Simplified approach for the estimation of the added resistance of ships in waves

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ABSTRACT:

The increasing problem of climate change has led to the creation of several laws to tighten restrictions on the fuel oil consumption of ships by maritime organizations. Private and public entities have therefore expanded the research being done on the effects between sailing ships and waves and how these phenomena translate to economic benefits. The wave added resistance is one of these phenomena, and it can contribute up to 30% to the total resistance a ship perceives, and consequently on the fuel consumption.

In this dissertation a thorough literature review was conducted to understand and evaluate the methods being used in the field of study of added resistance in waves and in its prediction through numerical simulations and regressions. An experimental database of tank test data was also created to later validate the theoretical and semi-empirical models and applied methodologies.

To test the theories several studies were conducted to obtain a methodology that would correctly predict the added resistance for different ship types and sea conditions. Through the computation of the added resistance with a combination of NTUA and Salvesen's methods, a record of observations of added resistance was made for a variety of sea conditions which is based on container ships specifically.

A linear regression model was developed which presented an adequate fit to the responses in the previously built database. It also, offers a quick and efficient method to predict the added resistance in irregular waves using a minimal amount of ship and ocean wave parameters. Other methods using machine learning algorithms are discussed as well as their applicability and future work is suggested noting the lack of public data regarding tank tests.

Keywords: Added resistance, Ocean Waves, Numerical Simulation, Machine Learning, Linear Regression Modelling

RESUMO:

O agravamento do problema das alterações climáticas tem levado a que as organizações marítimas responsáveis pelo regulamento da indústria tenham criado normas e leis, dando origem a restrições ao nível do consumo de combustível dos navios. Diversas entidades públicas e privadas responsáveis, têm levado a cabo investigações relativas aos efeitos provocados pelos navios enquanto navegam em ondas. Um destes fenómenos, é a resistência provocada pelas ondas, podendo representar 30% do total da resistência que o navio sofre.

Nesta dissertação uma minuciosa revisão bibliográfica foi conduzida para melhor perceber e avaliar os métodos e técnicas que estão a ser usados nesta área de estudo e sobre simulações e previsões numéricas feitas através de algoritmos e regressões. Foram também criadas bases de dados de experiências feitas com modelos de navios em tanques de ondas para validar os métodos teóricos e semi empíricos usados para o cálculo da resistência causada pelas ondas. Através do cálculo da resistência de ondas utilizando as metodologias estudas, foi criada de uma lista de observações da resistência em ondas para todas as condições de ondas e que tem por base navios porta contentores.

Foi desenvolvido um modelo através de regressão linear que apresenta uma correlação adequada às observações registadas. O modelo apresenta ainda uma forma eficaz e rápida de calcular a resistência de um navio, em ondas irregulares, utilizando parâmetros simples e de fácil obtenção, quer ao nível do navio, quer ano nível das ondas. Outros métodos que utilizam algoritmos de otimização são também discutidos, bem como as suas possíveis e futuras aplicabilidades. São também sugeridos métodos para uma futura continuação de trabalho e, também registada uma falta de dados experimentais por parte da comunidade científica, em geral.

Palavras-chave: Resistência Adicional, Ondas Oceânicas, Simulação Numérica, Algoritmos de Otimização, Modelação de Regressão Linear

TABLE OF CONTENTS

LIST OF TABLES:

LIST OF FIGURES:

ACRONYMS:

EEDI	Energy Efficiency Design Index
MEPC	Marine Environment Protection Committee
CFD	Computational Fluid Dynamics
MARIN	Maritime Research Institute of Netherlands
NMRI	National Maritime Research Institute of Japan
UNCTAD	United Nations Conference on Trade and Development
STA-Wave	Sea Trial Analysis-Wave formula
SPA-Wave	Sea Performance Analysis-Wave formula
STA-JIP	Sea Trial Analysis Joint Industry Project
NTUA	National Technical University of Athens
JONSWAP	Joint North Sea Wave Project
ISSC	International Ship Structures Congress
ITTC	International Towing Tank Conference
WADIC	Wave Directional Measurement Calibration
MLR	Multiple Linear Regression
ANN	Artificial Neural Network
IST	Instituto Superior Técnico
CENTEC	Centre for Marine Technology and Ocean Engineering, IST
KRISO	Korea Research Institute for Ships and Ocean Engineering
DTC	Duisburg Test Case
KVLCC2	KRISO Very Large Crude Carrier 2
CAD	Computer-Aided Design
A.P.	Aft peak
F.P.	Front peak
KCS	KRISO Container Ship
SE	Standard Error
RMSE	Root Mean Square Error
EDoF	Error Degrees of Freedom

SYMBOLOGY:

Lpp	Length between perpendiculars
m	Meters
deg	Degree (angular unit)
B	Beam or Breadth of a ship
T	Summer draught of a ship
k_{yy}	Radius of gyration around the y-axis
Cb	Block coefficient of the ship
χ	Wave heading angle
θ	Inclination of the waterline to the centerline of the ship
s	Inclination of the vector tangent to the waterline
$\overline{F_n}$	Normal average force per unit length
r	Density of salt water
g	Gravitational acceleration
z_a	Wave amplitude
ω_0	Circular wave frequency
V	Ship's speed
n_i	Generalized normal directions
$\bar{F}_i$	Mean drift forces (where $\bar{F}_1$ is the added resistance)
Fn	Froude number
η_j	Displacements
R_{aw}	Added resistance in waves
k_w	Wave number
F_j^I	Froude-Krilov exciting force and moment
F_j^D	Diffraction exciting force and moment
R_7	Added resistance in waves
ϕ_0	Incident wave potential
N	Two-dimensional outward unit normal vector

ω_e	Frequency of encounter
b_{ij}	two-dimensional section damping coefficient
R_{AWM}	Added resistance in waves due to motions effects
R_{AWR}	Added resistance in waves due to diffraction effects
λ	Wavelength
Vc	Groupe velocity
T_a	Draught at aft of ship
T_f	Draught at fore of ship
E1	Entrance angle of the ship at the waterline
E2	Exit angle of the ship at the waterline
S_i	Segments of waterline
a_{T^*}	Draft coefficient
T_p	Peak period
Hs	Significant height
χ_m	Mean Wave Direction
w_p	Peak wave frequency
Sw	Wave spectrum
Sw_{1D}	One-dimensional wave spectrum
T_{ep}	Peak period of encounter
w_{ep}	Peak encounter wave frequency
g	JONSWAP spectrum peakedness parameter
Φ_{AW}	Added resistance operator
∇	Volume displacement of ship
y	Model response
x_k	Model regressors
β_k	Regression coefficients
σ	Sample standard deviation
n	Number of samples
tStat	t-statistic test
p	number of coefficients in the model

$\hat{y}_i$	Model predicted response
y_i	Recorded response
$\bar{y}$	Mean of the observed responses
R^2	R-squared
R^2_{adj}	Adjusted R-squared
$R_{aw_{nd}}$	Added resistance in waves non-dimensional
FStat	F-statistic test

1. INTRODUCTION

1.1 Background and motivation

The sustainability of an industry is reliant on the advancement and development of new tools and skills. This constant innovation and research are what makes available the growth and expansion of the field. In ship engineering this is no different and in recent years the research has focused its attention on the growing problem of climate change. This conservation mindset created a push towards a more environmentally conscious industry, in which the shipping industry productivity has seen a subsequent improvement in the operational and financial efficiency of ships.

The world's ever growing population (United Nations, 2019) creates a constant demand for the increase in size and scale of the worldwide supply market. The shipping industry responsible for more than eighty percent of the total share of goods and cargo transported by sea worldwide (UNCTAD, 2020), is then understandably challenged to expand. In order to grow and keep up with the demand and at the same time lower costs, larger operations have to be developed in which bigger ships play a fundamental role. These so called economies of scale are then set up, but the entire industry will face challenges since it must be renewed and updated. With the increase in scale, it is also faced with the increase in operating and managing costs.

Fuel consumption is one of the main expenditures when it comes to shipping, both economically and environmentally. The increase in fuel costs along with climate change constraints, have introduced the need to build ships that consume less fuel. Shipping lines have adapted their operations to mitigate fuel oil consumption along with other measures to decrease the pollution in marine environments. This pollution can be originated from many sources, but the ones that have been the most prevalent over the years are related to exhaust gases, water pollution (ballast water, noise, etc.) and some pollutant cargos or wastes (Clark et al., 1989). Increasing concerns have been raised about the adverse environmental impacts caused by cargo movement in international trade by stakeholders and international government bodies. The waste produced in the shipping processes can put a burden on the environment which can lead to resource depletion. Many shipping companies have taken the initiative to find ways to minimize the environmental harm of their activities while improving their efficiency in order to better protect the marine ecosystem. The problem is set to deteriorate due to the growing of trade´s globalization. It is also important to increase awareness and improve shipping industry´s public perception which can lead to funding and new stakeholders' interest. As such, the marine industry continues to push for the development of ships that can comply with ever stricter regulations and laws that prevent the aggravation of the climate situation.

The development of maritime guidelines and regulations has an important role in enforcing the necessary measures in preventing the stagnation and deterioration of the environment situation. One of these benchmarks that has been implemented in recent years to regulate the new designs fuel consumption is the Energy Efficiency Design Index (EEDI). EEDI was established and made mandatory for new ships at the Marine Environment Protection Committee (MEPC), 62 in July of 2011 (MEPC, 2011). It can be considered one of the most important measures to be implemented since it will be incrementally tightened over the years for each category of ships to promote the development and research being done on the efficiency of machines and equipment on board. The ultimate goal of this measure is to lower the emissions of carbon dioxide to follow global trends in more efficient technology.

With the advent of the EEDI, the maritime industry has searched for ways to optimize every aspect of a ship, from design to operation. In this dissertation, a method to optimize the performance and efficiency of a ship in operation will be analyzed, in particular the speed loss in real sea conditions. Speed loss is derived from the resistance the ship encounters when sailing through waves and winds. The wave aspect will be further detailed in this study. The factor that is the most relevant however to this phenomenon is the hull form. Hull form optimization performance in a design phase can be optimized to make the full project meet EEDI criteria. Therefore, it is important to gather more knowledge about this subject and make models that can be used as a reliable tool to easily find the wave added resistance for a specific ship hull.

When it comes to the study of resistance in ships, extensive studies have been made which take into account the hydrodynamic capacities of the ship sailing in water. However, in the past, due to the limitations in models, specifically when it is necessary to take into account the navigation in waves, only calm water conditions were computed. Nowadays, the full range of drafts and speeds must be considered in real operational profiles. The detail in these studies must give reliable results to prove that the final ship will always be able to meet the criteria for fuel consumption.

The need for a dependable method to calculate ship's speed loss at real sea conditions, has contributed to the growth of the research in the field of added resistance in waves. This is obvious when looking at the increase in number of publications on this topic in Figure 1, especially the jump after 2011 after EEDI was implemented.

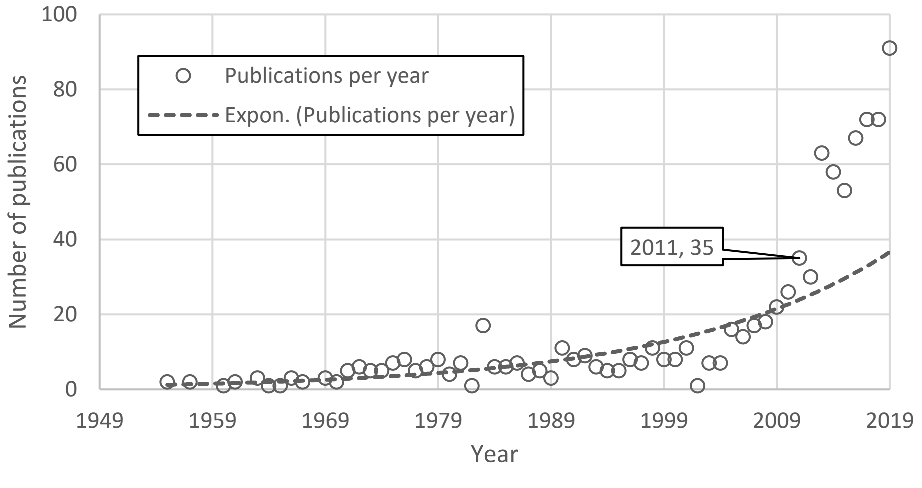

Figure 1 - Number of publications per year according to Google Scholar metrics for the keywords "added resistance of ship in waves" (visited on December 4th, 2020).

The complexity of this problem along with the increase in publications, suggests that a plateau has not yet been reached for the efficiency of ships operating in real conditions and that more research needs to be conducted in order to get a better grasp of the reality.

Following the extensive research that has been done over the years, this thesis will study and bring together some of the more accepted models by the scientific community. This will allow for a full range of scenarios to be predicted in accordance to ship specific parameters.

1.2 Objectives

The main objective of this thesis will be the creation of a meta-model that will evaluate the added resistance of a ship in real sea conditions. This meta-model must be comprised of different theories in order to fulfill the goal of calculating the added resistance for a full spectrum of wavelengths and ship characteristics. In order for this objective to be accomplished, some preceding investigation must be prepared to better understand the subject and the state of the art. This is important to gain the knowledge of how to apply each method for each specific condition.

The first step is then the study of the available techniques to calculate the added resistance in head and oblique waves. As shown in Figure 1, the availability of numerous publicly available research on this topic indicates that there is a plethora of methods to apply in different conditions. This initial study will help clarify the applicability of each method and the history, development, and new findings. Also, the compilation of experimental data open to the public, will make it possible to create a database to validate the end results.

After this preliminary study, it is important to do a comparison analysis of the different methodologies and evaluation of the benefits and weaknesses. Having the insight on how these techniques produce results and the reality of outputs for several different ships is of extreme importance to have a discussion on what the final model will aim to achieve and on what can be left behind.

The model chosen after a balanced compromise of pros and cons will compute results for the added resistance in waves. To validate the model, it is necessary to compare its results with available experimental data. The objective of this validation is to have proof that the model corresponds to a real case (ideally done with full scale experimental data) to grant scientific support for the thesis.

Having validated the model, the implementation of a code to calculate the added resistance in different sea-states should be made. As such, the estimation of the contribution of the wave added resistance on the total resistance can be computed. This is useful for the identification of such proportion depending on the encountered sea-state, which can be expressed as the coefficient of added resistance, useful in calculating the EEDI and other fuel consumption factors.

Finally, the application of the method for different ships enables the creation of a database that contains the characteristics of different ships, of different sea states and the added resistance for each case. This database creates the opportunity to define simplified equations for the estimation of the added resistance on a specific sea-state derived from the statistical analysis of the previous results.

1.3 Structure of the thesis

The thesis is organized in five chapters and one appendix. The layout follows closely the sequence of work that was done in order to enable the reader to get a better understanding of the subject and the steps taken to achieve the results.

Chapter 1 is the introduction of the topic to be discussed, the related background and the goals and structure of the work. This chapter will show the evolution of research that was done on ship added resistance in waves and how the current work is relevant in this context.

Chapter 2 contains the literature review. In this chapter the state of the art will be described for the topic of the added resistance in waves in detail. All the theories regarding the topic and the different approaches of solving and simplifying the problem of the added resistance in waves will be described.

Chapter 3 details the methodology and describes the methods that were deemed relevant to the current application and tests how these behave under different conditions. The preliminary results in this section will help predict which methods to be used to obtain the results.

Chapter 4 details the case study along with the results and a discussion. The final model is presented, and its advantages and disadvantages are presented in a clear manner. A validation of the model is also done as proof of concept through the scaling of three hulls. Finally, a different option of regression was done to establish the idea that different models have different uses.

Chapter 5 is the final chapter, and it details the conclusions that were reached through this work. It summarizes if the main objectives were achieved and also describes how the work can be further developed and improved for other applications.

2. LITERATURE REVIEW

2.1 Added resistance of a ship advancing in waves

A ship sailing in a real seaway is subject to external forces that cause it to have an involuntary speed loss. This speed loss is considered as the reduction in the power output of the ship, and therefore its performance. In other words, the difference of speeds when the ship is sailing in calm waters and when it is sailing in a real marine environment. This change is mainly attributed to the resistance from wind and waves. Preliminary studies of a ship hull take into account the thrust required to push the ship in calm waters. An allowance is then added to make sure there is enough power to counteract the sea effects, the "Sea Margin", which is usually around 15-30% of the thrust necessary in calm water. This value is usually obtained from observation of different ships on the same route (Pérez Arribas, 2007). A more realistic value of the Sea Margin may be determined from the computation of the added resistance. Since the calm water resistance and the wind resistance are nowadays easily estimated from established empirical formulas (Fujiwara et al., 2009; Holtrop & Mennen, 1982) and along with the recent economic and environmental emphasis on building more efficient ships, research focus has been shifted to the added resistance in waves.

Knowing that the added resistance in waves comes from the differential in energy expenditure when compared to calm water, then it is important to understand how this loss is originated. When ocean waves interact with a body there is a transference of energy to the adjacent water. From this transference there are three main components that can be considered from a dissipation of energy standpoint.

