Definition of the waves action on offshore structures

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Abstract

Offshore structures, even though they have originally been developped for oil exploitation platforms, are used nowadays on a wide variety of areas, such as eolic energy platforms, aquaculture and artificial islands. The actions they are submitted to, besides self-weight and additional loads, wind, accident action and earthquake, are related with the actions that the sea water causes on the structures.

This dissertation proposes to study the different types of force that an harmonic wave has on a structure, whether fixed or floating, and how the relevance of this type of force is related with the ratio between the structure's characteristic dimensions and the wave lenght. Furthermore, the structure's response to the action of the wave, is also analysed. In offshore structures vertical motions can be critical. This way, the dissertation studies the vertical translation motion and rotation movement perpendicular to the flow axis direction as a response to the different types of force caused by the action of an harmonic wave.

KEYWORDS: offshore structures, Froude-Krylov force, diffraction force, radiation force, Morison equation.

1 Introduction

Offshore structures were initially developed at the beggining of the XX century for the oil industry. Currently, this technology has many other uses such as renewable energies, aquaculture exploitation and even floating islands. The offshore structures are submitted to different actions such as self-weight, waves, wind, currents, accident (such as ships impact or explosions) and, depending on the location, they can also be submitted to an eartquake.

The offshore structures can be classified as fixed or floating structures, depending on the way the balance is guaranteed when they are submitted to different actions.

Fixed platforms are founded directly on the sea bed and the balance is guaranteed by transmiting applied forces directly to its foundations. This way, their behaviour is similar to inland structures, but they are submitted to a different action: the sea waves. Jackets, jack-ups and gravity platforms are examples of fixed structures.

Floating structures are fixed to the sea bed by cables and have less restrictions to rigid body movements than fixed structures. Semi-submersibles are examples of this kind of structures that can move in all the degrees of freedom. The number of degrees of freedom can be reduced by restraining, for example, the vertical movements of the structure by pre-tensioned cables. One example of this kind of structure is a TLP (Tension Leg Platform) where the horizontal translation and rotation movements are allowed but the vertical movements of translation and rotation are restrained. Floating structures have a dynamic behaviour when they are submitted to waves action. The equation of dynamic balance is used in order to get the transfer function that caracterizes the structure.

2 Definition of waves

The elevation of the free surface is defined by

$$z = \zeta(x, y, t), \tag{2.1}$$

where ζ is the wave amplitude. The referential is chosen so that $z = 0$ coincides with the free surface. The depth h is the distance between the free surface, $z = 0$, and the sea bed, $z = -h$. The wave height, H, is the difference between the wave crest and the wave trough. The wave amplitude, ζ, is half of the wave height, i.e., $\zeta = H/2$ (Figure 2.1). Waves are harmonics with wave length λ and period T. Assuming that the wave doesn't change in the transversal direction y, the flow can be considered bidimensional and the elevation z depends only of x.

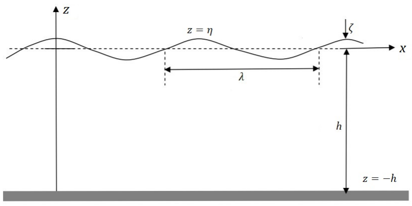


Figure 2.1: Wave definition (adapted from [11]).

The wave propagates over the positive x axis, as shown in Figure 2.1.

The action that the sea waves exert over a structure depends on the ratio between the section's characteristic dimensions and the wave length. In a small volume structure or *slender structure*, it's assumed that the sectionťs characteristic dimensions are small when comparing with the wave lenght λ. For example, for a circular element, if the diameter D is such that $\lambda > 5D$, there is a very small alteration of the incident waves when they pass through the structure [12]. In this case, the incident waves behave as undisturbed waves, which means they don't deform when they pass through the structure. The term *large volume structures* is used for offshore structures with dimensions D on the same order of magnitude as typical wave lenghts λ of ocean waves (2 s (4 s) to 8 s(10 s)). In this case, usually $D > \lambda/6$ [6].

In linear theory (or Airy theory), it is considered that the wave amplitude ζ is very small in comparison with the wave lenght, depth and section's caracteristic dimensions. It means that the wave-induced motions and load amplitudes are linearly proportional to ζ [12].

The sea water is assumed to be incompressible and inviscid, and the fluid motion is irrotational. The velocity vector of a water particle $\vec{u}(x, y, z, t) = (u_x, u_y, u_z)$ at time t at the point $\mathbf{x}= (x, y, z)$ in a cartesian coordinate system fixed in space can be defined as a velocity potencial Φ,

$$\vec{u} = \vec{\nabla}\phi = \frac{\partial \Phi}{\partial x}\vec{e_x} + \frac{\partial \Phi}{\partial y}\vec{e_y} + \frac{\partial \Phi}{\partial z}\vec{e_z}$$

where $\vec{e_x}$, $\vec{e_y}$ e $\vec{e_z}$ are unit vectors along the x, y and z axis, respectively. The fluid is incompressible if there is no volume variation through time, i.e., $\vec{\nabla} \cdot \vec{u} = 0$ (continuity equation). The analysis of the flow of an incompressible fluid can be deducted from conservation mass law that leads to the continuity equation. A flow is irrotational when $\vec{\nabla} \times \vec{u} = 0$. From these two conditions it follows that the velocity potential has to satisfy the Laplace equation

$$\vec{\nabla}^2\Phi = 0 \Leftrightarrow \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0. \tag{2.2}$$

The complete mathematical problem of finding a velocity potential of irrotational, incompressible fluid motion consists of the solution of the Laplace equation with relevant boundary conditions on the fluid [12].

