

HEAT TRANSFER AND FLUID FLOW IN PIPES IN THE TRANSITIONAL FLOW REGIME AND HIGH HEAT FLUXES FOR AN AUTOMOTIVE HEAT EXCHANGER

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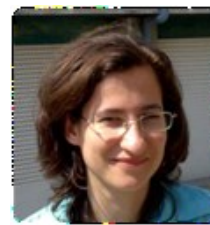
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ABSTRACT

The present work addresses an experimental investigation that focuses on the study of the heat transfer and developing fluid flow in smooth horizontal tubes in the transitional flow regime. An experimental set-up was developed and validated against literature correlations. Heat transfer at high heat fluxes (92 kW/m^2) and pressure drop measurements were taken at Reynolds numbers between 530 and 6050. Water was used as the working fluid. The test section was a stainless steel horizontal smooth circular tube with 5.35 mm internal diameter with heating length of 200 mm. The friction factor data demonstrates that there is a good agreement between experimental values and the correlations for developed fluid flow. However, for developing fluid flow the friction factor presents higher values than expected. Friction factor values are higher in diabatic flow than in adiabatic flow for laminar regime, but are similar for turbulent regime. In laminar flow regime the Nusselt number values are higher than those predicted by various correlations reported in literature due to the secondary flows which increase the amount of mixing in the tube. In contrast, in the transition regime the Nusselt number values are over predicted by available literature correlations.

Keywords: Fluid flow in pipes, transitional flow, adiabatic and diabatic flow, high heat fluxes, friction factor, Nusselt number.

1. INTRODUCTION

Despite the progresses in improving Internal Combustion Engines (ICE) efficiency, much work is still required to recover wasted thermal energy. Among the different waste heat recovery (WHR) options, the Rankine cycle system has shown very promising results, Domingues *et al.* (2013). A key element in the Rankine cycle waste heat recovery (RC-WHR) system is the heat exchanger (HEX) used to recover the thermal energy, Santos *et al.*, (2016). In this context, previous work (Santos *et al.*, 2016) has been carried out to design and implement a RC-WHR system, with particular emphasis on the customization of the HEX that was introduced in the vehicle exhaust line. The investigated HEX is a cross-flow heat exchanger, simple and robust, with the working fluid circulating inside the tubes.

Previous studies have demonstrated that in a practical application, the working fluid flow in the channels of the HEX is in the laminar or transition flow regime. Furthermore, during the design process it was identified that the best compromise between high heat transfer coefficients and relatively low pressure drop is usually in the transitional flow regime, for which theoretical and empirical relations are still scarce. According to a recent review paper by Meyer (2014), flow in the transitional flow regime has mainly been investigated by Professor Ghajar and his co-workers that were the first to investigate the heat transfer and pressure drop in the transitional flow regime, Ghajar and Tam (1991, 1994), Tam and Ghajar (1997) and Tam *et al.* (2010, 2013).

In the present practical application the fluid flow occurs in the transitional flow regime under both hydraulic and thermal developing flow conditions. The literature studies considered either fully developed flow (Tam *et al.* 2013), or average measurements of developing flow across a tube length (Meyer 2014). Therefore, the heat transfer and pressure drop characteristics of developing flow in the transitional flow regime have not yet received the required attention.

Within this scope, the present work addresses and contributes towards the design, test and construction of a modular test section that will allow studying the fundamental transport phenomena in the laminar and transition regime, which occurs within the channels of the heat exchanger.

2. LITERATURE SURVEY

The present section presents a literature survey on friction factor and Nusselt number correlations for laminar, transitional and turbulent flow.

