A Multi-Objective Nurse Scheduling Model with Preferences: Case Study in a Portuguese Public Hospital

Maria Lopes Martins
Paper to obtain the Master of Science Degree in Industrial Engineering and Management
Instituto Superior Técnico

September 15, 2020

Abstract

The Nurse Rostering Problem is a well-known and widely studied operations research problem. It consists of the allocation of a set of nurses, to a set of shifts and days in a planning horizon, meeting the demand and subject to a set of operational and legal constraints. An efficient nurse rostering is essential in an hospital, due to its direct correlation with the service level. Also, it is desirable to incorporate nurses' preferences as it directly impacts their satisfaction thus improving their efficiency at work. This thesis intends to develop an automated scheduling process while considering nurses' preferences for nurse units in the Serviço Nacional de Saúde spectrum. To this end, a multi-objective mixed integer linear programming model is developed aiming at minimizing 6 objectives: weekly overworked nights, under- and overtime, overstaffing, same specialty overstaffing and unfulfilled positive and negative preferences. Results show that it is not feasible to comply with all the scheduling requirements, albeit it is possible to incorporate nurses' preferences in an automated manner with the developed approach.

Keywords: *Nurse Rostering, Preference scheduling, Multi-Objective Programming, Operations Research for Health Care*

Introduction

Staff scheduling is an essential step in ensuring satisfactory health care for patients, ideally provided profitably, implying an efficient use of available resources, including nurses [21]. In recent years, the functioning of health systems, both in public and private sector, has undergone drastic changes due, on the one hand, to technological development and, on the other, to the increase in average life expectancy [1]. According to [3], automation in the creation of acyclic work rosters (as in the case of most nursing services) is recent and despite being computationally edited it is often generated manually. The Portuguese population has an high aging index, with high incidence of long-term or chronic diseases, which leads to an overload of the care providers of the *Serviço Nacional de Saúde* (SNS), the governmental agency responsible for guaranteeing the provision of health care to virtually all the Portuguese population. Thus, an effort should be made to generate balanced rosters considering the distribution of undesired shifts, rest days or weekends and avoiding excessive fatigue which contributes to the reduction of errors and accidents and improvements in the service provided [14] as well as perceived justice by nurses. This is especially important; according to [4], content workers are more productive, which consequently leads to an increase in service levels. This satisfaction is related not only to fairness aspects related to general preferences, but also to the particular preferences of each nurse, which can more easily be incorporated into an automatically generated roster. In this regard, in recent years, many works have incorporated personal preferences into their models: [1] [12] [23] are just a few examples, from 2018. According to the research conducted in this thesis, there is no mentioning in literature, at the date of delivery of this work, an optimization nurse scheduling model adapted to the Portuguese case. Thus, this master thesis addresses the development of an optimization model capable of:

- Automatically generating nurse rosters
- Increasing nurses' satisfaction by including general and personal preferences in the roster.

Literature Review

The Nurse Rostering Problem is one of the most complex and investigated personnel scheduling problems in recent decades [3]. Rostering is defined as the allocation of resources at certain times and subject to restrictions to optimize some objectives or simply to obtain feasible allocations [22]. Some characteristics of a quality roster are reflected in the respect for legal provisions, balance of the distribution of work among employees and throughout the planning horizon, guarantee of demand satisfaction and, more recently, incorporation of preferences [3, **?**]. A quality schedule increases the satisfaction of workers and, therefore, their efficiency in the tasks they perform [3, 16]. Nurses are qualified, indispensable and irreplaceable employees in any healthcare unit thus, the quality of their rostering directly impacts the quality of services provided to patients.
Since Dantzig's comment to Edie's work, in which he discusses for the first time a linear programming approach for scheduling toll workers [6, 8], operational research has been studying – alongside computing science – better and more effective approaches to scheduling and rostering problems

as well as methods for solving them, whether they are exact – mathematical programming – or approximate – metaheuristics – [3].

The Nurse Rostering Problem Nurse rostering consists of the assignment, subject to restrictions, to certain shifts on certain days, creating a work schedule depicted in Figure 1, in which each gray cell matches a shift worked. For n nurses, d days and t shifts, each roster appears as a two-dimensional matrix, with n rows and $d * t$ columns. A very simple scheduling problem may depend only on T_n and E_{dt}, which represent, respectively, the total shifts worked by each nurse n and the total number of nurses in each shift t of day d. However, in a hospital set, it is essential that rosters are designed to meet the needs of patients, which translate into the search for health professionals with different skills, availability and preferences, highly restricted by legal provisions and hospital rules. [**?**] defines 6 criterion

D	1						2						…	d						Tot.
T	F	1	2	3	…	t	F	1	2	3	…	t	…	F	1	2	3	…	t	
E_1																				T_1
E_2																				T_2
E_3																				T_3
…																				…
E_e																				T_e
Tot.	E_{1F}	E_{11}	E_{12}	E_{13}	…	E_{1t}	E_{2F}	E_{21}	E_{22}	E_{23}	…	E_{1t}	…	E_{dF}	E_{d1}	E_{d2}	E_{d3}	…	E_{dt}	-

Figure 1: Nurse roster example for e nurses, d days, and t shifts

to evaluate a nurse roster quality:

- **Coverage** Related to the number of nurses in each shift;
- **Quality** Related to the evaluation provided by nurses of their own individual roster, taking into account their preferences;
- **Stability** Related to the consistency of rosters, taking into account the provisions they abide by;
- **Flexibility** Related to the system's behaviour when a change occurs;
- **Fairness** Related to the nurses opinion on how the roster is influenced by them;
- **Costs** Related to the resource consumption when creating a roster.

