Capítulo 10

2

1 a) $U(s) = \dfrac{0{,}4}{s}$

$$s = \frac{-21 \pm \sqrt{21^2 - 80}}{2} = \frac{-21 \pm 19}{2} \begin{cases} -1 \\ -20 \end{cases}$$

$$Y(s) = \frac{0{,}4 \times 10}{s(s^2+21s+20)} = \frac{A}{s+1} + \frac{B}{s+20} + \frac{C}{s} = \frac{A(s^2+20s)+B(s^2+s)+C(s^2+21s+20)}{(s+1)(s+20)s} =$$

$$= \frac{s^2(A+B+C)+s(20A+B+21C)+(20C)}{s(s^2+21s+20)} \Rightarrow \begin{cases} A+B+C=0 \\ 20A+B+21C=0 \\ 20C=4 \end{cases} \begin{cases} B=-A-C \\ 19A+20C=0 \\ C=0{,}2 \end{cases} \begin{cases} B=\frac{4}{19}-0{,}2 \\ A=\frac{-4}{19} \\ C=0{,}2 \end{cases}$$

$$\lim_{t \to +\infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{4}{s^2+21s+20} = \frac{4}{20} = 0{,}2$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \underbrace{Ae^{-t} + Be^{-20t}}_{\text{transiente}} + \underbrace{0{,}2}_{\text{e.e.}}$$

$$\lim_{t \to +\infty} y(t) = 0+0+0{,}2$$

b) $U(s) = \dfrac{2}{s^2}$

$$Y(s) = \frac{2 \times 5}{s^3+0{,}1s^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+0{,}1} = \frac{A(s^2+0{,}1s)+B(s+0{,}1)+Cs^2}{s^2(s+0{,}1)} =$$

$$= \frac{s^2(A+C)+s(0{,}1A+B)+0{,}1B}{s^3+0{,}1s^2} \Rightarrow \begin{cases} A+C=0 \\ 0{,}1A+B=0 \\ 0{,}1B=10 \end{cases} \begin{cases} C=-A=1000 \\ A=-10B=-1000 \\ B=100 \end{cases}$$

$$\lim_{t \to +\infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} \frac{10}{s^2+0{,}1s} = +\infty$$

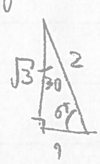
$$y(t) = \mathcal{L}^{-1}[Y(s)] = \underbrace{A+Bt}_{\text{e.e.}} + \underbrace{Ce^{-0{,}1t}}_{\text{transiente}}$$

$$\lim_{t \to +\infty} y(t) = A + \infty + 0 = +\infty$$

c) $U(s) = 1$

$$Y(s) = \frac{s}{s^2+s+1}$$

$s = \dfrac{-1 \pm \sqrt{1-4}}{2} \notin \mathbb{R}$; $\omega^2 = 1$, $2\xi\omega = 1 \Rightarrow \xi = 1/2$, $\Xi = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$, $\phi = \text{arctg}\,\dfrac{\sqrt{3}/2}{1/2} = \text{arctg}\,\sqrt{3} = 60°$



$$\lim_{t \to +\infty} y(t) = \lim_{s \to 0} \frac{s^2}{s^2+s+1} = 0$$

$$y(t) = \underbrace{-\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{3}\right)}_{\text{transiente}}$$

$$\lim_{t \to +\infty} y(t) = \underbrace{0}_{\text{e.e.}}$$

P

d) $U(s) = \frac{0{,}4}{s}$

$Y(s) = \frac{0{,}4\,s}{s^3+s^2+s} = \frac{0{,}4}{s^2+s+1}$ $\qquad \omega = 1,\ \xi = \frac{1}{2},\ \Box = \frac{\sqrt{3}}{2},\ \phi = \frac{\pi}{3}$

$$\lim_{t\to+\infty} y(t) = \lim_{s\to 0} \frac{0{,}4\,s}{s^2+s+1} = 0$$

$$y(t) = \underbrace{0{,}4 \times \frac{2}{\sqrt{3}}\, e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)}_{\text{transiente}} \qquad \lim_{t\to+\infty} y(t) = \underbrace{0}_{\text{e.e.}}$$

e) $U(s) = \frac{0{,}4}{s}$

$Y(s) = \frac{2{,}8}{s^2}$

$$\lim_{t\to+\infty} y(t) = \lim_{s\to 0} \frac{2{,}8}{s} = +\infty$$

$y(t) = 2{,}8t$

$$\lim_{t\to+\infty} y(t) = +\infty$$

2) a) polos $-2, -10$ <0 estável

b) polos 5 >0 instável

c) polos $2{,}5 \pm 3j$ $\xrightarrow{\text{parte real}}$ $2{,}5 > 0$ instável

d) polos -1 (duplo), $-2, -3$ <0 estável

e) polos 0 (duplo), $-3, -50$ instável (polo sobre eixo imag. c/ multipl. maior que 1)

f) polos -2 (quadr.) <0 estável

6 a) $s^4-2s^3-13s^2+14s+24$

s^4	1	-13	24
s^3	-2	14	
s^2	$\frac{26-14}{-2}=-6$	24	
s	$\frac{6\times14+48}{-6}=14-8=6$		
1	31		

troca de sinal ($s^4 \to s^3$)

idem ($s^2 \to s$)

2 polos no SCD

instável

b) idem, a eq. car. é igual

c) $s^2(s^4-2s^3-13s^2+14s+24)$

2 polos no SCD

2 polos na origem

instável (qualquer um destes basta)

d) s^4+4s^3+4s+5

s^4	1	0	5
s^3	4	4	
s^2	$\frac{0-4}{4}=-1$	$\frac{20}{4}=5$	
s	$\frac{-4-20}{-1}=24$		
1	5		

troca ($s^3 \to s^2$)

troca de sinal ($s^2 \to s$)

2 polos no SCD

instável

e) $2s^3-6s+4$

s^3	2	-6
s^2	0 aliás ε	4
s	$\frac{-6\varepsilon-8}{\varepsilon}$	
1	4	

2

ε ← se $0<\varepsilon\ll1$

troca

$\frac{-6\varepsilon-8}{\varepsilon}$ ← então $\frac{-6\varepsilon-8}{\varepsilon}\simeq-\frac{8}{\varepsilon}\ll0$

troca

4

logo há 2 trocas de sinal, 2 polos no SCD

instável