The pressure p follows from Bernoulli's equation

$$\rho\frac{\partial \Phi}{\partial t} + \frac{\rho}{2}|\vec{u}|^2 + p + \rho g z = C, \tag{2.3}$$

where $C = C(t)$ is an arbitrary function of time. Generally this constant can be taken as the atmospheric pressure p_0 on the fluid's free surface [5]. Neglecting the convective velocity term, the Bernoulli equation is obtained in linearized form. It follows therefore that the local and time-dependant pressure is determined as the sum of dynamic and hydrostatic pressure, i.e.

$$p - p_0 = -\rho\frac{\partial \Phi}{\partial t} - \rho g z, \tag{2.4a}$$

with the dynamic pressure

$$p_D = -\rho\frac{\partial \Phi}{\partial t}. \tag{2.4b}$$

The kinematic boundary condition expresses impermeability ($\frac{\partial \Phi}{\partial n} = 0$). The boundary condition of dynamic balance is simply that the water pressure is equal to the constant atmospheric p_0 on the free surface. These two boundary conditions are non-linear, however they can be simplified by linearizing the free surface conditions in order to obtain

$$\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = 0. \tag{2.5}$$

When the velocity potential Φ is oscillating harmonically in time with circular frequency ω, equation (2.5) can be written as

$$-\omega^2\Phi + g\frac{\partial \Phi}{\partial z} = 0 \text{ em } z = 0. \tag{2.6}$$

By assuming an horizontal sea bed and a free-surface of infinite horizontal extent, linear wave theory can be derived for propagating waves. The free-surface condition (2.6) is used together with the Laplace equation (2.2) and with the sea bed boundary condition

$$\frac{\partial \Phi}{\partial z} = 0 \qquad \text{on } z = -h, \tag{2.7}$$

in order to obtain the solutions to the potential Φ. The general solution is given by

$$\Phi(x, z, t) = \frac{\zeta_0 g}{\omega}\frac{\cosh(k(z + h))}{\cosh(kh)}\cos(\omega t - kx), \tag{2.8}$$

where k is the wave number given by

$$k = \frac{\omega^2}{g}. \tag{2.9}$$

The horizontal water particle velocity function is given by

$$u = \frac{\partial \Phi}{\partial x} \tag{2.10}$$

and the horizontal water particle accelaration function is given by

$$\dot{u} = \frac{\partial \vec{u}}{\partial t}. \tag{2.11}$$

In deep water ($h/\lambda > 0.5$ [11]), is valid the simplification

$$\frac{\cosh(k(z+h))}{\cosh(kh)} = \frac{e^{k(z+h)}}{e^{kh}} = e^{kz} \tag{2.12}$$

and the velocity potential for deep water is defined by

$$\Phi(x, z, t) = \frac{\zeta_0 g}{\omega} e^{kz} \cos(\omega t - kx). \tag{2.13}$$

The total pressure is the sum of the hidrostatic pressure with the dynamic pressure,

$$p = -\rho g z + \rho g \zeta_0 e^{-kz} \sin(\omega t - kx). \tag{2.14}$$

3 Fixed structures

Fixed structures are usefull when itťs important to limit the motions' amplitude caused by the sea waves action. This is the case of production and oil extraction technologies as well as eolic offshore structures. Usually fixed structures are used in finite water depths and also if they are to be permanent. The incident forces on the structure are directly transmitted to the foundations. Normally these are *slender structures.*

The effects of waves action on offshore structures are obtained by overlapping different hydrodynamic forces that act individually on different elements of the structure. To calculate hydrodynamic forces, it is necessary to integrate the pressure field over the wetted surface of the structure. The main forces that act in an offshore structure are:

i) Froude-Krylov force: pressure effects due to undisturbed incident waves;

ii) hydrodynamic 'added' mass and potential damping force: pressure effects due to relative acceleration and velocity between water particles and structural components in an ideal fluid;

iii) diffraction force: pressure effects due to disturbance of incident waves on the structure;

iv) viscous drag force: pressure effect due to relative velocity between water particles and structural components.

Figure 3.1 shows that forces on *slender structures* ($D < 5\lambda$) correspond to zones I, III, V and VI ($\pi D/\lambda < 0.6$). In these areas diffraction force is neglectible, and the inertia forces and drag forces are much more relevant, depending on the ratio between H e D, where H is the wave height and D a characteristic dimension of the structure.

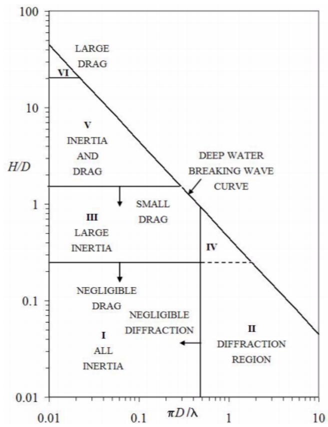


Figure 3.1: Different wave force regimes, where D is a characteristic dimension of the section, H is the wave height and λ is the wave length [6].

