Orbital Collision Warning and Avoidance

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Abstract

The growth of the number of objects in space increases the probability of collisions that will generate debris. This work will contribute for increasing the SSA of D-Orbit and to tackle the problem of space debris by mitigating it. The aim is to develop a collision warning tool to be integrated in the mission control software developed by D-Orbit. Based on TLEs retrieved from SpaceTrack and on SGP4 theory, the positions of every space objects for several time instants were calculated. Filtering techniques were implemented to select only the objects that may cause a collision with the object of value. The calculation of these positions have associated uncertainties, these were estimated in the form of a covariance matrix and a probability of collision was calculated, allowing conjunction analysis and assess if an avoidance manoeuvre is needed or not. The developed filtering techniques reduced the execution time of the code, allowing to predict conjunctions, for D-Orbit's satellite. The covariances estimation revealed large uncertainties for LEO objects especially in the along-track direction, due to the limitations of SGP4 theory in drag modelling. Analysis and comparisons with other collision warning services demonstrated that a more refined position calculation, with more accurate data, is needed. With TLEs it's possible to develop a collision warning tool, however with high uncertainties. The SSA of D-Orbit was increased and the objective completed.
Keywords: Space Debris, TLE, SGP4, Covariance, Probability of Collision, Conjunction Analysis

1. Introduction

Since 1957, with the launch of Sputnik, man-made space debris has been increasing, becoming a danger for the continuity of spaceflight. In 2009, a collision between Iridium-33 and Cosmos-2251 occurred, forming a large cloud of debris and resulting in a wake-up call for the space community to solve the problem of space debris. There are two main approaches to solve the problem: debris mitigation and debris removal. The first is concerned about establishing guidelines for operations in space. The second is finding ways and technologies to remove debris from space.

USA lists orbital information of about 20 000 objects, maintaining and updating this information daily in a GP (General Perturbations) catalog. This catalog was created on the 60s and it's processed by analytical approaches, namely the SGP4 (Simplified General Perturbations 4) theory used to generate TLEs (Two-Line Elements) and propagate them [12] to calculate space objects positions. If one knows where the objects will be in the future, it is possible to know if there will be close approaches between them. If the positional errors are known, it is possible to calculate a PC (Probability of Collision) for conjunction analysis. This helps the operator to decide if an avoidance maneuver is needed or not. Collision warning and avoidance is considered part of the mitigation approach to solve the debris problem.

Comparing the operational aspects of several agencies, some depend on CDMs (Conjunction Data Messages) provided by 18th SPCS (Space Control Squadron), others create their own CDMs through TLEs or tracking data not publicly available [11], resulting in different algorithms and implementations. In this work, TLEs will be used to create CDMs. The catalog of TLEs is available on Space-Track website. This work has the objective of developing a collision warning tool to be integrated into the mission control software of D-Orbit PT, increasing its SSA (Space Situational Awareness) and contributing to debris mitigation. This tool aims at a reduction of the execution time to detect conjunctions by implementing filtering techniques and allow the operator to assess a conjunction by analysing the calculated parameters, such as: TCA (Time of Closest Approach), MD (Miss Distance), relative velocity, covariance, scaling factor, PC and PC_{max}.

2. Selection of Dangerous Objects

2.1. Background

The objective is to find all conjunctions of the satellite of value for the operator (primary object) against all the other objects in the TLE catalog (secondary objects). A conjunction is defined as an event where both object's centers of mass are within a specified distance from one another. When propagating all objects for different time instances the program becomes computationally heavy. Thus, to turn the task of conjunction detection into a more manageable one, pre-filters and sieves are used to filter out secondary objects that cannot possibly collide with the primary. The difference between pre-filters and sieves is that the first only have to be applied per object pair once, while the latter has to be applied per object pair in each time instance.

2.1.1 Perigee/Apogee Filter

This filter discards pairs of objects that do not cross each other's altitude and so can never collide. Being q the larger perigee of the objects pair and Q the smaller of the two apogees, if

$$q - Q > D_{th} \ , \tag{1}$$

then the pair is ruled out.

2.1.2 Smart Sieve

In this method, the state vectors of the objects are computed for several time instances equally spaced in time, by a defined time step, during a certain time interval. If the distance between a pair of objects is higher than a threshold distance, R_{th}, the pair is eliminated from further consideration. Another distance should be defined, a critical distance, R_{cr}. This is the distance that defines a conjunction. If an object is inside this critical distance, then a conjunction is happening.

To define R_{th}, it's used the fact that objects orbiting Earth can't exceed the escape velocity, v_{esc}, of 11.186 km/s. Thus, the relative velocity between two orbiting objects can never exceed twice the escape velocity, 22.3726 km/s. Hence, during a time step duration, Δt, the secondary object cannot enter the critical volume if it's at a larger distance than R_{th} and so this object can be discarded. R_{th} is defined as

$$R_{th} = R_{cr} + \frac{d_{esc}}{2} = R_{cr} + v_{esc}\Delta t \ , \tag{2}$$

where d_{esc} is the distance travelled during Δt with maximum relative velocity of $2v_{esc}$. The overall smart sieve process is described in [10], including the XYZ sieves, step skipping, r^2 sieve, minimum distance sieve, r^2 fine sieve and fine conjunction detection.