The first are the viscous effects related to the energy consumed by the viscous friction, which is the smallest contribution to the added resistance (Faltinsen et al., 1981). Therefore, most procedures of calculating the added resistance ignore these effects and are based on potential theory. This is important because the added resistance becomes a non-viscous phenomenon and allows its scalability from tank experiments to full-scale scenarios by Froude's law. According to this the real ship's added resistance is computed by the cube of the scale of the experimental added resistance (Strom-Tejsen et al., 1984). The second consideration is the diffraction effect, which is more relevant in the short wave region. The diffraction induced added resistance occurs when the ship hull reflects the incident wave. The third and final component to consider is called the drift force, produced from the ship radiated waves. Dominating for the long wave region where the ship motions are larger this component makes for the largest contribution to the added resistance (Salvesen, 1978).

The interaction between the ship radiated waves, the reflected waves and the viscous effects could effectively be occurring at one certain instant of time. For this reason, it is difficult to discriminate each component as an individual part when observing the occurrence. It was found by Maruo that these interactions between each element of the added resistance are simply additive. This means that in order to compute the total of the added resistance, a sum of all of the parts is necessary. This is what is called the principle of superposition (Maruo, 1957).

Maruo also deduced that the additional resistance is proportional to the square of the wave height. His procedure to compute the added resistance and from other authors are based upon the linear wave induced motions and loads which are a first order problem approximation. Taking the mean of the longitudinal second order force will allow the calculation of the added resistance. This suggests that the added resistance must be proportional to the square of the wave amplitude and is non-linear. The assumption was tested several times and ultimately validated through experimental results (Gerritsma & Beukelman, 1972; Strom-Tejsen et al., 1984). This approximation is obviously valid in the region of moderate wave steepness and should be re-evaluated when considering steeper or breaking waves.

Having these considerations and deductions in mind, it is easier to understand the methods that have been applied to compute the problem of added resistance throughout time. These solutions will be described according to the chronological timeline of the progress of the research in this area as well as with the different theories.

2.2 Methods to calculate added resistance in regular waves

Extensive research of the added resistance has been done to understand the different methods to predict the loss of power of a ship sailing in waves and in which region of wavelengths each method is best employed. The research usually focuses on numerical studies based on potential flow theory or Computational Fluid Dynamics (CFD) of full-scale ships and validates these results with the experimental tank tests. However, CFD experiments and sophisticated numerical tools have yet to reach full maturity, which has been observed in recent benchmark performance assessments (Shigunov et al., 2018). Since CFD methods are also usually more expensive and time consuming, other methods based on slender-body theory or 3D panel methods have also been developed to estimate the added resistance.

As mentioned on the previous section, the loss of energy from the added resistance can be originated from the interaction between waves and ship. It is therefore necessary to compute the energy transmitted to the ship surrounding waters. The most established methods to analytically calculate this transference are derived from the theory of potential flow and are mainly based on either computing the energy of the wave in the far field or in the near field regions.

2.2.1 Theoretical formulations

The first attempts at calculating the added resistance in waves were made by Havelock who initially calculated the mean pressure of waves on a fixed object by diffraction theory (Havelock, 1940). From his report he concluded that the effect of diffraction only contributes to a small portion to the total of the added resistance and that the ship motions would be integral in the solution of this problem. On this assumption, Havelock would later compute the added resistance based on the integration of the longitudinal pressure over the wetted part of the oscillating hull (Havelock, 1942) using the Froude-Krylov approach to calculate hull pressures. This article would allow the first estimation in the correct order of magnitude of the drifting force of a ship sailing in head waves, since the experimental data at the time was insufficient in this field to validate the results. It would also contribute to the understanding of the relationship between the phase of the ship motions and the exciting waves and how the added resistance is a product of the energy expenditure to keep ship motions in a constant phase with the waves through the radiated waves and the fluid friction. The final aspect of importance from Havelock's work is the deduction that the maximum of the added resistance is seen in the resonance region of the vertical motions which directly translates to the importance of building a ship with good seakeeping abilities.

The integrated pressure method would also later be studied by Boese who relied on strip theory to further develop this method. In strip theory singularities are used to describe the entire hull form in sections as well as the velocity potential of the wave field. Since in strip theory one of the conditions is that there are not any longitudinal effects and therefore there is no longitudinal drift force to compute directly and a mean value has to be obtained for each strip. This is what Boese calculated to get the added resistance through the integration of this mean force (caused by the heave and pitch motions and waves) over the waterplane area (Boese, 1970).

Another important method to the analytical potential flow problem of the added resistance was developed by Maruo (Maruo, 1957). His research on added resistance focused on the principal of momentum and energy conservation based on a far field method. By employing the same method used for strip theories nowadays, Maruo evaluated the wave energy and momentum flux generated by a ship around a control volume of infinite radius surrounding the ship. By dividing the velocity potential into incident waves, radiated waves and diffracted waves he obtained a boundary value problem which solution lies in the calculation of the radiated and diffracted potential (since the incident wave potential is already known) and then calculating the forces acting on the ship hull. Through simplifications such as the slender-body assumption the problem became solvable however later other authors would further develop it. The Kochin function included in Maruo's formulation also has a great impact on the prediction of the solution, as it is composed of the radiated waves and the scattering waves. Joosen would expand Maruo's formula and concluded that the drift force is dominated by the potential of the radiated waves, except at the small wave region where the wave diffraction becomes more relevant (Söding & Shigunov, 2015). Another, development in Maruo's formula came from Newman who also analyzed the stability of the ship in oblique waves by computing the wave-induced steady yaw moment (Newman, 1967).

The final method of interest when considering analytical solutions to the added resistance numerical problem is the radiated energy method. This is a far field method which computes the energy necessary to create waves when the ship oscillates for one period, in head seas. Gerritsma and Beukelman developed this method with a base on strip theory to describe the form of the sections of the hull (Gerritsma & Beukelman, 1972). As other authors had previously concluded, it can be taken from this paper that the influence of surge can be neglected since the heave and pitch motions effects dominate the added resistance.

As mentioned in the methods before strip theory was becoming the most prevalent method to calculate the added resistance. This was also seen during the 1970s when Salvensen, Tuck and Faltinsen introduced a method based on linear 2-D strip theory which was more reliable (Salvesen et al., 1970). This method calculates the wave induced motions based on a generalized Haskind relation to evaluate the wave exciting moments and forces, which does not require the solution of the diffraction problem and give a more reliable result. According to (Salvesen, 1978) most formulas gave a 30% difference for the maximum of the added resistance. Salvesen would continue to develop this formula and found that it generally gave a better accuracy with the experimental results, for a wider range of ship forms and speeds.

With the development of several methods that focused on the main contribution to the added resistance, the radiation problem, it would become necessary to solve the diffraction problem to find the added resistance for all wavelengths. In the low wavelength region, where the ship motions effect can be neglected, not much research or experiments were being performed due to its complexity. Faltinsen was one of the main authors who contributed to the solution of this problem (Faltinsen et al., 1981). The development of an asymptotic theory for short waves based on a near-field method, was an update to previous theories as it introduced the forward speed effect. In Faltinsen's theory the ship hull is replaced with an infinite wall that has the cross-section of the original waterline since the waves close to the hull have small wavelengths. For this reason, Faltinsen's formulation is a very appealing method to use since it is a very simple physical model and only requires the shape of the waterline. This theory proved to give good agreement for blunt ships and low Froude numbers. However, when compared with experiments with fine hull forms and high Froude numbers the agreement was not so satisfactory since the method was not developed for such cases. Later Faltinsen made an attempt to develop a new formula that would be acceptable in these parameters and compared the numerical results with the experimental tests of a cargo ship (Faltinsen, 1983). However, the results were once again not satisfactory. Later on, other authors would further develop Faltinsen's formula to make it applicable in more cases. Ohkusu proposed a new formula that accounted for the interaction of the incident waves with the stationary flow that surrounds the ship hull (Ohkusu, 1986). In more recent years the work of Yang et al. is of interest since it also the tackles two of the most commonly accepted weaknesses of Faltinsen's asymptotic formulation (Yang et al., 2018). These are the effects of local steady velocity and the shape of the hull above still water level. In addition to these the finite draught of ships is also added to the new formulation. In terms of the forward speed effect, which accounts for a significant contribution to the discrepancy in Faltinsen's method, it has also more recently been the subject of focus of researchers (Kashiwagi et al., 2010).

2.2.2 Semi-empirical formulations

It was also during the 1970s that empirical formulations started to be developed and published. With the advent of new theories and an ever-growing publicly available database of experimental tests, it would become easier to derive formulas from these datasets. One of the most recognized semi-empirical formulas is the one developed by Fujii and Takahashi. In their formulation of the added resistance the ship motions and the reflection at the bow are considered and it is compared with their own experimental results for the S175 ship (Fujii & Takahashi, 1975). The reflection at the bow was developed from the solution of Ursel who developed the formula for waves reflecting on a vertical wall of a certain depth from the free surface (Ursell, 1947). This draft coefficient has become an important factor in semi-empirical formulas and was also studied by Kuroda et al. (Kuroda et al., 2008) who also followed this approach of using Bessel functions. Later on, Liu et al. (Liu et al., 2016) used the reasoning of Kwon (Kwon et al., 1981) who adapted the exponential decay of Smith to the wave amplitude and therefore to the decay of the added resistance with the square of the exponential function. The method of Fujii and Takahashi was developed for the National Maritime Research Institute of Japan and is therefore commonly mentioned as the NMRI formula.

At the time of development of the formula by Fujii and Takhashi, Jinkine and Ferdinand also made their contribution to the solution of the added resistance in long waves (Jinkine & Ferdinande, 1974). Their approach is fully empirical and is intended to be valid for fast cargo ships. It is dependent on a single factor related to the ship form which must be obtained experimentally. From this study it is observed that the frequency where maximum of the added resistance is located, is solely a function of the Froude number and of the radius of gyration around the y-axis and not on any ship form parameter. Although the authors mention their method is valid for the entire range of wavelengths, only the motion induced added resistance is calculated.

The formulations of Fujii and Jinkine have been established for some time and have more recently been combined to obtain a formulation that can cover all wave lengths. The Maritime Research Institute of Netherlands (MARIN) worked on this type of formulation and made it approved for the evaluation of the EEDI (ITTC, 2014). This led to the proposal of the formulas known as STA-Wave1 and STA-Wave2. The first computes the added resistance based on reflection, in irregular head waves. The former on the other hand is a more general formula as it includes the motion-induced added resistance component. Later MARIN found it important to find the added resistance in a real service condition and therefore all wave directions should also be considered. This new formula, SPA-WAVE, was developed within the Sea Trial Analysis Joint Industry Project (STA-JIP) and is based on wave added thrust and is no longer modeled after Fujii and Jinkine formulations. It was tested extensively and was validated with 41 different ships (Grin, 2014).

Another example of the update on the formulations of Fujii and Jinkine came from Lang and Mao (Lang & Mao, 2020). With each update on these formulas their formats become more flexible and accurate to predict the speed loss in any sailing condition. With the research of Lang and Mao, a correction factor was proposed for the significant wave height which considers the effect of coupled motions in high waves to predict the added resistance in irregular waves.

To conclude the subject of the development of semi-empirical formulations over the years, it is also of significance to mention the research that Liu and Papanikolaou have been developing and the NTUA method. This approach was initially developed using Maruo's far field theory combined with Faltinsen's approach (Liu et al., 2011) and would over the years be reexamined and was recently updated with the development of the formulas of Fujii and Jinkine (Liu & Papanikolaou, 2020). It has offered a fast and easy approach to the estimation of the added resistance in the design stage for engineers who want to predict the added resistance of a ship sailing in a representative seaway (Liu & Papanikolaou, 2017). When compared to STAWAVE2 it shows an improved agreement with model experiments, except for some ships and in very small wavelengths.

Although rich experimental databases are nowadays possible to compile due to publicly available results, tank tests measurements are still limited. The number of experiments with oblique waves is still small when compared to head waves. Also, due to the difficulty of measuring added resistance in short waves these measurements are sometimes not reliable or simply not available. In order to conduct a study on added resistance it is therefore necessary to compile some results of experimental tests in order to assess how well the numerical results match these. A review of some public results was done in order to create a database that can be used to validate analytical results (Table 1).

Table 1 - Public published available model experiments.

Ship type	**Name**	**Lpp [m]**	**Froude number**	**Wave Heading [deg]**	**Reference**
Container ship	S175	175	0.15 / 0.20 / 0.25 / 0.275 / 0.30	180 / 150 / 120 / 90 / 60 / 30 / 0	(Fujii & Takahashi, 1975; Kim et al., 2017; Nakamura & Naito, 1979)
Container ship	DTC	355	0.0 / 0.052 / 0.139	180 / 150 / 120 / 90 / 60 / 30 / 0	(El Moctar et al., 2012; Shigunov et al., 2018; Sprenger et al., 2017)
Container ship	KCS	230	0.26	180 / 135 / 90 / 45 / 0	(Sadat-Hosseini et al., 2015; Simonsen et al., 2013; Yasukawa et al., 2019)
Crude Carrier	KVLCC2	320	0.0 / 0.05 / 0.09 / 0.142	180 / 150 / 120 / 90 / 60 / 30 / 0	(Guo & Steen, 2011; Park et al., 2016; Shigunov et al., 2018; Sprenger et al., 2017)

The necessity of validation of numerical results against full scale measurements in actual sea conditions is also an important point to take into consideration. The deficiency in openly accessible results of real ships causes the research in this are to have some gaps which cannot be substantiated.

2.3 Added resistance in irregular waves

Although research focus on finding rational models for regular waves, actual sea states are comprised of a set of numerous regular waves with different parameters. Irregular sea states are in other words, the superposition of waves with different wavelengths, amplitudes, and directions. Therefore, the same can be applied to the added resistance of a ship in waves. To calculate the total of the added resistance, each component must be calculated first and then, according to the principle of superposition, they can be combined to find the total. The combination of a set of regular waves is what is called a wave spectrum. Wave spectra is the representation of the distribution of wave energy at a certain point in the ocean. The waves of different heights, periods and directions create a set which can produce a certain amount of energy and this is what is described in a spectrum.

This energy will obviously differ according to different climates and it is important to establish a relevant connection between spectra and the research being conducted. The system must be analyzed for the prediction of responses on the random seas it will operate in. For this reason, several experiments and extensive data acquisition has been done to collect and compute spectra for different parts of the ocean, including commonly operated ship routes and offshore rigs locations.

This information on random seas must then be compiled into a formulation which can be useful to predict wave-induced forces, motions and bending moments. The development of wave spectra formulations is a diverse field and some of the earliest proposals include the one of Neumann, who developed his formulation based on the visual observation of wave data (Neumann, 1953). Later the advent of better wave recorders allowed the development of the Pierson-Moskowitz spectral formulation, which measured data in the North Atlantic on board of weather ships (Pierson Jr & Moskowitz, 1964). Another relevant spectrum is the formulation which has its basis on the measurement program known as the Joint North Sea Wave Project (JONSWAP). This spectrum is the product of the Pierson-Moskowitz spectrum with a parameter called the peak enhancement factor (Hasselmann et al., 1973). This parameter regulates the maximum spectral energy density and is a random variable.

As mentioned before, spectra must be applied in accordance with the purpose of research. It was observed that for the case of the calculation of wave loads and ship motions, it is recommended by the International Ship Structures Congress (ISSC) and by the International Towing Tank Conference (ITTC) to use a spectrum which represents fully developed seas (Guedes Soares, 1984). The ISSC spectrum is an example of this, as it is based on the Pierson-Moskowitz spectrum and is a more peaked spectrum to represent rising or falling seas and as well as fully developed seas (ISSC, 1964).

Most studies of added resistance in waves, however, tend to simplify the problem by disregarding the direction of waves in a spectrum. The assumption that wave energy will mostly propagate in the same direction as the wind is not comprehensive enough, as the wave energy usually spreads over various directions, even if the majority of it does in fact propagate in the same direction as the wind (Ochi, 1997). It is therefore necessary to introduce the concept of directional spectrum in order to make a more accurate assumption of irregular waves. The spreading status of energy has also been the focus of some international projects such as the Wave Directional Measurement Calibration (WADIC) project. With these, data formulations of spread functions were made to find the directional spectrum.

The procedure to calculate the added resistance in waves is now a simple calculation of the added resistance for a wave with a given frequency and direction which is integrated for the entire spectrum, chosen according to the conditions that the system will operate in. Finding the added resistance in waves allows to find the power necessary the ship will need to sail in.

2.4 Regression analysis and artificial neural networks

The added resistance of ships is a problem of great difficulty to solve. The use of numerical methods has been the constant solution since a realistic physical model has proved too difficult to conjecture. A theoretical correct solution would include many inputs related to the vessel and ocean specific characteristics (Rawson & Tupper, 2001). Since there is no linear solution as of yet, the use of semi-empirical formulas has proved to be a reliable method of finding an estimate for the added resistance response given only a limited number of inputs. This is especially useful for engineers who need to get a result for the added resistance in an efficient fashion.

However, with the ever-increasing public database of tank model tests, attention has been diverted to models that instead of relying on the theory are based on regressions of the experimental data. The study of regressions has become more relevant with the creation of modern technologies that make accessible this kind of computing capabilities. The creation of regression based models through machine learning and artificial intelligence show signs of being highly accurate inside a broad spectrum of ship hull forms and waves (Cepowski, 2007).

