Finite burn losses in spacecraft maneuvers revisited

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Abstract

In this work, an evaluation of losses due to gravity and steering is done for high thrust spacecraft maneuvers, which are normally approximated as instantaneous maneuvers on preliminary studies. For this purpose an optimization method was used, the so called direct shooting. Three steering cases were studied: constant thrust direction; constant rotation and thrust aligned with velocity. Then, the various steering cases are studied in function of the specific impulse and thrust over weight. The case with the least performance is the case of constant thrust direction, as expected, while the results obtained show some surprises, which are discussed. The results are also compared to a known approximation of the literature where the losses are upper bounded. At the end as an alternative to direct maneuvers, it is studied the consequences of multiple apogee raising maneuvers to minimize the gravity losses.
Keywords: Gravity losses, Multiple maneuvers, Direct shooting

1. Introduction

1.1. Objectives and Motivation

In this thesis the problem of determining gravity losses for a generalized spacecraft maneuver is analysed. Heuristics that estimate these losses are reviewed to confirm their validity by comparison with numerical calculations and a procedure to minimize losses with a multi-impulse maneuver is presented.

1.2. Spacecraft maneuvering

In preliminary mission design a first approximation of the trajectories must be determined, while the spacecraft hardware is yet to be completely developed. Since high accuracy is not required, some simplifications and assumptions can be used. These assumptions involve simplifying orbital maneuvers to determine a delta-v budget, in which an error is introduced that must be evaluated and taken into account [24]. It is important to have good initial guesses for mission design, as better approximations can yield faster convergence of the orbit design and other systems such as the propulsion system.

1.3. Types of maneuvers

Orbital maneuvers can be divided in two types: high thrust maneuvers, which typically last a few minutes and are eventually approximated as instantaneous, and the so called continuous thrust maneuvers, where the thrust is very small but the thrusters are highly efficient.

The high thrust maneuvers are produced by chemical energy, where the engines can be solid, liquid (bipropellant or monopropellant) or hybrid. Other options, such as nuclear thermal engines, which are being studied but still have not been used. Typically, these engines perform maneuvers such as: Hohmann transfers, apogee raisings, apogee maneuvers or interplanetary injections. Continuous thrust maneuvers are propelled by electric energy (for example: ion gridded thrusters, arcjets, resistojets and others) generally for long maneuvers where the spacecraft are small and higher efficiency is important. This work focus on finite high thrust maneuvers produced by chemical thrusters.

1.4. Impulsive Approximation

In preliminary mission analysis, high thrust maneuvers, are often approximated as an instantaneous change in the velocity. This is convenient because it separates the problem of the maneuver from the problem of specifying the spacecraft, but implies an error, usually fairly small, as the actual burns typically occur only during a small part of the orbital period [24]. This approximation considers that the spacecraft changes from one state to another instantly, simply connecting two ordinary two-body problems (another approximation), making the problem description simple, with only two orbits and a change of the velocity vector in a point in space, Δv.The calculated Δv can be used to estimate the propellant mass needed for a given thruster and payload mass [4].

Nevertheless, the impulsive approximation is only valid to a certain degree. The burn always occurs in a finite interval of time and the real maneuver is different from the instantaneous one. The Δv from both is therefore different and so will be the predicted propellant for the maneuver.

1.5. Literature review

The real finite burn maneuvers can be determined numerically or analytically with an optimization method that can be direct or indirect.

In [11] it is theorized the modelling, design and optimization of finite burn maneuvers for a generalized trajectory. The issues associated with controlled engine burn maneuvers of finite thrust magnitude and duration are presented in the context of designing and optimization for a wide class of finite thrust trajectories. Although the link between a theoretical optimal thrust solution and its feasibility is debatable, as it depends on each specific spacecraft system design, the state of the art optimization method can be considered the optimal numerical solution for a finite-burn maneuver. It has been implemented in Copernicus [10], a trajectory design and optimization system, that unfortunately is not available to the wide public.

On the other hand, an open source software [7], General Mission Analysis Tool, developed by NASA, private industry, public, and private contributors can be used to simulate the environment. However the results are subject to the tool available numerical and optimization methods (shooting methods) and to the weak costumer support it has. Nevertheless, some representative results can be obtained using this software and can be used for code validation.

A Matlab script of optimal Finite-burn for interplanetary injection from Earth orbit [5] is used as the basis for the main script computing the finite burns. The original script uses SNOPT optimizer [6] to compute optimal thrust angles for given segments of the burn via direct shooting. The modified version, uses different settings of thrust and different optimization constraints.

Analytic approaches can be used, but most studies determine a trajectory under some assumptions that limit the application of the final expressions, which make them unhelpful for this work.