2.2. Implementation

The general conjunction detection method was implemented with the following processes:

- Catalog Download and Reading - The catalog with the more recent TLEs is downloaded from SpaceTrack and a dictionary of TLEs is created.
- Out-of-date TLE Filter - The objects whose TLEs epoch is older than 20 days are discarded.
- Perigee/Apogee Filter
- Propagation - All objects that passed previous filters are propagated, generating ephemerides for each object in the time interval analysed.
- Smart Sieve - The ones implemented were the step skipping method, the XYZ sieves, the r^2 sieve, and the r^2 fine sieve. The minimum distance sieve was not used because it was found that it would only be valid for small time steps, in the order of 10 seconds. It is intended to have a large time step, in the order of 180 seconds, to reduce the time it takes to propagate. The fine conjunction detection was also not used because it's not described in detail in [10].
- Generation of Conjunction Data - For one conjunction event, there is more than one time instance where the secondary object is at a relative distance smaller than R_{th} and only the data where the conjunction reaches the minimum distance is needed. Moreover, the secondary object may have more than one conjunction with the primary during the time interval analysed, because in the next revolution the objects may have a conjunction again. Only state vectors with minimum distance between them are selected. Note that this is not the true minimum distance due to discretization of time in propagation, it will be a distance in a time instance near to the one found.
- Refined Propagation and TCA - In order to find the time where the real minimum distance (TCA) occurs, it is used a Taylor series approximation to extrapolate the motion of the objects, see Eqs. (3) and (4) [6],

$$\mathbf{r} = \mathbf{r_0} + (t - t_0)\dot{\mathbf{r_0}} + \frac{1}{2}(t - t_0)^2\ddot{\mathbf{r_0}} + \dots \ , \tag{3}$$

$$TCA = t_0 - \frac{\mathbf{r_0} \cdot \dot{\mathbf{r_0}}}{\dot{\mathbf{r_0}} \cdot \dot{\mathbf{r_0}}} \ , \tag{4}$$

where $\mathbf{r}$ is the relative position vector and the variables with the subscript 0 are the ones found during the propagation that have a minimum relative distance. To obtain MD, it's necessary to replace t by TCA, in Eq. (3). This is only valid for small time steps, so to turn around this problem, a refined propagation of both objects around the TCA found is performed, with a time step of 1 second. This process is done in a loop always with smaller time steps until the TCA calculated differs less than 0.001 seconds from the previous TCA. If the MD found is larger than the R_{cr} defined before, then the object is discarded. If MD is smaller than R_{cr} then a conjunction was found.

2.2.1 Tuning the Threshold Distance of the Perigee/Apogee Pre-Filter

In the perigee/apogee filter, the desired is to discard as much objects as possible in order to reduce the computational time propagating the objects that pass this filter. To achieve this, it's necessary to tune the threshold distance D_{th} presented in Sec. 2.1.1. If D_{th} is too large then almost certainly that any discarded object will cause a conjunction but objects that don't need further analysis will be propagated, increasing the processing time. If D_{th} is too small, then some objects that will cause a conjunction in the future can be discarded, which is not desired. A good way to choose a value for D_{th} is to estimate the variation of the perigee and apogee during a certain time interval *a priori*, meaning that this estimation is not included in the process of conjunction detection and analysis, it will only be used to define the value of D_{th}.

All objects in the catalog with a TLE epoch not older than 20 days are propagated during a certain time, obtaining the perigee and apogee of each object's orbit at a certain time instant. The maximum variation of the perigee and apogee for each object was calculated and plotted against variables present in TLEs. This will allow to define D_{th}. The definition of D_{th} is not just one value, but several values, depending on the eccentricity of the object and the propagation time interval chosen, see Table 1.

Table 1: Definitions of D_{th} for each propagation time interval and each eccentricity interval.

Propagation Time	Eccentricity	D_{th} [km]
≤ 1 day	$0 \leq e < 0.2$	150
	$0.2 \leq e \leq 1$	200
> 1day, ≤ 3 days	$0 \leq e < 0.2$	400
	$0.2 \leq e \leq 1$	500
> 3 days, ≤ 6 days	$0 \leq e < 0.03$	900
	$0.03 \leq e < 0.2$	500
	$0.2 \leq e \leq 1$	1000
> 6 days,≤ 10 days	$0 \leq e < 0.03$	1500
	$0.03 \leq e < 0.2$	900
	$0.2 \leq e \leq 1$	1500

2.3. Validation of Filters Implementation

A brute force method and SOCRATES, will be used to compare the conjunctions found with the filters implemented in this work.

2.3.1 Brute Force Method

A brute force method was used to validate the filters implementation and assure that no false negatives or false positives were introduced. In this method, no filters are used and a time step of 1 second is chosen to assure that, during the propagation, there are no time instances with a conjunction not detected. This time step is very small, making this method extremely expensive computationally, which is why it will only be used for validation purposes.

For a propagation time interval of 7200 seconds, the execution time is 28 629 seconds, almost 8 hours, see Table 2. This shows that, from the operational point of view, a brute force method is useless. Imagine a conjunction happening in 2 hours, if it takes 8 hours to detect it, then the conjunction would only be detected after it happened. This confirms that the problem of conjunction detection is only practical if filters and sieves are used.

For the same propagation time intervals used in the brute force method and with a time step of 60 seconds, the filters and sieves method was implemented, see Table 3. It's possible to notice the importance and power of the perigee/apogee filter. It only takes 1 second regardless of the duration of the analysis interval because it is applied directly to the catalog and not to the ephemerides generated by the propagation.

