An hydrofoil control system for the SR02 vessel in the scope of the Técnico Solar Boat Project

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***Abstract*—This paper focuses on the design, implementation and testing of a control system for the hydrofoil vessel SR02, develop under the Técnico Solar Boat project. In this project solar-powered vessels are built and therefore aim to be as light and efficient as possible. For this purpose it is intended to design and implement a system with *hydrofoils*, wings that are inserted underneath the boat becoming submerged and creating sufficient lift force for the hull not to touch the water. This way it is possible to reduce the drag of the boat and advance in a more energy efficient way.**

A control system is developed that can regulate the lift force generated by the *hydrofoils* and thus stabilize the vessel at a certain height in relation to the water line.

The controller analysed is based on an LQR (Linear Quadratic Regulator), where reference tracking, integral action, anti wind-up and a delta implementation are introduced. Once the controller has been designed, the best gains for the system are sought, both for roll control and for heave control.

***Hardware in the loop* tests are carried in the microcontrollers. The results of these tests are compared with the data from the simulations to verify that the implementation in the final system is well done.**

With all this the vessel will proceed to the water and the results will be analyzed. Based on the tests and their results, some improvements are suggested, tested and analyzed. Such as the implementation of a speed control system and an active reference on roll control.

***Index Terms*—Hydrofoils, LQR, Solar Boat, Solar Competitions**

I. Introduction

Técnico Solar Boat was created by Naval engineering students of Instituto Superior Técnico (IST) in 2014. The team has since expanded and is now a group of around 30 students from a wide range of undergraduate, MSc and Phd programs.

The main goal of the team is to work on the development of a solar powered boat to participate in worldwide university competitions.

During all the years of existence of the Técnico Solar Boat team there was always one goal in mind, to have the best, most efficient and fastest boat for competitions. In order to make this happen there needs to be a special feature on the boat: hydrofoils.

Hydrofoils allow a boat to get elevated over the water line in such a way that the hull doesn't touch the water. This is one way of reducing the drag forces exerted on the boat, thus allowing it to consume less energy for the same horizontal speed.

The control was planned to be done using a sensor to measure the boat's distance to water and another to measure roll, pitch, angular velocities and x, y, z accelerations, then small stepper motors would adjust the angle of attack according to the desired pitch, roll and height above the water.

This paper addresses the modeling and control of the SR02 solar boat using hydrofoils, obtaining a smooth and thus comfortable ride.

This paper's final goal is to get the SR02 vessel to smoothly ride on hydrofoils, so the model will be made, the controller will be designed, and after validation it will be implemented on the real boat.

II. State of the Art

One of the first successful hydrofoil boat was ridden in 1894 [1]. According to [1], the reason for not having seen earlier boats successfully using hydrofoils was due to the lack of suitable structural materials.

Since then, hydrofoils have been used in different applications, mainly in the military [2] and in commercial [3] boats. Some have been made for public transports, in Holland for example [2].

In the context of this project there are many different types of hydrofoils systems implemented. Some teams have surface piercing foils, others have fully submerged foils with electrical sensor based control, and some use mechanical control systems.

Electronic hydrofoil control systems have already been used in the military back in 1980, using gyroscopes, accelerometers, height sensors and hydraulic servo actuators [4].

There has been many different kinds of automatic control systems used. Initially, most were with mechanical control of the angle of attack. Then, manual control systems were used, there was a man-in-the-loop concept where someone used a joystick to control the vessel stability. Many systems are described in depth in [5].

The first electronic control known to have been used was around 1953 on a vessel called Lantern. After some time with the fall of prices for electronic parts, electronic systems started to be more dominant [5].

After concluding some research and exchanges with other teams it's possible to say that most of the electronic control systems that have been developed are based on PID controllers.

In this thesis we explore the use of a linear quadratic regulator (LQR), which basically allows us to use any variable that may have an influence on the parameter that needs to be controlled, heave.

III. Background

The planned designs for TSB's vessels have always been a system with 3 foils, 2 at the front and one at the rear at the bottom of the rudder, which also has the propeller incorporated. With these 3 foils there is enough lift to carry the boat out of the water and having them far from the center of mass creates a relatively stable system.

The hydrofoils used in this project were designed by students of Aerospace Engineering who are also part of the Técnico Solar Boat project. They used data from *airfoiltools* to design the foils considering the SR01 vessel, which is similar to the SR02. These foils have the NACA 63-412 profile.

The *airfoiltools* is a data base of known foil profiles where one can find the lift and drag coefficients of each one of them. These lift coefficients are then used to compute de lift and drag forces of the foils.

Having the lift coefficient for the wings to be used it was linearized for the expected used range of AoA, considering a speed of 7 m/s. This is the cruise speed of the SR02 vessel.

The resulting linear regression is described by

$$C_L = b_1 \cdot \alpha + b_0 \tag{1}$$

where b_0 and b_1 are constants.

The same process was done to get the drag coefficient of the foil. For the drag the approximation is not linear but can be a 6^{th} degree polynomial.

The resulting curve approximation is represented by

$$C_D = a_6 \cdot \alpha^6 + a_5 \cdot \alpha^5 + a_4 \cdot \alpha^4 + a_3 \cdot \alpha^3 + a_2 \cdot \alpha^2 + a_1 \cdot \alpha + a_0 \tag{2}$$

where $a_{1...6}$ are constants.

IV. Model

A model needs to have its references defined in order to have equations that are consistent. First we define two reference frames, one is the body frame, which has its origin at the center of mass of the vessel and is composed by the axis $\{X_b, Y_b, Z_b\}$. The second frame can be fixed anywhere on earth and will not move with the motion of the vessel, it is composed by the axis $\{X_i, Y_i, Z_i\}$.