The Froude-Krylov forces are due to the pressure field of the undisturbed incident wave. If $\vec{n}$ is the normal vector on the body surface pointing outwards into the fluid, then, by considering all components, the Froude-Krylov force is obtained by integrating the pressure field of the undisturbed incident wave over the wetted surface S and is given by

$$\vec{F} = -\int_S p\vec{n} dS = -\int_V \vec{\nabla} p dV. \tag{3.1}$$

As in linear wave theory, the convective term of the acceleration is neglected. Furthermore, for slender structures, the local acceleration, changing only slightly in the region of the structure, is replaced by the water particle acceleration at the component x. With these assumptions, the Froude-Krylov force is obtained as a product of displaced water mass and local water particle acceleration,

$$\vec{F} = \rho V \frac{\partial \vec{u}}{\partial t}. \tag{3.2}$$

A trivial physical interpretation may be derived from this result: the pressure force on a small fluid element of arbitrary shape and mass ρV in a wave field leads to a motion of this element with the particle acceleration $\partial\vec{u}/\partial t$ [5].

In the general case, the dynamic pressure in (3.2) is replaced by the unsteady Bernoulli equation (2.3). Thus the generalized Froude-Krylov force is expressed in terms of the velocity potential Φ of the incident wave, i.e.,

$$\vec{F} = \int_S p\vec{n}dS = \rho \int_S \frac{\partial \Phi}{\partial t}\vec{n}dS. \tag{3.3}$$

For fully submerged slender components, this expression is transformed into (3.2). Note, that for all components of fixed or floating structures which penetrate the water surface, the pressure integration has to be performed over the wetted surface S according to (3.3) [5].

The hydrodynamic mass force acting on a body is obtained by integration of the pressure field arising from relative acceleration between structural component and fluid over the wetted surface. The hydrodynamic mass is the sum of the structure mass with a virtual water mass called *added mass*. The added mass that moves with a certain acceleration depends on the shape of the structure and the motion direction [5].

Diffraction force is due to the waves action over the structure when the incident waves are disturbed by the presence of the structure. In slender structures, this force is irrelevant because the waves don't deform when they pass through the structure. This force is the product of the added mass by the water particle acceleration.

Viscous force is due to relative velocity between water particles and the structure.

Typically fixed structures are slender structures. In this section, it is assumed that structure dimensions are small in comparison with wave length λ and, in this case, the incident waves behave as undisturbed which means that they don't deform when they pass through the structure. In the same way, diffraction phenomenon is not relevant. To calculate horizontal forces on slender structures, Morison proposed an empiric equation validated by experimental results. Morison's theory says that the horizontal force on a strip of a vertical cylinder is the sum of an inertia force and a drag force. The inertia force includes the Froude-Krylov force and the hydrodynamic force. Drag force corresponds to viscosity forces associated with pressure due to relative velocity between the water particles and the structure. The hydrodynamic horizontal force in a slender cylinder, according to Morison theory, can be written as

$$\vec{F} = (\vec{f_m}+\vec{f_d})dz = \rho\pi\frac{D^2}{4}C_m\dot{\vec{u}}dz+\frac{\rho}{2}C_dD|\vec{u}|\vec{u}dz. \tag{3.4}$$

Positive force direction is in the wave propagation direction, ρ is the water density, $C_m = 1 + C_a$ is the hydrodynamic mass coefficient (C_a is the added mass coefficient), C_d is the drag coefficient, $\vec{u}$ is the water particle velocity and $\dot{\vec{u}}$ is its acceleration. Coefficients C_d and C_m are empirically determined and it is assumed that for a slender cylinder $C_m = 2$ (half of the contribution comes from Froude-Krylov force and the other half from diffraction force) and $C_d = 1$.

To illustrate Morison's theory, a vertical cylinder fixed at the sea bed was modeled at Usfos [11] and the results obtained in the program were compared with analytic calculations. The analysis also aimed at verifying the influence of the diameter of the cylinder and the wave length, and the forces that act over it. In the analytic calculus for the horizontal force it was assumed $C_d = 1$ and $C_m = 2$ in the Morison equation. The cylinder was 12 m height and was fixed at the sea bed at 10 m depth and was submitted to an harmonic wave with wave length $\lambda = 12$ m and height $H = 1.5$ m. The chosen diameters were $D = 0.1$ m ($\lambda/D = 120$), $D = 1.5$ m ($\lambda/D = 8$) and $D = 5$ m ($\lambda/D = 2.4$). The two first diameters were slender structures and Morison's theory was applicable ($\lambda > 5D$); the last one was considered a large volume structure and would perturb the flow when it passed through the structure. Analysing Figures 3.2 and 3.3 it can be observed that drag forces are more relevant in this case since it is the structure with the biggest ratio λ/D in this study. For bigger diameters, but still within the slender structure range where Morison's theory is valid, and for a cylinder with $D = 1.5$ m, the forces that most contribute to the horizontal force are inertia forces (Figure 3.3). In this last case, inertia forces almost coincide with the total force, being the drag force almost null.

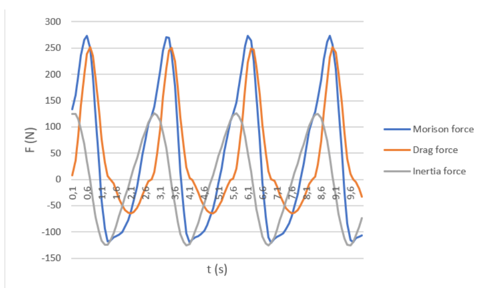
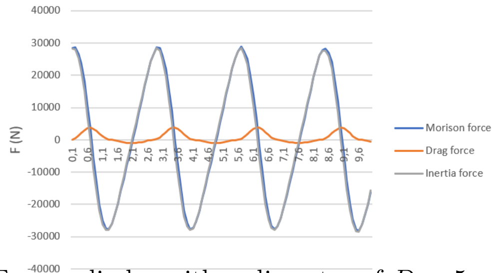


Figure 3.2: Horizontal force at the cylinder $D = 0.1$ m.