The propagation time is much smaller than the one of brute force because there are less objects to propagate due to the perigee/apogee filter and the time step used is sixty times larger. For a duration of 2 hours, the brute force method takes almost 8 hours, while the filter and sieves method takes only 1 minute approximately. For each propagation time interval, the number of conjunctions detected and their conjunction data is the same, this means that no object that will cause a conjunction is discarded by the filters and sieves method and the assumptions made in their development and implementation are valid.

2.3.2 Comparison with SOCRATES

The parameters calculated by SOCRATES [8] will be compared to the ones obtained by the developed warning tool. SOCRATES uses a conjunction threshold of 5 km hence, for comparison reasons R_{cr} will be set to 5 km as well. Object 46 274 (D-Orbit ION) will be analysed as the primary satellite to compare with the conjunction data of SOCRATES.

A run was performed with the catalog of 09/09/2020 and the conjunctions presented by SOCRATES were collected. It was verified that TLEs used by SOCRATES were the same as the one in the 09/09/2020 catalog. The number of conjunctions and conjunction data obtained was exactly the same as SOCRATES as it can be seen in

Table 2: Number of conjunctions obtained for several analysis intervals with the brute force method. The primary object is 20 510 and it was used a R_{cr} of 26 km. The catalog epoch is 26/08/2020.

Analysis interval [s]	# Conjunctions	Executing Time [s]				
		Catalog Reading	Propagation	Brute Force	Conj Data Gen	Total
180	1	1	158	42	0.01	202
600	4	1	539	168	0.03	708
1800	9	1	2511	1390	0.2	3902
3600	13	1	7286	4473	0.3	11761
7200	40	1	18614	10004	0.4	28620

Table 3: Number of conjunctions obtained for several analysis intervals with a time step of 60 seconds, with the filters and sieves method. The primary object is 20 510 and it was used a R_{cr} of 26 km. The catalog used was the one of 26/08/2020.

Analysis interval [s]	# Conjunctions	Executing Time [s]					
		Catalog Reading	Peri/Apo Filter	Propagation	Sieves	Conj Data Gen	Total
180	1	1	1	2	0.5	0.3	5
600	4	1	1	4	1	1	9
1800	9	1	1	11	3	2	18
3600	13	1	1	22	6	5	35
7200	40	1	1	43	10	9	65
43200	233	1	1	256	66	52	376
86400	463	1	1	519	148	116	786

Fig. 1 and Table 4. These comparisons suggest that the implementation of the filters and sieves method is correct.

NORAD Catalog Number	Max Probability / Start (UTC)	Dilution Threshold (km) / TCA (UTC)	Min Range (km) / Stop (UTC)	Relative Velocity (km/sec)
31304	1.666E-06	0.638	0.978	15.037
46274	2020 Sep 09 16:28:51.060	2020 Sep 09 16:28:51.386	2020 Sep 09 16:28:51.712	
46274	5.348E-07	0.729	2.669	8.149
44862	2020 Sep 09 20:24:39.502	2020 Sep 09 20:24:40.021	2020 Sep 09 20:24:40.540	
46274	1.580E-07	2.153	3.059	15.202
39313	2020 Sep 11 09:13:57.715	2020 Sep 11 09:13:57.976	2020 Sep 11 09:13:58.236	
46274	2.900E-07	0.949	3.782	5.519
46198	2020 Sep 11 21:03:17.472	2020 Sep 11 21:03:18.064	2020 Sep 11 21:03:18.657	

Figure 1: Conjunctions detected for ION by SOCRATES.

3. Covariance Estimation Using TLEs

3.1. Background

TLEs don't have information about the covariance of the state vector, so this section will cover the estimation of it. When the state vector of an object is calculated, it always have an associated uncertainty due to the kind of data used and also to the algorithms used to propagate those positions. Those uncertainties are represented in a matrix, called the covariance matrix [13]. The covariance matrix is an important information because it allows the operator of the satellite to know the confidence in its SSA. These uncertainties will allow, in Sec. 4, to perform conjunction analysis and calculate the PC.

Considering the vector $\mathbf{X} = [X_1, ..., X_N]$, the covariance matrix, P, of dimensions $N \times N$, is an extension of the meaning of covariance but for multidimensional variables,

$$P_{ij} = \text{cov}(X_i, X_j) = E\Big[\big(X_i - E[X_i]\big)\big(X_j - E[X_j]\big)\Big], \tag{5}$$

so the elements in the diagonal of the matrix, where $i = j$, are the standard deviations of the elements in vector $\mathbf{X}$. In this case, $\mathbf{X}$ is a vector of dimension 6 (3 components of position and 3 components of velocity) and P a matrix of dimension 6×6.

The covariance matrix can be divided into two matrices, the position matrix and the velocity matrix. If one only considers the positional covariance, it is possible to relate these values with the probability of finding the object in a certain region, called the covariance ellipsoid. This ellipsoid is the 3D representation of a covariance matrix and the uncertainties in position. It's the surface of a 3D Gaussian distribution where σ has three dimensions, the along-track, radial, and cross-track.

3.1.1 Methods for Covariance Estimation in TLEs

There are three methods to obtain the covariance for a TLE, these are: TLE differencing, TLE comparison with tracking data and TLEs as pseudo-obervations [13]. The first will be used in this work.