2.1 Friction factor

Regarding to hydrodynamic phenomena there is an important distinction between entry and fully developed regions. The hydraulic entry length depends on whether the flow regime is laminar or turbulent (Incropera 2006), and so that are given by Eq. (1) and Eq. (2) respectively:

$$z_{f,d,h_{lamin}} = 0.05 \cdot Re \cdot D_h \quad (1)$$

$$10 \cdot D_h \leq z_{f,d,h_{turb}} \leq 60 \cdot D_h \quad (2)$$

The friction factor was determined as follows:

$$f = \frac{D_h}{L} \cdot \frac{2 \cdot \Delta p}{\rho \cdot v^2} \quad (3)$$

where D_h is the hydraulic diameter, L is the channel's length, v the mean cross-sectional velocity, Δp the pressure drop along the channel, and ρ is the specific mass. For fully developed laminar flows, friction factor can be obtained analytically ($f = 64/Re$), Incropera (2006). However for developing laminar flow, not only friction effects are taken into account, but also entry region phenomena. Shah (1978) developed a correlation to determine the friction factor in both entry and fully developed regions:

$$f_{app} = \frac{64}{Re} \left(\frac{3.44}{(z^+)^{1/2}} + \frac{0.31/z^+ + 16 - 3.44(z^+)^{-1/2}}{1 + 0.00021(z^+)^{-2}} \right) \quad (4)$$

where:

$$z^+ = \frac{z/D_h}{Re} \quad (5)$$

Recently, Tam *et al.* (2013) developed a correlation based on Shah's given by:

$$f_{app} = \frac{64}{Re} \left(16 + \frac{0.00314}{0.00004836 + 0.0609(z^+)^{1.28}} \right) \quad (6)$$

valid for: i) $799 < Re < 2240$ e ii) $3 < z = D_h < 200$, with a precision between -26.1% and +28.1%. For fully developed turbulent flow, Blasius (1913) and Petukhov (1970) developed the correlations given in the Eq. (7) and Eq. (8), respectively:

$$f = 0.3164 Re^{-0.25} \quad (7)$$

$$f = (0.79 \ln(Re) - 1.64)^{-2} \quad (8)$$

Both correlations were developed for $Re > 3000$.

Regarding transitional flow Tam *et al.* (2013) developed the correlation:

$$f_{app} = \frac{64}{Re} \left(\left[1 + (0.0049 Re^{0.75})^{0.52} \right]^{\frac{1}{0.52}} - 3.47 \right) \quad (9)$$

obtained for: i) fully developed flow; ii) $2026 < Re < 3257$; and iii) $3 < z / D_h < 200$. Tam *et al.* (2013) also developed a correlation which includes a correction factor that accounts for entry region effects.

$$f_{app} = \frac{64}{Re} \left((1 + (0.0049 Re^{0.75})^{0.52})^{1/0.52} - 3.47 \right) \left(1 + \left(\frac{4.8}{z/D_h} \right) \right) \quad (10)$$

The application range for this correlation is: $2019 < Re < 3257$ and $3 < z / D_h < 200$. The correlations depicted in Eq. (9) and Eq. (10) have a precision of -8.6% to +7.9% and of -25.9% to +21.9%, respectively, Tam *et al.* (2013).

2.2 Nusselt number

Regarding the heat transfer phenomena, as in hydrodynamic mechanisms, two distinct regions can be distinguished: thermal entry region and thermal fully developed region. For laminar and turbulent flow (Incropera 2006), the thermal entry length is given by the Eq. (11) and Eq. (12) respectively:

$$z_{fd, \text{lam}} = 0.05 \cdot Re \cdot D_h \cdot Pr \quad (11)$$

$$z_{fd, \text{turb}} = 10 \cdot D_h \quad (12)$$

The Nusselt number for a circular pipe is defined as follows:

$$Nu = \frac{h}{k} D_h \quad (13)$$

For fully developed laminar flow with constant heat flux boundary condition in a circular pipe, the Nusselt number assumes a constant value of 4.364, Incropera (2006). Churchill and Ozoe (1973) obtained a correlation for both developing and fully developed conditions:

$$\frac{Nu_z}{4.364(1 + (Gz/29.6)^2)^{1/6}} = \left(1 + \left(\frac{Gz/19.04}{(1 + (Pr/0.0207)^{2/3})^{1/2} \times (1 + (Gz/29.6)^2)^{1/3}} \right)^2 \right)^{1/3} \quad (14)$$

where the local Nusselt number, Nu_z , and Graetz number Gz , are given by Eq. (15) and Eq. (16), respectively:

$$Nu_z = \frac{q_s'' \cdot D_h}{k(T_s(z) - T_m(z))} \quad (15)$$

$$Gz = \frac{\pi}{4z_+} = \frac{\pi}{4} \cdot \frac{Re \cdot Pr}{z/D_h} \quad (16)$$

Eq. (14) has an average deviation of 5%, for $0.7 \leq Pr \leq 10$, when compared against numerical results.

