Instituto Superior Técnico / Tecnical University of Lisbon

Departament of Bioengineering

Master on Biomedical Engineering

Digital Signal Processing in Bioengineering

2nd Semester de 2012/2013

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Test 2 □ / Exam 1 □

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Name : Number:

The duration of the test is 1h30m and of the exam is 3h. The score of each item in the test is 2 when right and -0.5 if wrong. In the exam the scores are half, 1 when right and -0.25 if wrong. Only one option can be selected in each question.

DON'T FORGET TO INDICATE THE QUESTIONS YOU ARE ANSWERING FOR THE SECOND TEST IN THE BOX LABELED WITH [*T:*□]. A maximum of 8 items are allowed in the test.

1. [**T:**□] Let $x(n) = [1, -1]^T$ and $y = [j, -j]^T$. What is the value of the inner product $< x, y >$?

   - □ a) 0
   - □ b) $2j$
   - ■ c) $-2j$
   - □ d) None

2. [**T:**□] Let $p = [p_1, p_2, ..., p_N]$ a complete, orthogonal and normed basis of a vector space S, were each element x can be obtained as the following linear combination, $x = \sum_{i=1}^{N} c_i p_i$. The norm of x in this space is

   - ■ a) $\|x\| = \|c\|$
   - □ b) $\|x\| \leq \|c\|$
   - □ c) $\|x\| < \|c\|$
   - □ d) None

3. [**T:**□] Consider a flame intensity, $x(t)$, captured by a photo diode (light sensor), $x(t)$. If a 4096 length FFT is used to obtain the spectrum what should be the sampling rate used, form the list below, in the digitalization process to have a spectral resolution smaller (better) than 100Hz?.

- □ a) 100kHz
- □ b) 250kHz
- ■ c) 500kHz
- □ d) None

4. [**T:**□] Consider the finite length sequence $x(n) = \{1, 2, 3\}$ and the sequence $y(n) = x((2-n)_5)$. What is the value of $y(3)$.

   - ■ a) 0
   - □ b) 1
   - □ c) 2
   - □ d) None

5. [**T:**□] Consider the complex signal $x(n) = \{1, 1-j, 0, 2-j, 3, -2+j, 2j\}$. What is the 8 length DFT value for $k = 8$, $X_8(8)$?

   - ■ a) $5 + j$
   - □ b) 0
   - □ c) 6
   - □ d) None

6. [**T:**□] What is the period of the signal $sin(0.01\pi n)$?

   - ■ a) 200
   - □ b) it is not periodic
   - □ c) 0.01
   - □ d) None

7. [**T:**□] The goal is to filter, in real time, an audio signal from a microphone with a 25 length impulse response FIR filter. The signal should be processed with a 500 sample length blocks and the convolution is performed by using a 512 length FFT algorithm. What is the number of overlapped samples of the input blocks?

   - □ a) 24.
   - ■ b) 12.
   - □ c) 0.
   - □ d) None

8. [**T:**□] With respect to the previous item how many processed samples should be added to the next output block if the corresponding invalid samples are set to zero?

   - ■ a) 24.
   - □ b) 12.
   - □ c) 0.
   - □ d) None

9. [**T:**□] Let $x(n)$ be a band limited signal, with a cut-off frequency of $\omega_c = 3\pi/4$. To change the sampling rate by a factor of $R = 0.75$, $f_s(new) = Rf_s$, where h is the impulse response of an ideal low-pass filter, what is the appropriated sequence of operations?

   - □ a) $T_{\uparrow 3}[T_{\downarrow 4}[h * x(n)]]$
   - □ b) $T_{\uparrow 3}[h * T_{\downarrow 4}[x(n)]]$
   - ■ c) $T_{\downarrow 3}[h * T_{\uparrow 4}[x(n)]]$
   - □ d) None

10. [**T:**□] What is the window that allows highest time frequency compression in the scope of time-frequency analysis?

    - ■ a) $e^{-t^2}/\sqrt{\pi}$.
    - □ b) $e^{-t}/\sqrt{\pi}$.
    - □ c) $sinc(t)$.
    - □ d) None

11. [**T:**□] Consider an infinite length discrete signal, $x(n)$. For this signal there is/are

    - □ a) a lower bound for scale resolution (coarser).
    - ■ b) a upper bound for scale resolution (finer).
    - □ c) no bounds for scale resolution.
    - □ d) None

12. [**T:**□] What is the polyphase decomposition of $H(z) = 1/(1 - 0.5z^{-1})$ for M=3?

    - □ a) $1/(1 - 0.5z^{-1})$.
    - ■ b) $(1 + 0.5z^{-1} + 0.25z^{-2})/(1 - 0.5^3 z^{-3})$
    - □ c) $z^{-2}/(1 - 0.5z^{-1})$
    - □ d) None

13. [**T:**□] A perfect reconstruction of a signal from the wavelet decomposition requires,

- □ a) the approximation and detail coefficients from all scales
- ■ b) the detail coefficients from all scales and the approximation coefficients of the coarser scale
- □ c) only the detail coefficients
- □ d) None

14. [**T:**□] Consider a 1024 length discrete signal, $x(n)$. How many decompositions scales is it possible to compute with a dyadic *discrete wavelet transform* (DWT)?

- □ a) 1024
- □ b) 102
- ■ c) 10
- □ d) None

15. [**T:**□] Consider the basis functions of the DFT, $\phi_k(n) = e^{j\frac{2\pi}{N}kn}$. What is the inner product $< \phi_k(n), \phi_r(n) >$? (where $\delta(n)$ is the impulse and N is the length of the signals to analyse).

- ■ a) $N\delta(k-r)$
- □ b) 0
- □ c) N
- □ d) None

16. [**T:**□] Consider the following prototype filter of a filter bank,

$$|H(\omega)| = \begin{cases} 1 - 4|\omega|/\pi & \text{if } |\omega| < \pi/4 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

How many replicas of this filter are needed do maximally decompose a real discrete signal (since the signal and filter are real consider only the interval $[0, \pi]$)?

- □ a) 6
- ■ b) 5
- □ c) 4
- □ d) None

Problems

Please present the details of your computations. For **test**,only one of these problem need to be solved.

1. **T:**□ Let $x(n)$ be a N length real discrete signal and

$$y(n) = \begin{cases} x(n) & \text{if n is even} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

   (a) How are related the N length DFT_Ns of $x(n)$ and $y(n)$?

   (b) **T:** Let $x(n) = [0, 1, 0, 2, 0, 3, 0, 4, 0, ...]$ be a 256-length discrete signal. What is the N length DFT of $y(n)$, $Y_{256}(k)$?

2. **T:**□Consider a *Linear Time Invariant* (LTI) filter described by the following difference equation

$$y(n) = x(n) + 0.5x(n-1) - 0.25y(n-1) + 0.5y(n-2) \tag{3}$$

   (a) Derive its transfer function, which is the Z-transform of its impulsive response, $H(z) = TZ(h(n))$.

   (b) Represent it graphically in the canonical *Direct Form II*, where the number of delay blocks is minimized.

   (c) Is this filter stable? Prove