For the body frame, the axis $\{X_b, Y_b, Z_b\}$ can be described as longitudinal, transversal and normal axis, respectively. The longitudinal axis is defined from stern to bow, the transversal from portside to starboard and the normal axis from top to bottom.

Figure 1 shows the 6 degrees of freedom (DOF) of a boat and the two reference frames, on the image called body-fixed and earth-fixed.

To model the behavior of the boat on hydrofoils the movements in *yaw* and *sway* will be ignored. *Sway* velocity will always be near zero, and all simulations will be considering the vessel going straight, so *yaw* constant. This simplifies the system to a 4 DOF model.

The forces that actuate on the boat when it's flying on the hydrofoils can be directly compared to the ones of an airplane. The main forces are the drag from the foil and the strut, the lift from the foil, the thrust from the propeller and the gravitational force. Figure 2 illustrates those forces on the SR02 vessel. The boat model designed is based on these forces.

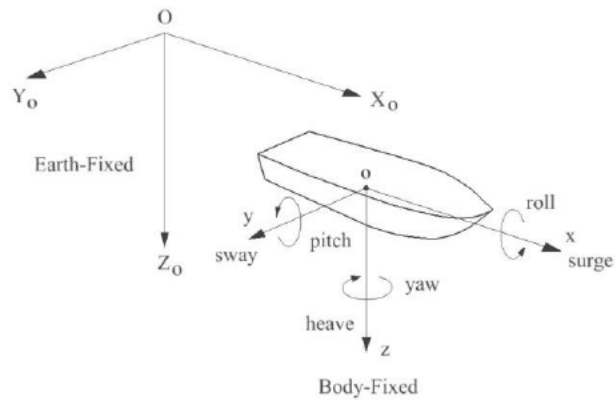

Fig. 1. The axis orientation and the movement definitions for boats. [6]

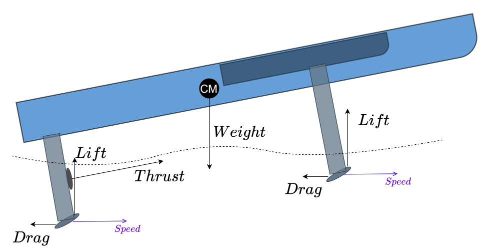

Fig. 2. Forces actuating on the boat when foilborne.

A. Equations

The lift and the drag equations are general equations that apply to hydrofoil lift just as they apply to airplane wings. Lift is represented by equation

$$L = \frac{1}{2} \cdot \rho \cdot u^2 \cdot A \cdot C_L \tag{3}$$

and drag by equation

$$D = \frac{1}{2} \cdot \rho \cdot u^2 \cdot A \cdot C_D \tag{4}$$

where A is the surface area of the foil, ρ is the density of the fluid surrounding the wing and u is the fluid velocity. C_L and C_D were defined in (1) and (2), respectively.

To the drag adds the drag force of the struts that support the foils. Equation (4) is used to compute this drag because they have foil like shapes.

Weight is computed based on the mass of the boat and g, earth's gravitational acceleration. Thrust is variable, and can be adjusted by the pilot.

Knowing all the forces that actuate on the foils and the boat, one can deduce the vertical acceleration, for heave, the horizontal acceleration, for surge, and the angular accelerations, for the pitch and roll angles.

Based on the references of figure 1 it's possible to illustrate all the forces on the XZ plane, then separate their projections on the body and inertial axis, as in figure 3 and figure 4.

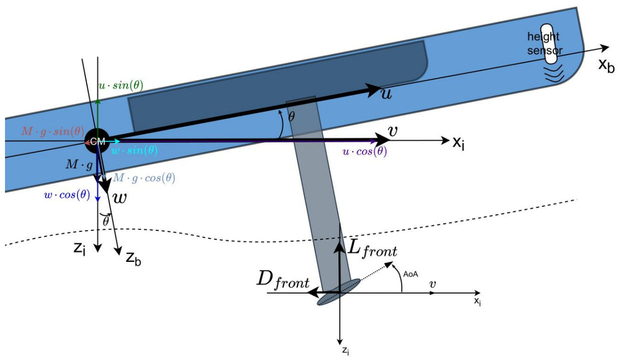

Fig. 3. Forces and velocities on the XZ plane and their projections.

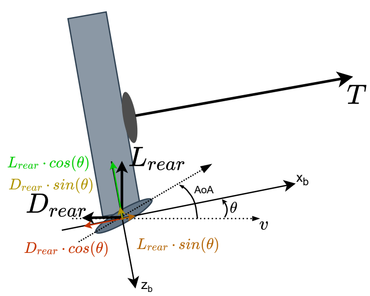

Fig. 4. Foil forces on the XZ plane, angle of attack and thrust force.

Having the projections of the forces it's possible to deduce the accelerations on the body axis, where $\dot{u}$ is defined as

$$\dot{u} = \frac{T - D_{rear} \cdot cos(\theta) - D_{front} \cdot cos(\theta)}{M} + \frac{L_{rear} \cdot sin(\theta) + L_{front} \cdot sin(\theta) - M \cdot g \cdot sin(\theta)}{M} \quad (5)$$

and $\dot{w}$, defined as

$$\dot{w} = \frac{-D_{rear} \cdot sin(\theta) - D_{front} \cdot sin(\theta)}{M} + \frac{-L_{rear} \cdot cos(\theta) - L_{front} \cdot cos(\theta) + M \cdot g \cdot cos(\theta)}{M} \quad (6)$$

Finally, the angular accelerations for roll and pitch need to be calculated. These accelerations are based on the lift and drag forces and the moments of inertia, I. The distances between each foil and the center of mass also need to be considered for the momentum.