For example, in [25] expressions that describe the trajectory of a high thrust maneuver are presented under the assumption that the thrust's direction is normal to the focal radius of the trajectory and the focal radius' change is small during the thrusting time interval. The expressions obtained are hardly useful for gravity loss analysis as the radial distance variation is considered to be small and the steering policy is adequate for thrust parallel to the LVLH frame, but nor for the types of maneuvers considered in this work.

Finite-thrust escape and capture trajectories are considered from either circular or elliptic orbit and the efficiency and certain parameters are determined [23]. The efficiency penalty due to finite-thrust is solved by applying a correction factor to the impulsive velocity increment. It considers tangential steering (thrust vector parallel to the velocity vector) and numerically integrates the maneuvers. The results are shown graphically for a wide range of jet velocity values and initial dimensionless acceleration and can then be used to apply to any particular similar case of interest — a specific planet, parking orbit, propulsion system and hyperbolic velocity. It is also shown that the difference between tangential steering and optimal steering is negligible. Nevertheless, as mentioned earlier, the specific constraints of the system are not always compatible with tangential steering, and this type of maneuver is not always optimal.

An intuitive analysis on the finite burn losses in which an analytic formula is determined for computing the losses is done in [14]. It considers that the finite thrust steering is of constant angular motion. It can be used for a general impulsive maneuver as long as the burn time is short enough. It must be mentioned that the expression needs numerical experimentation, which has not been found in the literature.

2. High thrust maneuvering

In this chapter, we will describe how maneuvers can be determined analytically and numerically and how spacecrafts perform them.

2.1. Problem definition

Real maneuvers take some time and while the burn is occurring the spacecraft goes through different instant Keplerian orbits. As a result, it is non trivial to obtain the exact orbit equal to the one from an instantaneous approximation. Generally the real maneuver is not planned to obtain exactly the instantaneous orbit but instead to accomplish certain objectives, such as the simple cases of reaching an apogee or energy level, these are simple cases. This raises the question of how accurately the finite burn losses of the finite maneuver can be compared to the instantaneous approximation. In this work maneuvers in which either the apogee or the asymptotic velocity V_∞ are the same on both maneuvers are considered.

To solve the maneuver problem variables such as burn duration and thrust direction need to be determined so that the objectives are accomplished. At the same time it is important that the Δv is minimized, thus an optimization method must be used.

The equations of motion that describe a thrusting arc on the main body inertial frame are [12]:

$$\dot{r_x}(t) = v_x(t), \tag{1}$$
$$\dot{r_y}(t) = v_y(t), \tag{2}$$
$$\dot{r_z}(t) = v_z(t), \tag{3}$$
$$\dot{v_x}(t) = -\frac{\mu}{r(t)^3} r_x(t) + \frac{T_x(t)}{m(t)}, \tag{4}$$
$$\dot{v_y}(t) = -\frac{\mu}{r(t)^3} r_y(t) + \frac{T_y(t)}{m(t)}, \tag{5}$$
$$\dot{v_z}(t) = -\frac{\mu}{r(t)^3} r_z(t) + \frac{T_z(t)}{m(t)}, \tag{6}$$
$$\dot{m}(t) = \frac{T}{g_0 Isp}, \tag{7}$$

where r is the radius to the main body, v is the velocity, μ is the standard gravitational parameter, r_x r_y r_z v_x v_y v_z T_x T_y T_z are the radius, velocity and Thrust in the inertial axes, Isp is the specific impulse and g_0 is the gravity acceleration at the surface of the Earth.

The following assumptions were made:

- Thrust magnitude is constant, being similar to bi-propellant liquid motors;
- Specific impulse is constant;
- Initial and final orbit are keplerian;
- maneuvers are two-dimensional, x and y plane

Because thrust and specific impulse are considered constant throughout the burn, the mass-flow rate is constant (7).

We consider three steering laws in this work inertially fixed thrust direction, inertially fixed constant thrust rotation and thrust parallel to velocity.

In the cases considered minimizing Δv is the same as minimizing the burn duration t because the mass flow rate and specific impulse are constant.

2.2. Analytic estimate and trajectories

It is possible to obtain an approximation or bounds for a Δv. In this work the analytic solution tested is the expression obtained in [14], (8). The expression explicitly gives a maximum value for what the finite burn losses can be. It is determined by adding to the instantaneous velocity change (of the usual instantaneous maneuver approximation) a position displacement compensation factor obtained from a maneuver done with thrust being applied at an arbitrary constant angular velocity.

This upper limit is

$$\text{FiniteBurnlosses} \leq \frac{1}{24}(w_s t)^2 \Delta v, \tag{8}$$

where

$$w_s^2 = \frac{\mu}{r^3} \tag{9}$$

(which is related to a maximum optimal angular rotation of the thrust), t is the burn time and Δv is the instantaneous approximation velocity change. The burn time must be computed previously for the specific impulse and thrust over weight.