Figure 3.3: Horizontal force at the cylinder $D = 1.5$ m.



For a cylinder with a diameter of $D = 5$ m, Morison's theory is no longer valid, since the structure behaves as a large volume structure and the diffraction forces assume particular relevance. Usfos considers horizontal forces acting in a large volume structure using a correction of Morison's theory - MacCamy & Fuchs theory. The results are shown in Figure **??**. In Figure 3.4 are the results of Morison theory, which are not valid in this case, and in Figure 3.5 are the results for a large volume structure (with MacCamy &Fuchs function activated in Usfos). It can be concluded that horizontal force in a large volume structure is smaller than the same force when calculated by Morison's theory. By using Morison's theory for large volume structures, the horizontal force is overestimated, which can lead to conservative results.

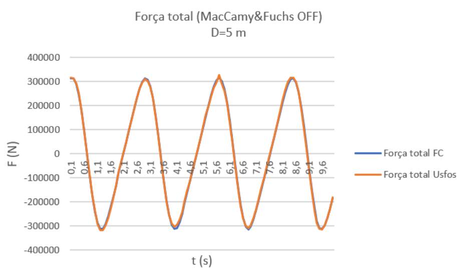


Figure 3.4: Força total calculada através da teoria de Morison.

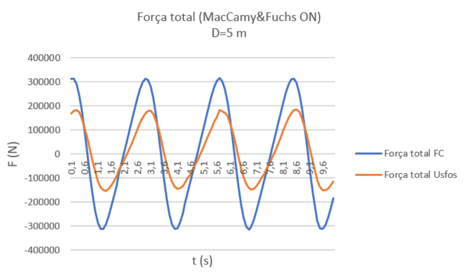


Figure 3.5: Comparação da força total calculada através da teoria de Morison com a solução de MacCamy & Fuchs.

4 Floating structures

Floating structures are the most used offshore structures solutions in deep water. They are also economically more appealing for temporary structures because they're cheaper than fixed structures [13]. Semi-submersibles are a typical example of floating structures.

This study aims to study the actions that the sea water has in a floating rigid body, and to identify the forces and rigid body movements that result from that action. To do that, balance dynamic equations are established in order to define the structure's motion in its different degrees of freedom. To characterize the motion transfer functions are used in the frequency domain. These functions are used to determine the answer of a structure to a signal. In this case, the waves are the signal to a system whose characteristics are linear, and the answer of the system is the motion of the floating structure.

To study the motions of a rigid body in the water it is important to define them first. The rigid body translation moves along x, y and z axis are defined as *surge*, *sway* and *heave*, respectively. The angular moves around x, y and z axis are defines as *roll*, *pitch* and *yaw* (Figure 4.1).

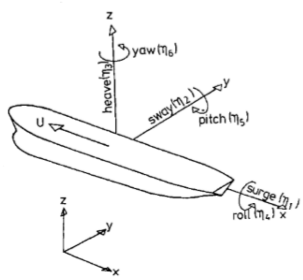


Figure 4.1: Definition of rigid body motion modes [12].

Floating structures can move along the six degrees of freedom and they are usually anchored with wires or chains [5]. Because such systems have low stiffness, the natural frequency is low and the structure can move in all six degrees of freedom. If the number of degrees of freedom is reduced, for example by restraining the structure with pre-tensioned cables in the vertical direction, vertical moves (heave, pitch and roll) will be restrained. This is the case of a TLP - Tension Leg Platform -, and the main difference between a TLP and a semi-submersible is precisely this vertical restraint. In floating structures, rigid body moves, especially in the vertical direction, can be critical.

In this study, the motions that will be analysed will be the translation vertical motion *heave* and rotation around y axis *pitch*, since these are the modes that represent motions in the vertical direction. *Roll*, also vertical, is neglectible, because the flow is considered do be bidimensional at xy plane.

The action that the sea waves exert over a floating structure can be defined by the overlap of two kind of forces:

i) the forces and moments on the body when the structure is restrained from oscillating due to incident harmonic waves. This hydrodynamic loads are the excitation forces and are composed by Froude-Krylov forces and moments and diffraction forces and moments. These are the forces that act on the structure as if it was fixed;

ii) the forces and moments on the body when the structure is forced to oscillate with the wave excitation in any rigid body motion mode. In this case, there are no incident waves, but waves generated by the body. This hydrodynamic loads correspond to radiation forces - inertia forces associated to added mass and potential damping - and by the elastic restoring forces.

Hydrostatic force depends on the body position in relation to $z = 0$ (free surface) so, by definition, is not affected by the waves that are no more than free surface disturbances [10]. It should also be referred the Archimedes' principle that establishes that a body totally or partialy immersed in a fluid is equal to the weight of the fluid that the body displaces. Due to linearity, the forces obtained in i) and ii) can be added to achieve the total hydrodynamic forces (Figure 4.2).

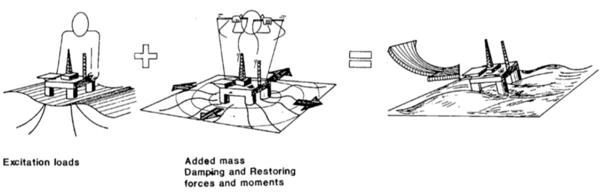


Figure 4.2: Superposition of wave excitation, added mass, damping and restoring loads [12].