With that it's possible to write the final equations for pitch acceleration $\ddot{\theta}$ as

$$\begin{aligned}\ddot{\theta} =& \frac{L_{front} \cdot sin(\theta) \cdot d_{z_{front}} + L_{front} \cdot cos(\theta) \cdot d_{x_{front}}}{I_y} + \\ &+ \frac{D_{front} \cdot sin(\theta) \cdot d_{x_{front}} - D_{front} \cdot cos(\theta) \cdot d_{z_{front}}}{I_y} + \\ &+ \frac{-D_{rear} \cdot sin(\theta) \cdot d_{x_{rear}} - D_{rear} \cdot cos(\theta) \cdot d_{z_{rear}}}{I_y} + \\ &+ \frac{L_{rear} \cdot sin(\theta) \cdot d_{z_{rear}} - L_{rear} \cdot cos(\theta) \cdot d_{x_{rear}} + T \cdot d_{z_{rear}}}{I_y}\end{aligned} \quad (7)$$

and roll acceleration $\ddot{\phi}$ as

$$\ddot{\phi} = \frac{L_{f_{left}} \cdot d_{y_{front}} - L_{f_{right}} \cdot d_{y_{front}}}{I_x} \quad (8)$$

B. Model

Having all the kinematic and dynamic equations of motion it's possible to describe the development of a simulation model in *Simulink*.

Before presenting the model some assumptions taken should be stated.

- The model is only valid for when the boat is foilborne,
- Only motion in the XZ plane is considered,
- The C_L and C_D of the foils are valid for a certain range of AoA and speeds,
- Yaw and sway are ignored,
- The surface area of each foil is assumed to be constant and independent of the AoA or roll of the vessel.

The model, here shown in figure 5, consists of a block that contains all the equations from section IV-A, integration blocks, and a block to get the x and z speeds in the inertial axis X_i, Z_i.

The block used to calculate the dynamics and kinematics of the vessel calculates the lifts and drags of every foil and then uses these results to calculate the roll, pitch, x and z accelerations, all this with the equations described previously. It takes the foils' angles as inputs, and also thrust, pitch and speed. Pitch is fed back into this block to correct the AoA of the foils. The real AoA, α, is defined as

$$\alpha = act + \theta \quad (9)$$

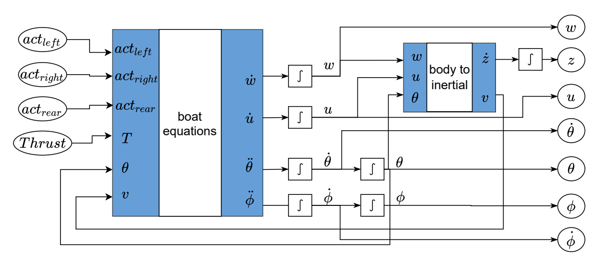

Fig. 5. Model architecture of the boat on foils.

C. Trimmed model

Having the model, it is possible to trim it in order to find an operating point of equilibrium. Trimming a model involves searching for a set of inputs and states that meet any specified conditions.

This is defined as a solution to the model's equations where all variables that define the system are time-invariant. Finding such equilibrium points will allow to find an ideal steady state about which the model will be linearized and the controller will be tuned

From the equations (5), (6), (7) and (8), one can understand some balances that must occur for an equilibrium to be reached.

For instance, assuming $\theta = 0$ and $\dot{u} = 0$, (5) results in

$$T = D_{rear} + D_{front} \tag{10}$$

then with $\theta = 0$ and $\dot{w} = 0$, from (6) we get

$$M \cdot g = L_{rear} + L_{front} \tag{11}$$

with $\theta = 0$ and $\ddot{\theta} = 0$, from (7) results

$$\begin{aligned} &T \cdot d_{z_{rear}} + L_{front} \cdot d_{x_{front}} = \\ &L_{rear} \cdot d_{x_{rear}} + D_{rear} \cdot d_{z_{rear}} + D_{front} \cdot d_{z_{rear}} \end{aligned} \tag{12}$$

and finally, with $\ddot{\phi} = 0$, from (8) we get

$$L_{f_{left}} = L_{f_{right}} \tag{13}$$

To find solutions of the system an iterative algorithm is used, the gradient descent algorithm. Gradient descent algorithm is used to find local minima of a function. It computes the gradient and takes steps proportional to the negative of that gradient towards the local minima.

To begin the search for the steady state the vectors $\boldsymbol{u_0}$ and $\boldsymbol{x_0}$ are set to

$$\boldsymbol{u_0} = \begin{bmatrix} act^0_{left} & act^0_{right} & act^0_{rear} & T_0 \end{bmatrix}^T \tag{14a}$$

$$\boldsymbol{x_0} = \begin{bmatrix} \phi_0 & \dot{\phi}_0 & \theta_0 & \dot{\theta}_0 & u_0 & w_0 & z_0 \end{bmatrix}^T \tag{14b}$$

where all values are the constants considered as the initial conditions.

Another restriction to be defined is the minimum and maximum values of each of these variables, these limits must be defined considering the real system limitations.

After having these conditions the model can be trimmed. There are a few ways to trim the model since we can freely choose whether or not to fix the rear foil AoA, and whether we want to specify a certain speed or a certain thrust. However, if we choose to fix act_{rear} the pitch angle θ will be zero only for a specific u speed. For other u speeds, the resulting pitch θ will vary.

Being able to trim the model with a fixed rear AoA is important in the context of this project. The rear foil is attached to the propulsion column and for now it's not electronically controllable.

The resulting steady state points with different fixed parameters are shown in tables I and II.

In table I we have the AoA needed in order to have a steady state for different speeds, with a fixed pitch $\theta = 0$. The table also shows the resulting thrusts needed to achieve these states.

