FICHA 4

18) $f: \mathbb{R} \to \mathbb{R}$ CONTÍNUA

$$\boxed{g(x) = f(x)\,\text{SEN}(x)}$$

MOSTRAR: $g'(0) = f(0)$

$$g'(x) = \cancel{f'(x)}\,\text{SEN}(x) + f(x)\cos x$$

$$g'(0) = 0 + f(0)$$

$$g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} \qquad g(0) = 0$$

$$= \lim_{x \to 0} \frac{f(x)\,\text{SEN}\,x}{x}$$

f CONTÍNUA: $\lim_{x \to 0} f(x) = f(0)$

$$g'(0) = f(0)$$

32) $f(x) = x + \sqrt{x^2+1}$

$$f'(x) = 1 + \frac{1}{2\sqrt{x^2+1}} \cdot 2x$$

$$= 1 + \frac{x}{\sqrt{x^2+1}}$$

$f'(x) = 0$ $\quad \frac{x}{\sqrt{x^2+1}} = -1$

$\Leftrightarrow \quad x = -\sqrt{x^2+1} \Rightarrow x^2 = x^2+1$

IMPOSSÍVEL

f' NÃO TEM ZEROS

$\forall_x \; f'(x) > 0$

f ESTRIT. CRESCENTE

$f'(x) = 1 + \frac{x}{\quad}$

$$\sqrt{1+x^2}$$

$$f''(x) = \frac{1\cdot\sqrt{1+x^2} - \dfrac{x}{\sqrt{1+x^2}}\cdot x}{1+x^2}$$

ZEROS:

$$f''(x)=0 \qquad \sqrt{1+x^2} = \frac{x^2}{\sqrt{1+x^2}}$$

$$1+x^2 = x^2 \quad \text{IMPOSSÍVEL}$$

$$f''(x) = \frac{\dfrac{1+x^2}{\sqrt{1+x^2}} - \dfrac{x^2}{\sqrt{1+x^2}}}{1+x^2}$$

$$= \frac{1}{(1+x^2)\sqrt{1+x^2}} = \frac{1}{(1+x^2)^{3/2}} > 0$$

f CONVEXA

ASSÍMPTOTAS

$$f(x) = x + \sqrt{1 + x^2}$$

$$m = \lim_{x \to +\infty} \frac{x + \sqrt{1 + x^2}}{x}$$

$$= \lim_{x \to +\infty} \left(1 + \frac{\sqrt{1 + x^2}}{x} \right)$$

$$= \lim_{x \to +\infty} \left(1 + \frac{\cancel{\sqrt{x^2}} \sqrt{1/x^2 + 1}}{\cancel{x}} \right)$$

$x > 0 \quad \sqrt{x^2} = x$

$$m = 2$$

$$\lim_{x \to -\infty} \frac{f(x)}{x} = 0 \qquad \text{Pois} \quad \sqrt{x^2} = -x$$

$$b = \lim_{x \to +\infty} \left(x + \sqrt{1 + x^2} - 2x \right)$$

$$= \lim_{x \to +\infty} \left(\sqrt{1 + x^2} - x \right) =$$

$$1 + x^2 - x^2 \qquad \frac{1}{\ } = 0$$

$$= \lim_{x \to +\infty} \frac{x}{\sqrt{1+x^2} + x} = \infty$$

~~À ESQUEL~~

À DIREITA: $y = 2x$

À ESQUERDA:

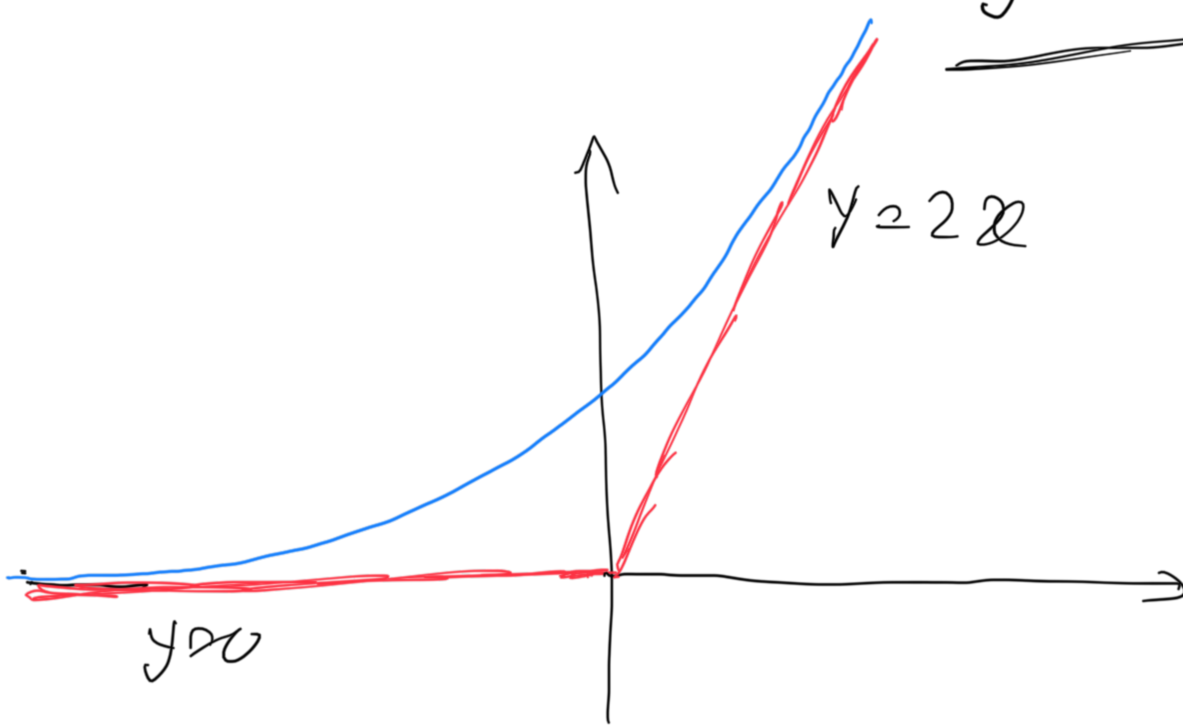
$m = 0$

$$b = \lim_{x \to -\infty} \left(\overset{-\infty}{x} + \overset{+\infty}{\sqrt{1+x^2}} \right)$$

$f(x) - 0$

$$= \lim_{x \to -\infty} \frac{x^2 - (1+x^2)}{x - \sqrt{1+x^2}} = 0$$

$y = 0$

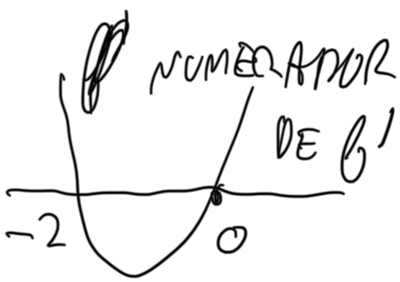


34 a) $f(x) = x + \frac{1}{x+1}$

$$f'(x) = 1 - \frac{1}{(x+1)^2} = \frac{(x+1)^2 - 1}{(x+1)^2}$$

$f'(x) = 0 \iff x+1 = \pm 1$

$x = 0$ ou $x = -2$



		-2		-1		0	
f'	$+$	0	$-$	$*$	$-$	0	$+$
f	$\nearrow$ $\cap$	MAX	$\searrow$ $\cap$	$*$	$\searrow$ $\cup$	MIN	$\nearrow$ $\cup$