TLE differencing is based only on historical TLEs and works as follows: take as much TLEs of an object as possible, propagate them to the same time instance and subtract the state vectors to obtain the residuals. [9]. The covariance matrix is extracted in the most recent TLE epoch or another desired time instance. A downside of this method is that considers that TLEs are unbiased and TLE generation is

Table 4: Conjunctions detected for ION by the developed warning tool.

Secondary ID	TCA [UTC]	MD [km]	Relative Velocity [km/s]
31304	2020-09-09T16:28:51.387	0.978	15.037
44862	2020-09-09T20:24:40.021	2.669	8.149
39313	2020-09-11T09:13:57.975	3.059	15.202
46198	2020-09-11T21:03:18.064	3.782	5.519

error free. The biggest advantage is that only relys on SGP4/SDP4 theory, so it can be used for any cataloged object in a simple and fast way, which is why it was used in this work. TLE comparison with tracking data relys on more accurate data [13], like GPS, not available for every objects and TLE as pseudo-observations require access to inverted tracked station models also not available. [5]

3.2. Implementation

TLEs will be downloaded for the objects involved in a conjunction for a period 20 days into the past. The more recent TLE will be called prime and the covariance will be calculated only at TCA.

Defining the state vectors by $\vec{X} = [\vec{r} \quad \vec{v}]^T = [r_x \quad r_y \quad r_z \quad v_x \quad v_y \quad v_z]^T$ and calculating the residuals between the state vectors obtained from propagated TLEs and the prime state vector,

$$\delta\vec{X}_{\mathrm{TEME}i} = \vec{X}_{\mathrm{propagated}_{\mathrm{TEME}}} - \vec{X}_{\mathrm{prime}_{\mathrm{TEME}}} \; , \quad (6)$$

where i refers to the i-th TLE propagated to TCA, in a set of N TLEs. Remembering Eq. 5, $P_{\mathrm{TCA}_{\mathrm{TEME}}}$, the covariance matrix at the TCA in the TEME frame is

$$P_{\mathrm{TCA}_{\mathrm{TEME}}} = E[(\delta\vec{X}_{\mathrm{TEME}} - m)(\delta\vec{X}_{\mathrm{TEME}} - m)^T] \; , \quad (7)$$

being m the vector with the mean of the residuals,

$$m = \frac{\sum_{i=1}^{N-1}(\delta\vec{X}_{\mathrm{TEME}})_i}{N-1} \; . \quad (8)$$

3.3. Results

In order to validate the implementation of the covariance generation process, it was tried to reproduce the results of Osweiler [9]. The results of the covariance are presented in a NTW referene frame, for validation purposes. For the time window 8 described in Osweiler as the time interval between 2004/10/06 and 2004/10/21, all TLEs, whose epoch is in this time interval, were retrevied for the LAGEOS satellite with the prime TLE shown in Fig. 2.

```
1 08820U 76039A   04294.65281357 -.00000002 +00000-0 +10000-3 0  9996
2 08820 109.8594 349.9486 0044244 196.5047 163.4332 06.38664775408443
```

Figure 2: Prime TLE used in Osweiller for the LAGEOS satellite and time window 8.

The covariance obtained by Osweiller is presented in Table 5. Table 6 shows the covariance matrix obtained by the developed code for covariance generation. Comparing the positional covariance in Tables 5 and 6, the difference obtained in the positional standard deviations is approximately 5.32 m, 0.09 m, 1.46 m in the in-track, normal and cross-track displacements, respectively. These are very small when compared to the standard deviations of LEO objects, so the discrepancies found between the implementation of Osweiller and the one developed in this work are not significant.

In Table 6, the in-track displacement has a covariance value of $\sigma^2 = 0.0416\,\mathrm{km}^2$ which gives a standard deviation of $\sigma = 203\,\mathrm{m}$. LAGEOS is a MEO satellite, so it won't suffer the drag effects that a satellite in LEO will. The normal ($P_{R_N R_N}$) and cross-track ($P_{R_W R_W}$) covariances lead to standard deviations of 52 m and 88 m, respectively, which are much lower than the in-track deviation. These comparisons were done for the other satellites and time windows in Osweiler, the results were similar to the presented case.

When integrating this covariance generation process in the collision warning tool, the difference in implementation is that covariance is not calculated for the prime TLE epoch but for TCA. To give an example of a covariance matrix in a conjunction, for the 01/10/2020 catalog and ION as the primary object, it was found a conjunction with object 44 414. The covariances at TCA were generated for these two objects. ION had an along-track standard deviation of 36 km, a radial standard deviation of 195 m and a cross-track standard deviation of 93 m. The secondary object had an along-track standard deviation of 25 km, a radial standard deviation of 115 m and a cross-track standard deviation of 39 m. The along-track deviations are very large, comparing with radial and cross track deviations, these are typical values for objects in LEO and an indicator of the limitations of SGP4 and TLEs in drag modelling.

4. Probability of Collision Calculation

4.1. Background

The PC is of outmost importance when analysing a conjunction because the MD parameter ignores position uncertainty and may lead to an exaggerated assessment of the true risk. The PC includes position uncertainty because it is obtained through covariance information.

There are several approaches for the PC calcula-

Table 5: Covariance matrix obtained by Osweiller for LAGEOS satellite and time window 8.