From table II we can see the steady states for different fixed values of rear AoA. These have all been found for a u speed of 7 m/s. As expected, when act_{rear} raises, then θ reduces to compensate the generated lift.

act_{front} $[deg]$	act_{rear} $[deg]$	T $[N]$	u $[m/s]$
5.9	5.5	115	6
3.5	3.2	144	7
1.9	1.7	186	8
0.8	0.7	235	9
0.1	0	289	10
-0.5	-0.6	350	11
-0.9	-1	416	12
-1.3	-1.3	488	13

TABLE I
STEADY STATES VALUES OF FRONT AND REAR FOIL'S AOA AND THRUST FOR DIFFERENT u SPEEDS

act_{front} $[deg]$	act_{rear} $[deg]$	T $[N]$	θ $[deg]$
-0.8	-1	145	4.2
0.2	0	144	3.2
1.2	1	144	2.2
2.2	2	144	1.2
3.2	3	143	0.2
4.2	4	144	-0.8

TABLE II
STEADY STATES VALUES OF THRUST AND PITCH FOR DIFFERENT FIXED REAR FOIL AOA AND A u SPEED OF 7 M/S

D. Linearization

The nonlinear continuous-time space state model is defined as a set of inputs, outputs and state variables related by first order differential equations,

$$\begin{aligned} \dot{x}(t) =& f(x(t), u(t), t) \\ y(t) =& g(x(t), u(t), t) \end{aligned} \tag{15}$$

where $x(t)$ is the state vector, $u(t)$ the input vector and $y(t)$ the output vector.

A linear space state model is represented by the equations

$$\begin{aligned} \dot{\mathbf{x}} =& A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} =& C\mathbf{x} + D\mathbf{u} \end{aligned} \tag{16}$$

The linearized model is valid in a small region about a given operating point. In the present case, the physical plant considered is the hydrofoil vessel. The full nonlinear model described before will be linearized about trimming (equilibrium) conditions defined by the variables listed in tables I and II.

The linearized space state model variables are defined as

$$\begin{aligned} \delta\mathbf{x} &= \mathbf{x} - \mathbf{x}_{steady} \\ \delta\mathbf{y} &= \mathbf{y} - \mathbf{y}_{steady} \\ \delta\mathbf{u} &= \mathbf{u} - \mathbf{u}_{steady} \end{aligned} \tag{17}$$

now replacing in (16) the linearized space state model is

$$\begin{aligned} \delta\dot{\mathbf{x}} =& A\delta\mathbf{x} + B\delta\mathbf{u} \\ \delta\dot{\mathbf{y}} =& C\delta\mathbf{x} \end{aligned} \tag{18}$$

where the matrices A, B and C are the Jacobians of the model.

This model will be separated in two, one that describes heave and pitch of the vessel and one that describes roll.

For each one of them, the foils' input can be reduced as one single input. For the heave and pitch model the front foils take the same common angle of attack, and for roll it's one differential angle.

Thrust is present in the model but it is always manually controlled by the pilot. Thus, for the model used to design the controller it can be removed.

Consider the operating condition determined by a desired speed of 7 m/s from table I, this is the expected endurance speed of the vessel. For 7 m/s, we get u_{steady}, x_{steady} to be

$$u_{steady} = \begin{bmatrix} 3.5 & 3.5 & 3.2 & 144 \end{bmatrix}^T \tag{19a}$$

$$x_{steady} = \begin{bmatrix} 0 & 0 & 0 & 0 & 7 & 0 & -0.3 \end{bmatrix}^T \tag{19b}$$

Considering the reduced model with $\mathbf{x} = \begin{bmatrix} \theta & \dot{\theta} & u & w & z \end{bmatrix}^T$ and $\mathbf{u} = \begin{bmatrix} act_{front} \end{bmatrix}$ the matrices for the heave and pitch model are

$$\begin{aligned} A_h &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1.38 & 0 & -4.49 & 0 & 0 \\ 4 \cdot 10^{-3} & 0 & -0.19 & 0 & 0 \\ -1.50 & 0 & -2.8 & 0 & 0 \\ -0.12 & 0 & 0 & 1 & 0 \end{bmatrix} \\ B_h &= \begin{bmatrix} 0 \\ 13.53 \\ -2.3 \cdot 10^{-3} \\ -0.5 \\ 0 \end{bmatrix} \\ C_h &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{20}$$

Then considering $\mathbf{x} = \begin{bmatrix} \dot{\phi} & \phi \end{bmatrix}^T$ and $\mathbf{u} = \begin{bmatrix} act_{left} & act_{right} \end{bmatrix}^T$ the matrices for the roll model are

$$A_r = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B_r = \begin{bmatrix} 0 & 0 \\ 124.1 & -124.1 \end{bmatrix} C_r = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{21}$$

and for the differential control we use $\mathbf{u} = \begin{bmatrix} act_{dif} \end{bmatrix}$ with B matrix as

$$B_r = \begin{bmatrix} 0 \\ 124.1 \end{bmatrix} \tag{22}$$

Now two models define the full behaviour of the variables that we wish to control. To make sure it's controllable we verify the controllability of the system. For that the controllability matrix C_o and the rank of the controllability matrix need to be known.

The controllability matrix is defined as

$$C_o = \begin{bmatrix} B & AB & A^2B & ... & A^{n-1}B \end{bmatrix} \tag{23}$$

where n is the dimension of the model. For the heave model $n = 5$ and for the roll model $n = 2$.

Given that, the model is controllable if the matrix C_o has full rank, which is $rank(C_o) = n$. Having full rank also means that the matrix does not have linearly dependent lines.

