Instituto Superior Técnico / Tecnical University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Signal and Systems in Bioengineering

1st Semester de 2013/2014

João Miguel Sanches

Test 2 / Exame 1

January 18, 2014

Name : Number:

The duration of the test is 3h. The score of each item is 1 when right and -0.25 if wrong. Only one option can be selected in each question.

Part 1

1. Consider the finite length sequence $x(n) = \{1, 2, 3\}$ and the sequence $y(n) = ((4-n)_6)$. What is the value of $y(3)$?

   - □ a) 1
   - □ b) 2
   - □ c) 3
   - □ d) None of the above

2. Let $g(x, y) = e^{d(x,y)}$ where $d(x, y)$ is a metric function. $g(x, y)$ is

   - □ a) a metric function because is strictly positive.
   - □ b) a metric because is convex.
   - □ c) a metric because is null if and only if $x = y$.
   - □ d) None of the above

3. What is the period of the signal $x(n) = cos(0.2\pi n)$?

   - □ a) 0.2.
   - □ b) 10.
   - □ c) 20.
   - □ d) None of the above

4. Let us consider an infinite signal, to be filtered by a 15 length impulse response FIR filter . To implement the filtering process by blocks with a 1024 length FFT algorithm, what should be the length of the input blocks to not have overlap of these blocks?

   - □ a) 1010.
   - □ b) 1024.
   - □ c) 1038.
   - □ d) None

5. Let $x(n) = [1, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0]$ be a periodic real sequence. What is the value of the 11^{th} coefficient, $X(10)$?

   - □ a) 2
   - □ b) -2
   - □ c) 0
   - □ d) None of the above

6. What is the value of the l_∞ norm of the vector $x = [-2, -1, 0, 1]$?

   - □ a) $\sqrt{6}$
   - □ b) 2
   - □ c) 4
   - □ d) None of the above

7. Consider the *Linear Time Invariant* (LTI) system

$$H(z) = \frac{1}{1 - 2\rho\cos(\theta)z^{-1} + \rho^2 z^{-2}}. \tag{1}$$

   What type of filter is this system for $\rho = 0.8$ and $\theta = \pi/4$?

   - □ a) High-pass filter .
   - □ b) Band-pass filter.
   - □ c) Low-pass filter.
   - □ d) None of the above

8. Consider a discrete signal sampled at a 8192 samples/s. What is the minimum spectral distance between components of the spectrum if a 4096 length FFT_{4096} is used?

   - □ a) 0.5Hz.
   - □ b) 1Hz.
   - □ c) 2Hz.
   - □ d) None of the above

Problem (4) Consider a finite length, N, signal $x(n)$. Let $y(n)$ be a M length sequence, obtained from $x(n)$, by sampling its Fourier transform in $M = N/2$ evenly spaced frequencies, including $X(\omega = 0)$.

1. What is the M length $y(n)$ signal? Derive it analytically by sampling the Fourier transform of $x(n)$.
2. If $x = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$ represent $y(n)$ for $M = N/2 = 5$.

Part 2

1. Let x and y two independent random variables with variances σ_x^2 and σ_y^2 respectively. What is the variance of the $z = 2x + y$?

   - □ a) $4\sigma_x^2 + \sigma_y$.
   - □ b) $2\sigma_x^2 + \sigma_y$.
   - □ c) $\sigma_x^2 + \sigma_y$.
   - □ d) None of the above

2. Consider the following LTI system

$$H(z) = \frac{1}{1 - 0.25z^{-1}} \tag{2}$$

   What is the mean value of the output signal if the input is white noise with mean $\mu = 2$ and variance $\sigma^2 = 1$?

   - □ a) 1.
   - □ b) 2.
   - □ c) 4.
   - □ d) None of the above

3. Consider the system (2). If the input is a zero mean ($\mu_x = 0$) white noise with variance $\sigma_x^2 = 2$, $x \sim N(0, 2)$. What is the *power spectral density* (PSD) of the output?

   - □ a) $\frac{0.75}{1.25 - 0.5cos(\omega)}$.
   - □ b) $\frac{1.5}{1.0625 - 0.5cos(\omega)}$.
   - □ c) $\frac{2}{1.0625 - 0.25cos(\omega)}$.
   - □ d) None of the above

4. What is the expected value of a random variable x with distribution $p(x) = 2(1 - x)$ for $0 \leq x \leq 1$?

   - □ a) 0.
   - □ b) 1/3.
   - □ c) 1.
   - □ d) None of the above

5. Let $H(z)$ be an ideal lowpass filter with cutoff frequency $\omega_c = \pi/4$ and $H(\omega)|_{\omega=0} = 1$. Consider the input signal $x(n) = 1 + \cos\left(\frac{\pi}{3}n\right) + \eta$ where η is zero mean white noise with variance $\sigma_\eta^2 = 1$. What is the power spectrum of the output, $P_y(\omega)$?

   - □ a) $P_y(\omega) = 2\pi\left(\delta(\omega) + 0.5\delta(\omega - \pi/3) + 0.5\delta(\omega + \pi/3)\right) + 1$.
   - □ b) $P_y(\omega) = 2\pi\delta(\omega) + 1$.
   - □ c)$P_x(\omega)$.
   - □ d) None of the above

6. Consider a LTI system with impulse response $h(n) = \delta(n) - \frac{1}{3}\delta(n-1)$ and an input signal with autocorrelation $\phi_{xx}(m) = \delta(m) + 1$. What is the mean of the output signal?

   - □ a) 4/9.
   - □ b) 2/3.
   - □ c) 0.
   - □ d) None of the above

7. The static error of a type 1 feedback system to an input ramp, $x(t) = tu(t)$, is

   - □ a) infinite.
   - □ b) finite not null.
   - □ c) null.
   - □ d) None of the above

8. A closed-loop system with complex conjugated poles is always

   - □ a) unstable.
   - □ b) stable.
   - □ c) overshot
   - □ d) None of the above

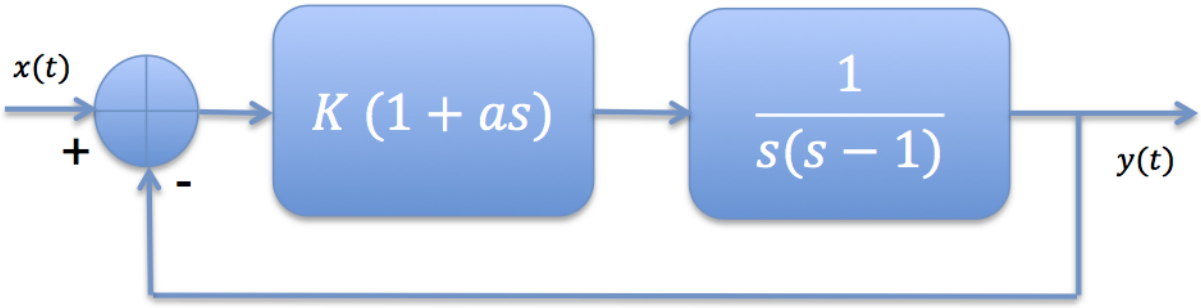

Figura 1: Unit feedback control system.

Problem (2) Consider the feedback system represented in Fig.1.

1. Is this system stable in open-loop (without feedback)? Why?
2. Draw the root-locus of the system for $K > 0$ and $\alpha = 0$. Is the closed-loop system stable? Why?
3. For $\alpha = 1$ what is location of the closed loop zero?
4. What is the value of K for $\alpha = 1$ that stabilize the system?
5. Draw the new root-locus for $\alpha = 1$ and $K > 0$).