Ghajar and Tam (1994) also developed a correlation for the laminar regime in both entry and fully developed regions:

$$Nu = 1.24 \left[\frac{Re \cdot Pr \cdot D_h}{z} + 0.025(Gr \cdot Pr)^{0.75} \right]^{\frac{1}{3}} \cdot \left(\frac{\mu}{\mu_s} \right)^{0.14} \quad (17)$$

This correlation was obtained using mixed and forced convection, with: i) $280 \leq Re \leq 3800$, ii) $3 < z / D_h < 192$, iii) $40 \leq Pr \leq 160$, iv) $1000 \leq Gr \leq 2.8 \times 10^4$ and v) $1.2 \leq \mu/\mu_s \leq 3.8$. This correlation represented the experimental data to within -16.9% and +15.4%. Recently, Gnielinski (2010) obtained the following correlation valid for laminar regime and both developing and developed regions:

$$Nu = \left(4.354^3 + 0.6^3 + \left(1.953^3 \sqrt{Re \cdot Pr \cdot D_h / L} - 0.6 \right)^3 + \left(0.924^3 \sqrt{Pr \sqrt{Re \cdot D_h / L}} \right)^3 \right)^{1/3} \quad (18)$$

For fully developed turbulent flow the Dittus-Boelter correlation is widely used (Bejan and Kraus, 2003; Incropera 2006):

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \quad (19)$$

The application range is: i) $0.7 < Pr < 120$, ii) $2500 < Re < 1.24 \times 10^5$ and iii) $L / D_h > 60$. Maximum error of this correlation in comparison with experimental data is approximately 40 %, Bejan and Kraus (2003). However, it is possible to find in literature other correlations with better accuracy, such as Gnielinski's correlation:

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \quad (20)$$

where f is the friction factor, which can be evaluated using Eq. (8). Gnielinski's correlation has an accuracy of 10 % for: i) $0.5 < Pr < 10^6$, ii) $2300 < Re < 5 \times 10^6$, Bejan and Kraus (2003). Gnielinski (1976) developed another correlation for the turbulent regime based on Eq. (20) and adding a correction factor to take into account entry region effects:

$$Nu = Nu_{eq20} \left[1 + (D_h/L)^{2/3} \right] \left(\frac{\mu}{\mu_s} \right)^{0.14} \quad (21)$$

Eq. (21) can be applied for: $Re > 3000$ and $0.5 < Pr < 2000$. Ghajar and Tam (1994) for both developing and fully developed turbulent flow derivate the following equation:

$$Nu = 0.023 \cdot Re^{0.8} Pr^{0.385} (z/D_h)^{-0.0054} (\mu/\mu_s)^{0.14} \quad (22)$$

whereas the correlation was compared against experimental data where the following parameters were evaluated: i) $7000 \leq Re \leq 49000$, (ii) $3 < z / D_h < 192$, (iii) $4 \leq Pr \leq 34$, (iv) and (v) $1.1 \leq \mu/\mu_s \leq 1.7$. The experimental data was correlated within -10.3% and +10.5%.

3. EXPERIMENTS

This study contributes to the development of an automotive heat exchanger (HEX) that will be included in a Rankine cycle waste heat recovery (RC-WHR) system. It is important to point out that the HEX must have a large quantity (more than 200) of short length tubes (200 to 400 mm) with internal diameter from 4 to 7 mm that are connected with return chambers. In the present work, the fundamental fluid flow and heat transfer mechanisms are investigated for a single smooth horizontal pipe with $L/D_h \approx 72$, which corresponds to the practical operating conditions of HEX integrated in the RC-WHR system. The fluid flow in the tubes of the HEX occurs mainly in transitional flow regime under developing flow conditions. Those conditions were replicated in the experimental set-up. Atmospheric pressure was considered at this early stage of the work, which minimizes design complexity.