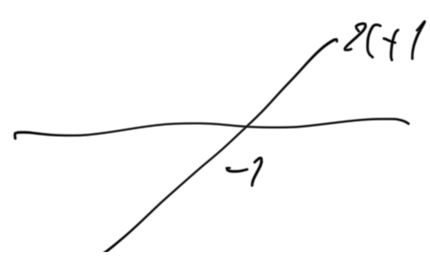
$$f''(x) = \frac{2}{(x+1)^3}$$

SINAL = SINAL DE $x+1$

$y = x$ ASSÍMPTOTA À DIREITA E À ESQUERDA

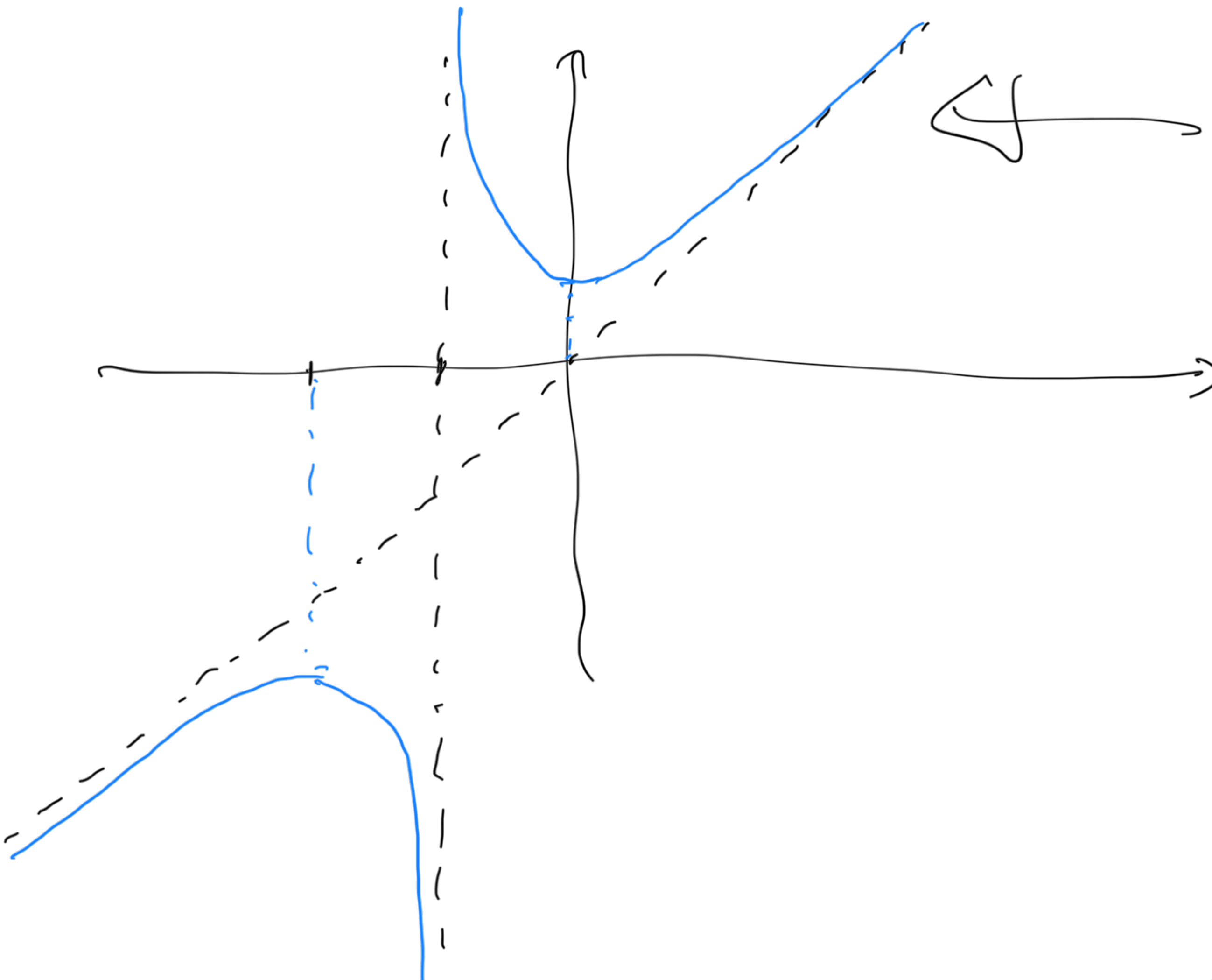
ASSÍMPTOTA VERTICAL EM $x = -1$

$$\lim_{x \to -1^+} \left(x + \frac{1}{x+1} \right)$$