The simplest form of creating this regression could be a logistic regression. A logistic regression analysis is commonly used when the dependent variable is binary, and an association must be evaluated with another independent variable. Therefore, this method produces results which have lower variances. After this method, the multiple linear regression (MLR) method would provide a more significant result in the estimation of the added resistance. Whereas the previous method could only compute a binary output, MLR methods allows the identification of the relationship between two continuous variables (Hellevik, 2009). For this reason, MLR methods could also be helpful in understanding effects that are undeveloped from theory.

The application of MLR methods to the added resistance is a relatively new concept. Nevertheless, some models are available and it is concluded that simple linear models are not so precise, and to correct this, non-linear variables should be added to the model (Alexandersson, 2009; Cepowski, 2018). Shortcuts and practical methods should be used in order to find models that are dependable.

More advanced methods to find the added resistance in waves, such as with the assistance of neural networks, have also been in the focus of recent studies. Although the use of MLR methods is clearer and concise, artificial neural networks (ANN) provide a higher level of confidence (Cepowski, 2018, 2020). Neural networks usually present models that rely in very large numbers of coefficients which will present good correlation with the input data. Simpler models derived from tank test data have been implemented and present an acceptable degree of pertinence in a real engineering scenario or in a preliminary design stage when computing EEDI for example (Cepowski, 2020).

Scientific research shows signs of underdevelopment in the creation and use of meta-models to compute the added resistance in real operating conditions based on real seas. This introduces an opportunity to explore how such a method could be established with a basis on theory and empirical methods. Building a model that has a basis on established theoretical and semi-empirical methods to create a database rather than experimental data, could also provide insight into the problem.

3. METHODOLOGY

In this chapter the methodology that was adopted will be explained in detail, using numerical formulas as a support. The numerical simulations done in this chapter were based on the added resistance methods from the references according to the applicability of each one and on the necessity to correctly predict the added resistance in the full range of wavelengths and ship characteristics.

For this reason, the numerical simulation was developed having its basis on an earlier implementation developed at CENTEC. The adaption of this proprietary code is necessary for future work on the topic of added resistance in waves and to correctly build a model to calculate the added resistance in waves. The development of the code was based on three methods to calculate the added resistance: Salvesen's far field method, Faltinsen's asymptotic method and Liu and Papanikolaou's semi-empirical method. To compute the real case scenario of the added resistance in waves, the calculation in irregular waves is done using a spectral analysis of the irregular waves.

Finally, to complete the main objective of the dissertation which is the regression to obtain the meta-model will also be clarified according to the theory described by the computer implementation in MATLAB.

The methodology to create and improve the final model will be explained as well as the relevant input and output files that are necessary for the successful handling of the code and of the meta-model to compute the added resistance.

3.1 Description of the main inputs for the calculation of the added resistance in regular waves

The setup of any numerical simulation should start with the clear definition of the main inputs which will be used to compute the final results. This is important since these variables and parameters will be the foundation of the final results. In the sea environment, both the ship and the sea waves have properties that should be detailed to the necessary extent of the desired final approximation.

3.1.1 Ship hydrostatic parameters

In order to simplify and validate the final results, a decision was made to choose ships of the same type to conduct the simulation. The chosen ship type was container ships. These vessels tend to use more slender hull forms and operate at higher velocities. Also, it was important to consider the public availability of experimental data which is higher for containerships (Liu & Papanikolaou, 2020). Three containerships are used, based on the availability as well as the range of different hull shapes and hull characteristics inside this category.

These three hulls are the S175 containership, the KRISO containership and the Duisburg Test Case (DTC) containership. In addition to these containerships, a crude carrier is also defined in order to benchmark some of the methods. For each hull, three input files are created which contain the calculation settings (to calculate the added resistance in regular waves), the hull data and finally the table of offsets for each hull.

The calculation settings include the ship's summer draught in meters, the trim in meters (positive when fore draft is greater than aft draft), the ship's speed in knots, the metacentric height in meters and other parameters related to ship and wave characteristics. As for the ship data, several variables are described to compute the added resistance in calm waters. The relevant variables for this application, however, are the length between perpendiculars in meters, the beam in meters and the radius of gyration around the y-axis.

Some hydrostatic ship parameters must then be known for the calculation of the added resistance in regular waves. These variables will be used as an input to create the database which will be applied to create the meta-model. These parameters take into account ships that are designed for service conditions, meaning that the added resistance will be calculated for a real operation situation. Taking this into account for the three hulls that were chosen, the parameters necessary for this computation are listed on Table 2.

Table 2 – Ship hydrostatic properties.

Parameter	Definition	S175	KCS	DTC	KVLCC2
L_{pp} [m]	Length between perpendiculars	175	230	355	320
B [m]	Beam	25.4	32.2	51	58
T [m]	Service draught	9.5	10.8	14.5	20.8
k_{yy}	Radius of gyration around y-axis	43.75	57.5	88.75	80
Cb	Block coefficient	0.5716	0.651	0.664	0.8098

3.1.2 Ship hull definition

The most intricate and equally important step in the definition of the ship hull, however, would be the definition of a table of offsets for each hull. The table of offsets indicates the spatial coordinates of the points represented in three dimensions. Two methods were employed to get these coordinates and correctly define the hull. The first method was straightforward and was simply through mapping the lines plan of each of the hulls. The lines plan for the chosen ships are shown in Figure 2, Figure 3 and Figure 4.

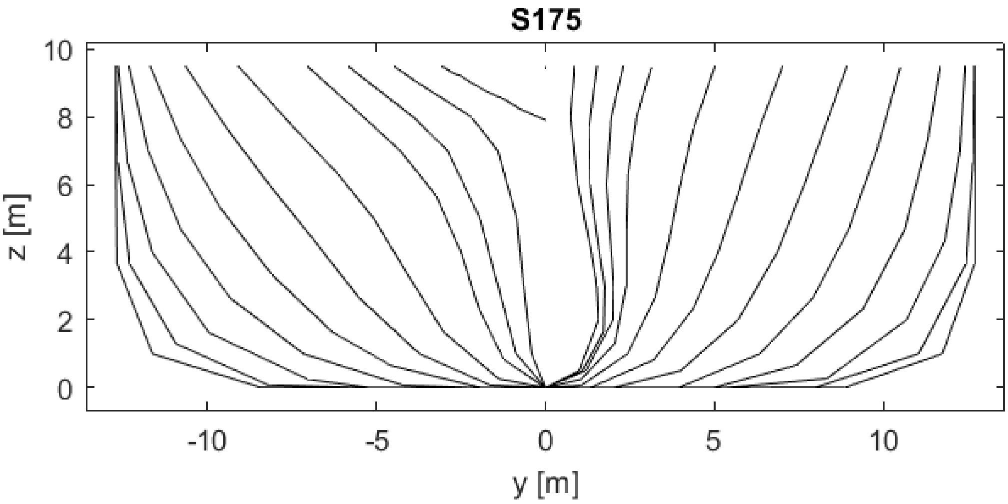

Figure 2 - Lines plan of the S175 ship.

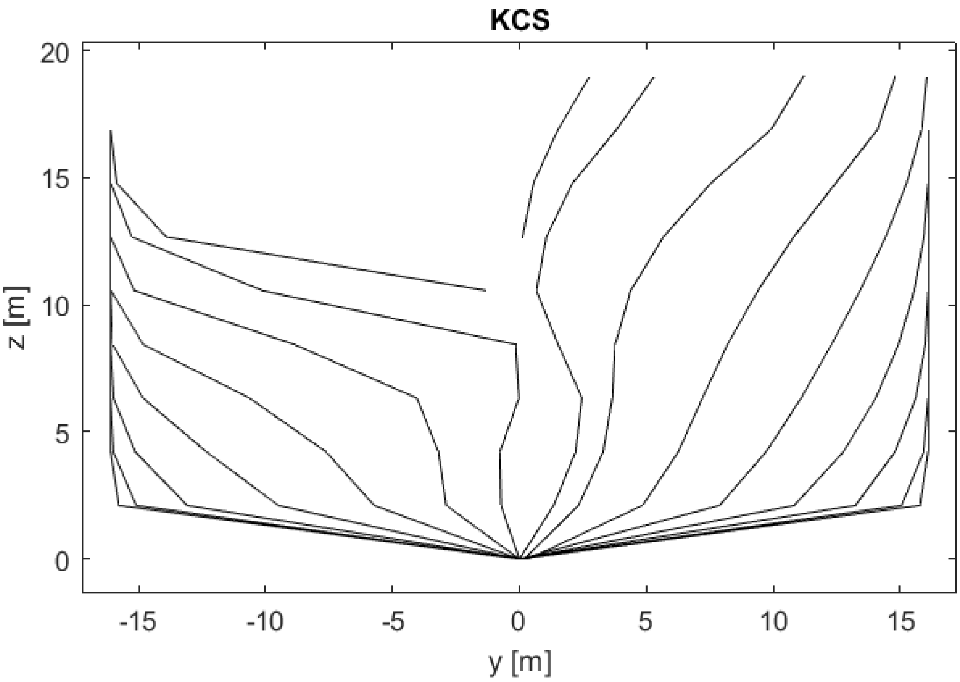

Figure 3 - Lines plan of the KCS ship.

The shapes represented in these figures were used for all the computations. Their definition was mostly performed with the assistance of Computer Aided Design (CAD) software, specifically in this case Rhinoceros 3D version 6. These hulls are commonly available in a 3d CAD format, so the process to get the points with the desired accuracy can be optimized in this sense through the marking of the waterline, buttocks, and section planes. The intersection of these planes defines points which are used to create the offsets table for the input file.

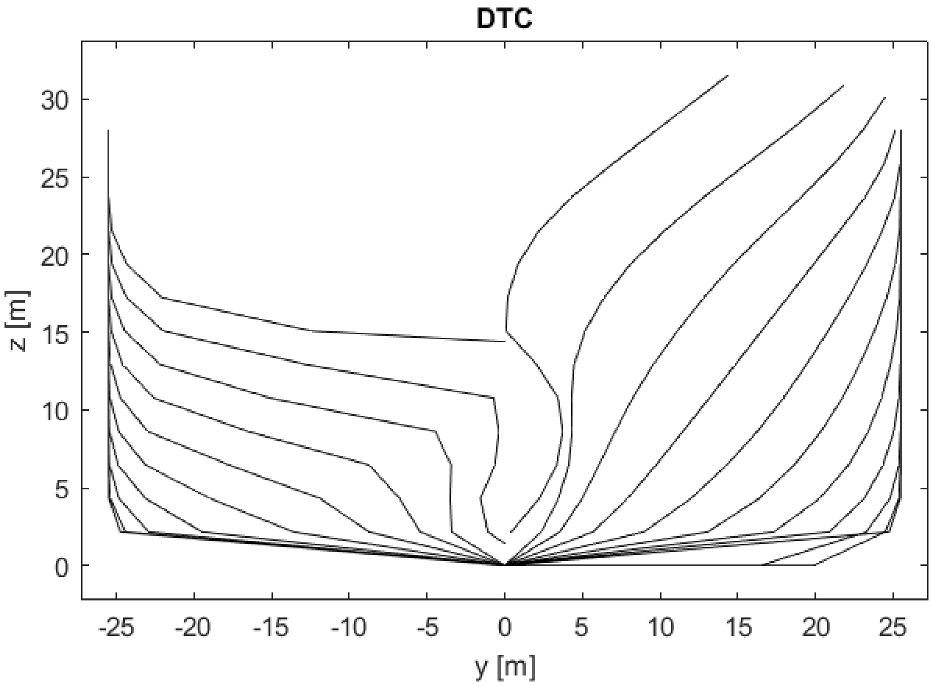

Figure 4 - Lines plan of the DTC ship.

3.2 Faltinsen's asymptotic formulation for added resistance

The study of hull hydrodynamics led to the study of the pressure of water waves on fixed objects in the nineteen forties by Havelock (Havelock, 1940), which showed the similarity of the results in a diffraction problem when considering a wall with vertical sides. The conventional theory of direct pressure integration would then be developed to propose a general theory to predict the added resistance of a ship in ocean waves taking into consideration the ship motions and energy conservation (Gerritsma & Beukelman, 1972; Maruo, 1957).

However, this theory was proved to be difficult to solve in the region of low wavelengths. The study of limiting behavior in wavelengths led to the research on asymptotic low wavelengths most notably by Faltinsen (Faltinsen et al., 1981), whose formula is an improvement on previous research since it includes the forward speed effect and is applicable to fuller hull shapes and all wave directions. It did not however, produce true results for finer hull shapes and high Froude numbers.

3.2.1 Theory

Faltinsen's formulation for the added resistance in the asymptotic low wavelength region takes into account regular waves that are incident on a ship. Knowing that the wavelength is small, it can be considered that it is much smaller than the draught of the ship, meaning only the region of the ship close to the waterline will influence the flow field and that there is only a small change in the waterline area along the wavelength. It can also be said that the small wave excitation forces on the ship will indicate that it is possible to neglect the effects of the ship motions induced by the waves, on the added resistance. Factoring these considerations, it is possible to replace the ship by an infinitely vertical and static cylinder which has the same cross section as the waterline of the ship. The diffraction problem then becomes similar to that of a wave hitting a vertical wall.

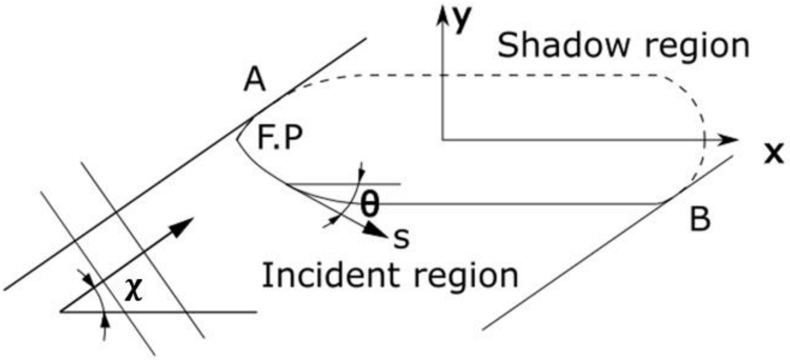

Figure 5 - Coordinates used in Faltinsen's formulation.

The normal average force per unit length on the wall, $\overline{F_n}$, is taken as:

$$\overline{F_n} \sim \frac{1}{2}\rho\, g\, \zeta_a^2 \left\{\sin^2(\theta+\chi)\frac{2\omega_0 V}{g}(1-\cos\theta\cos(\theta+\chi))\right\} \tag{1}$$

To then calculate the mean drift force components and yaw moment an integration is done along the region of the ship's waterline which the wave is directly incident on, the incident region as defined on Figure 5. This integration is given as:

$$\bar{F}_i = \int_L \overline{F_n} n_i dl \tag{2}$$

In the case of the added resistance, which should be equal to the mean drift force in the direction of the x-axis, i is taken as 1, and $n_1 = \sin\theta$. The motions of the ship are represented in Figure 9, and the correct coordinates used in this method in Figure 5.

3.2.2 Implementation in programming language

The implementation of this formulation was written in the programming code language MATLAB. This software allowed the creation of a script to facilitate the calculation of the added resistance for a different number of wave headings in a significant wavelength region.

The code starts by establishing the different variables necessary, and by calling for a function that will plot the waterline for the desired draught, from the table of offsets of the selected ship. The function ("waterline.m") does a linear interpolation between the waterlines that are above and below the draught that the user requests. It also adds a point in the stern transom to facilitate some calculations later.

The script then will enter a loop to calculate the added resistance for each wave heading and for each l/L. In this *for* loop, it starts by defining the region where the integration will happen, the incident region.

This is done by calling a function (“shadow_region.m”), that simply finds the inclination of the waterline to the centerline of the ship, θ, and then enters a restriction where if the $\sin(\theta + \chi) > 0$ it means that it is included in the incident region and all the other points of the waterline are then deleted. This agrees to the conclusions of Sakamoto and Baba (Sakamoto & Baba, 1986). The points are then ordered in a clockwise manner to then do the integration described in the theory section. This allows to finally calculate the added resistance for all the headings necessary.

3.2.3 Preliminary results

The results obtained from the code were compared to results of implementations by other published authors (Liu et al., 2015; Vitali et al., 2016) for wave headings in the region of 180º (head seas) to 90º (beam seas). This region of wave propagation direction showed very similar results to those published, as it is shown in Figure 6. Some small variations are possible due to the definition of the waterline not being similar to other authors in this implementation.