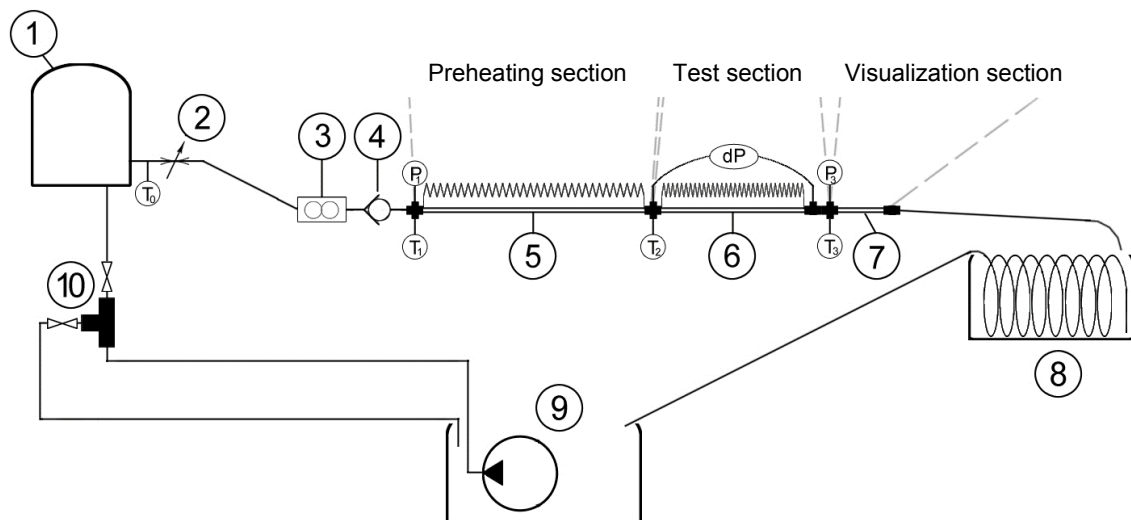


Fig. 1 – Schematic representation of the experimental set-up.

Fig. 1 depicts a schematic representation of the experimental set-up, which includes: 1) a reservoir; 2) a fine adjustment valve; 3) a volumetric fluid flow meter; 4) a non-return valve; 5) a preheating section; 6) a test section; 7) a visualization section; 8) a condenser; 9) a pump; 10) a T connecting valve. The pre-heating section consists of an AISI 304 stainless steel channel with 2000 mm length and 6 mm internal diameter. Heat flux is imposed at the wall by means of a heating wire with 35.5 Ω . Temperature and pressure were measured at the inlet of the pre-heating section using a K-type thermocouple and an absolute pressure transducer (MPX4250A). The test section also consists of an AISI 304 stainless steel channel with 412 mm length and 5.85 mm internal diameter. The heating wire has 170.4 Ω and is coiled around 200 mm of the channel's length. Temperature was measured at the inlet and outlet of the test section by two K-type thermocouples. Pressure was measured at the outlet using an

absolute pressure transducer (MPX4250A). Pressure drop along the channel was measured using a differential pressure transducer (Honeywell 26PC series).

All uncertainties were calculated within the 95% confidence level using the method described in Dunn (2014). Reynolds number uncertainty is almost constant (about 7%) for every tested flow conditions. Friction factor's uncertainty decreases with Reynolds number, whereas for $Re > 4000$ the friction factor uncertainty is approximately constant (about 12%). Furthermore in diabatic conditions the friction factor's uncertainty is slightly larger than that for adiabatic flow. The friction factor uncertainty varied between 10.8% and 24.9% while the Nusselt numbers uncertainties varied between 10.3 and 25.8%.