The Tsiolkovsky rocket equation [21]:

$$m_p = m_i \left(1 - e^{\frac{-\Delta v}{Isp g_0}}\right), \tag{10}$$

where m_p is the propellant mass spent on a maneuver with a certain Δv, m_i is the initial mass, Isp is the specific impulse and g_0 is the gravitational acceleration at the surface of Earth. Knowing the mass flow rate (7), which is constant as a result of the thrust and specific impulse being considered constant in this work, we can determine the time it would take to spend that mass:

$$t = \frac{g_0 I_{sp} m_p}{T} = \frac{I_{sp}\left(1 - e^{\frac{-\Delta v}{Isp g_0}}\right)}{\frac{T}{W_0}}, \tag{11}$$

where W_0 is the probe weight, and T is the thrust magnitude, related by

$$\frac{T}{W_0} = \frac{T}{g_0 m_i}, \tag{12}$$

2.3. Numerical determination of high thrust maneuvers

The keplerian orbit is described by the 6 classical orbit elements which can be determined by the position and velocity at a given point in time. The instantaneous approximation is just the difference of the final minus the initial velocities, whereas in the real finite maneuver it becomes a problem of optimization or targeting. The parameters involved, such as the burn time and direction, are obtained to meet the constraints.

2.4. Optimization methods

Trajectory optimisation methods can be either direct or indirect. In the next paragraphs we will follow [8].

Indirect methods are based on a variational calculaus principle: the Pontryagin minimum principle. It is possible to formulate a two-point boundary problem with a set of adjoint variables and a switch function, the solution of which will yield a history of the time-dependent controls, the so called Hamiltonian system.

Direct methods differ from the indirect in that the time-dependent controls are described by a finite set of parameters. The effect of such a limitation on the optimal solution in most problems is negligible. In this thesis the direct method single shooting is used since it is simpler and there is available in literature a script that uses it [5].

The so called single shooting is conceptually the simplest method [8]. The initial state vector values are specified and the trajectory is progagated forward in time, with the control variable values obtained from the parameterised representation. At the end of the propagation, some of the required constraints may be met and others not. Because both the system and the constraints are non-linear, non-linear programming is used to solve the minimisation problem.

This work uses SNOPT [6] and YUKON (from GMAT [7]) as optimization algorithms for the non-linear programing problem, both use sequential quadratic programming which is an iterative procedure that models the non-linear problem by iteration with quadratic programming sub problems (where the objective function is a quadratic function arranged with information obtained from the differentiation of the problem, the gradient and the Hessian) [15].

Types of constraint can be applied to the trajectory. Regarding the control variables, such as the burn duration, the initial angle of the Thrust or the rotation, they are constrained within certain values.

For the case of escape maneuver we use the characteristic energy C_3, which is equivalent to the asymptotic velocity. For a Hohmann transfer to higher orbit we use the apogee.

2.5. Typical approaches to real maneuvers

In Mars Observer mission for example [3], the orbit insertion by the upper stage was planned to occur at a constant radius, velocity and flight path angle. Such strategy simplified the system guidance software. Meanwhile, the satellite performed all maneuvers by pitching at a constant rate about an inertial fixed axis.

In the Mars Global Surveyur mission the spacecraft used a "pitch-over" steering strategy for the Mars orbit insertion in order to maximize the efficiency of the burn. "Pitch-over" works by using the attitude control thrusters to slew the spacecraft at a constant rate during the 20 minute to 25 minute burn in an attempt to keep the thrust tangent to the trajectory arc [17].

Considering the above practical examples, the choice of the three steering cases are within the real maneuvers.

2.6. Typical spacecraft systems

In Table 1 some important parameters are shown for different types of spacecraft.

Table 1: Propulsion and payload values parameters for different types of missions

Spacecraft	Isp, s	T/W_0	Utility	ref
Fregat	332	0.2687	Upper stage	[1]
Centaur	450	0.4387	Upper stage	[22]
VEO	317	0.0339	Interplanetary probe	[18]
MGS	318	0.0582	Interplanetary probe	[16]
Cassini	305	0.0080	Interplanetary probe	[9]
INSAT-3D	Na	0.0212	Geostationary probe	[19]

For apogee maneuvers in Geo Stationary satellites generally the engines have typically 490 N of thrust and 310 Isp [13].

Considering the above examples of spacecrafts, for this work we used a range for T/W_0 of 0.03 to 0.5, for specific impulse of 300s, 400s and 600s.