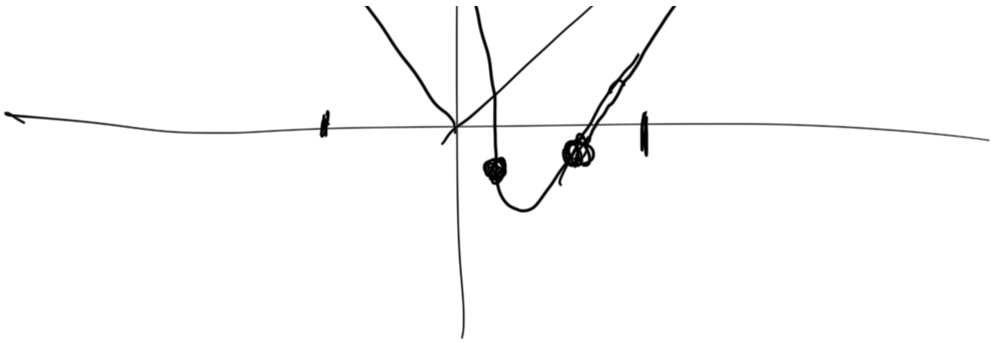


$$= -1 + \frac{1}{0^+} = +\infty$$

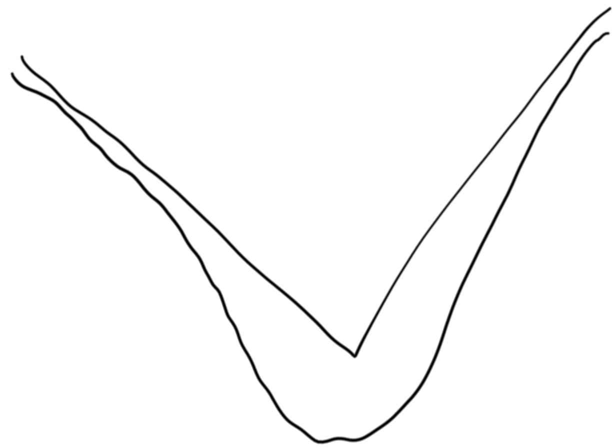
$$\lim_{x \to -1^-} \left(x + \frac{1}{x+1} \right) = -1 + \frac{1}{0^-} = -\infty$$



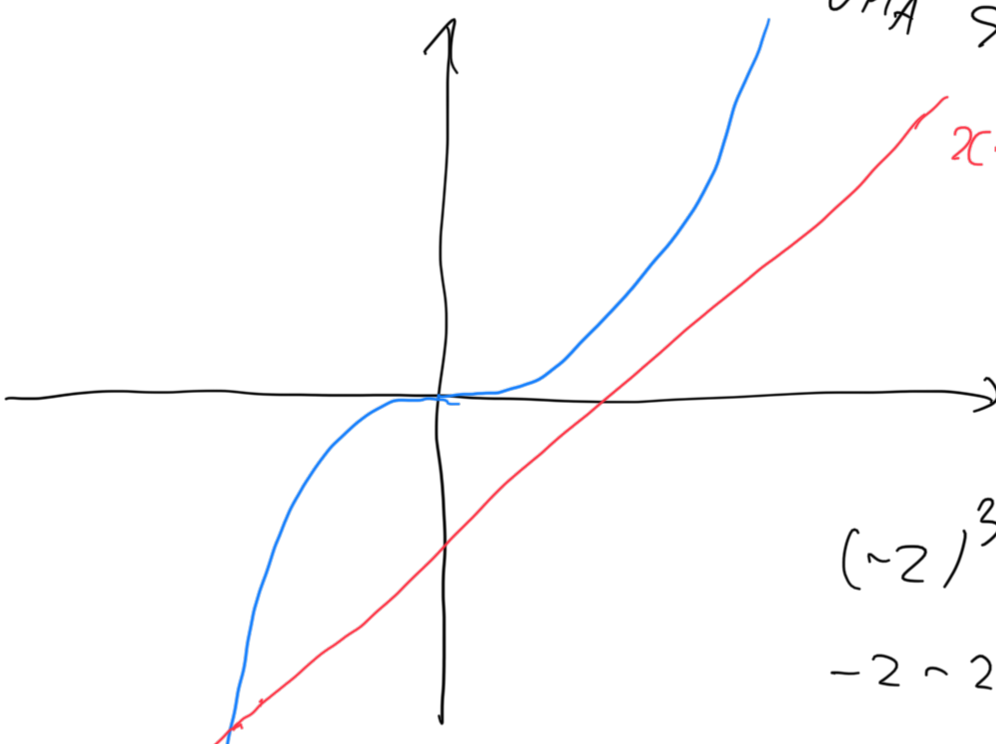
33)



2a)