For the heave model we get

$$rank(C_o) = 5 \tag{24}$$

and for the roll model we get

$$rank(C_o) = 2 \tag{25}$$

Having control over the front hydrofoils and since they can influence pitch, height and roll, the result is as expected, the models are controllable.

V. Controller

A. Linear Quadratic Regulator

Linear Quadratic Regulator (LQR) is a modern control technique [7] for linear space state models in the form

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} &= C\mathbf{x} + D\mathbf{u} \end{aligned} \tag{26}$$

with a cost function defined as

$$J = \int_0^\infty (\mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}) dt \tag{27}$$

where Q is a real symmetric positive-definite matrix and R is a semi positive-definite matrix that penalise the state variables and the inputs, respectively.

Under suitable stabilizability and detectability conditions, there is a unique feedback stabilising solution to the minimisation problem obtained from the unique positive definite solution P to the algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{28}$$

from which the optimal state feedback gain K is given by

$$K = R^{-1} B^T P \tag{29}$$

which will itself set the gains for the state feedback law as

$$\mathbf{u} = -K\mathbf{x}. \tag{30}$$

Figure 6 shows the block diagram for a system with a simple LRQ feedback control loop.

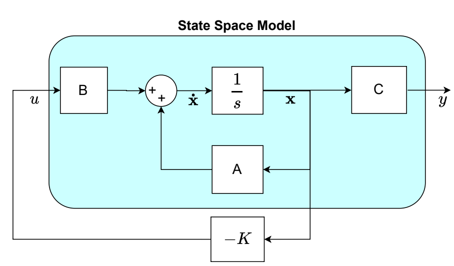

Fig. 6. Classic LQR block diagram.

B. Reference following

In this system the output of the heave model needs to follow a reference value. Wherever a non-zero value is desired for a certain state variable, that state variable can be replaced by

$$x - x_d \tag{31}$$

where x_d is the desired value for the state variable x.

C. Integral action

For any controller based on a simplified model there will be uncertainty on the accuracy of the model, and even with this the goal is to have zero steady state error between the system's output and its reference. For this purpose, an integral action can be added to the controller which will guarantee the desired reference will be followed and the error will always tend to zero [8].

The idea of integral action is to add a new state, here x_i, that will be the integral of the error between the desired reference and the actual value of the state, with dynamic described by

$$\dot{x_i} = ref - z. \tag{32}$$

With this new state the initial system from equation 16 is extended to

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x_i} \end{bmatrix} = \begin{bmatrix} A\mathbf{x} + Bu \\ ref - z \end{bmatrix} = \begin{bmatrix} A\mathbf{x} + Bu \\ ref - C\mathbf{x} \end{bmatrix} \tag{33}$$

so the extended model with new state x_i is

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x_i} \end{bmatrix} = \begin{bmatrix} A & \mathbf{0} \\ -C & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} ref \tag{34}$$

and the new controller feedback is

$$\mathbf{u} = -\mathbf{K}\mathbf{x} + k_i x_i \tag{35}$$

where k_i is the feedback gain for the integrated output error.

Having this new model now the weight matrix Q also needs to be adapted due to the new state

$$Q_i = \begin{bmatrix} Q & \mathbf{0} \\ \mathbf{0} & q_i \end{bmatrix} \tag{36}$$

D. Anti wind-up

When the controller calculates the input value necessary to go towards the desired reference output it should always stay within the limits of the actuator. This creates non-linearities that can create problems in the control loop.

When the controller sets the input of the system as a value above the limit of the actuator an effect called windup happens in the integrator of the controller.

When the windup effect happens the performance of the system is deteriorated with higher overshoot and longer settling times [9].

To solve this, the difference between the saturated and unsaturated signals is fed back into the integrator through the integral state variable x_i. This allows the saturation to discharge the integrator as if the integration was turned off until the input comes back to an acceptable range. With this the integral state variable is now

$$\dot{x_i} = ref - z - (u - \overline{u}) \tag{37}$$

and the new system input computed by the controller is

$$u = -\mathbf{K}\mathbf{x} + k_i \int_0^t (ref - z - (u - \overline{u}))dt \tag{38}$$

E. Delta implementation

There is an improvement that can be made to this controller which consists in having the integrator after all feedbacks, so not only integrating the error signal and saturation but all feedbacks. This allows for a better performance when a controller is designed based on a linearization of a non linear model, such as this controller, as it eliminates any biases present in the measurements of the states. This is know as delta implementation [10].

Since we linearized the model around an equilibrium point in section IV-D, this controller is made to control the system around this steady state. If the steady state has non zero state values, when applied to the non-linear model, the equilibrium points have to be subtracted from the state measurements before being multiplied by the gains K. Given that, the feedback equation becomes

$$u = -\mathbf{K}(\mathbf{x} - \mathbf{x_{steady}}) + k_i \dot{x_i} \tag{39}$$

where $\mathbf{x_{steady}}$ is the state vector at the equilibrium point used for the linearization. Delta implementation eliminates this part.

This method also allows to easily decide the initial input u of the system by setting the initial condition of the integrator. Without this implementation there can be larger initial oscillations at the start of the controller.

To integrate all feedback signals the solution is to derivate the signals that were not originally being integrated. This way we can implement the scheme seen in figure 7, and we get our final controller.

The integration was moved just before the saturation block and the feedback signals $\mathbf{K}\mathbf{x}$ goes through a derivative block.

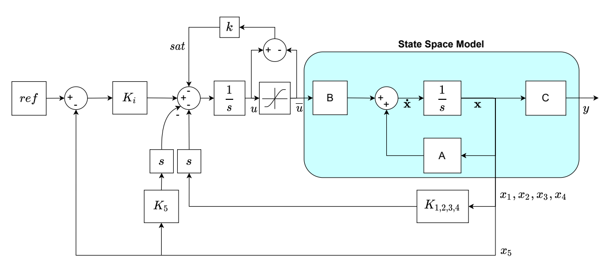

Fig. 7. LQR model with delta implementation on heave.