2.7. Multiple apogee raising maneuver

We can use the technique of apogee raising to minimize the burn losses. By dividing the direct Δv in smaller ones, each burn will be shorter and closer to the main body, taking advantage of the Oberth effect. Whereas a direct maneuver would last longer and the burn would take place in a trajectory that would eventually be further away from the main body. Thus multiple apogee raising maneuvers save Δv and have a significant impact on mission planning as in the mission chandrayaan-2 [20].

2.8. Cases studied

In this work the maneuvers determined start from a circular parking orbit of 200 km altitude, with ideal Δv ranging from $1\,\mathrm{km\,s^{-1}}$ to $5\,\mathrm{km\,s^{-1}}$ with a step of $0.5\,\mathrm{km\,s^{-1}}$, for T/W_0 from 0.03 to 0.5 with a step of 0.01 for a specific impulse of 300 s, 400 s and 600 s for the three steering laws.

Then as an alternative to direct maneuvers, the multiple apogee raising maneuver is studied for a specific impulse of 300 s and $T/W_0 = 0.15$ for a transfer orbit to Mars and for a geostationary transfer orbit (GTO).

3. Numerical implementation of the direct shooting

In this chapter the details and process of computing the maneuvers numerically are presented. There are two types of algorithms developed, one that determines single maneuvers (developed in MATLAB) and other that computes multiple maneuvers (developed in GMAT and MATLAB).

3.1. Single maneuver and multiple maneuvers

For both single and multiple maneuvers we used only the direct shooting method. In single maneuvers, since more than one steering law is studied it was developed in MATLAB, because it allows for more flexibility in programming the steering mechanisms. For multiple maneuvers, for the case where thrust is parallel to the velocity it was developed in

GMAT because of its simplicity of implementation, whereas for the other steering laws it was implemented in MATLAB.

3.2. Single maneuver: Algorithms implemented

To determine the optimum single maneuver we used the optimization parameters:

- Thrust duration
- Initial inertial angle for thrust direction
- Inertial rotation of the thrust direction

The inputs are: the orbital elements of the initial orbit, the type of steering law and the spacecraft parameters (initial mass, specific impulse and the desired T/W_0 which then determines the thrust magnitude). The outputs are the final velocity, radius and mass (which are then converted to orbital elements), the optimal steering variables and the burn duration.

The algorithm takes the initial guesses for the optimization variables and integrates the equations of motion of the thrusting arc and returns the final values of the constraints (apogee or C3) and the objective function (which is the minimization variable, burn duration) to the optimizer. By iteration it attempts to obtain the optimal values that met the constraints.

3.3. Propagator

The integration of the equations (1) to (7) were done with ODE45 function of MATLAB, which uses explicit Dormand-Prince method of order 4.

3.4. Optimizer

The Optimization problem was solved using direct shooting with the package SNOPT.

The shooting method is very sensible to the initial optimization variable guesses, thus various initial guess methods were tested. The final results are a mixture of optimal results obtained with from different techniques.

3.5. Multiple Maneuvers — MATLAB algorithm

The algorithm was further enhanced to perform multiple maneuvers. It was done for the purpose of studying a mission to Mars where the spacecraft could be large and have low maneuverability, having inertial fixed thrust steering.

The algorithm was arranged by adding a new optimization variable, the true anomaly in which the new burn starts, the burn duration, the angle direction and the angle rotation for each extra maneuver.

3.6. Multiple Maneuvers — GMAT algorithm

GMAT is used for multiple apogee raising maneuvers with thrust aligned with velocity.

We considered that the burn times of each maneuver are the variables. Starting from a circular orbit it propagates the first maneuver and then propagates to the new perigee where two variables are used: one for backward propagation defining the initial point of the new burn and the other the burn time. The boundary end constraint is the semimajor axis and it determined after the final maneuver. The semimajor axis is equivalent to the orbit's energy, and so to the asymptotic velocity. The minimization variable is the sum of the time of the maneuvers performed, which is again equivalent to minimizing Δv, since the thrust and specific impulse are modelled constant.

3.7. Code test and validation

We compared the program developed with the one offered by GMAT for various T/W_0 for the case of thrust parallel to velocity and the results were 0.001 % different which is acceptable. This is probably due to the GMAT optimizer not reaching the exact target, having higher tolerance values.

4. Results for real maneuvers

In this chapter the finite burn losses and effects on the final orbit are determined and discussed. The three steering cases are compared to each other and to the heuristic from literature [14] and effects on the perigee are shown.