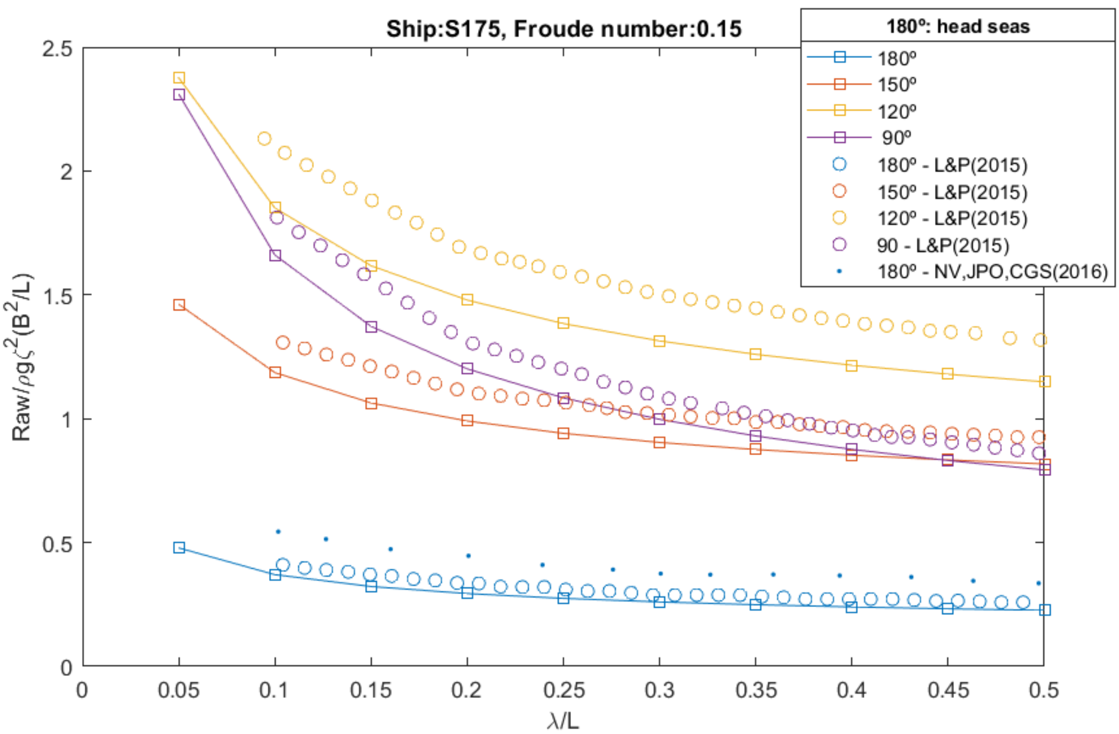

Figure 6 - Added resistance in short waves of S175 at Fn=0.15.

The results from wave headings ranging from beam seas to following seas were not plausible. There is also a limitation in comparing these results because the few data points publicly available are not so reliable since experimental procedures in low wavelength regions are quite difficult to accurately predict (Fujii & Takahashi, 1975). Therefore, the results that the formula produces for following waves are not acceptable to use as a basis of the model. The results are shown in Figure 7, on the beginning of the next page.

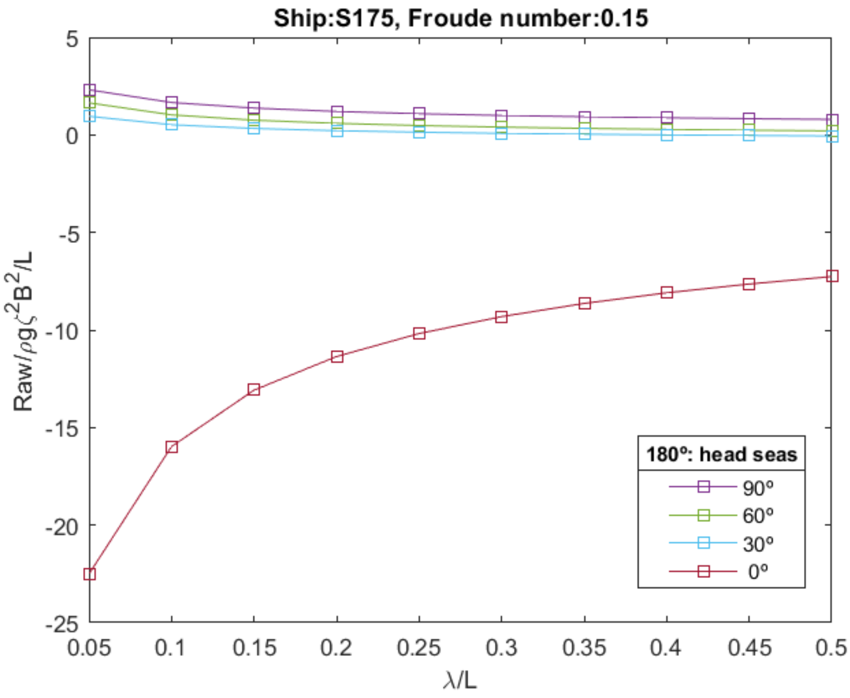

Figure 7 - Added resistance in short waves of S175 at Fn=0.15.

The results were also taken for a fuller hull shape, in this case the KVLCC2 ship. The results are shown on Figure 8, represent the comparison of the implementation against the implementation by a published author (Vitali et al., 2016). For this case, the agreement is satisfactory since the formula is more straightforward for these kind of hull shapes. It also validates the idea that Faltinsen's asymptotic formula produces consistent and reliable results for head waves.

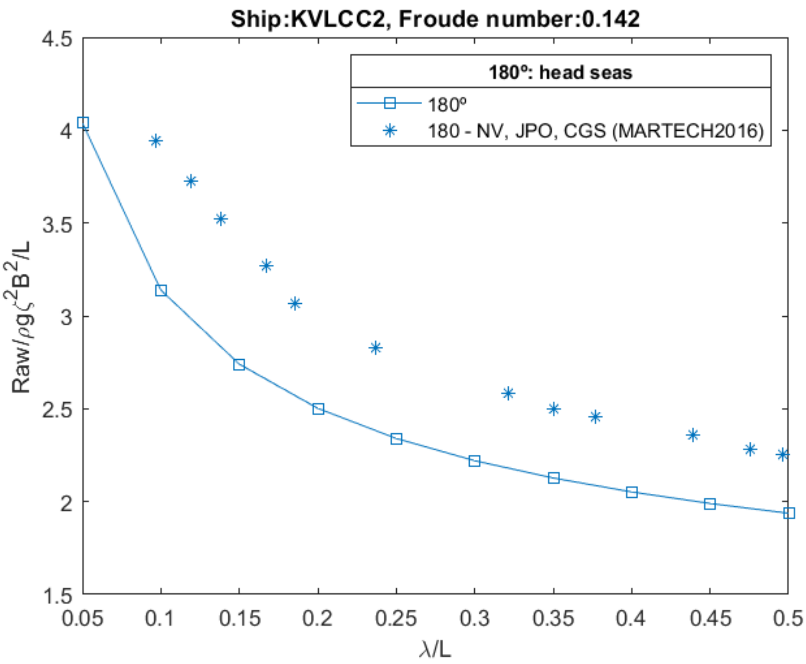

Figure 8 - Added resistance in short waves of S175 at Fn=0.15.

Although Faltinsen's formulation has been regarded as one of the most accepted methods to calculate the added resistance in short waves, it still has some faults which research has been trying to resolve. It is said in Faltinsen's paper that the formulation is applicable to every wave direction, however it can be seen from the results it computes that following seas results are not trustworthy. It is also important to highlight the fact that more experimental data is needed in the low wavelength region for scientific research to find a formula to better match the real solution.

3.3 Salvesen method

This method had already previously been implemented on the in house code of CENTEC, so an implementation was not made for it. Still, the knowledge of the theory behind it must be understood to better adapt it and to adapt the code to compute a better estimate of the added resistance. In this section, a brief description of the method will be done.

The Salvesen, Tuck and Faltinsen linear theory (Salvesen et al., 1970) is applicable to both the method of Faltinsen and the method known as the Salvesen method. The theory of this method assumes a ship with a slender hull form with lateral symmetry advancing at a constant mean forward speed in sinusoidal waves with an arbitrary heading. This method considers a right-handed coordinate system fixed on the ship. The coordinate system and the ship motions are represented on Figure 9.

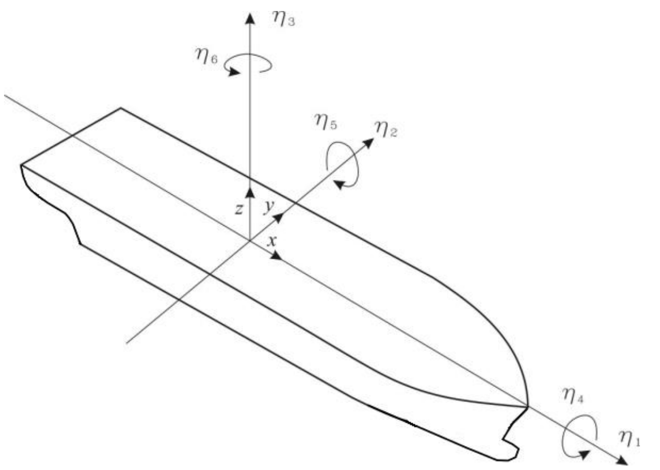

Figure 9 – Representation of the ship motions.

From these considerations it is possible to define Salvesen's formula for added resistance which is based on the Froude-Krylov approach to calculate hull pressures (Salvesen, 1978). The formula is then expressed as:

$$R_{aw} = -\frac{i}{2} k_w \cos\chi \sum_{j=3,5} \eta_j \left\{ \left(\hat{F}_j^{I^*} \right) + \left(\hat{F}_j^{D} \right) \right\} + R_7 \qquad (3)$$

In this equation, $\hat{F}_J^{I^*}$ is the complex conjugate of the Froude-Krylov part of the exciting force and moment and $\hat{F}_J^{D}$ represents the diffraction force and moment but using the complex conjugate of the incident wave potential. The formulas are represented as:

$$
\begin{aligned}
F_3^{I^*} &= i\rho\omega \iint_S N_3 \phi_0^* \, dS \\
F_5^{I^*} &= i\rho\omega \iint_S x\, N_3 \phi_0^* dS \\
\widehat{F_J^D} &= \int_L \widehat{h_3}(x) d\, x \\
\widehat{F_J^D} &= -\int_L \left(x + \frac{iV}{\omega}\right) \widehat{h_3}(x) d\, x
\end{aligned}
\tag{4}
$$

Where,

$$
\widehat{h_3}(x) = -\rho k_w \int_C \Psi_3 (N_3 + iN_3 \sin\beta) \phi_0^* d\, l \tag{5}
$$

$$
R_7 = -\frac{1}{2}\zeta_a^2 \frac{\omega^2}{\omega_e} k_w \cos\chi \int_L e^{-2k_w ds} (b_{33} + b_{22}\sin^2\chi) d\, x \tag{6}
$$

It should be noted that Salvesen method overestimates the added resistance in shorter waves when compared to Faltinsen's method (Matulja et al., 2011). A better approximation is then necessary to compute a better approximation in this range of wavelengths and for a bigger range of ship hull forms.

3.4 NTUA semi-empirical method

As mentioned in chapter 2.2.2 one of the more recent methods being developed from the combination of development of previous theories and empirical formulas is the so called NTUA method developed by the National Technical University of Athens. Throughout the years this method has been updated and improved and the results are very promising (Liu & Papanikolaou, 2020).

3.4.1 Theory

This method follows the principle of superposition where the added resistance in waves is divided in the added resistance caused by the motions of the ship, R_{AWM}, and the diffraction effects, R_{AWR}.

$$
R_{AW} = R_{AWM} + R_{AWR} \tag{7}
$$

The motion effect is based on the theory developed by Jinkine and Ferdinand (Jinkine & Ferdinande, 1974). The formula is rather complex, and it is detailed in the next equations.

$$R_{AWM} = 4\rho g \zeta_A^2 \frac{B^2}{L_{pp}} a_1 a_2 a_3 \overline{\omega}^{b_1} e^{\frac{b_1}{d_1}(1-\overline{\omega}^{d_1})} \tag{8}$$

$$\overline{\omega} = 2.142 \sqrt[3]{k_{yy}} \sqrt{\frac{L_{pp}}{\lambda}} \left[1 - \frac{0.111}{C_B}\left(\ln\frac{B}{T_{max}} - \ln 2.75\right)\right]\left(\frac{C_b}{0.65}\right)^{0.17} \cdot \cdot \left[(-1.377Fn^2 + 1.157Fn)|\cos\ \chi| + \frac{0.618(13+\cos\ 2\chi)}{14}\right] \tag{9}$$

$$a_1 = \begin{cases} \left(60.3C_b^{1.34}(4k_{yy})^2\left(\frac{0.87}{C_b}\right)^{-(1+Fn)\cos\chi}\left(\ln\frac{B}{T_{max}}\right)^{-1}\frac{(1-2\cos\chi)}{3} \text{ for } \frac{\pi}{2} \leq \chi \leq \pi \right. \\ \text{interpolation between beam and following wave cases for } 0 < \chi < \frac{\pi}{2} \\ f(U, V_c) \text{ for } \chi = 0 \end{cases} \tag{10}$$

$$a_2 = \begin{cases} 0.0072 + 0.1676Fr \text{ for } Fn < 0.12 \\ Fn^{1.5}\exp(-3.5Fr) \text{ for } Fn \geq 0.12 \end{cases} \tag{11}$$

$$a_3 = 1.0 + 28.7 \text{ata n} \frac{|T_a - T_f|}{L_{PP}} \tag{12}$$

$$b_1 = \begin{cases} 11.0 \text{ for } \overline{\omega} < 1 \\ -8.5 \text{ elsewhere} \end{cases} \tag{13}$$

$$d_1 = \begin{cases} 566\left(\frac{L_{pp}C_b}{B}\right)^{-2.66} \text{ for } \overline{\omega} < 1 \\ -566\left(\frac{L_{pp}}{B}\right)^{-2.66} \times \left(4 - \frac{125\ \text{atan}\ |T_a - T_f|}{L_{pp}}\right) \text{elsewhere} \end{cases} \tag{13}$$

Due to the intricacy of this formula, it is important to further explain it. Since the formula aims at calculating the added resistance for all wave headings, a method was adopted to estimate the resistance in following waves. Specifically referring to the variable a1, which in order to be calculated for following waves must have a specific Froude number. Therefore, for this variable, the results of the study show that for following waves when:

1. The Froude number equals zero, the formula for head waves is applied with Froude number equal to zero and the added resistance will be negative as expected (added thrust), since the ship will drift with the waves.
2. The ship's speed is equal to a quarter of the group velocity, Vc, the added resistance caused by the ship motions is equal to zero.
3. The ship's speed is equal to half of the group velocity, the added resistance due to radiation effect can be calculated as the added resistance of the ship at Froude number zero in head waves.
4. The speed of the ship is bigger than half of the group velocity the formula for head waves is applied using a relative Froude number of $(U - Vc/2)/\sqrt{gLpp}$.

5. The speed of the ship is smaller than a quarter of the group velocity, Vc, and bigger than zero, a linear interpolation is computed between the cases 1 and 2.
6. The speed of the ship is smaller than half of the group velocity and bigger than a quarter of the group velocity, again a linear interpolation must be done between the cases 2 and 3.

On the other hand, the theory employed for the calculation of the added resistance caused by diffraction effects in waves was adapted from Fujii and Takahashi's semi-empirical method (Fujii & Takahashi, 1975) which is based on Faltinsen's method (Faltinsen et al., 1981). Since the theory for this method has been previously explained, the formulas can be seen below. The adaption of the theory of Faltinsen involved using a new coordinate system, shown in Figure 10 and some parameters to easily describe the waterline, in Figure 11.

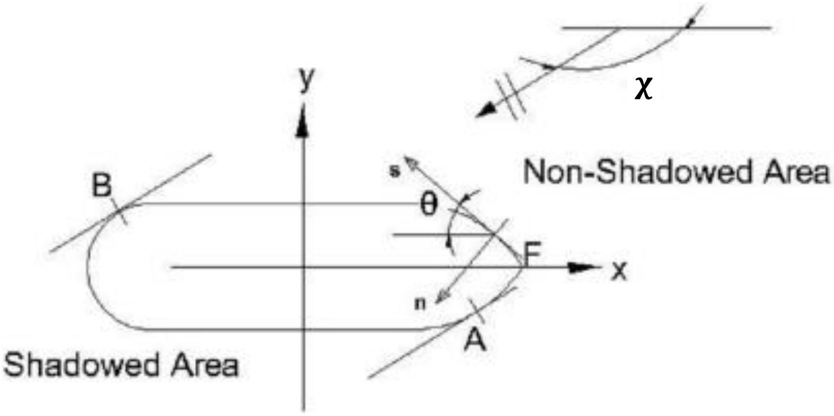

Figure 10 - Coordinate system for the calculation of the added resistance due to diffraction effects (Liu & Papanikolaou, 2020).