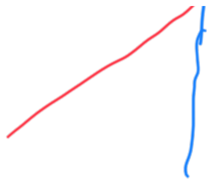


a) $x^3 = x-2$ PELO MENOS UMA SOLUÇÃO



$(-2)^3 = -8$

$-2-2 = -4$



$[-2, 0]$

$$x^3 = x - 2$$

$$\underbrace{K(x) = x^3 - x + 2}_{\text{CONTÍNUA}} = 0$$

$K(-2) = -8 + 2 + 2 = -4 < 0$

$K(0) = 2 > 0$

T. BOLZANO : $\exists\, c \in]-2, 0[\quad K(c) = 0$

$$c^3 = c - 2$$

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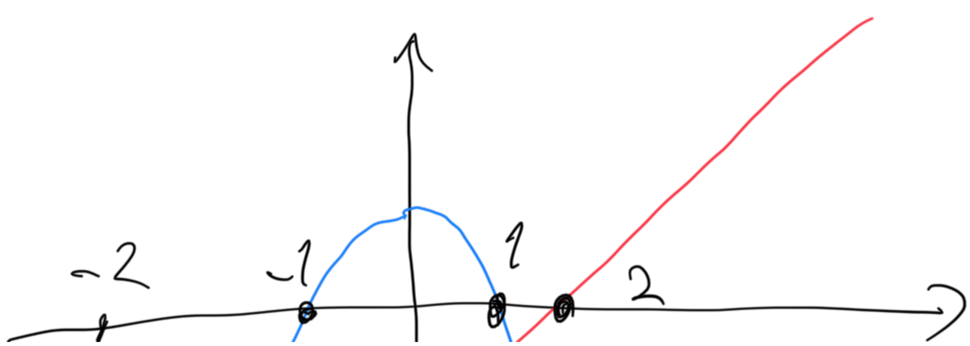
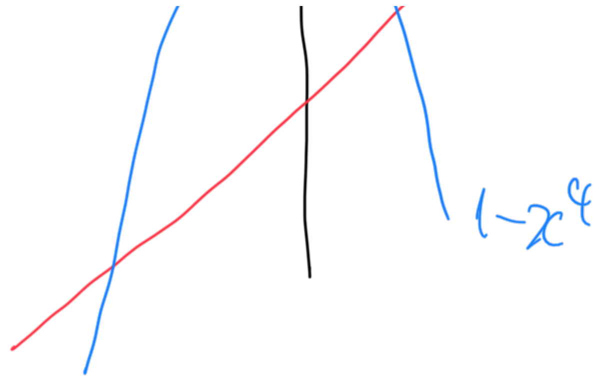
b) $x - 2 = 1 - x^4$

TEM PELO MENOS 2 SOLUÇÕES.





$[1, 2]$

$x^4 = 1$

$x^2 = \pm 1$

$x = \pm 1$

$-x-2 \quad \cancel{=} \quad -2-2=-4$

EXISTE UM ZERO EM $]-2, 1[$

E OUTRO ZERO EM $]1, 2[$

MOSTRAR QUE $x-2 = 1-x^4$

TEM EXACTAMENTE 2 SOLUÇÕES

$$K(x) = x-2-1+x^4 = x-3+x^4$$

$\underline{K'(x)} = 1+4x^3 = 0 \qquad x^3 = -1/4$

$$x = -\sqrt[3]{1/4}$$

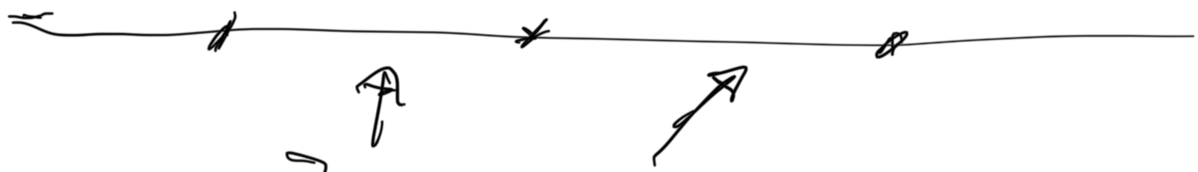
K' TEM EXACTAMENTE UM ZERO

$\Downarrow$

K NÃO PODE TER MAIS DE 2 ZEROS

K' NÃO PODE TER MAIS DE 2 ZEROS

ZEROS DE K:



$\exists$ ZERO DE K'