Excitation force due to the wave action over a structure has two components: Froude-Krylov force and diffraction force, which are determined assuming that the body is restrained from oscillating. Excitation force F_i is the sum of these two forces,

$$F_i = -\int\int_S (pn_i dS + A_{i1}a_1 + A_{i2}a_2 + A_{i3}a_3)\,, \quad (4.1)$$

where p is the undisturbed pressure (2.4b) and $\vec{n} = (n_1, n_2, n_3)$ is the unit vector normal to the body surface defined to be positive into the fluid. The integration is over the average wetted surface of the body; a_1, a_2 and a_3 are acceleration components along the x, y and z axis of the undisturbed wave field and are to be evaluated at the geometrical mass centre of the body; A_{ij} are the terms of the added mass in the three directions and on the six modes.

The first term in equation 4.1 is the Froude-Krylov force. The other terms, physically, represent the fact that the undisturbed pressure is alterated due to the presence of the body - diffraction force.

Radiation forces are the result of waves generated by body motions; they are associated with a pressure field and represent the water resistence to the body motions. They are materialized by terms of added mass and damping. There are no incident waves, but the forced motions generates waves. This results in pressures on the structure surface that, when integrated, originate resulting forces and moments on the body. These are the radiation forces.

By defining the force components in the x, y and z direction by F_1, F_2 and F_3 and the moment components along the same axis as F_4, F_5 and F_6, the hydrodinamic added mass and damping loads due to harmonic motion mode η_j can be written as

$$F_k = -A_{kj}\frac{d^2\eta_j}{dt^2} - B_{kj}\frac{d\eta_j}{dt} \quad (4.2)$$

where A_{kj} and B_{kj} are defined as added mass and damping coefficients, respectively. There is a total of 36 added mass coefficients and 36 damping coefficients. However, there are ways to simplify the problem. For example, half of the coefficients are zero if the structure has zero speed and the submerged part has one vertical symmetry plane. A_{kj} and B_{kj} are functions of body form, frequency of oscillation and the forward speed. Other factors like finite water depth will also influence the coefficients. If the structure has zero forward speed and there is no current it can be shown that $A_{kj} = A_{jk}$ and $B_{kj} = B_{jk}$.

As an example of a vertical motion, a buoy in an harmonic wave field will be considered in order to derive the equations of motion. For the other degrees of freedom, the deduction is similar if the referencial is properly chosen (so that the motions are decoupled).

Considering the vertical motion of a a buoy submited to the action of a resulting vertical force in an harmonic wave field, the Froude-Krylov force follows from the pressure field of the undisturbed incident wave

$$F = -\int\int_S (pn_i)\, dS = \rho \int\int_S \left(\frac{\partial \phi}{\partial t} n_i\right) dS. \quad (4.3)$$

Under the effect of this force, the vertical motion of the buoy is given by

$$\eta_3 = \eta_0 e^{-i(\omega t - \varphi)}, \quad (4.4)$$

where ω is the frequency and φ is the phase of the motion in relation to the wave action. Or, in the same way

$$\eta_3 = X \sin(\omega t) + Y \cos(\omega t). \quad (4.5)$$

The term in cosine has to be considered because, in general, the answer in a system with damping is not in phase with the loads [15]. The vertical velocity in the buoy is

$$\dot{\eta}_3 = X\omega \cos(\omega t) - Y\omega \sin(\omega t) \quad (4.6)$$

with vertical acceleration

$$\ddot{\eta}_3 = -X\omega^2 \sin(\omega t) - Y\omega^2 \cos(\omega t). \quad (4.7)$$

As a consequence of this motion, the reaction forces are:

- hydrostatic restoring force

$$F_R = -C_{33}\eta_3 = -\rho g A_0 \eta_3,$$

- inertia force

$$F_T = -\rho V \ddot{\eta}_3,$$

as well as the hydrodynamic pressure forces which depend on relative acceleration and velocity, i.e.

- the hydrodynamic inertia force

$$F_{3m} = -A_{33}(\ddot{\eta}_3 - \dot{u}),$$

- the linearized drag force

$$F_{3dl} = -B_{33}(\dot{\eta}_3 - u),$$

where u and $\dot{u}$ are the verticle water particle velocity and acceleration respectively. The restoring coefficient C_{33} follows from the specific weight ρg multiplied by the waterplane area A_0 (area in the water plane that intersects the structure); A_{33} is the added mass and B_{33} is the damping coefficient; ρV is the mass of the fluid that the body displaces. The equilibrium of forces, with the Froude-Krylov force according to (3.3) results in

$$\rho \int\int_S \frac{\partial \phi}{\partial t} n_3 dS - C_{33}\eta_3 - \rho V \ddot{\eta}_3 \\ -A_{33}(\ddot{\eta}_3 - \dot{u}) - B_{33}(\dot{\eta}_3 - u) = 0.$$

By separating the variables, the linear equation of vertical motion is obtained:

$$\begin{aligned}(\rho V + A_{33})\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 \\ = \rho \int\int_S \frac{\partial \phi}{\partial t} n_3 dS + A_{33}\dot{u} + B_{33}u \\ = F_{3a}\sin(\omega t). \end{aligned} \quad (4.8)$$