4.1. Baseline Solution — real maneuvers

We study a maneuver equivalent to an impulsive maneuver of the stated Δv. The Δv is increased from $1\,\mathrm{km\,s^{-1}}$ to $3\,\mathrm{km\,s^{-1}}$, where the orbit after the maneuver is elliptical, and can be though as a transfer orbit of a Homann transfer. Above $3\,\mathrm{km\,s^{-1}}$, the resulting orbit is hyperbolic and can be though of as an escape maneuver, resulting in a certain C3. The raise of Δv was though to cover many interesting cases from apogee raising maneuvers to GTO transfers and escape trajectory to Mars. We study the Δv losses as a function of T/W_0 for a specific impulse of 300 s, (but also for 400 s, 600 s) deemed a representative value for modern technology.

For a specific impulse of 300 s, figure 1 shows the results for thrust parallel with velocity (vnb), figure 2 with fixed direction (c), and figure 3 with inertial fixed rotation (w):

All cases present the same pattern where the finite burn losses highly increase bellow a certain T/W_0, whereas for higher T/W_0 the curves are quite close to each other, as the maneuvers duration are shorter and the steering law has less effect on the losses. It can also be seen that figures 1 and 3 look similar. For $T/W_0 = 0.05$, for the lowest Δv, the finite burn losses on both cases are around $75\,\mathrm{m\,s^{-1}}$, while on figure 2 they are around $250\,\mathrm{m\,s^{-1}}$. In other words, a comparison of figures

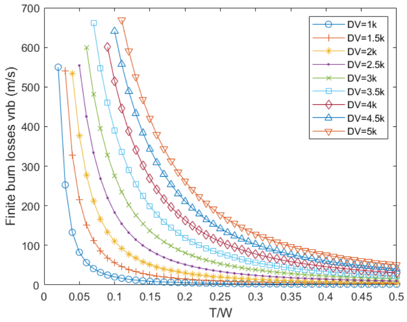

Figure 1: Finite burn losses with direction of the Thrust parallel to the velocity (vnb) Isp = 300 s

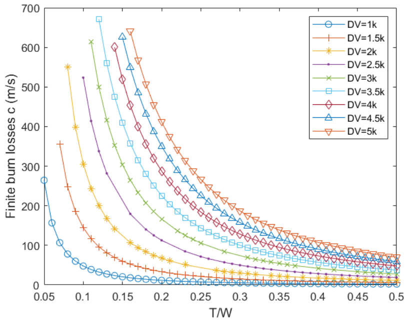

Figure 2: Finite burn losses with inertial fixed direction (c) of the Thrust Isp = 300 s

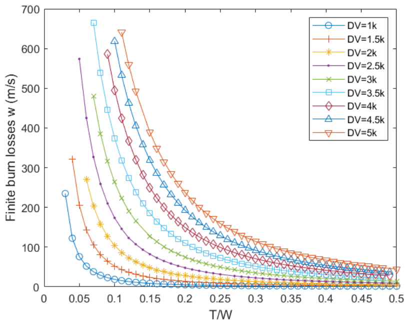

Figure 3: Finite burn losses with inertial fixed rotation (w) of the Thrust Isp = 300 s

1 and 3 shows that the difference in burn losses between the cases of Thrust parallel to velocity and thrust with constant rotation is very small. On the other hand, the performance of thrust with constant direction is clearly worst.

4.2. Comparing losses with different specific impulse

We can compare the burn losses of the cases of thrust parallel to velocity between the specific impulses 400 s and 300 s. Contrary to what we could expect, the burn loss is higher for the higher specific impulse as shown in figure 4 (the same happens for the other steering cases).

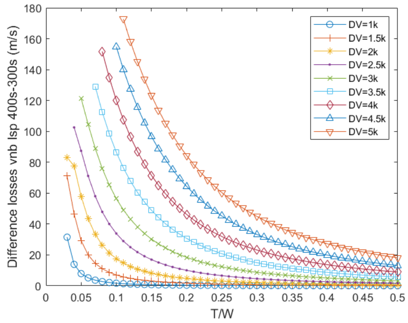

Figure 4: Difference of the finite burn losses between Isp = 400 s minus Isp = 300 s

This result is unexpected because intuitively it seems that a higher specific impulse should lead to lower losses. However with a higher specific impulse there is a lower mass flow rate for the same magnitude of Thrust. Since the acceleration increases while the spacecraft's mass decreases, if less fuel is spent, the spacecraft mass will not decrease as much and the acceleration increase rate will be lower, resulting in a longer maneuver. This is confirmed on figure 5 where the duration of the burn is compared between the two specific impulses and it is larger for a specific impulse of 400 s.

Despite the longer burn time, the fuel carries more energy (because of the higher specific impulse) and at the end of the maneuver the total mass is now larger. This means that the higher specific impulse maneuver is still more efficient, as it can accelerate more payload mass. That would be clear if we start with the same dry mass in both cases.