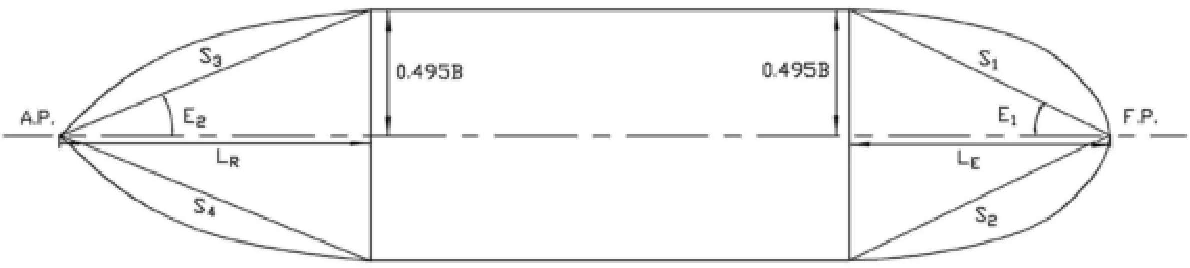

Figure 11 – Waterline and waterline parameters definition (Liu & Papanikolaou, 2020).

$$R_{AWR} = \sum_{i=1}^{4} R_{AWR}^{i} \quad (14)$$

(15)

$$\text{when } E_1 \le \chi \le \pi :$$

$$R_{AWR}^1 = \frac{2.25}{4}\rho g B \zeta_A^2 a_T \cdot \left\{\sin^2(E_1-\chi) + \frac{2\omega_0 V}{g}\left[\cos E_1 \cos(E_1-\chi) - \cos\chi\right]\right\}\left(\frac{0.87}{C_b}\right)^{(1+4\sqrt{Fr})f(\chi)}$$

$$\text{when } \pi - E_1 \le \chi \le \pi :$$

$$R_{AWR}^2 = \frac{2.25}{4}\rho g B \zeta_A^2 a_T \cdot \left\{\sin^2(E_1-\chi) + \frac{2\omega_0 V}{g}\left[\cos E_1 \cos(E_1+\chi) - \cos\chi\right]\right\}\left(\frac{0.87}{C_b}\right)^{(1+4\sqrt{Fr})f(\chi)} \tag{16}$$

$$\text{when } 0 \le \chi \le \pi - E_2 :$$

$$R_{AWR}^3 = -\frac{2.25}{4}\rho g B \zeta_A^2 a_T \cdot \left\{\sin^2(E_2+\chi) + \frac{2\omega_0 U}{g}\left[\cos E_2 \cos(E_2+\chi) - \cos\chi\right]\right\} \tag{17}$$

$$\text{when } 0 \le \chi \le E_2 :$$

$$R_{AWR}^4 = -\frac{2.25}{4}\rho g B \zeta_A^2 a_T \cdot \left\{\sin^2(E_2-\chi) + \frac{2\omega_0 U}{g}\left[\cos E_2 \cos(E_2-\chi) - \cos\chi\right]\right\} \tag{18}$$

Where,

$$f(\chi) = \begin{cases} -\cos\chi\,\pi\ , & -E_1 \le \chi \le \pi \\ 0\ , & \chi < \pi - E_1 \end{cases} \tag{19}$$

For S1 and S2 segments:

$$T^* = T_{max} \tag{20}$$

And for S3 and S4 segments:

$$T^* = \begin{cases} T_{max}\left(4+\sqrt{|\cos\alpha|}\right)/\,5, & C_b \le 0.75 \\ T_{max}\left(2+\sqrt{|\cos\alpha|}\right)/\,3, & C_b > 0.75 \end{cases} \tag{21}$$

$$a_{T^*} = \begin{cases} 1 - e^{-4\pi\left(\frac{T^*}{\lambda} - \frac{T^*}{2.5L_{pp}}\right)}, & \lambda/_{Lpp} \le 2.5 \\ 0, & \lambda/_{Lpp} > 2.5 \end{cases} \tag{22}$$

Where a_{T^*} is the draft coefficient.

3.4.2 Preliminary results

Through the implementation of the formula in MATLAB programming language, it was possible to validate the results presented in the study of Liu and Papanikolaou. The following images will show the comparison between the results from the implementation and those present in the article for the S175 containership sailing with Froude number of 0.25.

On Figure 12, the results computes are shown in a plot where the comparison with the results of the method of Liu and Papanikolaou (Liu & Papanikolaou, 2020). The implementation is fully functional for head waves, and the results showed agreement in this range. Promising results are also seen for beam seas (90º) where results follow the line very accurately. These results are also in agreement with the experimental data from the S175 containership, which is proof that this method is reliable in head seas.

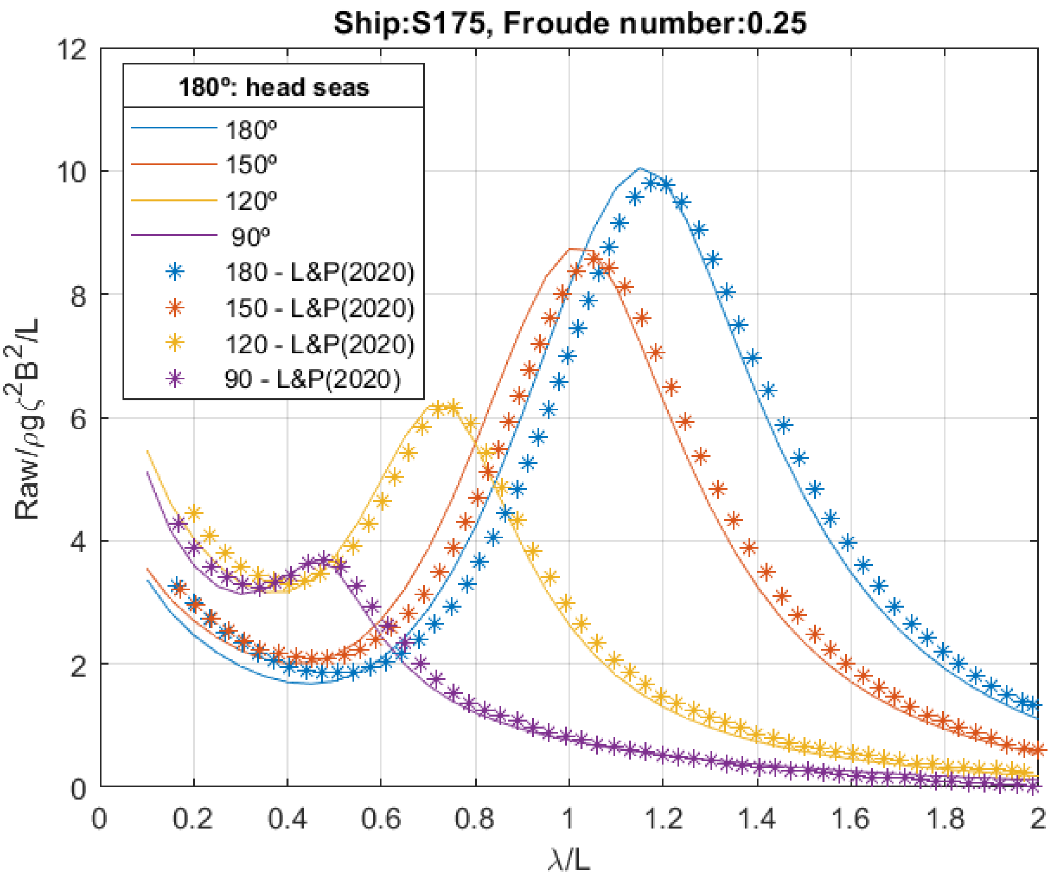

Figure 12 - Comparison of results for head, aft quartering, and beam seas.

Where the implementation of the method does not agree with the results given in the article is on fore quartering and following waves. In this case, plotted on Figure 13, the original results for following waves maintain the maximum of the added resistance in the same position as the maximum for head waves. This is described in the article however, the results produced do not match this. Instead, the peak is at a lower wavelength which seems to be more in accordance with the real case. Similarly, for the peaks for quartering fore seas which do not match the results from the article. After discussion with the authors of the article, it was disclosed that the full method would be publicly released soon so the results can be further validated when a fully working code is available.

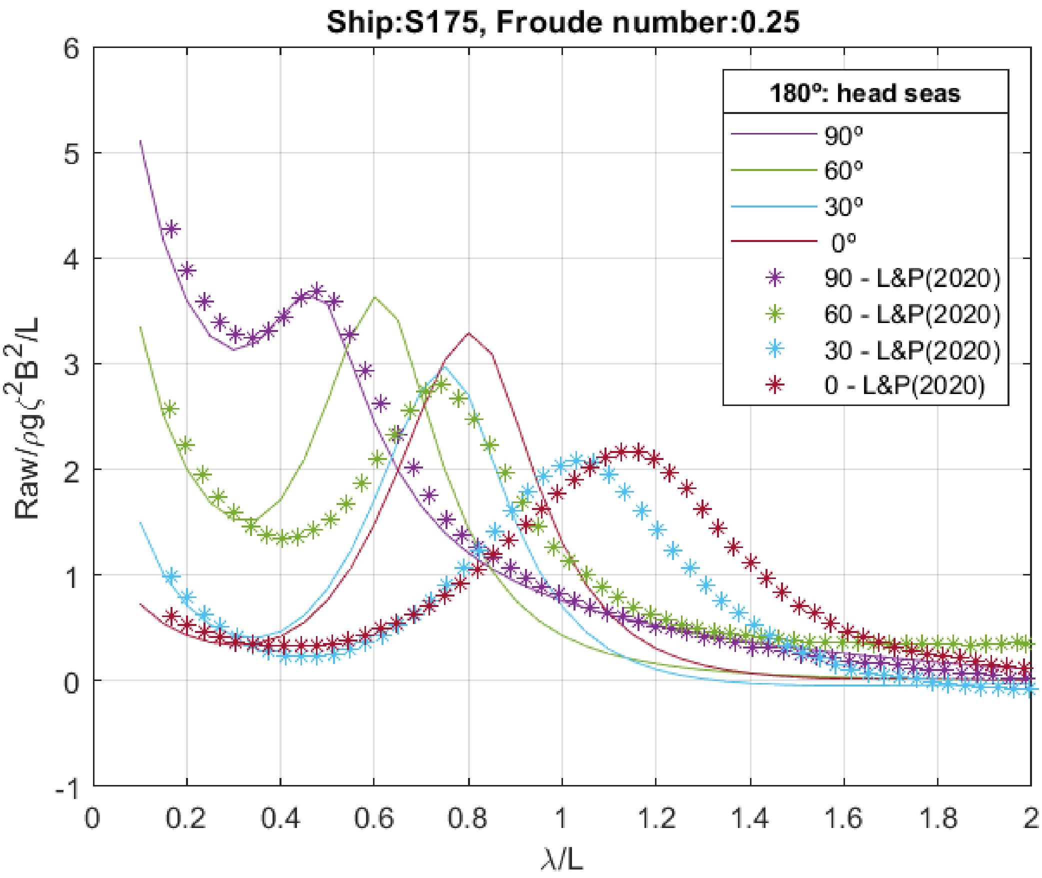

Figure 13 - Comparison of results for following, fore quartering and beam seas.

With the results of these three methods a choice was done to build a working system to compute the added resistance in waves. A database with these results should be compiled for the creation of the meta-model.

3.5 Added resistance in irregular waves

Irregular waves are defined as an array of waves described by three parameters. Therefore, to make the analysis of a wave group these characteristics are described as:

- Wave peak period, T_p, which describes the wave period with the highest wave energy
- Wave significant height, H_s, describing the mean of the of the highest third of wave height
- Mean wave direction, χ_m, which represents the mean of the individual wave directions

To calculate the added resistance for a group of waves the distribution of the wave energy must be computed. This was achieved through a wave spectrum. This spectrum should be a two-dimensional array function of frequencies, ω, and directions, χ, to encapsule the three parameters. The wave spectrum should then be given as:

$$Sw(\omega, \chi) = Sw_{1D}(\omega) \cdot \; spread \tag{23}$$

To compute the one-dimensional spectrum, the ISSC spectrum was chosen (ISSC, 1964). This spectrum is a modified Pierson-Moskowitz spectrum. It represents fully and partially developed sea states since the need for only fully developed seas is too limiting. It is represented in formula as:

$$Sw_{1D}(\omega) = 0.313\; H_S^2 \frac{\omega_p^4}{\omega^5} e^{-1.25\left(\frac{\omega_p}{\omega}\right)^4} \tag{24}$$

Since it is a two-parameter formulation, containing the wave frequencies and the significant height, it is very useful when applied in the context of added resistance in real seas and there is a large family of spectra that derived from this formulation to characterize different sea severities. The comparison with other spectra, such as the JONSWAP, in Figure 14 allows for the confirmation, that in fact the spectrum must be carefully chosen depending on the characteristic wave climate of the area that is being studied.

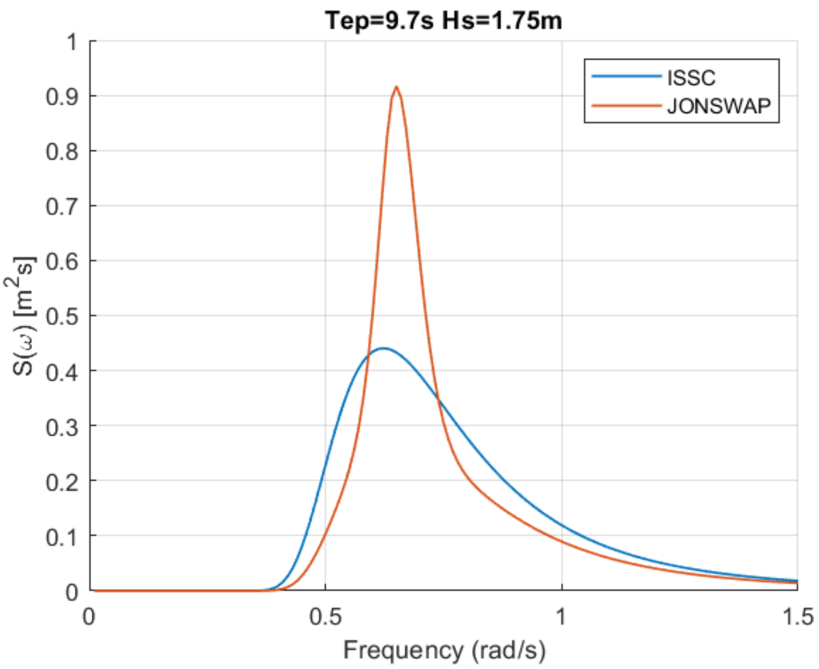

Figure 14 – Comparison between spectrums (ISSC and JONSWAP (g=3.3).

To compute the relation between the one-dimensional spectrum and the direction of the energy distribution of the wave group it is necessary to introduce the wave energy spreading function. One of the most common spreading formulations is the cosine-square formula. This formula simply assumes that the spreading is proportional to the cosine square of the wave heading. It is given as follows:

$$\text{Spread} = \begin{cases} \frac{2}{\pi}(\cos(\chi_m - \chi))^2 & \text{for, } \chi_m - \frac{\pi}{2} < \chi < \chi_m + \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

Remarking that when, $\chi_m - \chi < 0$, the argument of the cosine should be $2\pi - (\chi_m - \chi)$.

Applying the energy spreading function to a point spectrum is the appropriate method to compute the ship motions or the added resistance in waves. Then, to calculate the added resistance in waves in irregular seas with the use of the previously implemented methods, that have as an output the added resistance operator, Φ_{AW}, the integration with the product of the spectrum should be used.

$$R_{aw} = 2\int \int S_w \circ \Phi_{AW} \, d\omega \, d\chi \quad (25)$$

The integration calculates the added resistance over half of the submerged hull, therefore a multiplication by two must be performed.

3.6 Regression analysis

The modeling of the correlation between variables is commonly done through statistical methods. Regression analysis is one of these techniques. Using this method, it is not intended to create a model that will establish a causality between variables, but instead it will give insight into the theory of the approach that generates the data, in this case the computation of the added resistance in waves.

3.6.1 Choice of parameters and dimensional analysis

Since it is important to create a model that can be generic enough to fit a large enough number of ships, within a chosen range, a dimensional analysis was conducted to arrange the variables in a proper fashion. The variables chosen to build the model came from the study of the methods and from the knowledge gained from its implementation. Also, some common dimensionless parameters are used, which have been standard in the marine industry and in ship design.

The investigation of which parameters to use and by converging on the recommendations of the variables proposed in the research of Liu and Papanikolaou (Liu & Papanikolaou, 2020), the final dimensionless parameters that will be used to build the final model are shown in Table 3.

These variables are simple in their essence, in order to quickly estimate the added resistance for any ship. They are commonly available for most ships with perhaps the exception of the ones regarding waterline angles. These could be inferred for a specific ship type if impossible to define the waterline of the ship (in the case a lines plan is not accessible, for example). Fuller hull shapes will have a higher angle, and the opposite for more slender forms. With these assumptions, the calculation of the added resistance in waves can be easily accessed through the presented meta-model.

Table 3 - Dimensionless numbers used in the meta-model.

Dimensionless parameter	**Definition**
Lpp/B	Ratio between length and width of ship.
B/T	Ratio between width and height of ship.
E1	Angle of entrance of the waterline.
E2	Angle of exit of the waterline.
Cb = $\frac{\nabla}{L \cdot B \cdot T}$	Block coefficient is the ratio of the underwater volume of ship to the volume of a rectangular block having the same overall length, breadth, and depth.
kyy/Lpp	Ratio between the radius of gyration (y-axis) and length.
Fn = $\frac{V}{\sqrt{g \cdot Lpp}}$	Froude number is the ratio between the inertial forces and gravitational forces.
Hs/λ_p	Significant height of the incident wave made dimensionless by the peak wavelength.
$\omega_{e_p} \sqrt{Lpp/g}$	Peak wave encounter frequency made non-dimensional by the square root of the length divided by the gravitational acceleration.
χ_m	Mean wave direction of the incident wave.
$Raw/(\rho g \zeta_a^2 B^2/Lpp)$	Dimensionless wave added resistance.