The first member of the equation characterizes a linear oscillatory system, while the second member is the harmonic wave excitation, that correspods to Morison equation linearized. Normalizing the motion equation with the total mass $(\rho V + A_{33})$ and adding the simplifications

$$\omega_n = \sqrt{\frac{C_{33}}{\rho V + A_{33}}} \quad \text{e} \quad \xi = \frac{B_{33}}{2(\rho V + A_{33})\omega_n}$$

where ω_n is the natural frequency of the structure for the vertical translation motion, ω is the wave frequêncy and ξ is the damping coefficient, the equation (4.8) can be written

$$\ddot{\eta}_3 + 2\xi\omega_n\dot{\eta}_3 + \omega_n^2\eta_3 = \frac{F_{3a}}{\rho V + A_{33}}\sin(\omega t). \quad (4.9)$$

Replacing η_3 and its derivates by the expressions (4.5), (4.6) and (4.7) in the equation (4.9) and isolating the terms in sine and cosine, and considering that β is the ratio between the wave frequency and natural frequency of the structure, i.e.,

$$\beta = \frac{w}{w_n},$$

it leads to an equation system

$$\begin{cases} -X\bar{\omega}^2 - Y\bar{\omega}^2\,(2\xi\omega) + X\omega^2 = \dfrac{F_{3a}}{\rho V + A_{33}} \\ -Y\bar{\omega}^2 + X\bar{\omega}\,(2\xi\omega) + Y\omega^2 = 0 \end{cases},$$

where the first equation represents the terms in sine and the second equation the terms in cosine. These two relations have to be satisfied individually because the sine and cosine terms vanish at different times [15]. Dividing both equations by ω^2 and regrouping terms, the equations can be written

$$\begin{cases} X = \dfrac{F_{3a}}{C_{33}} \dfrac{1-\beta^2}{(1-\beta^2)^2 + (2\xi\beta)^2} \\ Y = \dfrac{F_{3a}}{C_{33}} \dfrac{-2\xi\beta}{(1-\beta^2)^2 + (2\xi\beta)^2} \end{cases}.$$

Replacing expressions X and Y at the expression (4.5), it's achieved

$$\begin{aligned}\rho &= \sqrt{X^2+Y^2} = \frac{F_{3a}}{C_{33}}\left(\left(1-\beta^2\right)^2+(2\xi\beta)^2\right)^{-\frac{1}{2}}\\ \eta_3 &= \frac{F_{3a}}{C_{33}}\frac{1}{\sqrt{\left(1-\beta^2\right)^2+(2\xi\beta)^2}}\sin(\omega t-\varphi),\end{aligned}$$

where ρ is the motion amplitude. The phase φ is given by

$$\varphi = \tan^{-1}\left(\frac{2\xi\beta}{1-\beta^2}\right).$$

The transfer function in *heave* is

$$\begin{aligned}H_3(\omega) &= \frac{\eta_3(\omega)}{\zeta(\omega)} = \frac{F_{3a}}{C_{33}\zeta_a}\left[\left(1-\beta^2\right)^2+(2\xi\beta)^2\right]^{-\frac{1}{2}}\\ &= \frac{F_{3a}}{C_{33}\zeta_a}D(\beta,\xi). \end{aligned} \tag{4.10}$$

where the dynamic magnification factor D is equal to

$$D(\beta,\xi) = \frac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}. \tag{4.11}$$

parameter (see Fig. 2.6). If the buoy is a floating cylinder with vertical axis, submerged to depth h_c, then the Froude-Krylov force in deep water, given by (3.1) and the pressure force of the incident wave on the waterplane area A_0 given by (2.4b), results in

$$F_{3a} = p_{z=-h_c}\times A_0 = A_0\rho g\zeta_a e^{-kh_c}.$$

Neglecting the small forces arising from hydrodynamic mass and drag, considering $C_{33}=\rho g A_0$ and $k=\omega^2/g$, transfer function can be written as

$$|H_3(\omega)| = \frac{\eta_a}{\zeta_a} = e^{-kh_c}\cdot D(\beta,\xi) = \left|\frac{e^{-\frac{\omega^2}{g}h_c}}{1-\frac{\rho\forall}{C_{33}}\omega^2}\right|, \tag{4.12}$$

i.e. the magnitude of the heave transfer function depends on the draft of the buoy, h_c, or on the decay of wave effects with non-dimensional depth $kh_c = \frac{\omega^2}{g}h_c$. In long waves, i.e. for low frequencies, both factors (the exponential depth parameter and the magnification factor) approach unity, and the buoy follows the wave.

The generalized equation of motion for the six degrees of freedom can be written as

$$\sum_{k=1}^{6}\left[(M_{jk}+A_{jk}(\omega))\ddot{\eta}_k + B_{jk}(\omega)\dot{\eta}_k + C_{jk}\eta_k\right] = F_j e^{-i\omega t}, \tag{4.13}$$

where M_{kj}, $j=1,...,6$ are the components of the generalized mass matrix of the structure and F_j are the complex amplitudes of the exciting forces and moment-components given by the real part of $F_j e^-i\omega t$ [12]. A_{kj} and B_{kj} vary according with the wave frequency ω. Using the dynamic balance equation (4.13), the motions on the other five degrees of freedom can be deducted in the same way as it was done for the vertical motion *heave* if a proper referential is chosen where the gravity centre coincides with the vertical axis of the referential. This way, the motions will be decoupled and can be studied individually.