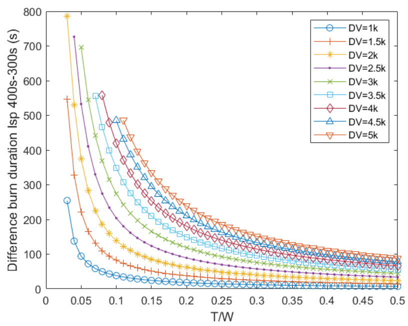

Figure 5: Difference of the burn duration between Isp = 400 s and Isp = 300 s

4.3. Comparison of the steering solutions for $Isp = 300\,\mathrm{s}$

The difference between the burn losses with direction of the thrust parallel to velocity and constant direction of thrust is shown on figure 6 and on figure 7 it is shown the difference between constant direction and rotation. We can see that inertial fixed thrust direction is outperformed by the other steering options specially for lower T/W_0 ratios where the burn is longer, the velocity direction throughout the burn undergoes bigger changes and the misalignment between velocity and thrust is large. The higher the T/W_0 ration (above 0.3), the smaller the difference (reaching values around 0.5 % of the impulsive approximation).

In figure 8 it is shown the comparison of the cases of thrust parallel to the velocity and thrust with fixed rotation. We can see a surprising result, where the rotation case performs slightly better above T/W_0 =0.1 but for lower ratios that have higher Δv, thrust parallel to the velocity has lower losses. In general both are very close to each other as the difference is lower than 1 % of the impulsive approximation, except around T/W_0 =0.05 where it is approximately 2 %.

This is an important observation because simulating a maneuver with thrust parallel to velocity is easier than using constant rotation. Since both are similar on certain ranges we may estimate the burn losses on preliminary design stages with thrust parallel to velocity.

We can observe with more detail the comparison of these two cases by viewing the orbits of $T/W_0 = 0.15$, figure 9 for a $\Delta v = 4.5\,\mathrm{km\,s^{-1}}$. An interesting effect occurs where the case with rotation starts pointing slightly to inside the orbit and stays closer to the Earth throughout the orbit and takes better advantage of the Oberth effect even though the thrust isn't aligned with the velocity (in which case there is no loss of energy to the velocity due to misalignment of the thrust and velocity vectors). The thrust vector starts by pointing slightly to the main body and ends up being aligned with the velocity, achieving both proximity to the main body and effectiveness in steering.

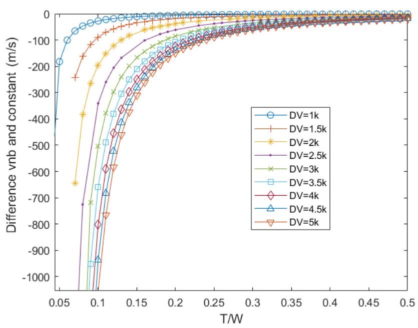

Figure 6: Difference between finite burn losses with direction of the Thrust parallel to the velocity and constant direction of Thrust

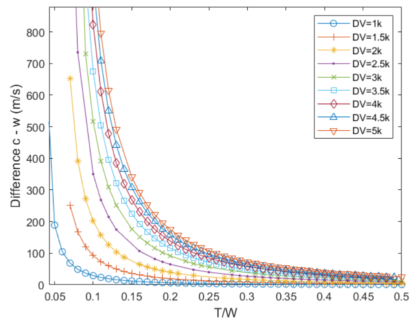

Figure 7: Difference between finite burn losses with constant direction of Thrust and inertial fixed rotation of the Thrust

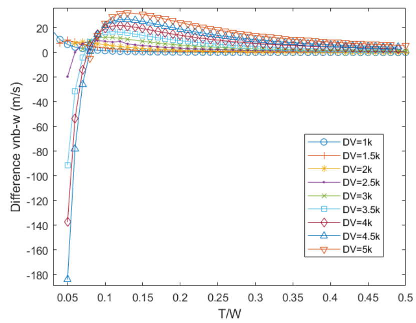

Figure 8: Difference between finite burn losses with direction of the Thrust parallel to the velocity and and inertial fixed rotation of the Thrust

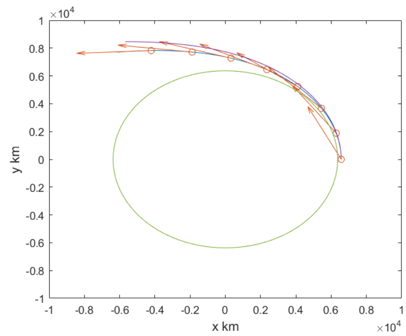

Figure 9: Orbits with angular rotation (the one with arrows that show the thrust direction) and with thrust aligned with velocity. The circle represents the Earth.

4.4. Effects of finite burn on the perigee

It is important to analyse what happens to the variables describing the real orbit, such as the new perigee.