With this controller and testing different simulations the final gains computed are

$$K_h = \begin{bmatrix} 4.7 & 0.6 & 6.9 & -8.4 & -13.5 & -10 \end{bmatrix} \tag{40}$$

with $K_{sat} = 1$

We can also analyse the frequency response of the closed loop linear heave and pitch system with the bode plot from figure 8. The cutoff frequency is at $\omega = 1.0825\ rad/s$, where the magnitude drops below -3 dB. Having a zero gain for low frequencies shows us the reference will be followed.

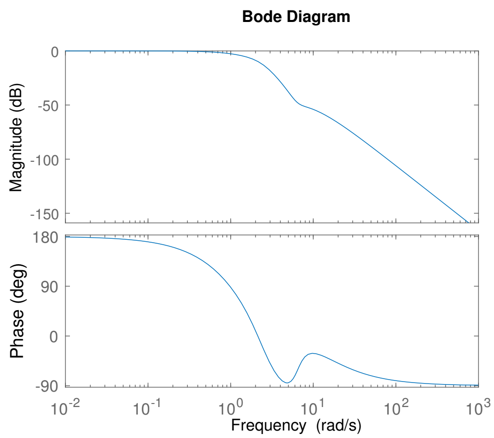

Fig. 8. Bode plot of the closed loop linear system for heave and pitch.

F. Final controller vs simple LQR

Now with the gains computed another simulation was done to see the influence of a perturbation on the velocity read by the sensor of the boat and compare the first LQR with the last with delta implementation. A bias of $0.5m/s$ was applied to the velocity, trying to simulate a case where sea current could be present. Figure 9 shows how the first LQR with simple reference following reacts when this bias appears at $t = 10s$. And figure 10 the reaction of the LQR with delta implementation to the same perturbation. As we can see the final controller which has an integrator has a deviation of less than $10cm$ and quickly goes back to the desired reference, while the simpler controller stabilizes at a completely different value.

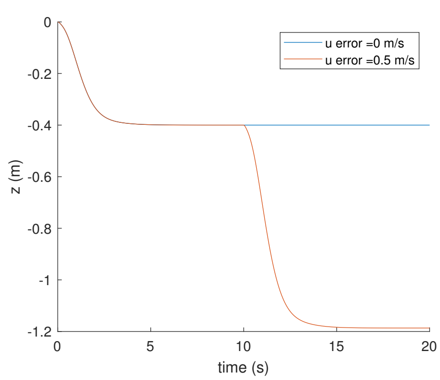

Fig. 9. Front foil step response with LQR control and simple reference following, with error on speed sensor value at $t = 10s$.

G. Controller for roll

All the simulations and equations mentioned above were done for the heave and pitch controller, which is very complex

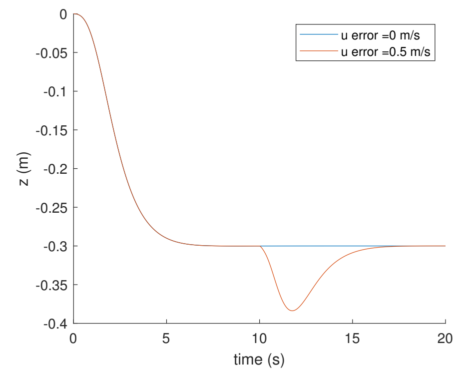

Fig. 10. Front foil step response with LQR control as in figure 7 (delta implementation) with error on speed sensor value at $t = 10s$.

and includes non zero reference following. Because of that a lot more focus and work was done on that controller compared to the roll controller. The roll controller only uses 2 state variables as inputs, roll and roll rate, and has to maintain the roll as close as possible to zero. Besides this, all the equations are the same, and the simulation to find the optimal gains were done in the same way.

After doing all the simulations, the gains computed for the differential output of the controller are

$$K_r = \begin{bmatrix} 2 & 0.25 & 1 \end{bmatrix} \tag{41}$$

and for the saturation, $Kr_{sat} = 1$.

H. Common and differential modes

As said in the linearization section IV-D the control is separated in two modes, common and differential. Common mode will control heave and pitch of the vessel by actuating with the same value on each front foil. Differential mode will actuate with symmetrical values on each front foil to maintain the vessel's roll stable.

Having those modes act_{left} is defined as

$$act_{left} = act_{com} + act_{dif} \tag{42}$$

and act_{right} as

$$act_{left} = act_{com} - act_{dif} \tag{43}$$

where act_{com} and act_{dif} are the common and differential deflections, respectively.

VI. Simulations

After having the model and the controller designed, a wide variety of simulations were done using the non-linear model of the hydrofoil vessel. The first simple simulations were done to see if the controller would hold the vessel at the steady state for which the controller was designed. Afterwards, simulations were done with variations in thrust, then with added noise extracted from real data from the sensors that have already been used in the vessel.

The simulations were mostly done looking at heave and pitch, then some more simulations were done for the roll analysis.

Using noisy data, the simulation in figure 11 was obtained. As we can see the vessel's heave stays close to the reference, but it oscillates about 3 or 4 centimeters over and under the reference. Pitch (as θ) on the image can be seen very stable.

Around time 15s and 16s there were outliers on the used data, they can be seen reflected on the common mode output of the controller, but it didn't jeopardize the vessel's heave.