The definition of the parameters related to the sea state are the significant wave height, the encounter peak frequency and the mean wave direction. These variables fall under the assumption of deep wave theory and therefore the dispersion relations can be applied with the consideration that the depth is much larger than half of the wavelength.

The mean wave direction is simply set to a range between head waves and following waves. Head waves are taken as 180º or p radians and following waves as 0º or 0 radians. Within an interval of 30º the remaining directions fall under the fore (30º and 60º) or aft (120º and 150º) quartering waves and beam seas (90º).

For the wave frequency, a range of periods was chosen between 2 and 15 seconds which are then converted into frequency and later into encounter frequency. The formulas for these frequencies are:

$$T_p = \frac{2\pi}{\omega_p} \quad (26)$$

$$\omega_{e_p} = \omega_p - {\omega_p}^2 \frac{V}{g} cos\, \chi_m \quad (27)$$

The significant height was defined as a function of the peak period, since there is a range of acceptable heights for each peak period. To find this function, the graph of recorded observations from the research published from the ITTC 2005 (Downie et al., 2005) was used. The Figure 15 shows this record of observations of significant heights for each peak period. Tracing a line from the origin to the point of the maximum of the 100-year prediction the formula for the maximum of the significant period in function of the peak period is achieved.

$$H_s = \frac{15}{16} T_p \quad (28)$$

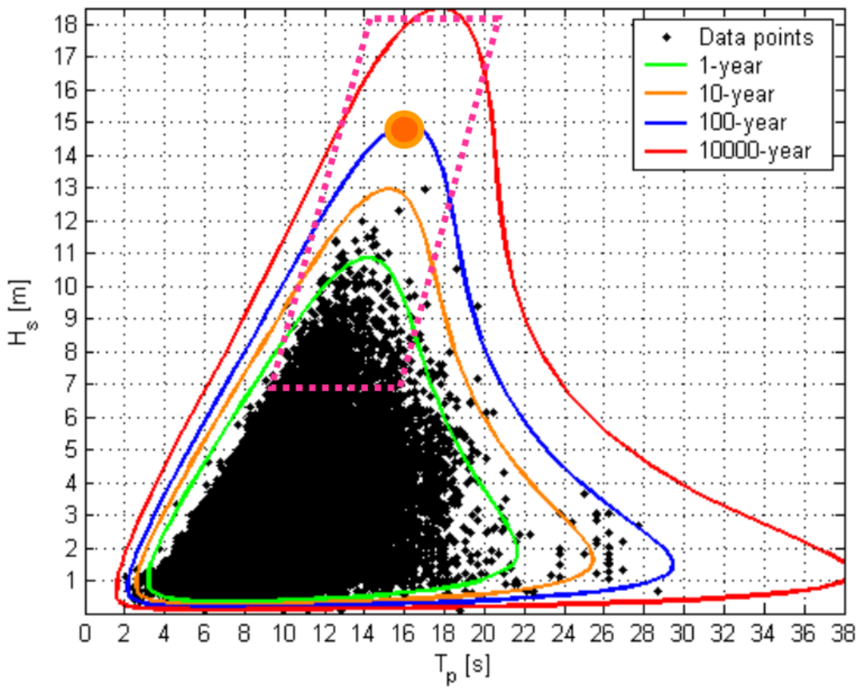

Figure 15 – Scatter diagram of occurrences registered in the Northern North Sea (Downie et al., 2005).

The significant wave height is then defined in a range from 0.5 meters to the maximum computed from the formula above. This range will allow the computation of added resistance in waves that fall within reasonable and acceptable values to use as an input in the creation of the model.

3.6.2 Database

Collecting data and the creation of a database are crucial in any regression analysis. One of the basic methods for assembling the data into one array is through a designed study based on historical data (Montgomery et al., 2012). In this specific case, the creation of the method to calculate the added resistance based on semi-empirical formulas that have the foundation on experimental data and theory allows for the collection of the necessary data. The creation of a database was therefore, based on the methods previously implemented for the computation of the added resistance in irregular waves.

The database is a collection of eleven variables (which were described on Table 3) divided in columns, where the last column will be the response variable, in this case the added resistance in waves. The final database should be extensive for the model to be able to predict the response for a multitude of different ship and ocean state parameters. The final database contains around seventy thousand records which were made using the computations of the added resistance on a numerical code in the programming language of MATLAB.

3.6.3 Linear regressions in MATLAB

The use of a programming code for the computation of the added resistance and for the compilation of data for the database was important to keep a consistency in the creation and production of compatible files and codes. To keep this consistency in the creation of the model with the use of linear regressions, MATLAB was also chosen to perform the regression analysis. The multitude of options, accessibility and strong computation power of this software made for a sensible choice in the decision of which computer software to use.

The simplest model that can be created is a simple linear regression model. This is a model that has a single regressor, x, which is related to a response, y, through a single line as in

$$y = \beta_0 + \beta_1 x \qquad (29)$$

Here β_0 and β_1 are representative unknown constants. This model is the simplified line formula and can be computed with the use of two single responses, which makes it really simple to fit to a small data collection. It is however rather poor when considering a large database of regressors and a single response. For this purpose, it is more appropriate to use a multiple linear regression model.

Multiple linear regression models are more difficult to fit, especially when a rather large amount of regressors is being used. The formula of a multiple linear regression is for example of the type

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k \qquad (30)$$

As it can be seen a multiple linear regression model can relate the response to k regressor or predictor variables. β_k are also often called the regression coefficients.

In order to increase the complexity of the structure of the model and also its ability to better model the response, two methods can be applied separately or simultaneously. The first would be the inclusion of polynomial models which make increase the order of the predictors such as for example

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2{}^2 + \ldots + \beta_k x_k{}^k \quad (31)$$

Polynomial models can increase the fit of a model without including additional predictor variables which is useful when the model formula is meant to be small with few regressor variables. However, it is also possible to include in this model the same regressor with two different powers.

After increasing the order, the other factor that allows for the increase in complexity of the structure of the model, is the inclusion of interaction effects between the regressor variables. An inclusion of interaction effects is for example

$$y = \beta_0 + \beta_1 x_1 + \beta_{12} x_1 x_2 \quad (32)$$

This method allows for an additional increase in the fitting of the model to the expected response. Using a combination of these two methods it is possible to create a more complex model. The fitted regression equation can be used to predict future responses or to estimate a mean response.

MATLAB has a number of functions to perform the creation of a linear model. Some allow for a more straightforward implementation, where the output formula of the model will be a representation of the user entered formula structure. Other formulas are able to further adjust the formula and to enter restrictions on which predictors to be used and how.

One of these functions is *fitlm* (Mathworks, 2020), which fits a linear regression model to the observed data. It returns a linear regression model using a matrix input data set. The model formula must include all the predictors that the user has set and unless stated otherwise, will be created without powers or interaction effects.

Another function which is more relevant to the fitting of a formula is *stepwiselm* (MathWorks, 2020b), that follows a straightforward method through the completion of several steps. A sequential approach to the fitting of the multiple linear regression formula following conditions attributed by the user will undoubtedly potentiate the creation of a better model.

This function, *stepwiselm*, starts the creation of the model from a constant model, including only the interception term. It will then look for the next parameter to add to the formula and by computing the p-value (defined in the next section) of this parameter and decides to include it or to exclude it from the model. This decision is by default made through an algorithm that follows two steps generally:

1. If the p-value of a predictor to add are smaller than 0.05, then the parameter is added to the formula
2. If the p-value of a previous predictor that had already been entered becomes greater than 0.10 after adding another predictor to the formula, then it is removed from the formula

After there are no more variables available to add to the model, the process is terminated and a final formula that fits the model is achieved. This model can be used to predict responses for future observations and to validate the model.

3.6.4 Interpreting the statistic outputs

The linear regression functions from MATLAB besides outputting the linear model, also compute and provide some statistical parameters in order to evaluate the feasibility of the function. These statistics are dependent of the type of model that the user will input and change in term of the type of regression being performed. For the most common cases of simple linear, quadratic or interaction effects regressions the statistical coefficients are computed for the regressors and for the overall formula. The parameters that the linear regression models output can be consulted on MATLAB's user manual (MathWorks, 2020a).

In the beginning of the computation of the model, the function will add or remove a predictor based on the p-value that it adds to the formula. The p-value is commonly defined as the probability of occurrence of a result at least as extreme as the observations in a test, considering that the null hypothesis is true. In other words, it states how often an observed difference could have happened from chance. The computation of the p-value in MATLAB is done for the predictors and for the model. For the predictors, it is done through a t-statistic of the hypothesis testing for each one if it is equal to zero. It is used through Student's t-test. For the model it is done through an F-test. The F-test is a statistical test that has an F-distribution under the null hypothesis. The F-distribution is a common continuous probability distribution usually for the analysis of the variance. The p-value is a good estimator of the behavior of the predictors and a lower value is always preferred. MATLAB uses as a default a 5% significance level for the coefficients of the predictors in the end model.

Other statistical parameters that will be given for each coefficient of the predictor include are the standard error and the t-statistic for each, tStat. The standard error, SE, is simply calculated by

$$SE = \frac{\sigma}{\sqrt{n}} \tag{33}$$

Where σ is the sample standard deviation and n the number of samples.

tStat is the t-statistic for each of the coefficients. It tests that the corresponding coefficient is zero, considering the null hypothesis, against the alternative for it to be different from zero, done considering the other predictors in the model. The computation of tStat is done as

$$tStat = \frac{Estimate}{SE} \tag{34}$$

Regarding the model itself, and besides the already mentioned p-value of the model, there are also some statistics which will be relevant in its analysis. MATLAB will output the number of observations that it registered (in case there is some problem with the predefined database) along with the error degrees of freedom, EDoF, given by

$$\mathrm{EDoF} = n - p \tag{35}$$

Where n is the number of observations, and p is the number of coefficients in the model, including the intercept. This is important in understanding how the model can fluctuate through the number of variables that are free to vary.

The more statistic oriented parameters of the model that allow for the better evaluation of the fit of the model are the *F-statistic vs. constant model, RMSE* and the *R-squared*. The first of these being a simple test statistic for the F-test on the regression model, called in MATLAB*, F-statistic vs. constant model*. This will evaluate the fit of the model when compared to a simpler model which consists only of a single constant term.

the root mean squared error, *RMSE*. The RMSE refers, as the name implies, to the square root of the mean squared error. This allows the estimation of the standard deviation of the error distribution. The formula is usually given as

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(\hat{y}_i - y_i)^2}{n}} \qquad (36)$$

Where $\hat{y}_i$ corresponds to the model predicted response and y_i to the recorded response. The RMSE is habitually used to compare recorded observations against a predicted model and is therefore a good indicator of suitability of the formula.

Finally, the *R-squared* (or R^2) followed by the *Adjusted R-squared* (or R^2_{adj}). R-squared is a statistical coefficient determining how the predictor variables will explain the variance of the response. The formula to compute the R-squared is

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2} \qquad (37)$$

Where $\bar{y}$ is the mean of the observed responses. The R-squared can easily be adjusted to provide a better result using more predictor variables however, this does not always imply a better fit.

The adjusted coefficient of determination is particularly important when assessing the over-fitting of the model. Its formula is

$$R^2_{adj} = 1 - \left[\frac{(1 - R^2)(n - 1)}{n - p - 1}\right] \qquad (38)$$

Therefore, it can be said that none of these parameters is good measure of the model by itself. An analysis of all of them in combination must be performed to evaluate the fit of the model. In order to prevent over-fitting, the model must have some sensitivity to noise and the number of predictors must reflect the general population of occurrences rather than a small portion, even though it would increase the R-squared to a value that would suggest a better fit, for example.

4. RESULTS AND DISCUSSION

To reach a conclusion, the methods described were applied into a case study to study the added resistance over a range of conditions and to build a linear regression model. Taking into consideration the preliminary results of these methods and the accessibility to some of them through computer software a decision was made on how to proceed to solve the problem of added resistance of a ship in ocean waves and on how to simplify this solution into a model.

An additional goal of this dissertation was the validation of the model. This would be done using scaled ship hulls to predict the added resistance in waves using the model. Through the comparison of the predicted results against the calculation of the added resistance in irregular waves through the usual computations, an effort towards an authentication of the results is performed.

This chapter explains the results and discusses their validity. An analysis for a more efficient model, and for a better regression is considered, as well as some different ways in which the regression could be done through machine learning. The debate on which procedure to use is explained in this manner and left as a suggestion for future work.

4.1 Results from the implementation of the added resistance and database creation

In order to create a database which will be used to train the model, the results from the methodology must be analyzed. Following this analysis, a decision on a range of methods and characteristics will be selected for the model which will give it its range of applicability. In other words, the limits, and restraints of the model.

Initially, the decision on a formulation to be used for the prediction of the added resistance in waves was made. A discussion was led based on the preliminary results from implementations of numerical simulations in MATLAB. The test of all the formulations detailed previously formulation supported the conclusion that two methods were better for this application. These two are Salvensen's method and the NTUA method which were picked for their reliable results in a multitude of different scenarios. Salvensen's method had been previously implemented on the in-house code of CENTEC to compute the added resistance in waves. This method is used to predict the added resistance for waves in the region of head to beam waves, and for higher wavelengths. Its main feature of having a higher peak is evident when comparing Figure 12 and Figure 16. For the rest of the headings and wavelengths the NTUA method is applied for a conservative estimate of the added resistance. The compilation of both codes resulted in an end product which is simple to use and gives reliable outputs.

The choice of ship type was based on how the formulations behave for the different ship characteristics. Fixing a ship type will allow for a more reliable result and training of the model. It does however restrict the model to this range of ships. For this context containerships were chosen. These ship types tend to have more slender hull forms and higher Froude numbers. Three ships were chosen to train the model, the S175 ship, the KCS ship and the DTC ship. The characteristics of these ships are described on Table 2. It is then obvious that the model is limited to this range of ship characteristics, but it also allows for more reliable results withing this group. The model is then trained for ships ranging from 175 meters to 355 meters and for Froude numbers of 0.18 to 0.25. There is a limit on how much these inputs can fluctuate outside these limitations. However, results were found to still be acceptable within a 10% factor (R-square larger than 0.7) of scale of the ship main parameters but for 20%, results become more divergent (R-square smaller than 0.7).

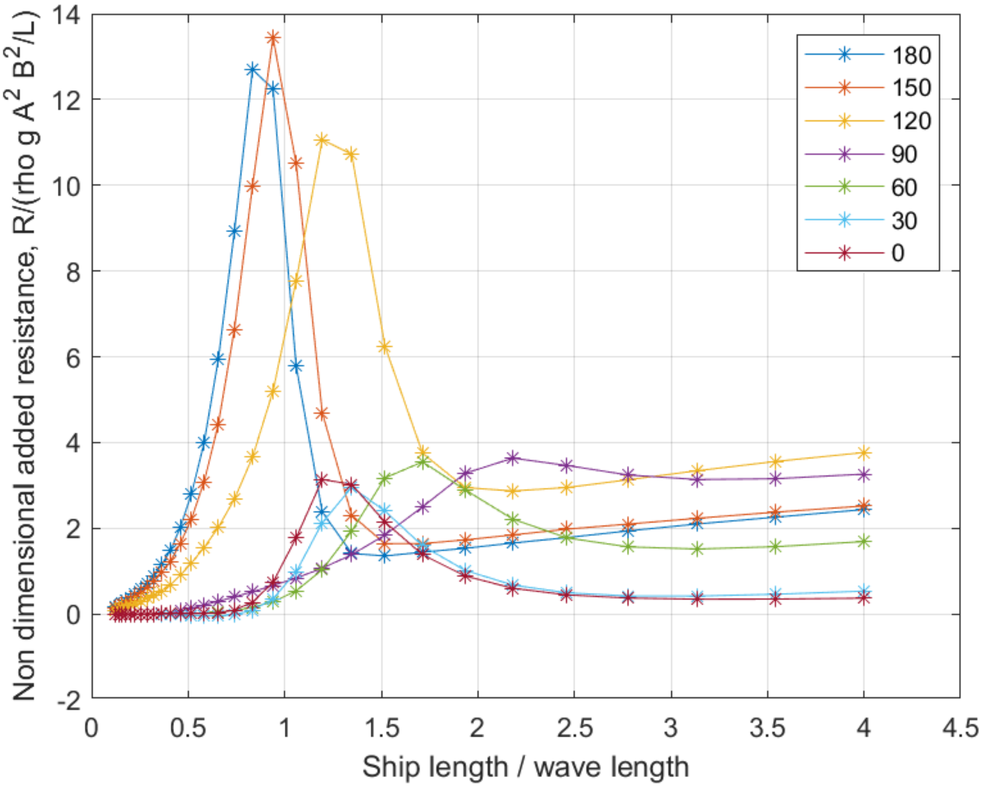

Figure 16 - Added resistance in waves of S175 ship at Fn=0.25, using a combination of Salvesen and NTUA methods.

