Analytical and Finite Element Models Applied to Forced Assemblies

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Abstract

For all industries components are manufactured, which independently or as part of a more complex structure, fulfill a specific function for which they have been designed, created and developed.

Nowadays, with the technological processes available, it is not yet possible to produce parts without dimensional variations that don’t compromise its manufacturing cost. It is necessary to prepare the existing assembly lines to deal, in the best way, with components with deviations of shape and dimension without compromising their performance and purpose.

In this work, an analytical model and a finite element model have been developed in order to be able to estimate the influence of the shape deviation of the components on the final assembly, particularly the internal stresses generated. The models formulated, aided by the tools created in Excel to facilitate the whole process of presentation and interpretation of results and modelling of the components, allowed us to estimate the efforts on two beams, in a fast and intuitive way, in both models. Although the models presented specifically represent the assembly of two beams, for both the methodology that has been established aims at an easy adaptation of the tools to different types of assemblies and components.

Keywords: Force Assembly, Shimming, Force Fit, Wing Box

1. Introduction

Every day engineers and technicians are challenged to create cheaper, lighter and more efficient components. In other words, components that meet specific design requirements but also have economic and mechanical advantages.

As far as the assembly of components is concerned, the existing dimensional variations influence the relative spatial position of the parts and their internal stresses. The dimensional variation of a part to be assembled influences influence not only a specific assembly stage but all subsequent ones. The main difficulty in this area of study is the determination of the interference of a certain dimensional variation in the final assembly of an assembly line [1].

Understanding the influence of dimensional variations in a final structure, particularly on the distribution of internal stresses, it is of great interest to establish a procedure or method that easily allows the calculation of the maximum internal stresses and final position of the structure.

When assembling a wing box the existence of dimensional deviations both in the reinforcement and in the metal sheet leads to the existence of gaps between the two components along the contact surface. In order to make the clearances disappear there is the possibility to manufacture wedges, or shims, to fill the existing clearances and therefore making the clearances disappear.

The aim is to determine, for each assembly, the influence of a certain clearances in a shimless assembly, effectively estimating the internal stress distributions on the components to be assembled caused by the forced assembly of a structure with clearances and without shims.

The main objectives of this work are to develop two representative models of a forced assembly applied to a particular case, present in the aerospace industry, and to compare the results obtained through these models. It is proposed the analysis, in both models, of a forced assembly and the presentation of the respective stress distribution for a specific clearance defined at the beginning of chapters 3 and 4.

2. Background

At an early stage of design, for an aircraft to be properly designed, it is first necessary to establish a purpose for it, one or more specific objectives that the aircraft must meet [2].

During the intermediate stages of an aircraft's
design, it is inevitable that structural problems will be solved, and it is necessary to define the dimensions of a given component and what physical properties it should have.

From the first aircraft made to the most recent ones, the wing has undergone great changes, made possible by the knowledge acquired through experience, the development of engineering and the appearance of materials with more advantageous mechanical and physical characteristics.

The need for aircraft with a greater carrying capacity, both cargo and people, has benefited from the possibility of building stronger wings. The efforts to which they are subject are therefore greater for this type of transport. Thus, a structure has been introduced, called the Wing Box, on the internal part of the wings, which is responsible for ensuring the shape of the desired profile and, at the same time, a structural rigidity capable of withstanding the application of high loads.

The Wing Box is the part of the wing responsible for transferring static and dynamic loads to the fuselage, and is also, essentially this structure, responsible for resisting all torsional, bending and cutting loads that are applied to the wings [3].

Throughout this work, and as mentioned at the beginning of this section, the stage in the construction of a wing box with the greatest interest in study, is the assembly of spars, stringers or ribs to the metal panels, also known as skins.

According to Rooks (2001) [4], and for perfect components, i.e. without dimensional variations, the following indications can be followed for the assembly of a rib, or reinforcements, and a panel. With the reinforcements fixed, the sheet metal to be assembled is placed against them and, with the use of pneumatic clamps, both components are held together.

A robot then pierces the two components to be assembled and inserts the fastener, which can be either screws or rivets. Usually, the fasteners are inserted sequentially. Finally, the clamps are released by completing this stage of manufacture.

In fact, all components that are part of the internal structure of a wing have large dimensions and restricted assembly relationships. However, due to manufacturing errors and assembly deformations, the components cannot fit tightly enough to prevent the emergence of gaps between the parts [5].

Thus, if there are dimensional variations, the assembly process shown above undergoes some necessary changes, with some intermediate steps having to be added. After positioning the panel under the reinforcements, clamps are used to apply a compressive force to the parts in the pre-tensioning stage. The gaps in the structure are then measured and a map or function is created to define the shape of the shims to be applied. Having machined all the necessary wedges and removed the clamps from the pre-tensioning stage, they are inserted between the two parts and the clamps are repeated to force the two components and the wedge to join. The remaining production steps are similar to the previous ones, where a robot drills through the whole set and inserts the clamp.