	R_T [km]	R_N [km]	R_W [km]	V_T [km/s]	V_N [km/s]	V_W [km/s]
R_T [km]	4.38E-02	5.47E-02	-9.35E-04	-2.55E-06	-1.76E-05	2.58E-06
R_N [km]	5.47E-03	2.69E-03	9.16E-04	-1.25E-06	-1.21E-06	5.64E-09
R_W [km]	-9.35E-04	9.16E-04	8.05E-03	-4.25E-07	2.72E-06	-2.94E-06
V_T [km/s]	-2.55E-06	-1.25E-06	-4.25E-07	5.78E-10	5.67E-10	-3.29E-12
V_N [km/s]	-1.76E-05	-1.21E-06	2.72E-06	5.67E-10	8.09E-09	-1.84E-09
V_W [km/s]	2.58E-06	5.64E-09	-2.94E-06	-3.29E-12	-1.84E-09	1.58E-09

Table 6: Covariance matrix obtained by the developed code for LAGEOS satellite and time window 8.

	R_T [km]	R_N [km]	R_W [km]	V_T [km/s]	V_N [km/s]	V_W [km/s]
R_T [km]	4.16E-02	5.36E-03	-6.77E-04	-2.50E-06	-1.66E-05	2.43E-06
R_N [km]	5.36E-03	2.70E-03	1.03E-03	-1.25E-06	-1.12E-06	1.65E-09
R_W [km]	-6.77E-04	1.03E-03	7.79E-03	-4.78E-07	2.59E-06	-2.78E-06
V_T [km/s]	-2.50E-06	-1.25E-06	-4.78E-07	5.79E-10	5.27E-10	-1.39E-12
V_N [km/s]	-1.66E-05	-1.12E-06	2.59E-06	5.27E-10	7.65E-09	-1.74E-09
V_W [km/s]	2.43E-06	1.65E-09	-2.78E-06	-1.39E-12	-1.74E-09	1.49E-09

tion, SOCRATES has a very conservative approach. It calculates the maximum PC, instead of the actual PC [3], setting a standard shape and orientation for the covariance and changing the size of it, until it reaches a maximum PC. This is done by setting radial, along-track, and cross-track values of 100 m, 300 m, 100 m, respectively [1], which is not a good assumption since this isn't the case for all conjunctions. In this work, a better approach will be used. The PC calculation is based on covariance information, obtained in Sec. 3. Then this covariance will be sized in order to have the maximum probability.

4.2. Implementation

The definition of the PC is shown in Eq. 9. It's the integration of a 2D Gaussian probability density function centered on the secondary object, over the circle of radius d centered on the primary object,

$$PC = \frac{1}{2\pi|Det(C)|^{1/2}} \iint_{x^2+y^2\leq d^2} \exp\left(-\frac{1}{2}(\mathbf{r}-\mathbf{r_{s/p}})^T C^{-1}(\mathbf{r}-\mathbf{r_{s/p}})\right)\mathrm{d}x\mathrm{d}y\,, \quad (9)$$

where C is the 2×2 projection on the conjunction plane of the combined 3×3 covariance at TCA (see Fig. 3), d is the sum of the two object radius also called HBR (Hard Body Radius), $\mathbf{r} = [x\ y]^T$ is any point in the conjunction plane that satisfies $x^2+y^2 \leq d^2$ and $\mathbf{r_{s/p}} = [r_{s/p}\ 0]^T$ is the position of the secondary relative to the primary, in the conjunction plane.

When calculating the PC, what is computed is not exactly the probability of collision, but the probability that two objects are less than a specified distance at TCA [2]. This approach is made because there is no information about size, shape and attitude of all catalogued objects, so a sphere of a certain radius around the objects is defined.

[1]T. S. Kelso. SOCRATES: Satellite Orbital Conjunction Reports Assessing Threatening Encounters in Space, Dec. 2019. Accessed on 2020/11/22. `https://celestrak.com/SOCRATES/`

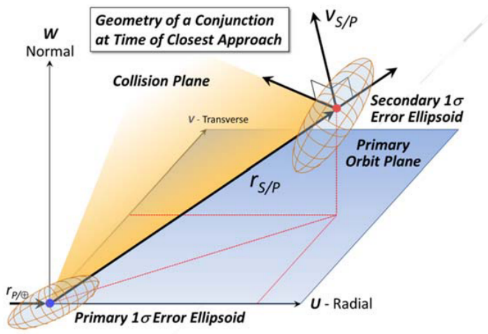


Figure 3: Conjunction plane, also called collision plane or B-plane, and the 1-σ error ellipsoids of each object. [2]

The data needed for the PC calculation is: the size of the objects; the TCA of the conjunction and the positional covariance of the objects. The last two parameters were obtained in Sec. 2 and 3, so it's only necessary to know the size of the objects. The DISCOS database [1] will be used because it has information about the sizes of cataloged objects. This database is organized by object type and if the object size is not present in this database, it will be used a pre-assigned conservative value for the dimension of that type of object (10 m for debris and mission related objects and 20 m for payload, rocket bodies and unknown).

The PC calculation is a 3D dynamic problem but this will be reduced to a 2D static problem by considering the following assumptions [2]: objects will be treated as spheres with conservative radius; the relative motion is considered rectilinear with constant velocities, during the conjunction; velocity covariance is negligible; positional covariance is constant during the conjunction, allowing to calculate the PC only at TCA; positional uncertainties can be described by a random Gaussian distribution; primary and secondary position uncertainties are independent, allowing to simply sum both covariances to form a combined covariance.