In terms of the wave characteristics, it is relatively easy to predict the added resistance for the entire range of periods, heights, and directions. But it will produce a wide range of results which will require a linear regression formula to have more predictors related to wave attributes.

The database is then the result of the computation of the added resistance for this range of parameters. It is compiled in a table with the main parameters of the ship and waves as described in Table 3. It was of extreme importance to make these parameters non-dimensional, in order for the model to have a wider range of applicability.

4.2 Numerical model

The creation of a numerical model that would satisfy the conditions to compute the added resistance in waves within the described range of limitations, would first start with a simple linear regression model without interaction effects or powers. In this way a better understanding of how to further develop the formula could be gained. The simple linear model was obtained running the function *fitlm* in MATLAB and is described by Table 4 and the following formula:

$$R_{aw_{nd}} \sim \beta_0 + \beta_1 \frac{L_{PP}}{B} + \beta_2 E_1 + \beta_3 F_n + \beta_4 \omega_{e\,p_{nd}} + \beta_5 \frac{H_s}{\lambda} + \beta_6 \chi_m \tag{39}$$

This linear model is not fully developed, and this was clear by the statistical parameters computed which were registered in Table 4. Although the statistical parameters for each of the predictor coefficients are quite promising, the overall model has a bad fit to the observed data. The R-squared (along with the adjusted R-squared) of 0.487 shows very bad correlation with the observed results. The correlation is even more evident when the plot of the observed response versus the predicted response as in Figure 17. This in combination with a value of the RMSE which would show a relatively acceptable prediction of response, allows for the conclusion that this model is insufficient in the prediction of the added resistance. In this case the *p-value* and *F-statistic vs. constant model* parameters represent acceptable values too but should be disregarded in the light of a compromising R-squared.

Table 4 - Linear model characteristics.

			Predictors statistics		
	k	**Estimate (b_k)**	**SE**	**tStat**	**pValue**
(Intercept)	0	-2.3214	0.063462	-36.58	3.27E-290
Lpp/B	1	0.35982	0.010383	34.656	5.94E-261
E1	2	-0.95443	0.054321	-17.57	5.85E-69
Fn	3	0.45025	0.01718	26.208	1.14E-150
$w_{ep\ nd}$	4	-0.0067063	7.12E-05	-94.22	0
Hs/lambda	5	-1.3275	0.013267	-100.06	0
χ_m	6	0.091301	0.00042609	214.28	0
Model statistics					
No. of observations					70224
Error degrees of freedom					70217
Root Mean Squared Error					0.104
R-squared					0.487
Adjusted R-Squared					0.487
F-statistic vs. constant model					1.11E+04
p-value					0

It is also possible to conclude from this model that some parameters are immediately irrelevant to the regression. These parameters are kyy/Lpp, E2, B/T and Cb. These are all parameters related to the ship and their contribution to the estimation of the added resistance in irregular waves is not as significant as those describing the wave field. An emphasis was therefore made to keep the predictors related to the ship to a minimum while increasing the number of those that can better reflect what is observed from the energy of the waves.

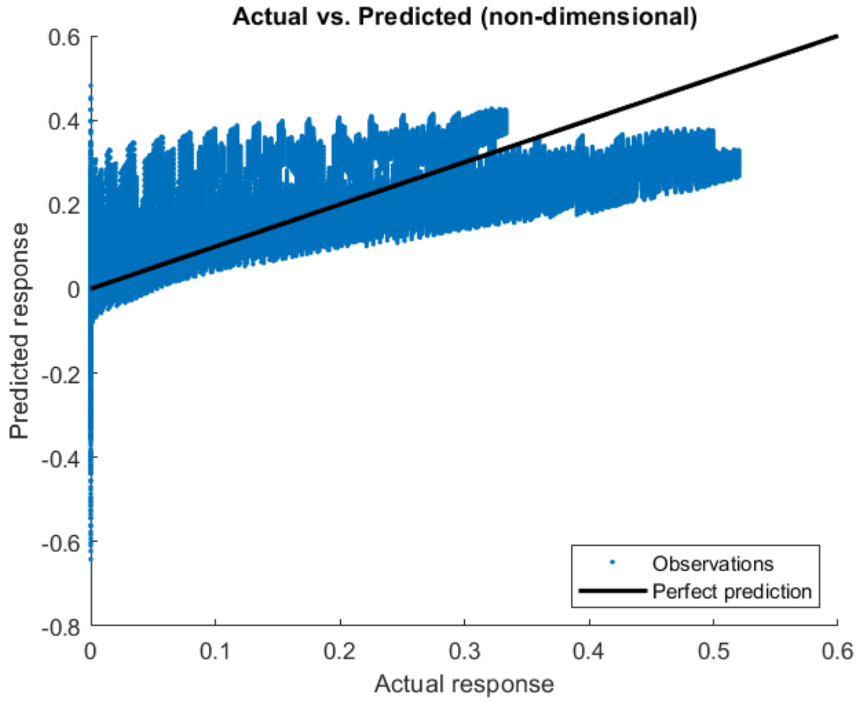

Figure 17 - Actual response plotted against the predicted response from the simple linear model.

The following iterations would then have these considerations taken into account. MATLAB's command *stepwiselm* allowed the creation of a linear regression through the definition of an upper limit to the complexity of the model's formula structure. This was important in defining which parameters to use and how many predictors to allow the formula to have.

The designation of which parameters to use for the predictors was made through troubleshooting the model with different predictors, with the goal of increasing the fit of the model. This is a limitation of this type of computation of a regression model in MATLAB. It is not possible to simply input the number of predictors that would give the best fit or the number of interactions or order of the formula intended by the user.

Ultimately it was found that for the ship parameters the interaction effects would produce the best results. It was also found that the fitting of the model increased when including the product of the block coefficient with the angle of entrance for one predictor and for another predictor the product of the length multiplied by the beam and Froude number. For the wave parameters better results were found when increasing the power of the predictors as well as when including interaction effects for the wave frequency and wave directions.

The final model is comprised of ten predictors including the interception term, including eight terms from the initial ten used in the database to train the model. The number of predictors was kept to a minimum since the inclusion of a large number of predictors could lead to the overfitting of the model. Therefore, by keeping the overfitting of the model under control, would allow for the model to have the best possible fit for ships withing the range of application. This would prove significant in the validation of the model using scaled hulls. The formula that was found optimal is then presented from the formula below and described in more detail along with the statistics in

Table 5.

$$R_{aw_{nd}} \sim \beta_0 + \beta_1 \frac{H_s}{\lambda} + \beta_2 (E_1\, C_b) + \beta_3 \left(\omega_{ep_{nd}}\, \chi_m\right) + \beta_4\, \omega_{ep_{nd}}{}^2 + \beta_5\, \chi_m{}^2 + \beta_6 \left(\frac{L_{PP}}{B} \frac{B}{T} Fn\right) + \beta_7\, \chi_m{}^3 + \beta_8 \left(\omega_{ep_{nd}}{}^2\, \chi_m{}^2\right) + \beta_9 \left(\omega_{ep_{nd}}{}^3\, \chi_m{}^3\right) \tag{40}$$

Table 5 – Final linear model characteristics.

			Predictors statistics		
	k	**Estimate (b_k)**	**SE**	**tStat**	**pValue**
(Intercept)	0	-0.09541	0.002128	-44.835	0
Hs / λ	1	-0.74541	0.009171	-81.278	0
E1 * Cb	2	1.0453	0.016739	62.448	0
$\omega_{ep_{nd}} * \chi_m$	3	-0.0149	6.05E-05	-246.45	0
$\omega_{ep_{nd}}{}^2$	4	-6.31E-05	2.31E-06	-27.287	4.28E-163
$\chi_m{}^2$	5	0.20843	0.000483	431.15	0
Lpp/B * B/T * Fn	6	0.007247	0.000409	17.722	4.04E-70
$\chi_m{}^3$	7	-0.05354	0.000144	-372.14	0
$\omega_{ep_{nd}}{}^2 * \chi_m{}^2$	8	0.000132	8.05E-07	163.65	0
$\omega_{ep_{nd}}{}^3 * \chi_m{}^3$	9	-2.67E-07	2.03E-09	-131.66	0

Model statistics	
No. of observations	70224
Error degrees of freedom	70214
Root Mean Squared Error	0.0651
R-squared	0.801
Adjusted R-Squared	0.8
F-statistic vs. constant model	3.13E+04
p-value	0

The predictors in this formula show an improved statistical fit to the observed responses. The p-value is maintained very low, being close to zero or approximately zero for all the predictors and the *tStat* is relatively acceptable for all the predictors. It can be seen that it is not quite good for the regressor which is composed of the product of three terms, which severely complicates the interaction effects.

Regarding the model itself, the fitting is very good considering the type of model. Both the R-squared (and adjusted R-squared) and the RMSE when analyzed together show significantly good values. When compared to the linear model which was the first iteration of the regression model, the results are much better. And comparing the plots of the predictions in Figure 17 and Figure 18 the fitting of the model is also considerably improved. The remaining statistical parameters also have the expected values although not being as relevant to the discussion of the behavior of the model.

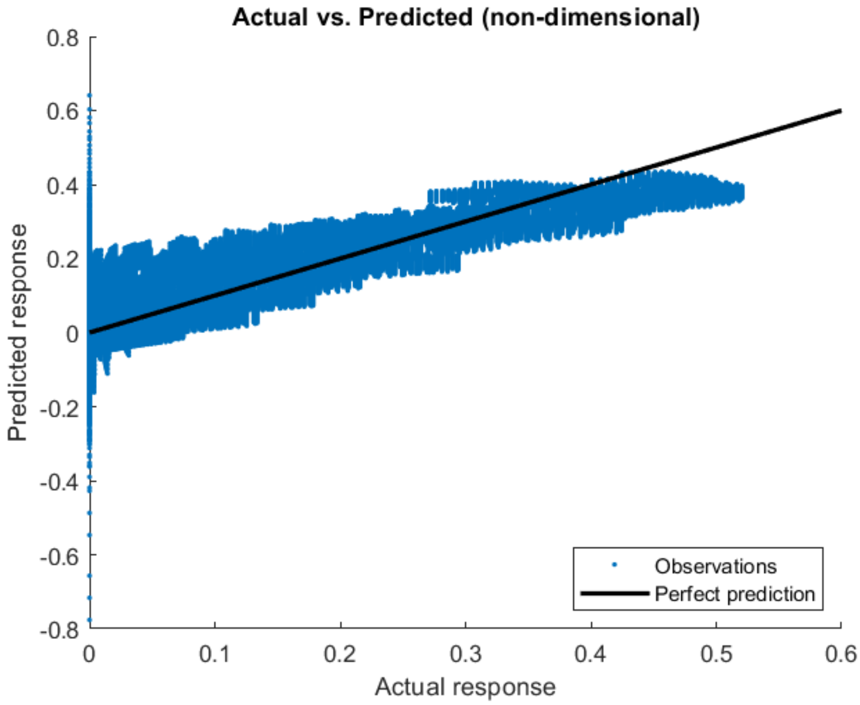

Figure 18 - Actual response plotted against the predicted response from the final linear model.

From Figure 18 and from the detailed analysis of the plots of the model (in Appendix 1) the more evident weaknesses of the model in estimating the added resistance in waves were studied. One of these shortcomings is the severe overestimation of the added resistance for low values compared to the peaks. Seen in Figure 18, the closer the actual responses are to zero, the bigger the variance there is in the model responses.

The method to eliminate this weakness would be to not include the lower regions of wave frequencies and wave heights. Some computed examples showed better fits, but since it does not give a full picture of the added resistance they were omitted.

4.3 Model validation

In order to fully validate the linear regression model, a test was conducted using hull forms scaled from those that were used to train the model. Changing the parameters of these hulls would test the range of the model and could reveal some limitations and strengths of the model.

The hulls that were chosen to scale were the S175 and the KCS. The scaling factors were applied to each axis using the table of offsets of each hull. These scaling factors are recorded in Table 6.

Table 6 – Scale factors applied to each ship.

	Ship	
Scale factor along:	**S175**	**KCS**
x-axis	1.1	0.9
y-axis	0.9	1.1
z-axis	0.8	0.8

Both were tested by calculating the added resistance in irregular waves through the same method used to build the database to train the model and comparing these results with the predicted values from the model. Both ships were tested for a high standard Froude number of 0.25 to check how the model behaves in limit conditions. The response plots are represented in Figure 19 and in Figure 20.

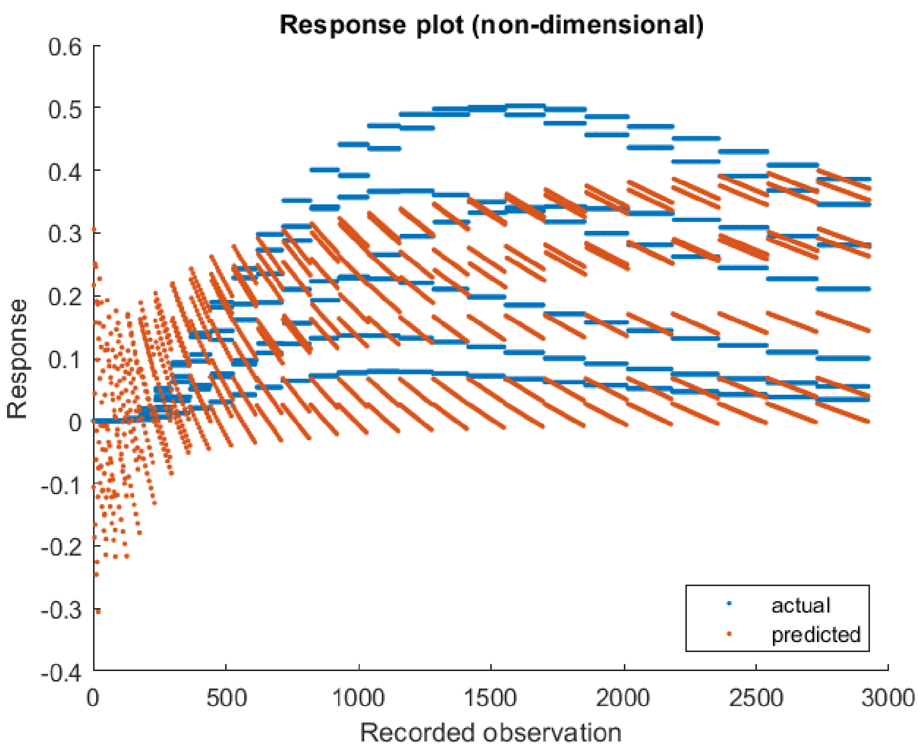

Figure 19 – Response plot of the scaled S175 ship at Fn=0.25 (R^2 = 0.75).

The analysis of these two figures show in a clear way the performance of the model. As mentioned, the weaknesses persist in the lower regions of the added resistance, which indicates an overfitting that is just not possible to overcome using linear regression models. This is carried through the remaining responses and although the overall fit does not seem so good, when excluding these low values, the added resistance predicted by the model becomes more acceptable.

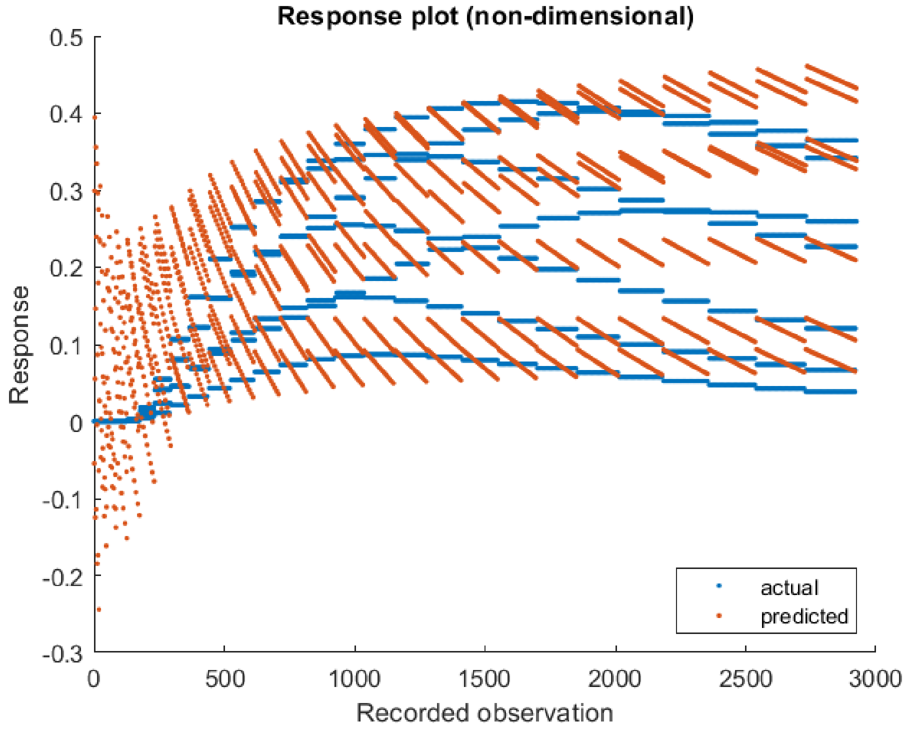

Figure 20 - Response plot of the scaled KCS ship at Fn=0.25 (R^2 = 0.70).

