ASTROPHYSICS EXAM

INTEGRATED MASTER IN ENGINEERING PHYSICS

Wednesday, 6$^{\text{th}}$ July 2016, 10h00m — 12h00m

Before you begin read these instructions carefully

- Write your ist-number and name on top of every sheet that you use to answer this exam.
- Clearly identify each sheet of the exam with an arabic number in the top right side.
- Start solving each problem on a new page.
- The duration of the exam is of 2h00m + 00h30m.
- Students can not give up the exam during the first 30 minutes.
- The use se mobile phones and calculators is **not allowed** during the exam.
- Always perform first the analytical calculations and only at the end replace the numeric values.
- Fundamental Constants, Formulae and additional material can be found in the last page of the exam.
- Good Luck !

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so.

1. The intensity at a frequency ν of a blackbody at temperature T is

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/(kT)}-1},$$

where h, c and k are the Planck constant, the speed of light and the Boltzmman constant, respectively. Accordingly, compute the following quantities:

(a) the Planck's law as a function of the λ, i.e, $B_\lambda(T)$.

(b) the total intensity $B(T)$ using $B_\nu(T)$.

(c) the integrated radiative flux for a star with a temperature of $12000K$ and a radius $2.3\ R_\odot$ at a distance of $3\ pc$. What can be said about the spectra of this star ?

(d) Consider a star with a surface temperature of 2500 K. Determine:

i. the wavelength λ_{max} at which the star emits the maximum radiation. What can be said about the spectra of this star ?

ii. the fraction of radiation (approximately) that this star is emitting in the range $[1.0\ cm, \infty]$.

2. Estimate the $\Delta\lambda$ related with the processes that broaden spectral lines:

(a) The lifetime of an electron in the first and second excited states of hydrogen is about $\Delta t = 10^{-8}\ s^{-1}$. What is the natural broadening of the H_α line of hydrogen, $\lambda = 656.3\ nm$?

(b) What is the Doppler broadening, for hydrogen atoms in the Sun's photosphere (T = 5777 K) ?

(c) Again, consider the hydrogen atoms in the Sun's photosphere, where the temperature is 5777 K and the number density of hydrogen atoms is about $1.5 \times 10^{23} m^{-3}$. What is the pressure broadening of the H_α line?

3. The virial theorem applies to a wide variety of bound gravity systems, such as single stars, clusters of stars and clusters of galaxies.

(a) Show that the virial theorem for a gravitationally bound system of N particles of equal mass can be written as

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2K + U, \tag{1}$$

where K is the kinetic energy, U is the gravitational potential energy and I is the "generalised" moment of inertia defined by $I = \sum_i^N m_i r_i^2$.

(b) For a gravitationally bound system in equilibrium, show that the total energy of the system is always one-half of the time-averaged potential energy. Clearly explain your simplifications.

(c) Calculate the potential gravitational energy of a spherical stellar cluster of constant density with a total mass M and radius R.

(d) Calculate the rms velocity for the previous stellar cluster. Assume that the stars in the cluster have equal mass.

(e) Consider a spherical stellar cluster with a radius of $8\ pc$ with one million stars each with mass of $0.6M_\odot$:

(i) Determine the gravitational potential energy of the stellar cluster.

(ii) Determine the rms velocity for the cluster.

4. Answer to the following questions of general knowledge in astrophysics:

(a) Explain what is the solar cycle. What are its main properties ?

(b) Explain the main characteristics of the solar dynamo.

(c) What is helioseismology ?

(d) What is a stellar cluster ? Indicate the known types of clusters. What are the principal characteristics of each type (age, metallicity, mass, location, $\cdots$) ?

(e) Describe the known structure of the Milky Way (use a scheme).

5. Choose **only one of the following topics** to discuss about in detail. If needed use diagrams, schemes and graphs to illustrate your points.

(a) The evolution of a low mass star like the Sun from the pre-main sequence to the red giant phase.

(b) The impact of the pp chain, CNO cycle and other nuclear reactions in the evolution of stars.

ASTROPHYSICS EXAM · INFORMATION SHEET

Physical and Astronomical Constants

mass of proton	$m_p = 1.67 \times 10^{-27}\ Kg$
cross section of hydrogen	$\sigma = \pi(2a_o)^2 = 3.52 \times 10^{-20}\ m^2$
velocity of light	$c = 2.999 \times 10^{10}\ cm\ s^{-1}$
Newton constant	$G = 6.6726 \times 10^{-8}\ dyne\ cm^2\ g^{-2}$
Planck constant	$\hbar = h/(2\pi) = 1.0546 \times 10^{-27}\ erg\ s$
ideal gas constant	$\mathcal{R} = k_b N_A = 8.3143 \times 10^7\ erg\ K^{-1}\ mol^{-1}$
Avogadro's number	$N_A = 6.022 \times 10^{23}\ mol^{-1}$
Boltzmann constant	$k_b = 1.38064852(79) \times 10^{-16}\ erg\ K^{-1}$
Stefan-Boltzmann constant	$\sigma_{SB} = 5.67 \times 10^{-5}\ erg\ cm^{-2}\ s^{-1} K^{-4}$
Wien's displacement constant	$b_w = 0.2897756\ cm\ K$
parsec	$pc = 3.086 \times 10^{18}\ cm$
astronomical unit	$AU = 1.496 \times 10^{13}\ cm$
Solar Radius	$R_\odot = 6.955 \times 10^{10}\ cm$
Solar Mass	$M_\odot = 1.989 \times 10^{33}\ g$
Solar Luminosity	$L_\odot = 3.839 \times 10^{33}\ erg\ s^{-1}$

Additional Formulae

Equation of state for an ideal gas	$P = \frac{\mathcal{R}}{\mu}\rho T = v_s^2 \rho$
Integral	$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$
H_α broadening formulae	$\Delta\lambda \approx \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$
-	$\Delta\lambda \approx \frac{\lambda^2}{c}\frac{n\sigma}{\pi}\sqrt{\frac{2kT}{m}}$
-	$\Delta\lambda \approx \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}}$