4.2.1 Maximum Probability and Covariance Sizing

The covariance realism is a concern when analysing a conjunction. The covariance obtained in Sec. 4.2 is also an estimation, hence it may be overestimated or sub-estimated. The size of the covariance may be changed, this means that the entries in the covariance matrix may be multiplied by a scaling factor, k, that maximizes PC. This process is called covariance sizing and it's used to obtain the maximum PC. The scaling factor can be found analytically, assuming that PC takes its maximum when the Gaussian distribution does, [4]

$$k = \sqrt{\frac{r_{s/p} C^{-1} r_{s/p}}{2}} \, , \tag{10}$$

where $r_{s/p}$ is the relative position vector and C the 2×2 projection on the conjunction plane of the combined 3×3 covariance at TCA (see Fig. 3). The scaled covariance is obtained by,

$$C_{\mathrm{PC}_{\max}} = k^2 C \; . \tag{11}$$

PC is calculated the same way as before but using the scaled covariance, $C_{\mathrm{PC}_{\max}}$, in order to obtain the maximum PC. If $k > 1$, then the uncertainties are small comparing to MD, if $k < 1$, it means that the covariance estimated is diluted and the uncertainties are larger than MD. If operating in this dilution region, the recommendation is to obtain better data to reduce the uncertainty and reassess the conjunction [3].

4.3. Results

Several cases will be studied, performing conjunction analysis in extreme and real cases, comparing with a LEOLabs conjunction and analysing the collision between Iridium-33 and Cosmos 2251. When a conjunction is detected, a conjunction analysis is made by the operator to evaluate the danger of the conjunction and decide if an avoidance manoeuvre is needed or not, building his decision upon the parameters calculated (TCA, MD, relative velocity, positional covariances, PC, HBR, Maximum PC and scaling factor).

4.3.1 Extreme and Real Cases

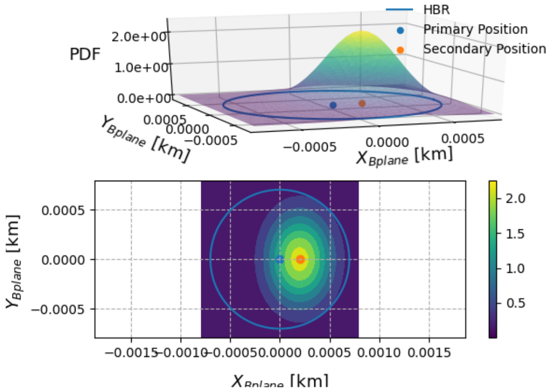
In this test, three cases will be studied: two simulated cases and one real case. The simulated cases represent the extreme cases in PC calculation, when PC equals 1 and when PC equals 0. The real case is a conjunction found for ION when running the code. For every case, HBR=0.7 m because ION largest dimension is 0.4 m and object 44 414 is 0.3 m. Figure 4 shows the conjunctions of the three cases, in the conjunction plane, with the respective PDF. The relevant parameters for each case are defined as:

- Case 1 - Simulated conjunction where MD was defined as 0.2 m and the projected combined covariance matrix was set as $\begin{bmatrix} 0.05 & 0 \\ 0 & 0.1 \end{bmatrix}$ m^2.
- Case 2 - Simulated conjunction where MD was defined as 100 m and the projected combined covariance matrix was set as $\begin{bmatrix} 100 & 0 \\ 0 & 10000 \end{bmatrix}$ m^2.
- Case 3 - Conjunction found between ION and the object 44 414 when running the catalog of 01/10/2020. MD is 3212 m and the projected combined covariance matrix is $\begin{bmatrix} 5.8 \times 10^8 & -3.0 \times 10^8 \\ -3.0 \times 10^8 & 1.3 \times 10^9 \end{bmatrix}$ m^2.

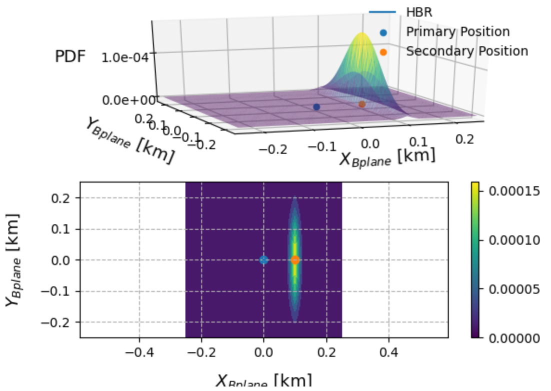
In case 1, the PC obtained was 0.96 which matched with the expectations. In Fig. 4(a), it's important to notice the shape and orientation of the PDF. The covariance matrix was defined as a diagonal matrix with standard deviations of 0.22 m in the x-axis and 0.32 m in the y-axis resulting in a projected combined covariance ellipse more stretched along the y-axis. Since it is a diagonal matrix, the ellipse is not tilted with respect to the conjunction frame as it is for example in case 3, Fig. 4(c). This case represents a high risk conjunction, so the operator should decide to perform an avoidance manoeuvre.

For case 2, the PC obtained was 6.5×10^{-26} which is a very small value, matching the expectations. A detail to note in Fig. 4(b) is that the HBR is barely noticed in the plot, because MD is much larger than HBR, so when MD increases, the PC will decrease because the PDF will have lower values in the HBR area. This case represents a safe conjunction, so the operator doesn't need to perform an avoidance maneuver.