4. RESULTS AND DISCUSSION

4.1 Adiabatic and diabatic friction factor

The validation of the experimental set-up was an important part of the present work. The friction factor and Nusselt number data were first validated against various literature correlations. In terms of pressure drop this was done by taking measurements in the pre-heating section. These experiments were taken with no heat flux imposed. Fig. 2 depicts a comparison of the adiabatic friction factor data as a function of the Reynolds number against literature correlations for adiabatic fully developed flow.

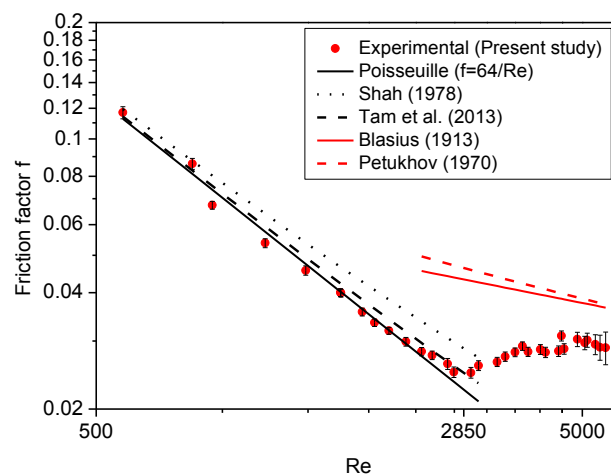


Fig. 2 – Adiabatic friction factor as a function of the Reynolds number, in the pre-heating section.

In the laminar regime, the adiabatic friction factors are very close to those predicted by the Poiseuille equation, within 4% mean relative error. For $Re > 2800$ the deviation from

experimental values against literature correlations is much larger since the flow had not reached fully turbulent conditions and was in the transitional regime.

In the test section, a series of experiments were performed without heating in order to evaluate the adiabatic friction factor. As it can be observed in Fig. 3 the experimental friction factor was considerably larger than the values predicted by different literature correlations even when compared with those that include entry region effects. These observations are consistent with those of Barlak *et al.* (2011). These authors studied the effect of the L/D_h ratio on the friction factor and concluded that for $L/D_h < 100$ in the laminar regime, the friction factor increases when L/D_h decreases. In the present study, the L/D_h ratio for the test section is about 70. Furthermore, about 70% of the data points correspond to an entry length that occupies at least 80% of the channel length. Fig. 3 also reveals that for $Re > 2500$, the friction factor data is larger than that predicted by Blasius and by Petukhov correlations.

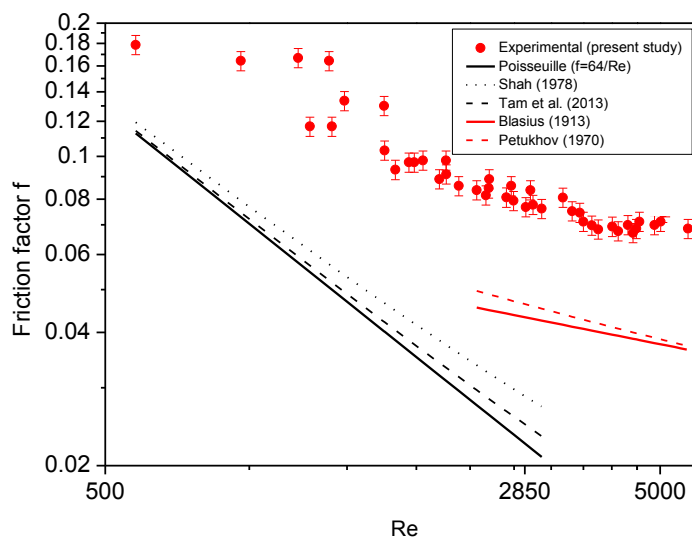


Fig. 3 – Adiabatic friction factor as a function of the Reynolds number, in the test section.

Fig. 3 reveals that the experimental data follows the same trend as the values predicted from the correlations. Indeed, Fig. 3 clearly shows that the experimental and theoretical curve's slope are very similar in both laminar and turbulent regimes, and the transition occurs smoothly instead of abruptly as usual. This phenomenon has also been observed by Barlak *et al.* (2011). Friction factor's uncertainty has an increase from 2 to 7% for a Reynolds number approximately equal to 2700, which has been stated to be a good indicator of transition, Meyer (2014).