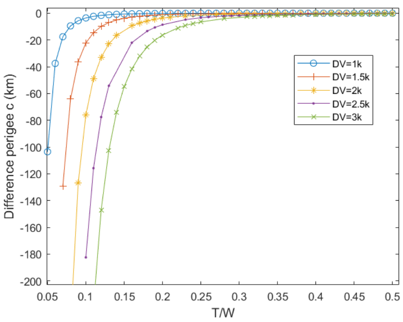

Figure 10: Perigee after burn for inertial constant thrust direction

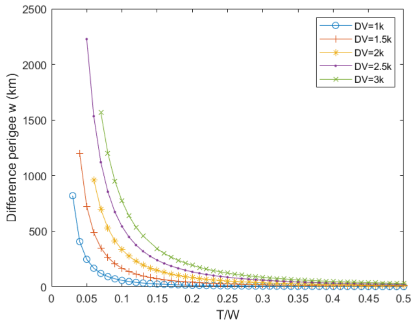

Figure 11: Perigee after burn for inertial constant rotation thrust direction

Figures 10 and 11 show the perigee after the burn for each steering mechanism except for thrust parallel to velocity which is similar to inertial fixed rotation. We can see that for higher T/W_0 (above 0.2) the dislocation of the perigee is small being around 1 to 10 km. For the cases of thrust rotation and parallel to velocity the perigee always increases, resulting in higher energy orbits since the apogee is the same for all the maneuvers so a higher perigee means a larger semi-major axis. On the other hand, if the thrust direction is constant the final perigee is lower; less energy was put into the orbit.

4.5. Analytic estimate

In the figure 12 the finite burn losses as foreseen by the solution developed from [14] and explained in chapter 2 is presented for 300 s.

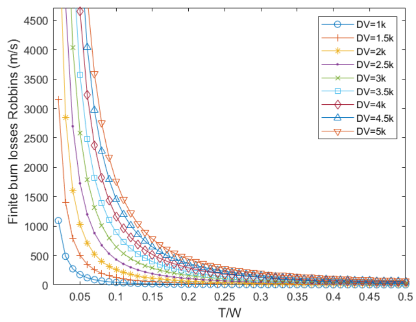

Figure 12: Finite burn losses from [14]

The burn losses curves are similar as observed in the real case, except for the case of constant direction where the losses are only predicted above T/W_0 0.1 for Δv higher than $3.5\,\mathrm{km\,s^{-1}}$. We can compare the percentage of the losses from the estimation (figure 12) with the real losses (figures 1, 2, and 3) using the relative difference of losses (13).

$$\text{RelativeBurnlosses } \% = \frac{\text{estimate} - \text{real losses}}{\text{real losses}} 100 \tag{13}$$

In figures 13 and 14 it is shown the results for the expression (13) for specific impulse of 300 s (thrust parallel to velocity is similar to the case of constant rotation). If the estimation is similar to the real losses, we would expect a behaviour where the predicted losses would be in percentage somewhat constant to the real losses. Instead in figure 13 we obtained similar results with a linear behaviour in some cases (for apogee targeting maneuvers) and on other cases exponential behaviour (for C3 maneuvers). This may suggest the expression, despite giving above estimates for those cases, around 125 %, it is not that reliable. The expression does not predict well for the case of constant thrust direction, where it is only good in a small region of T/W_0 (0.1 to 0.25).

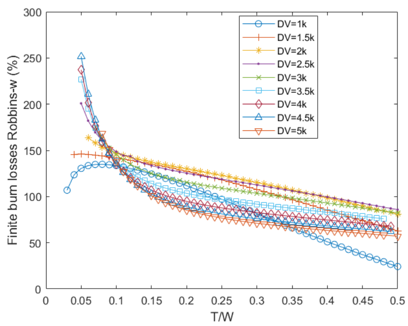

Figure 13: Difference between analytic estimation and numerical finite burn losses for intertial constant rotation thrust direction

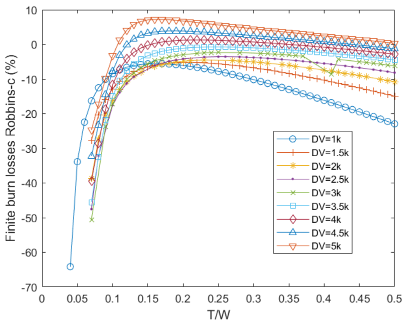

Figure 14: Difference between analytic estimation and numerical finite burn losses for inertial constant thrust direction

4.6. Multiple apogee raising maneuver — Mars transfer orbit

A maneuver to Mars at the starting orbit requires an impulsive approximation Δv of $3.6127\,\mathrm{km\,s^{-1}}$ to achieve the elliptical Hohmann transfer orbit. A specific impulse of 300 s and $T/W_0 = 0.15$ was used to obtain these results because they are conservative according to the existing technology from the Human Exploration of Mars Design Reference Architecture 5.0 [2]. On this Mars mission design, the trans-Mars insertion maneuver is planned to be done with 2 burns (1 extra burn in relation to a direct maneuver).