To illustrate the behaviour of floating structure, the vertical motions (heave and pitch) of a semi-submersible submitted to an incident wave in the x direction will be studied. Tranfer functions will be obtained in each mode and excitation forces and moments, F_3 and F_5 will also be analyzed in order to obtain these transfer functions. The referential origin is in the gravity centre of the structure.

The semi-submersible in analysis is a structure that consists of two pontoons and four columns, and it is assumed that there is no damping. The pontoons are responsible for the flutuability of the structure and the columns guarantee the hydrostatic stiffness of the structure. The structure is symmetric and operates in deep water (Figure 4.3)

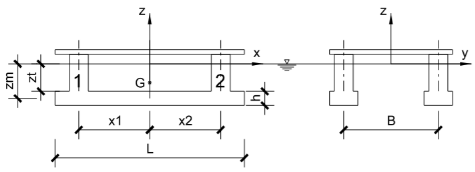


Figure 4.3: Semi-submersible with two pontoons and four columns (adapted from [12]).

The undamped equation of vertical motion can be written as

$$(\rho V + A_{33})\ddot{\eta}_3 + C_{33}\eta_3 = F_3(t). \tag{4.14}$$

The pontoons and the columns contribute to vertical excitation force. Considering only the contribution of the pontoons,

$$F_3^p(t) = -\rho g\zeta_a\sin(\omega t)e^{kz_m}\cos\left(\frac{kB}{2}\right)k\left(V_p+\frac{A_{33}}{\rho}\right),$$

where B is the distance between the pontoons axis and V_p is the pontoons volume (Figure 4.3). This force is the sum of the Froude-Krylov force with the diffraction force. The Froude-Krylov force was calculated considering that all the pontoons are submersed, so the expression (3.1) could be used. In terms of the columns, the contribution to the vertical excitation force is given by the product of the pressure at the centre of the section with the area of each column ($A_0/4$), so that the pressures on the columns on the left side of the gravity center G given by p_1 and the pressure on the columns on the right side of G are given by p_2 (Figure 4.3), i.e.,

$$\begin{aligned} p_1 &= \rho g \zeta_a e^{kz_t}\sin(\omega t + kx_1) \quad \text{and} \\ p_2 &= \rho g \zeta_a e^{kz_t}\sin(\omega t + kx_2) \end{aligned}$$

where x_1 is the distance from the left column axis to the vertical referential axis, x_2 is the distance from the right column to the vertical referential axis and z_t is the vertical distance from the free surface to the top of the pontoons.

The contribution from the columns to the vertical excitation force is given by

$$F_3^c(t) = \rho g \zeta_a \sin(\omega t) e^{kz_m}\cos\left(\frac{kB}{2}\right) A_0 e^{k(z_t - z_m)},$$

where A_0 is the waterplane area and z_m is the vertical coordinate of the geometric centre of the pontoon. It is assumed that the free surface elevation at the center of the structure is $\zeta = \zeta_a \sin(\omega t)$ and that $k(z_t - z_m)$ is small (λ is big in relation with the structure dimensions). F_3 can now be written as the sum of the two contributions, $F_3(t) = F_3^p(t) + F_3^c(t)$.

By analogy with an oscillatory linear system,

$$\begin{aligned} &\left(-\left(\rho V + A_{33}\right)\omega^2 + \rho g A_0\right)\eta_3 = F_3(t) \quad \text{and} \\ &\eta_3 = \eta_{3a}\sin(\omega t), \end{aligned}$$

i.e.,

$$\frac{\eta_{3a}}{\zeta_a} = e^{kz_m}\cos\left(\frac{kB}{2}\right)\left(1 - \frac{\frac{\omega^2 z_m}{g}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right),$$

where the natural frequency on the vertical translation mode is

$$w_n = \sqrt{\frac{\rho g A_0}{M + A_{33}}}, \tag{4.15}$$

where M is the submerged mass of the semi-submersible.

The transfer function for vertical motion is given by

$$H_3(\omega) = \frac{\eta_{3a}}{\zeta_a} = e^{kz_m}\cos\frac{kB}{2}\left(1 - \frac{kz_m}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right). \tag{4.16}$$

This example can be illustrated with values. Considering an harmonic wave of $T = 10$ s that propagates along the negative x-axis; the dimensions are $L = 100$ m, $B = 50$ m, pontoons have a section of 15×7 m^2, the columns diameter is 10 m, the draught is 22 m, $x_1 = -37.5$ m, $x_2 = 37.5$ m; $z_t = -15$ m and $z_m = -18.5$ m. Assuming that $A_{33} = 2LA_{33}^{(2D)}$ [12], where L is the length of a pontoon and $A_{33}^{(2D)}$ is the vertical bidimensional added mass for the vertical motion of a pontoon. It is also assumed that $A_{33}^{(2D)} = 2.3\rho A_p$ [12], where A_p is the area of a pontoon section. It follows that

$$M + A_{33} = \rho(2LA_p + |z_t|A_0) + 2\times 2.3\rho A_p L = 7.59\times 10^7 \text{ kg}.$$

The restoring coefficient

$$C_{33} = \rho g A_0 = 3.16 \times 10^6 \text{ kg s}^2. \tag{4.17}$$

The vertical excitation force is given by the sum of the two contributions and replacing with values,

$$F_3 = F_3^p + F_3^c = -5.9 \times 10^6 \sin(\omega t) \quad \text{(N)}.$$

Regarding the undamped equation of motion, knowing the values of $M + A_{33}$ and C_{33} and replacing these values on the equation (4.14), the vertical motion η_3 is given by

$$\eta_3 = 0.2\sin(\omega t).$$

Expression (4.16) is used to write the transfer function in heave, since the vertical excitation force and vertical motion are known. Tranfer function H_3 is represented in Figure 4.4. The natural frequency of the structure is given by

$$w_n = \sqrt{\frac{\rho g A_0}{M + A_{33}}}. \tag{4.18}$$

The stiffness is given by the columns. The natural frequency of the semi-submersible is $\omega_n = 2.0$ rad/s. By analysing Figure 4.4, it can be observed that the graphic has a vertical asymptote for $\omega = 2.0$ rad/s as expected, since there is no damping.