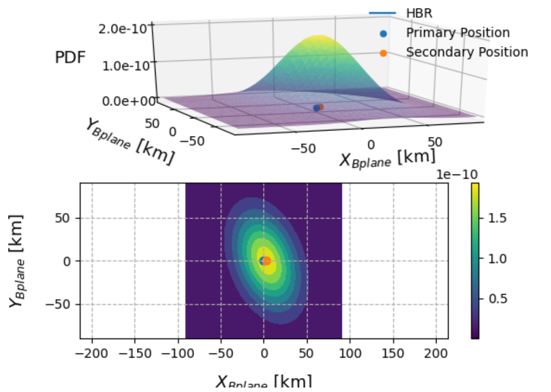
In case 3, the PC was 3.7×10^{-10}, which is larger than case 2, as expected. The values of PC, in most conjunctions found when running the code, will be similar to this one, because the standard deviations are larger than in the simulated cases. In case 3, the decision of perform an avoidance maneuver or not is difficult because, even if MD is large and PC is small, the uncertainties in position are large, so it's not clear if the conjunction is safe or unsafe. This is where $\mathrm{PC}_{\max}$, and scalling factor, k, can help in the decision. The $\mathrm{PC}_{\max}$ and k obtained was 1.39×10^{-8} and 0.1, respectively. Since k is smaller than 1, then the covariance used for PC calculation is diluted, i.e., the uncertainties are larger than MD. More accurate data is needed to better evaluate the collision and decide if an avoidance maneuver is needed.



(a) Case 1



(b) Case 2



(c) Case 3

Figure 4: Conjunctions in the conjunction plane and frame defined in Fig. 3, with the respective PDF in the Z axis.

4.3.2 Comparing with LEOLabs conjunction

Another test was a comparison with the parameters obtained by LEOLabs, in a high risk conjunction, detected on the 13th October 2020. The conjunction parameters found by LEOLabs are shown in Tables 7 and 8. [2]

Table 7: Data of the high risk conjunction found by LEOLabs.

TCA [UTC]	MD [m]	Rel. Vel. [km/s]	PC
2020-10-16T00:56:40.726	25	14.659	0.038

Table 8: Data of the objects involved in the collision.

	Primary Object	Secondary Object
Norad ID	19826	36123
σ_x[m]	5	4
σ_y[m]	80	82
σ_z[m]	8	10

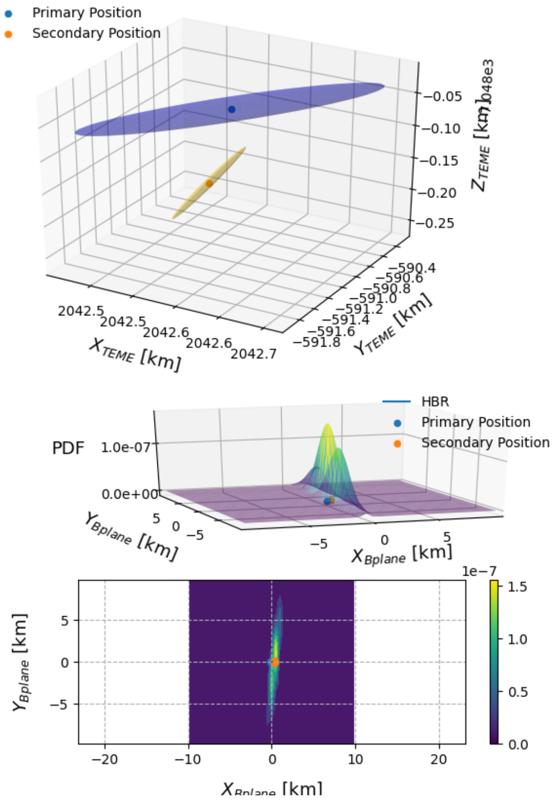
When running the developed code with the catalog of 14/10/2020 11:30:46 [UTC] and object 19 826 as primary object, it was found a conjunction with object 36 123. The conjunction data found by the developed collision warning tool is shown in Tables 9 and 10. Figure 5(a) shows the plots in the conjunction plane for the PC calculation as well as a plot to better visualize the conjunction geometry. Analysing the results obtained in Table 9 and Fig. 5(a), the TCA and relative velocity are very similar to the ones of LEOLabs. The scaling factor, k, is higher than 1 so, the PC is not in a diluted region, meaning that the uncertainties are not too high.

The MD obtained, in Table 9, is much larger than the one obtained by LEOLabs, Table 7. Hence, the PC will be smaller comparing to the one of LEOLabs. The PC obtained is between 1.48×10^{-13} and 2.22×10^{-5} and the one of LEOLabs is 3.80×10^{-2}. This also happens because the covariance matrices estimated in PC calculation are different. The standard deviations obtained by the developed method are present in Table 10, which are much larger than the ones used by LEOLabs, in Table 8. The differences found between both tools in the conjunction data are due to the type of data used. The developed collision warning tool uses TLEs and LEOLabs uses data of their own radars.