The table 2 shows the burn losses of multiple apogee raising maneuver to Mars. From table 2, we see that for multiple maneuvers, the more apogees raising are performed, the steering losses become more and more negligible. The initial difference of a direct maneuver of $4047\,\mathrm{m\,s^{-1}}$ versus $3824\,\mathrm{m\,s^{-1}}$ became just of $9\,\mathrm{m\,s^{-1}}$ at three extra maneuvers. In fact, the case of constant direction gains a lot more from multiple maneuvers than the other cases, reaching a 10 % efficiency with three extra maneuvers in relation to the direct maneuver, whereas the thrust parallel to velocity case gains close to 4 % and is stabilizing. This indicates that it is a good orbit design option to make multiple maneuvers for a crewed Mars mission, especially if constant thrust direction is used (a single extra burn already saves 8 % Δv). Of course the more burns the more complex the orbit design becomes, and as planned on the reference 5.0 mission design [2] one extra maneuver may be enough for saving sufficient Δv.

Table 2: finite burn losses of multiple apogee maneuvers to Mars

Burns	Constant dir.	Constant rot.	T//V
Direct	$4047.12\,\mathrm{m\,s^{-1}}$	$3811.91\,\mathrm{m\,s^{-1}}$	$3824.95\,\mathrm{m\,s^{-1}}$
1 extra	$3719.67\,\mathrm{m\,s^{-1}}$	$3667.94\,\mathrm{m\,s^{-1}}$	$3676.68\,\mathrm{m\,s^{-1}}$
2 extra	$3660.45\,\mathrm{m\,s^{-1}}$	$3638.03\,\mathrm{m\,s^{-1}}$	$3644.90\,\mathrm{m\,s^{-1}}$
3 extra	$3640.58\,\mathrm{m\,s^{-1}}$	n/a	$3631.34\,\mathrm{m\,s^{-1}}$

4.7. Geostationary Transfer Orbit

It was also determined multiple apogee raising maneuver for Geostationary Transfer Orbit (GTO). The GTO requires a impulsive approximation Δv of $2.3357\,\mathrm{km\,s^{-1}}$, the results obtained are presented on table 3. The aim of studying a GTO was to observe a case that is not an escape orbit and thus does not require a large maneuver at the end.

Again, constant direction with just one extra maneuver reduces the relative losses significantly in relation to the the others cases, by 5 % of the direct value whereas thrust parallel to velocity reduces around 1.4 %.

Table 3: finite burn losses of multiple apogee maneuvers to Geostationary orbit

Burns	Constant dir.	Constant rot.	T//V
Direct	$2512.33\,\mathrm{m\,s^{-1}}$	$2405.45\,\mathrm{m\,s^{-1}}$	$2408.79\,\mathrm{m\,s^{-1}}$
1 extra	$2378.76\,\mathrm{m\,s^{-1}}$	$2354.75\,\mathrm{m\,s^{-1}}$	$2373.03\,\mathrm{m\,s^{-1}}$
2 extra	$2359.16\,\mathrm{m\,s^{-1}}$	$2345.08\,\mathrm{m\,s^{-1}}$	$2347.08\,\mathrm{m\,s^{-1}}$
3 extra	$2347.41\,\mathrm{m\,s^{-1}}$	n/a	$2345.86\,\mathrm{m\,s^{-1}}$

5. Conclusions

As expected the worst performing steering law is the one with constant thrust direction while the other two cases have similar performances. Thus, thrust parallel to velocity, which is the easiest to compute, can be used to estimate the Δv in the early stages of orbit design since it has similar performance in relation to thrust with fixed rotation. Higher specific impulse does not result in lower losses but spends less fuel mass. The attempt to predict the finite burn losses in a simple way is found to be nontrivial. The analytical estimate from [14] does not predict the losses reliably, especially for thrust with constant direction. Although, it is still better than a blindfold guess, being around 125% of the finite losses value for the cases of thrust parallel to velocity and inertial fixed rotation. The multiple apogee raising maneuver lowers significantly the losses particularly for the case of constant thrust direction (where it is saved 10% of the direct maneuver losses with three extra burns), that may be useful for a future mission to Mars.

5.1. Future Work

The analytic estimate (8) may have other approaches and solutions where we could have a better prediction for the burn time of the maneuver. Another SQP based algorithm could be used. Furthermore, it would be interesting to do a similar study but starting on an elliptical orbit because it allows for more degrees of freedom.

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