As far as gap measurements are concerned, these should be taken in the direction of the fastener and also in the transverse and perpendicular direction of the fasteners. Measurements can be made manually by handling calibrated plates or, in some cases, by using 3D scanners applied to robotic arms, also used in the automotive industry [5].

The process of filling the gaps with shims it’s called shimming. The whole shimming process involves many additional steps to the original process. The manufacturing time is severely hampered by the need to manufacture shims, which in turn adds to manufacturing costs [6].

With the aim of minimising the effect of dimensional variations on the structure to be assembled, particularly in relation to the gaps between components, the appearance of internal stresses and their influence on the final position of the assembly, innovative methods have recently emerged which attempt to predict and reduce manufacturing time and cost.

Being aware of the need to ensure the manufacture of a structure capable of meeting the design requirements and, at the same time, of great importance in reducing production costs, in chapter 3 and 4 two models will be proposed that aim to estimate the internal stresses caused by the dimensional variations of the components.

3. Analytical Model

In this chapter, an analytical model applied to the Excel software has been developed, which is expected to be able to represent a simplified forced assembly and to present the obtained results in a quick and easy to interpret way.

Looking now at figure 1 a representation of a forced mounting between three beams and a plate can be seen by means of rivets or bolts.

With regard to the structural components and as explained in the previous chapter, there are numerous possible combinations of different structures that can be applied to a forced assembly, however, in the formulation of this model only the presence of two structural elements will be considered, which mostly support transverse loads. Therefore, in order to make the analytical model simpler and more reliable, it is proposed to consider only the beams to represent the structural elements of
the whole, not considering an assembly between beam and plate as presented in figure 1.

In the figure 2, where the two black lines represent two components to be assembled and the vertical lines in green represent the fasteners, a simplified representation of two phases of a forced assembly is made.

It should be noted that in 2a the initial phase, prior to assembly, where the parts are still physically separated and without applied loads, and in 2b the final phase where the parts are in contact, bearing the loads applied by the fasteners and in a position of equilibrium is presented.

3.1. Dimensional Variations and Gap

As mentioned, at this stage of the study the deviations in form, \( \delta(x) \), will be defined as known mathematical functions. It is also known that any continuous function can be defined by means of Fourier Series.

\[
f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_n \cos(\omega_n t) + b_n \sin(\omega_n t) \right)
\] (1)

In order to simplify the function that defines the deviation in form, some simplification will be made below in relation to the function. It is proposed that the functions \( \delta(x) \) can be defined by cosine functions and only by a natural frequency with a value of \( 2\pi L \), where \( L \) represents the length of the beams. A maximum value for the function will also be established at half the total length of the beam, \( \delta(x = L/2) = \delta_{max} \).

The simplified equation of a function with the above conditions is as follows:

\[
\delta_i(x) = \frac{\delta_{max}}{2} - \frac{\delta_{max}}{2} \cos(\omega_n x).
\] (2)

According to the established orientation of the axes, the displacement value, \( Y_i(x) \), can be defined as a function of the distance to the equilibrium position in the following way:

\[
Y_1(x) = -\delta_1(x) \quad \text{and} \quad Y_2(x) = \delta_2(x).
\] (3)

The formulas 4 represent the relationship, according to Euler-Bernoulli's Beam Theory, between the displacement of the elastic line of a beam with the distributed load applied, the transverse effort and the bending moment.

\[
\begin{align*}
\frac{\partial^2 Y_1}{\partial x^2} &= \frac{M_1(x)}{E_1 I_1} \\
\frac{\partial^2 Y_2}{\partial x^2} &= \frac{V_1(x)}{E_1 I_1} \\
\frac{\partial^4 Y_1}{\partial x^4} &= -\frac{w_1(x)}{E_1 I_1}
\end{align*}
\] (4)

With two displacement functions, one for each beam, and three equations of Euler-Bernouilli’s Beam Theory, six equations are obtained corresponding to the transverse load, bending moment and distributed load for each beam. The equations obtained which concern the Beam I and Beam II are presented in the systems of equations 5 and 6 respectively.

\[
\begin{align*}
M_1(x) &= -\frac{\delta_{1max}}{2} \left( \frac{2\pi}{L} \right)^2 E_1 I_1 \cos \left( \frac{2\pi x}{L} \right) \\
V_1(x) &= \frac{\delta_{1max}}{2} \left( \frac{2\pi}{L} \right)^3 E_1 I_1 \sin \left( \frac{2\pi x}{L} \right) \\
w_1(x) &= -\frac{\delta_{1max}}{2} \left( \frac{2\pi}{L} \right)^4 E_1 I_1 \cos \left( \frac{2\pi x}{L} \right)
\end{align*}
\] (5)
ated by their assembly. For that, the PBEAM beams will be used to model two beams and analyse which stress and displacement fields are created by their assembly. For that, the PBEAM beam elements will be used to discrete Beam I and Beam II, being the number of elements, used to model them, one of the variables of the model.