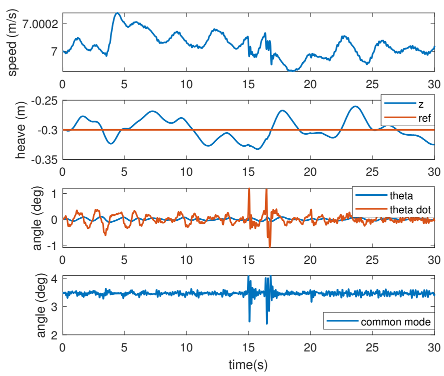

Fig. 11. Simulations with noisy data to simulate reality.

VII. Implementation

A. Hardware

The system is composed of

- 3 stepper motors with optical encoders, 2 for the front foils and one for the rear foil (for future use)
- 3 motor controllers
- one Attitude and Heading Reference System sensor
- one Teensy USB-based microcontroller development board
- a PCB which connects all sensor and actuators to the Teensy microcontroller development board
- one GPS antenna
- one ultrasonic sensor
- one RaspberryPi for remote firmware updates

The Teensy board is where all the code is implemented. This includes the LQR controller, the communication with the motor controllers, the AHRS, the ultrasonic sensor and the pilot's dashboard.

Since the ultrasonic sensor will be located at the bow and the controller considers heave at the center of mass of the vessel, a small correction had to be done considering the boat's pitch. The sensor is at about 3 meters ahead towards the bow compared to the center of mass of the vessel, so the compensation for state z was computed as

$$z = US - 3 \cdot sin(\theta) \tag{44}$$

where US is the sensor reading.

B. Hardware in the loop test

After programming the microcontroller with all the code necessary for the LQR controller, test data were generated using *Simulink*. These data were fed into the microcontroller and the outputs generated from the *Simulink* and the microntroller were then compared.

Figure 12 shows the result of these tests. The data follows the same behavior on both platforms even though there is a small bias, but since the test on the microcontroller didn't have feedback this could be expected.

Since the code for the roll controller had the same structure as the heave controller this test was only done for the heave controller and considered acceptable for both controllers.

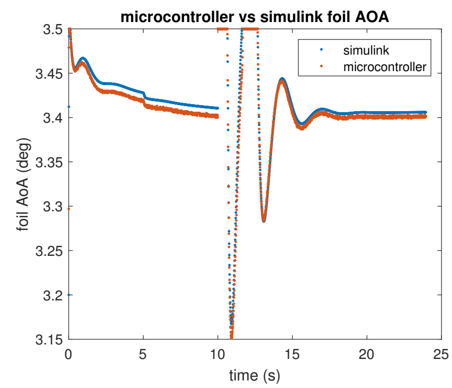

Fig. 12. Comparison between LQR controller output on the microcontroller and *Simulink* for heave control.

VIII. Results

A. Roll controller tests

After having everything implemented tests were done with the vessel in the water.

Testing showed that the gains had to be larger than what we initially used. But even with higher gains we couldn't do tight turns. So we added one more degree for the differential angle.

Our final gains for the roll controller were

$$K_r = \begin{bmatrix} 0.8 & 0.25 & 1 \end{bmatrix}. \tag{45}$$

Figure 13 shows the results of these gains for the roll controller, while doing a relatively tight loop. The data shows that the boat was going close to its cruise speed, maintaining a steady roll.

B. Heave and pitch controller tests

The heave and pitch control tests were very interesting and made us progress along the different times we had test days. We had about 10 full days of tests, spread over 3 months, dedicated to this.

When the vessel took off the first times there was a speed problem. When a certain speed was reached the boat took off and kept accelerating until there was too much lift on the rear foil and the vessel would "bow dive". This can be seen very clearly in the images of figure 14.

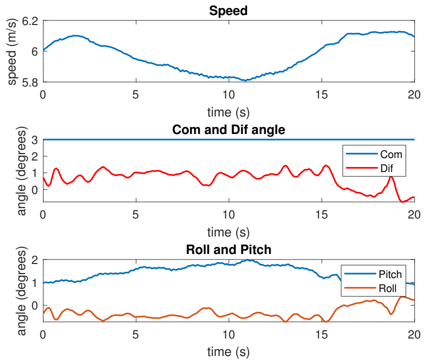

Fig. 13. Roll controller active and turning at cruise speed.

(a) Flight just after takeoff and before "bow diving". (b) Moment where the boat "bow dives".

Fig. 14. Sequence where the boat can be seen flying and then "bow diving".

Because of this we implemented a simple PID that would adjust the power to the motors automatically to maintain a constant speed.

To automatically adjust the power a PID was used between the throttle and the motor controller. The throttle's command was now interpreted as the desired speed, which was compared to the real speed of the boat and the PID computed the adequate power to send to the motor controller.

A library for the used microcontrollers was used and implements the PID controller that can be seen in figure 15. This library uses a different method to prevent wind-up effect. It simply limits both the output, and the integral term to the limits chosen by the user [11].

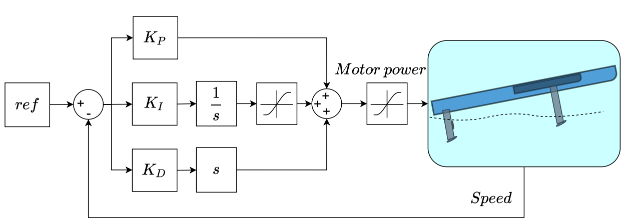

Fig. 15. PID feedback loop that was implemented to keep a constant speed.

To find the best PID gains we first tried gains that were sound to use considering the order of magnitude of both the power command and the desired speeds. After implementing the PID and running quick tests on dry land we tested the PID and fine tuned the gains. After a few runs the gains were adjusted to

$$K_P = 35 \qquad K_I = 5 \qquad K_D = 15. \tag{46}$$

With this strategy we obtained the results from figure 16. We can see that the speed stays very close to the desired speed, with some oscillations. Furthermore, by looking at the height and pitch data we can see that the speed oscillations match the ups and downs of the boat. These ups and downs are due to the fact that the common mode angle is very close to its upper limit, and so sometimes the lift at the front is not sufficient.

