the signals,is

   - ☐ a) $\delta(k - r)$.

- ☐ b) 0.
- ☐ c) 1.
- ☐ d) None

8. What is the period of the signal $y(n) = cos(n)$?

- ☐ a) 1 sample.
- ☐ b) 2π rad/sample.
- ☐ c) 1 second.
- ☐ d) None

Problem (4) Consider the following model describing noisy observations

$$y(n) = x(n) + \eta(n) \tag{4}$$

where $\eta(n)$ is additive white Gaussian noise with normal distribution, $\eta(n) \sim \mathcal{N}(0, \sigma^2)$. $\mathbf{y} = [y(0), y(1), ..., y(N-1)]^T$ are the noisy observations and $x(n)$ **the unknown signal to estimate**.

The *maximum likelihood* (ML) estimation of $\mathbf{x} = [x(0), x(1), ..., x(N-1)]^T$ can be computed by minimizing the following energy function,

$$J = \|(A\mathbf{x} - \mathbf{y})\|^2 \tag{5}$$

where A is a known $N \times N$ matrix modelling the blur effect.

1. Derive the close form solution for the ML estimate of $\mathbf{x}$ from the observation $\mathbf{y}$ vector.

2. The minimization of (5) is an *ill-posed* problem. To regularize the solution a modified energy function is proposed,

$$J = \|(A\mathbf{x} - \mathbf{y})\|^2 + \alpha \sum_{n=1}^{N-1} (x(n) - x(n-1))^2 \tag{6}$$

   Derive the close form solution of the minimizer of (6).