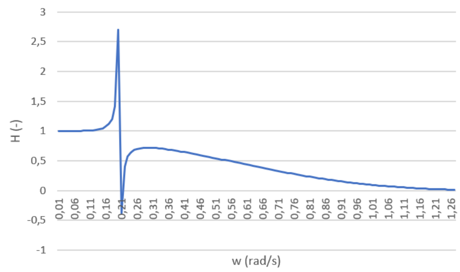


Figure 4.4: Transfer function for the vertical motion (*heave*) of the semi-submersible.

The pitch equation of the semi-submersible is given by

$$(I_5 + A_{55})\frac{\partial^2 \eta_5}{\partial t^2} + C_{55}\eta_5 = F_5(t). \tag{4.19}$$

The pitch inertia moment I_5 is assumed to be numerical equivalent to A_{55} coefficient, to which the pontoons and the columns contribute [12] and the expression was derived in the dissertation [19] The resul is

$$A_{55} = 4.1 \times 10^{1}0 \text{ kg m}^2.$$

To calculate C_{55} the moment in the center of the pontoon is considered,

$$M_p = \rho g \times \frac{1}{2} \times \frac{B}{2} \times \frac{B}{2} \times \theta \times L \times \frac{2}{3} \times \frac{B}{2} \times 2.$$

Replaced by proper values (ρ, g, B and L), it is obtained $C_{55} = 2.8 \times 10^9$ kg m^{2}s^{-2}. In the expression of the wave exciting pitch moment F_5 [19] the two terms from contributions from the horizontal forces on the columns can be neglected. In this way, after replacing by poper values, the wave exciting pitch moment is given by

$$F_5 = 2.3 \times 10^8 \cos(\omega t) \quad \text{(Nm)}.$$

From equation (4.19) it can be derived the pitch motion

$$\eta_5 = -0.008 \cos(\omega t) \quad \text{(rad)}.$$

With the motion and forces equations (η_5 and F_5) it is simple to determine the transfer function for the rotation around y axis (*pitch*) since it's the ratio between the motion amplitude η_{5a} and the wave amplitude ζ_a,

$$H_5(\omega) = \frac{\eta_{5a}}{\zeta_a} = \frac{F_5}{\zeta_a \sin(\omega t)\left(-\omega^2(I_5 + A_{55}) + C_{55}\right)}.$$

The transfer function H_5 is represented in Figure 4.5 once the values of the example are replaced. A cancellation period can be observed when the excitation force on the columns cancels the excitation force from the pontoons.

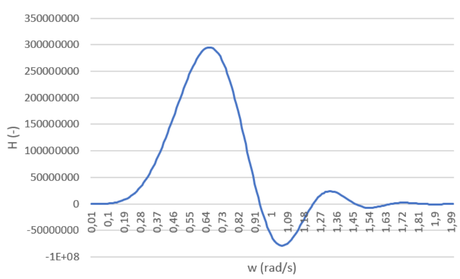
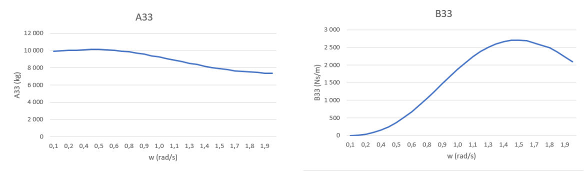


Figure 4.5: Transfer function for the rotation motion around y axis (*pitch*) of the semi-submersible.

5 Numeric model of a barge

The numeric results of a floating body submitted to an harmonic wave is presented in this chapter. The body is a barge with geometry: $L = 2$ m (x), $B = 4$ m (y) and $D = 2$ m (z). The structure was modelled on Nemoh [16]. This program calculates the hydrodynamic coefficientes $A_{kj}(\omega)$ and $B_{kj}(\omega)$ that depend on the wave frequency.

The program gives hydrodynamic coefficients accordingly with the frequency, and it also provides the resulting forces, at the referential's origin, of Froude-Krylov force and diffraction force. Radiation pressures are given for each panel of the mesh and the resulting radiation force must be calculated. In the dissertation the vertical motion (*heave*) was discussed as well as the rotation around the y axis (*pitch*) [19]. In Figure 5.1 the hydrodynamic coefficients for the vertical motion are presented as an example. The analysis of the results of forces and motions are presented in the dissertation [19].



(a) $A_{33}(\omega)$. (b) $B_{33}(\omega)$

Figure 5.1: Hydrodynamic coefficients for the vertical motion determined in Nemoh.

6 Conclusions

The work was important to understand how the sea waves act on an offshore structure, and the structure's response to such forces, bearing in mind that the structure's size and the wave lenght must both be taken into account at the same time. This knowledge is fundamental to support the design of an offshore structure.

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