As far as the fasteners are concerned, it is expected to be able to reproduce a load similar to that applied by the fasteners and thus ensure that there is no slack when assembling the two beams. It is therefore proposed to use an infinitely rigid element, called RBE2, which would be responsible for applying a load on the two beams that is sufficiently high to reduce the existing clearance.

This model will consider two different sets of border conditions, called CF1 e CF2. In the first, CF1, border conditions equivalent to the Analytical Model will be imposed, i.e., the translation in X and Y and the rotation in Z will be fixed in the four end nodes and the remaining rotations to one of the nodes, in order to ensure the model’s static stability. These conditions are aimed at the analysis of a forced assembly in circumstances as close as possible to the circumstances present in the analytical model, so that the comparison of results between the two models can be made.

4.1. Excel Program
As the object of the study is the forced assemblies and these can assume various differences between them, either in relation to the properties of the materials, the dimensions of the existing clearances or even the number of fasteners, the time necessary for the modelling of the various models necessary for a study of this type is of extreme importance. Thus, realizing the usefulness of the existence of a tool capable of making the entire modeling process more intuitive and faster, similar to the work developed in section 3.5 for the analytical model, a program was created, in Visual Basic for Applications (VBA) in Excel, a tool capable of designing a .dat file with the necessary information to define a model in Nastrana language from data previously entered in Excel.

In order to make modelling and especially post-processing processes more coherent and more intuitive to interpret, some rules have been established regarding the ID of each node and each element.

- elements with the ID’s between 1 and 1998 are part of the Beam I
- elements with the ID’s between 10001 and 19998 are part of the Beam II
- elements with the ID’s between 20001 and 29998 are part of the fasteners.
- beam elements are defined by nodes with consecutive ID’s, the element ID being equal to the lowest ID of these ID’s

\[
\begin{align*}
M_2(x) &= \frac{\delta_{2,\text{max}}}{2} \left( \frac{2\pi}{L} \right)^2 E_2 I_2 \cos \left( \frac{2\pi x}{L} \right) \\
V_2(x) &= -\frac{\delta_{2,\text{max}}}{2} \left( \frac{2\pi}{L} \right)^3 E_2 I_2 \sin \left( \frac{2\pi x}{L} \right) \\
w_2(x) &= \frac{\delta_{2,\text{max}}}{2} \left( \frac{2\pi}{L} \right)^4 E_2 I_2 \cos \left( \frac{2\pi x}{L} \right)
\end{align*}
\] (6)

It is proposed that an equilibrium condition exists for the analytical model which relates the values of the load distributed in the Beam I, \(w_1\), and in the Beam II, \(w_2\). Respecting the relationship between the direction of the loads distributed for each of the beams it is proposed that:

\[w_1 = -w_2.\] (7)

Solving a system of equations containing \(5\), \(6\) and \(7\), in order of the maximum values of the displacement functions, \(\delta_{1,\text{max}}\) and \(\delta_{2,\text{max}}\), one deduces the following equation:

\[\delta_{1,\text{max}} = g_{\text{max}} \frac{E_1 I_1}{E_1 I_1 + E_2 I_2}\] (8)
\[\delta_{2,\text{max}} = g_{\text{max}} \frac{E_2 I_2}{E_1 I_1 + E_2 I_2}\] (9)

4. Finite Element Model
In order to create a tool capable of adapting to a greater variety of forced assembles and thus obtain results closer to the real ones, capable of carrying out not only linear as well as non-linear analyses and also allowing the visualisation of the efforts, not only in the equilibrium position, but also in any intermediate instant, it was decided to create a finite element model. Finite Element Models (FEM) have long been a tool used to assist technicians or engineers in the development stages of an engineering project. This tool is very versatile in several areas of study. The development of various software capable of containing all the methodology and mathematics behind FEM, makes the formulation of models and analysis much more practical and easy to apply.

For this was chosen FEMAP. Finite Element Modeling And Postprocessing, from Siemens PLM Software to simulate the assembly models, since it is frequently used by companies linked to the aerospace sector and is available, free of charge, to university students, and there is enough online documentation for consultation.