Fig. 4 depicts the experimental friction factor against the Reynolds number for adiabatic and diabatic flow. The straight line corresponds to the analytical solution for the laminar regime,

while the dotted line represents the friction factor obtained with Blasius equation for turbulent flow Eq. (7). Fig. 4 reveals that the friction factor is larger for diabatic flow, in comparison with adiabatic flow, for laminar and transitional regimes, while for the turbulent regime, friction factor has the same magnitude in both cases. This observation is in good agreement with the results reported in the literature (Ghajar and Tam 1994; Meyer, 2014). Those authors stated that the difference found between diabatic and adiabatic flow in the laminar regime was due to secondary flow effects that usually occur for small fluid mean velocities inside the channel. Furthermore, when the imposed heat flux increases, shear stress at the wall increases, increasing the friction factor, Meyer (2014). The diabatic friction factors increased in the laminar and transition regimes due to the effect of secondary flow. Heating influences the start and end of transition region and transition is delayed for increasing heat fluxes, Meyer (2014).

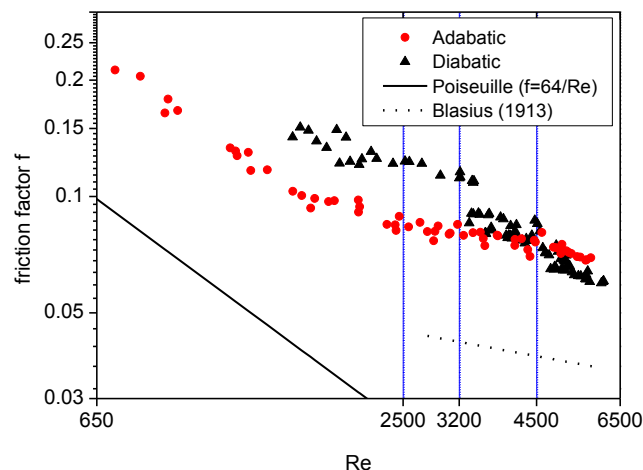


Fig. 4 – Experimental friction factor as a function of the Reynolds number, in the test section.

Meyer (2014) also refers that in the turbulent flow regime, the friction factor is independent of the inlet geometry and secondary flow. Fig. 4 shows that for $Re > 3300$ adiabatic and diabatic friction factors are very similar since no secondary flow effects are present.

4.2 Nusselt number

Average Nusselt number was evaluated in the test section with an imposed heat flux of 92 kW/m^2 . Fig. 5 compares the experimental data against different literature correlations for laminar developing flow. Experimental values were in better agreement with Eq. (18) - Gnielinski (2010) with a mean relative error of 16.3%.

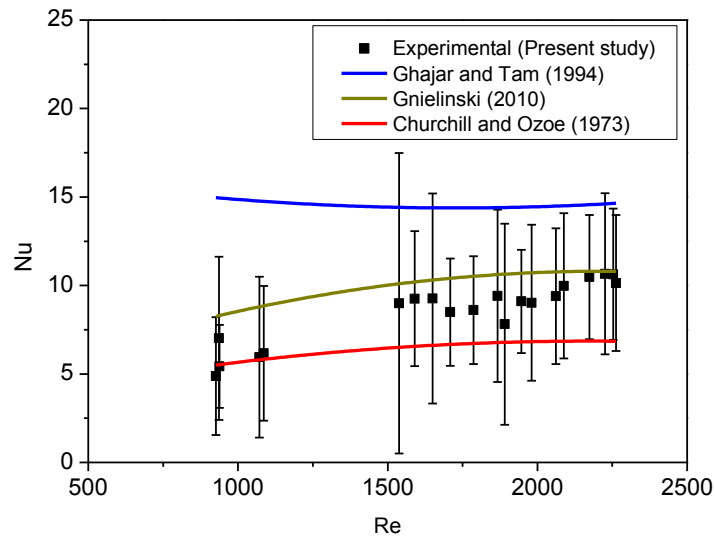


Fig. 5 – Average Nusselt number as a function of Reynolds number for laminar regime, in the test section.