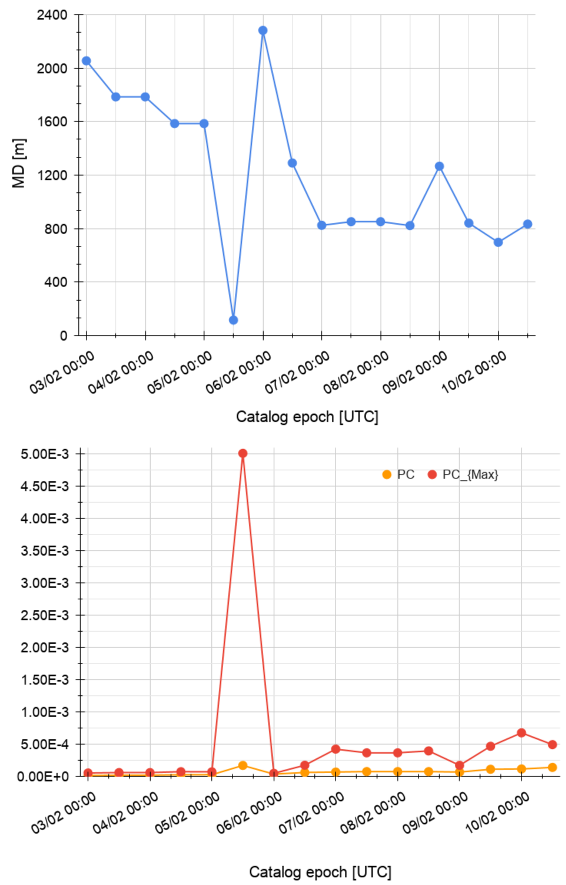
Table 9: Conjuntion data found by the developed collision warning tool.

TCA [UTC]	MD [m]	V. Rel. [km/s]	PC	PC_{max}	k
2020-10-16T00:56:40.761	351	14.659	1.48E-13	2.22E-05	4.82

[2] LEOLabs. This visualization shows our latest information on the event. Published on 2020/10/15. Accessed on 2020/11/24, `https://twitter.com/LeoLabs_Space/status/1316822400131104768/photo/1`



(a) Above, error ellipsoids in the RTN frame and objects positions in the TEME frame. Below, conjunction plane with PDF for PC calculation, for the case of Sec. 4.3.2.



(b) Above, evolution of MD with time. Below, evolution of PC and PC_{max} with time, for the case of Sec. 4.3.3.

Figure 5: Plots obtained for Sec. 4.3.2 and Sec. 4.3.3.

Table 10: Data of the objects involved in the conjunction.

	Primary	Secondary
Norad ID	19826	36123
Type of Object	Payload	Rocket Body
Largest Dimension [m]	2.1	7.5
TLEs used	33	29
σ_x [m]	69	55
σ_y [m]	794	176
σ_z [m]	143	19

4.3.3 Collision between Iridium-33 and Cosmos 2251

In this test, it will be analysed the collision between Iridium-33 and Cosmos 2251 in 2009, in order to have an idea of the PC threshold to consider a conjunction as a high-risk conjunction, as well as study the evolution of the conjunction data with time.

The collision between these satellites happened at 2009/02/10 16:55:59 [UTC] [7]. The TLEs emitted before the collision are available on SpaceTrack, hence, these were retrieved and fed into the developed collision warning tool. Several conjunction analysis were made, two times per day, starting from seven days before the collision, using the more recent catalog at noon and midnight of each day. The TCA, relative velocity, scaling factor (k), MD, PC and PC_{max} were calculated and the evolution with time of the last three is shown in Fig. 5(b).

The TCAs obtained vary between 2009/02/10 16:55:59.662 [UTC] and 2009/02/10 16:55:59.928 [UTC]. The relative velocities vary between 11 646.66 m/s and 11 647.33 m/s, which are very small variations and, in the case of TCA, similar to the one when the collision occurred. Hence, the TCA and the relative velocity are reliable parameters, even for predictions of seven days.

In Fig. 5(b), the MD varies between 117 m and 2282 m. In spite of being possible to identify a descendent behaviour of MD with time, there are some outliers with large and small MDs. Since MD is related to PC, intuitively, when MD decreases, PC increases and that is shown by the outlier of 05/02 12:00, where MD decreases to 117 m, causing a peak in PC and PC_{max}. PC varies between 1.12×10^{-5} and 1.68×10^{-4} and PC_{max} between 4.51×10^{-5} and 5.01×10^{-3}.

The variations of these parameters are due to the different TLEs that were emitted along that week, revealing low consistency for a good conjunction analysis. The scaling factor was always smaller than 1 for each simulation, indicating that more accurate data needs to be used to reduce the uncertainties in position. However, this analysis suggests that even for PCs in the order of 10^{-5}, the conjunctions can be considered as high-risk conjunctions.

5. Conclusions

The initial objective of generating a warning between a chosen object and a threatening object was achieved, as well as all the processes needed to select the dangerous objects, generate covariances and calculate PCs allowing the operator to analyse conjunctions and assess the risk of collision.

The filtering techniques studied were implemented with success, allowing to run the developed code in a reasonable time and predict the ION conjunctions for the next 7 days. The covariance generation process implemented allowed to calculate a PC for conjunction analysis. The implementation of the PC calculation allowed to perform several conjunctions analysis and assess if the conjunction is dangerous or not. The extreme cases studied to test the implementation of PC gave the expected results and the comparison with LEOLabs conjunction revelead the limitations of using TLEs. The detection of the collision between Iridium-33 and Cosmos 2251 allowed to set a PC threshold (10^{-5}) to consider a conjunction has a high risk conjunction.

TLEs are the largest source of inaccuracy in the developed tool. Future work would involve to change the source of data from TLEs to ephemerides in an Special Perturbations Catalog that is provided by SpaceTrack to satellite operators. This will involve to work with orbit determination processes and numerical propagators, that can be much more accurate than TLEs and SGP4.

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