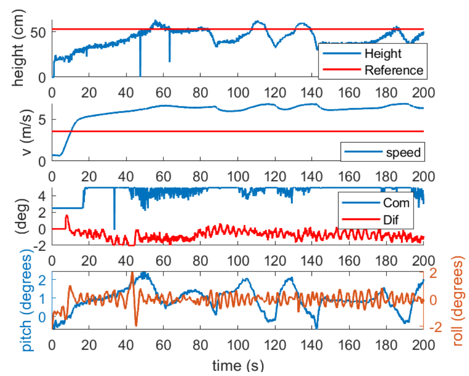

Fig. 16. Flight data with PID speed control.

After all the implementations and tests the Técnico Solar Boat team did a tour of Portugal, Odisseia 2020, to show the vessel to the public. Some more tuning was done that allowed the vessel to stay foilborne for periods of 10 minutes non-stop, when the wheater conditions were good. Figure 17 shows the data from one of those moments.

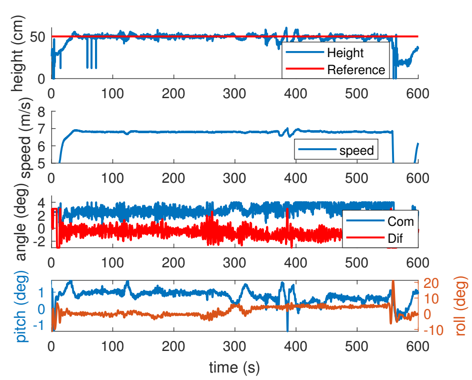

Fig. 17. Long flight during the Odisseia 2020 event.

C. Active roll reference

When we managed to get reasonably stable flights we started to test the limits of curvature.

Because the roll controller would try to maintain the boat level the wind could get under the side panels that support the solar modules. This triggered the idea of using a non-zero roll reference. Changing the roll reference could allow the boat to lean towards the inside of the curve.

To implement an active roll reference we need to know when the boat is turning. For this, and to not have to add sensors to the boat, we can use the yaw rate, ψ, that comes from AHRS sensor that is present on the vessel. To adjust the sensitivity of the reference the yaw rate is multiplied by a gain, as in equation

$$ref = K_{ref} \cdot \psi \tag{47}$$

Figure 18 shows the adapted feedback loop for the new addition on the roll controller.

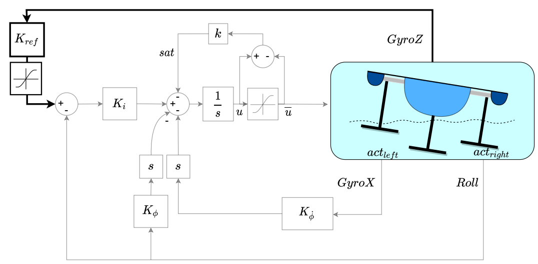

Fig. 18. Upgrade done on the roll controller with yaw rate (here GyroZ) fed back to the reference.

Before implementing this on the vessel, real data was used to adjust the gain K_{ref}. Simulations were done to see if the controller would follow the reference.

The results of the simulation can be seen in figure 19. The roll can be seen following the reference very closely with the gains previously chosen in (45).

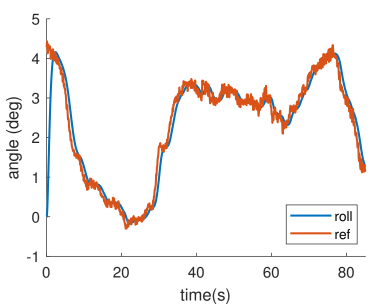

Fig. 19. Active roll reference simulation with filtered yaw rate as reference.

With the simulation done to validate the control this was tested on the vessel. After some trials the gain was set to

$$K_{ref} = 0.8 \tag{48}$$

and the maximum roll set to 5 degrees.

This control worked well and can be seen on figure 20. The data shows the roll reference being followed while doing loops starting at time 300s.

IX. Conclusions

A controller for the deflection of hydrofoils on a solar powered boat was designed, simulated and successfully tested.

The controller was based on an LQR with some improvements. First a reference following for heave was implemented, then integral action, anti wind-up, and finally a delta implementation. All these improvements were done to make the controller as robust as possible. Having the full implementation of

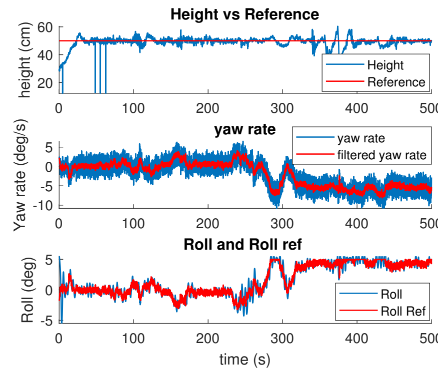

Fig. 20. Real data with active roll reference implemented on the vessel and doing loopings at cruise speed.

the controller, it was compared to the classical LQR controller and proven to be better at following a reference when a bias was applied to one of the state variables. The controller was then tested by simulating close to real conditions in *Simulink*.

The results were positive and proved that the vessel was well designed. Some limitations were found on the mechanical part of the hydrofoils' deflection mechanism, however the controller worked on the current conditions of the vessel.

For future work an improved mechanism for the deflection of the foils could allow to test the vessel at higher speeds and in conditions with more waves.

Another major improvement to be made is to have a controllable rear foil and implement the controller for such vessel.

Finally, the model could be improved to consider movement in yaw and sway to simulate the vessel turning.

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