Fig. 6 shows the experimental and predicted Nusselt number as function of the Reynolds. The correlations used for comparison correspond to the Eq. (18) - Gnielinski (2010) for the laminar regime and Eq. (21) - Gnielinski (1976) for the turbulent regime. Both correlations include entry region effects. The results depicted in the Fig. 6 were obtained by a series of 130 experiments with an imposed heat flux of 92 kW/m^2 .

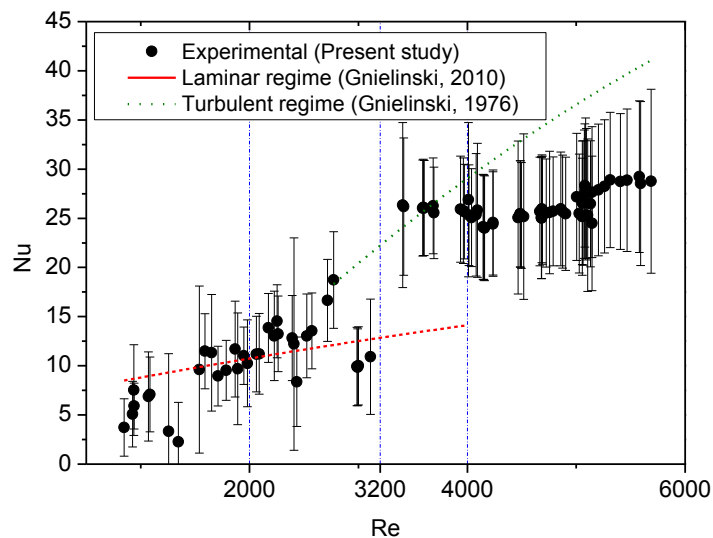


Fig. 6 – Experimental Nusselt number as a function of the Reynolds number ($800 < Re < 6000$).

As expected, for laminar flow, the Nusselt number is larger than the analytical value ($Nu = 4.364$). This is mainly due to the: i) developing thermal boundary layer along the whole channel; and ii) secondary flow, which increase the amount of mixing in the tube. These observations are consistent with those of Meyer (2014), which refers that the Nusselt number

for laminar flow regime is strongly influenced by secondary flow effects and increased with increasing heat flux.

Fig. 6 reveals that the Nusselt number increases considerably for $Re \approx 3200$ which is a good indicator of transition. It is important to point out that analogous to the friction factor, the Nusselt number in the turbulent flow regime are independent of the inlet geometry and secondary flow, Meyer (2014). Furthermore, it is well known that in the turbulent regime the Nusselt number increases with the Reynolds number, as it can be observed in the Fig. 6 for $Re > 4500$. The correlation derived by Gnielinski (1976) is for the turbulent regime, in fact turbulent conditions are not reached in the test section and flow is characterized by fluctuations between laminar and turbulent regimes (transitional regime). Hence, the Nusselt number in the transition region is overpredicted by the Gnielinski (1976) correlation, which was also observed by Meyer (2014). This author refers that correlations for turbulent regime usually over predict the Nusselt number for Reynolds number between the critical value (onset of transition) and about 10000.

5. CONCLUSIONS

The experimental adiabatic friction factor in the test section is larger than that predicted by the literature correlations for both laminar and transition regimes. However the experimental friction factor represented as a function of Reynolds number has the same trend as the different correlations revised.

Laminar friction factors obtained with heat flux imposed on the channel's wall were larger than those obtained without heat flux due to the secondary flows increasing the amount of mixing in the tube. However they presented similar values in the transitional regime, i.e. for $Re > 4500$, since secondary flow effects could be neglected.

In the laminar regime, Nusselt numbers were higher than those predicted for fully developed flow, which is due to the mixed convection mechanism and the developing thermal boundary layer. On the other hand, in the transitional regime Nusselt number results were always over-predicted by the various correlations from the literature that were developed for turbulent conditions, not reached in the present study.

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