The ease of use of use of the formula, in combination with the simplicity in modifying and testing the regression and its adaptability are the strong points in favor of linear regression models. The disadvantages will obviously be carried through the model however, this is the simplest way of finding a formula that can rapidly and efficiently calculate the added resistance with parameters easily available for any ship.

4.4 Sensitivity analysis

The analysis of which parameters are relevant in the formula can indicate some knowledge into the formulations used to build the database to create the regression model. Knowing that the parameters unused in the final model are the angle of exit at the waterline and the radius of gyration along the y-axis it can be said that the methods used to estimate the added resistance are not too reliant on these parameters.

MATLAB automatically decides the inclusion or exclusion of these parameters through an analysis of the p-value. The default values of adding or removing a parameter were maintained. These are respectively a p-value of 0.05 and a p-value of 0.1. Using this analysis makes sure that the regression formula estimates are relevant, and the fitting of the model is done correctly. The steps taken for the addition of the new regressors to the formula and their statistical contribution to the model as a whole were computed as follows:

1. Adding ${\omega_{ep_{nd}}}^2$, FStat = 1963.3507, pValue = 0
2. Adding Hs/λ, FStat = 7816.6754, pValue = 0
3. Adding ${\chi_m}^2$, FStat = 17161.8063, pValue = 0
4. Adding ${\chi_m}^3$, FStat = 50860.9676, pValue = 0
5. Adding $(\omega_{ep_{nd}} * \chi_m)$, FStat = 27107.2546, pValue = 0
6. Adding (E1*Cb), FStat = 2647.3798, pValue = 0
7. Adding $({\omega_{ep_{nd}}}^2 * {\chi_m}^2)$, FStat = 9761.2465, pValue = 0
8. Adding $({\omega_{ep_{nd}}}^3 * {\chi_m}^3)$, FStat = 17048.4074, pValue = 0
9. Adding (Lpp/B*B/T*Fn), FStat = 314.066, pValue = 4.041492e-70

It is important to note the order of the addition of new parameters as it also gives some insight about what parameters bring the most value to the formula and how they contribute to the regression. Wave parameters were typically recorded as entering the formula first, which would mean they play an important role in the fitting of the model.

4.5 Alternative regression method

Through MATLAB it is rather easy to test other regression models that, although are not in the scope of this dissertation, could still be useful at hinting improvements for futures works. The application *Regression Learner* included in the software make it accessible to easily calculate a new regression model using the previously described database. Even though all the features of this application will not be too explored, the example of the fine tree was registered.

The fine tree regression model works by portioning the data into branches and leaves. After the data is split into these sets a prediction is made based on the mean of the response in the training set for a particular set. The fine tree simply creates more division than a course tree would and therefore is extremely accurate. Although a simple formula is not given as in the linear regression model, a set of responses is given for the entire set and therefore the model is quite large and can only be used in the context of the software.

To prove that this kind of regression model could work in this application, a model was created, and results can be analyzed through the statistics and through plots in Figure 21 and Figure 22. The R-squared for this model is 1.00 and the root mean squared error is 0.0091. Comparing these values to those of the linear regression model it is possible to say that this kind of regression produces a better fit of this data.

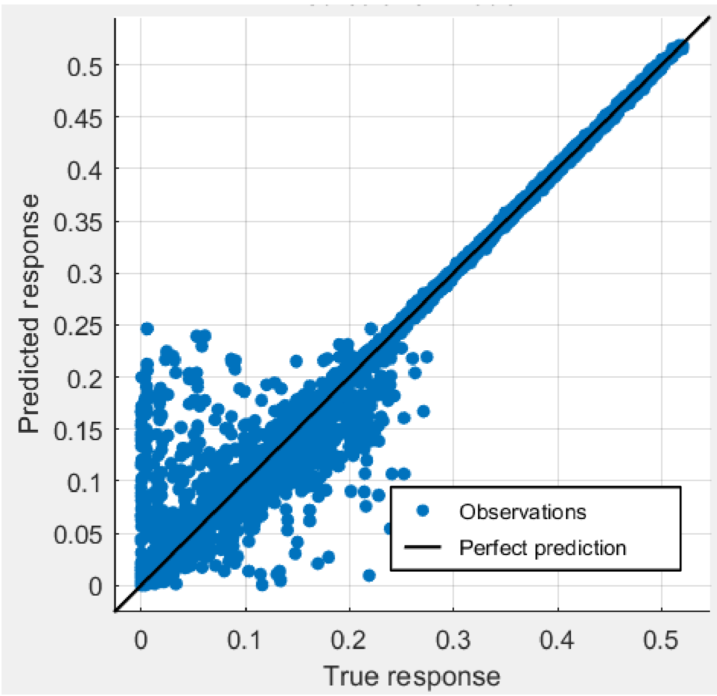

Figure 21 – True response vs. predicted response by the fine tree regression model.

From the analysis of the plots, it is possible to conclude that indeed the tree regression predicts the responses with greater accuracy but some of the deficiencies uncovered in the linear model are still present in this model. Particularly, as it is evident from the plots, the prediction of the added resistance for lower values which has been explained before. This behavior of both of the regression models gives better insight on the system used to create the database, which could potentially indicate some anomalies for these lower regions of peak wave frequencies and significant heights.

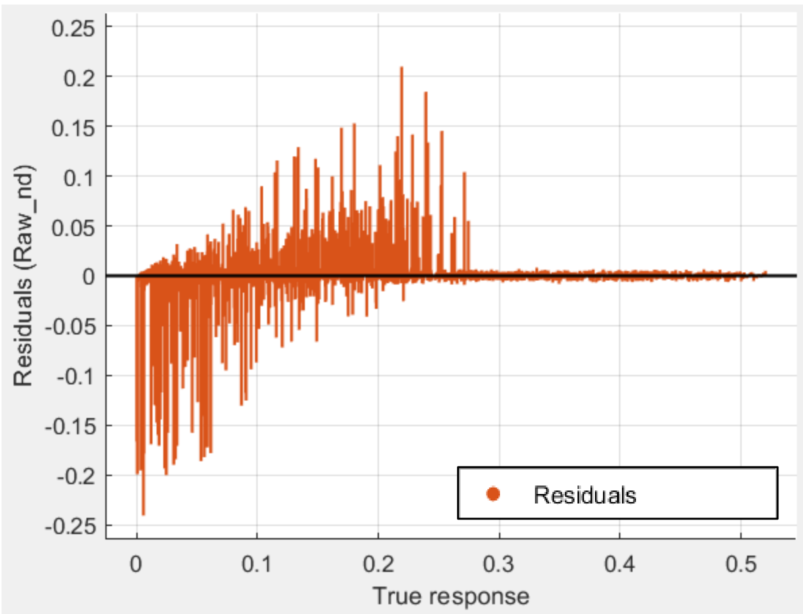

Figure 22 – Residuals plot of the fine tree regression model.

5. CONCLUSIONS

5.1 Achievements

The main objective of this dissertation was the creation of a meta-model through a linear regression that would simplify the calculation of the added resistance of a ship in waves. For this purpose, a database was created for the identification of values of the added resistance computed from methods that were analyzed and tested for the solution of this problem.

The extensive literature review defined the history and precedents of methods and techniques used to calculate the added resistance in waves. From this study it was possible to clearly isolate the two main areas of focus on the theory for the solution of the added resistance, which are divided into theoretical and semi-empirical formulations. Having the understanding of the considerations made by each formulation, allowed for the choice of which methods to select for the preliminary testing and decision making of which concepts to further combine for the computation of the final solution.

The study of the theories behind the solution of the added resistance problem should be made in conjunction with the research of the regression models that have been applied to this problem. The detection of a gap in the literature was clear in this area, where there are only a few developed models. This gap was concluded to be related to the lack of knowledge regarding machine learning in the maritime engineering field where it is just starting to become more appealing due to the alternative it provides from the extensive and complex formulations. There will be a public need and demand for more experimental data regarding tank tests and full-scale analyses motivated by the development in regression theory.

Through the analysis of the state of the art, it was possible to determine some methodologies to be applied to the problem that would give the necessary results for the implementation of a regression model. The methods of Faltinsen, Salvesen and NTUA were implemented into programming language and duly tested. Faltinsen's method was deemed unnecessary in this context since it had several limitations in the type of ships and directions it could reliably compute. This conclusion is also supported by the fact that the NTUA method is an extension of Faltinsen's asymptotic formulation for short waves. Salvesen's method had previously been implemented in CENTEC's numerical code for the added resistance and also proved to be a reliable method. Although it is known that this method overestimates the peak of the added resistance, it is still reliable enough to predict the added resistance for different ship types in head or fore quartering waves.

Regarding the regression theory, a decision was made to create a linear regression model that would simplify the solution of the problem and could give some more background into the inner workings of the methods used to compute the added resistance in waves. The linear regression model focused on the calculation of the added resistance in irregular waves. For this reason, a database was created, composed of eleven dimensionless terms, computed from the integration of the added resistance in regular waves and the chosen ISSC spectrum.

The process of creating a linear regression model was an iterative procedure. Both in the quantity and form of the terms to add to the model. Using this method, it was possible to achieve the best fit for the regression using a formula that would be simple enough to quickly predict the added resistance of a ship sailing in waves. This formula can calculate the added resistance with quite good reliability (eighty percent or higher) except in the regions where the added resistance becomes quite low, where it overestimates the responses.

However, the main goal of finding a formula that simplifies the problem and meets the needs of quickly predicting the added resistance is met. This was validated with the use of scaled ship hulls which predicted the added resistance with an acceptable degree of accuracy and resulting in a R-squared generally higher than zero point seven which is considered to be very effective taking into account the type of regression. Having this main objective in mind it is possible to conclude that the statistics of the model indicate a good fit for the needed formula, especially considering the phenomenon of the added resistance which is strongly dependent on interactions and effects between ship and waves.

The model presented is capable of computing responses for a wide array of wave parameters, but it is more selective when it comes to ship parameters. The definition for the limit range of the model presented in this dissertation is:

- Ships must be containerships or of similar block coefficient to common containerships (less than 0.7 usually)
- Dimensions not larger than 10% of the maximum and smaller than the minimum of those from the hulls used to train the regression model

This model is valuable since it is defined through a formula that is simple to use and allows for the wide and efficient access to the added resistance in waves. This is especially important in engineering and project scenarios. However, computing a model that has a better fit to the recorded observations and has a wider range of operation than the one presented in this thesis is possible.

In conclusion, it is important to take note that regression models should not be used to make hypotheses about the physical theories or models but rather to gauge how the methodology to the computation of the responses of the added resistance in waves functions. And also, that a regression model must be chosen based on a clearly defined goal, and the statistics must be interpreted to clearly state if that goal was met within some guidelines and considerations.

5.2 Future work

The necessity of simplification of the regression model was in part due to the long formulations that are usually applied in this field and in part due to the lack of experimental data to validate these formulations. The overall lack of information and development that is being done on added resistance in waves will certainly help further build the knowledge base and future works can rely on past experiences and more reliable data.

The regression model was constructed from the database which was lacking variety in terms of ship characteristics which made it overfit these parameters. More ships in the model's database would contribute to the linear regression model. An option of scaling these three hull forms could be adopted, however these would not characterize real or common ships. For this same reason, the validation of the formula using scaled hull forms could be made in more depth with ships whose parameters fall just outside the range described for the model.

As studied in the literature review several methods have been the focus of study for the added resistance in regular waves. Most of these methods have been developed during the seventies and eighties and nowadays most methods rely on an incomplete database of ships. Other methods that are appropriate to apply in the computation of the added resistance could be studied and developed with a basis on formulations developed through regression and machine learning methods. This will contribute to a more realistic approach to better estimate the added resistance in waves.

More complex models and the use of advanced software to customize models to the need of the user would highly contribute to the development of the regressions due to the high complexity of the physical phenomena behind the system. It is also important to educate students on regression techniques and their applicability in all fields of engineering, specifically in this case to the study of added resistance.

The foundation of this dissertation and of the accredited and qualified semi-empirical methods used to develop models and formulations to solve this problem are resting on experimental databases that have simply not been built to a necessary extent. The scarcity of test tank data for different headings, mainly over following waves causes some unusual behaviors in formulas that cannot be validated. This is also the case in the small wavelength region where the data is more difficult to obtain due to the low accuracy. Research and development teams working on this area will continue to invest in a public database to develop the field and be able to meet the demands of a growing industry.

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APPENDIX 1 – SIGNIFICANT PLOTS OF THE LINEAR REGRESSION MODELS

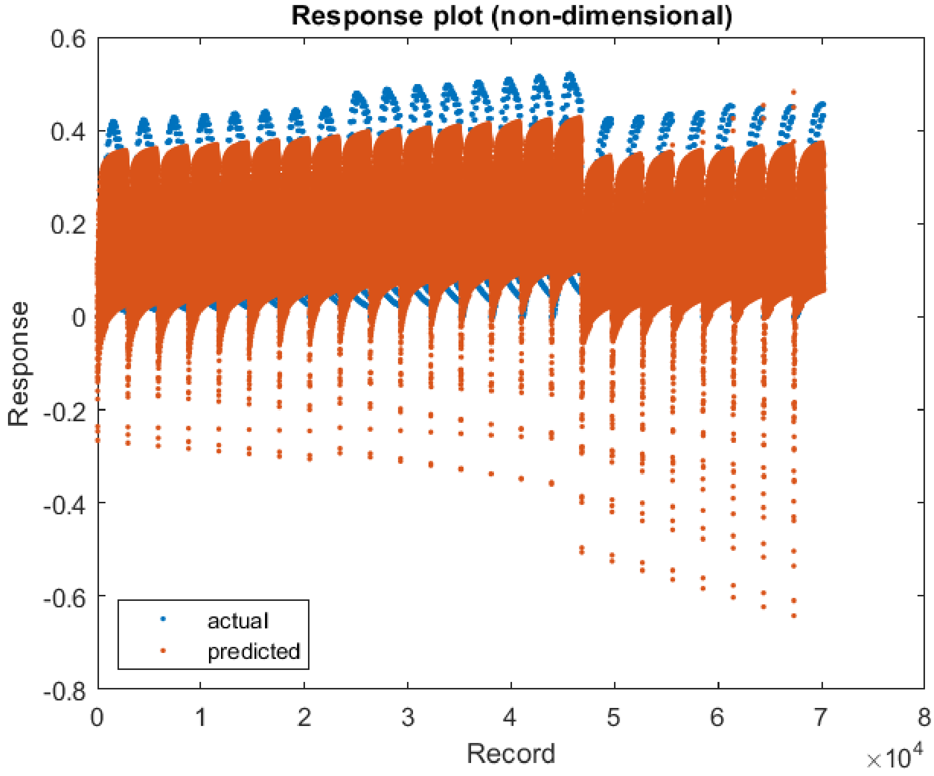

Figure 23 – Initial linear model predictions and recorded observations.

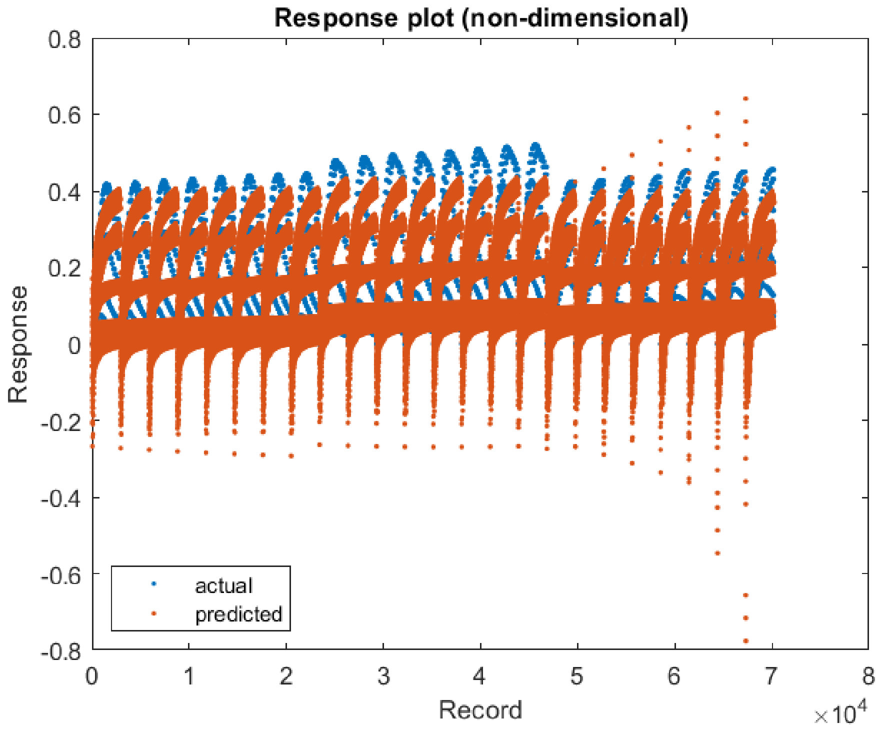

Figure 24 - Final linear model predictions and recorded observations.