In a similar way to the proposed Analytical Model, presented in chapter 3, the finite element model will be used to model two beams and analyse which stress and displacement fields are created by their assembly. For that, the PBEAM beam
Table 1: Physical properties and cross-section dimensions attributed to the PBEAM elements that discrete the beams

<table>
<thead>
<tr>
<th></th>
<th>M. Young [E]</th>
<th>Poisson [ν]</th>
<th>Thickness</th>
<th>Width</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam I</td>
<td>7000 daN/mm²</td>
<td>0.33</td>
<td>20 mm</td>
<td>60 mm</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Beam II</td>
<td>7000 daN/mm²</td>
<td>0.33</td>
<td>20 mm</td>
<td>80 mm</td>
<td>1000 mm</td>
</tr>
</tbody>
</table>

- fasteners elements are define by nodes with ID's 10000 and 20000 lower than the fastener ID.

Having established these rules in the numbering of nodes and elements and as mentioned above, it becomes much easier to locate an element or model node through its ID, and thus quickly know to which component it belongs or which nodes define it.

4.2. Simulation

Having finished the modelling part, it is then necessary to establish the type of analysis to be performed, since the modelling part is fully defined by the file created by Excel.

In this work, two types of analysis will be carried out only, static linear and nonlinear static. For a Linear Analysis, the necessary changes to the Analysis Set Manager will be presented. Listing the tab, section and information to be inserted (shown in bold) in the Analysis Set Manager menu.

- Analysis Set → Analysis Program → 36.Simcenter Nastran
- Analysis Set → Analysis Type → 1.Static
- Nastran Bulk Data Options → Rigid Element Method → 1.Lagran
- Boundary Conditions → Constrains → 1.Cond. Fronteira 1
- Boundary Conditions → Temperatures → 2.Fastner_Temperature

5. Results

For both models the physical dimensions and properties presented in the table 1 and the shape and clearance deviations shown in table 2 were considered.

Table 2: Dimensional Variations imposed on Beam I and II.

<table>
<thead>
<tr>
<th>Shape Deviation</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam I</td>
<td>2.86 mm</td>
</tr>
<tr>
<td>Beam II</td>
<td>2.14 mm</td>
</tr>
</tbody>
</table>

6. Conclusions

With regard to the definition of the shape deviation through the Fourier series, it was concluded that they guarantee a continuous dimensional variation...
Table 3: Maximum values for internal effort obtained from the Analytical Model and the FEM and relative errors.

<table>
<thead>
<tr>
<th>Analytical Model</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{max}$ [daN mm]</td>
<td>$V_{max}$ [daN]</td>
</tr>
<tr>
<td>15791.36</td>
<td>99.02</td>
</tr>
<tr>
<td>Error [%]</td>
<td>0.783</td>
</tr>
</tbody>
</table>

along the length of the beams and allow setting shape deviation values in the transverse direction for discrete values of $X$.

Thus, although it is mathematically possible to guarantee a continuous deviation that respects the cross-sectional deviations measured for a finite number of points, the trigonometric functions do not ensure the smoothest dimensional variation close to reality, since, between the points along the length for which the deviation was measured, it may assume different behaviour.

Analysing table 3 along with chapter 5 of the thesis, where the distribution of effort obtained from the two models formulated is presented, it is possible to conclude that for a clearance of 0.5% of the length between the components to be assembled the difference in maximum stresses and loads presents an error of less than 1.5% (See table 3). The maximum stresses and loads in the components have also been compared for assembly configurations with different length values and also maximum clearance, but no conclusion could be drawn.

As expected, with the increase in the maximum clearance value for the same length the differences between the maximum loads tend to increase, which indicates a distance from the results obtained from the two models for clearance values to be divided by the larger length. This phenomenon can be explained by recalling that the analytical model was formulated considering small displacements, not representing correctly the distribution of the efforts in the bending of the beams.

As regards the tool developed and presented in the chapter 3 it contains the entire formulation of the analytical model, allows an intuitive configuration of the characteristics of a forced assembly and the calculation and rapid presentation of the results obtained and at the same time allows an easy interpretation of these same values, thus fulfilling the initial objectives that were proposed.

In the case of the tool developed in the section 4 to create the Nastran file that defines the complete modelling of an assembly, this is of great value as it has provided the simulation of a greater variety of configurations, as it significantly reduces the time required for their modelling.

As the speed of obtaining results is one of the main objectives of the work, the models developed, aided by the tools developed, make it possible to quickly estimate the internal tensions in the components and consequently to determine their influence on their lifetime, supporting in an intuitive, practical and immediate way the design choices that the technician and engineer have to adopt.

As far as future work is concerned, the experimental validation of the results obtained for both models is considered fundamental in order to be able to apply them with confidence and security in real project situations. The models under study should also be simulated for a greater variety of forced assembly configurations, so that it is possible to predict the influence of different characteristics and geometries on the final results.

References