Given the plethora of articles found on the Nurse Rostering Problem [7] proposes an $\alpha|\beta|\gamma$ classification based on the [11] homonymous classification for scheduling problems. This allows for a systematic review of articles according to Figure 2 adapted from [7].
Correctly modelling a problem by translating its characteristics into mathematical relationships is the basis for its adequate implementation. [2] proposes a four step methodology to do so:

- Treatment and analysis of the data collected;
- Constraint definition – both hard and soft
- Objectives definition
- Mathematical model formulation

$\boldsymbol{\alpha}$ Personnel Environment	Personal constraints	**A** Availability **S** Sequences **B** Balance **C** Chaperoning
	Skill interactions	**2, 3, . . .** Fixed number **N** Variable number **I** Individual skill definition
$\boldsymbol{\beta}$ Work characteristics	Coverage constraints	R range **T** Time intervals **V** Fluctuating
	Shift type	**2, 3, . . .** Fixed number **N** Variable number O Overlapping
$\boldsymbol{\gamma}$ Optimization objective	Objective	P Personnel constraints **L** Coverage constraints **X** Number of personnel **R** Robustness **G** General
	Mode	**M** Multi-objective

Figure 2: $\alpha|\beta|\gamma$ classification for Nurse Rostering Problems

The type of model chosen depends on the characteristics of the problem and also on the solution approach one wishes to implement. Mathematical programming can be categorized based on restrictions and objective function linearity, variables' domain, number of objectives, and accommodation of uncertainty in the model. When a model presents linear restrictions and objective function it is known as a Linear Programming Model, as it is the case of both [12] and [13] work. If on top of this, all variables are integer, the model is classified as Linear Integer Programming as it is the case of [23].
If more than one objective is to be optimized, like in the works of [18] and [1], the model is defined as Multi-Objective Integer Linear Programming Model.
Depending on the model developed, different solution approaches can be pondered:

- Exact Approaches: when solving a model to optimality through exact algorithms like the Simplex method or Branch-and-Bound, respectively found in [10] and [20];
- Approximate Approaches: heuristic or meta-heuristic approaches which are solution search algorithms for when dealing with models for each an optimal solution cannot be found in a reasonable time, found in the work of [13] and [17].

Problem Description

Currently in Portugal there is a constitutional guarantee of access to a tending-towards-free healthcare to all citizens [19]. Thus it is necessary that there is a network of healthcare providing institutions, financed by national taxes: the (SNS). The *Hospital do Espírito Santo Hospital de Évora* (HESE) is part of this group of institutions, acting as a general public hospital providing its users with various specialized medical services. It serves the Alentejo region, specifically in two areas of demographic influence [5][1]:

[1]Data from 2017.

- Direct Influence: NUTS III of Alentejo Central (Évora District) - 155,372 patients;
- Indirect Influence: NUTS III of Alentejo Litoral, Alto Alentejo and Baixo Alentejo - 320,302 patients.

In order to respond to the demand described, HESE has a team of 1,501 dedicated professionals. These include 492 nurses spread over eighteen services that encompass twenty-six distinct medical specialties. It is believed that 600 nurses would be needed to respond to the demand described [9].

Nurses work in most services on a continuous basis, with 3 8-hour and 30-minute shifts with a fixed schedule and no overlapping [2]: a morning shift starting at 8:00 am, an afternoon shift starting at 4:00 pm and an evening shift starting at 00:00. Shifts can be allocated on a rotation basis or in an acyclical manner; in the first case there is a predefined sequence of shifts that must be completed for each nurse or team of nurses whereas for the last one there is no defined sequence. This is the most common way of allocation shifts in HESE.

HESE uses a human resources management software called *Sisqual Operation Cockpit* which allows you to define a *a priori* desired rotation, applying it to the service in question. In other cases, the software works as an aid to manual scheduling, allowing for each day and each nurse to be assigned a type of shift, day off, vacation or other type of presence. The existence of teams implies that the schedule of the nurses who belong to it is identical, so their scheduling is equivalent to that of a single nurse.

The approach to the problem should be generic and capable of dealing with any nursing service integrated in the SNS network [3] as long as it presents continuous and acyclical shift patterns. Thus, of the existing 18 services in HESE, 2 were selected: Medicina 1 (Med 1) and Ortopedia (Orto). The choice is related to the differences in the characteristics of the problem, the degree of complexity and its size. The incorporation of nurses' preferences greatly affects scheduling: in services where this possibility is given, Head Nurses spend a great amount of time in the attempt to incorporate them in the roster.

Model Formulation

Problem Statement The problem addressed consists of scheduling a set of nurses, belonging to the same service, to a set of shifts that take place 24 hours a day, 7 days a week, in a time frame of 4 weeks. The set of existing shift types is:

- Morning (M) – from 8:00 to 16:30 ;
- Afternoon (T) – from 16:00 to 00:30;
- Night (N) – from 00:00 to 8:30;
- Rest (D) - fictional shift that represents the nurse's absence related to time off;
- Out (A) - fictional shift that represents the absence of a nurse for reasons other than a break (e.g. vacation, illness).

In order to guarantee the adequate provision of health care, each type of work shift - M, T, and N - have an ideal, minimum and maximum number of nurses allocated to it [4]; this allocation should be made according to the hours available for rostering, with each nurse ideally meeting the contract hours - which may differ between nurses - within a limit under- and overtime in each planning period. Still on the aspect of the adequacy of the care provided, attempts should be made to distribute nurses from different specialities, with each one having a maximum number of nurses of the same speciality to allocate to the same shift.
On each day, it is possible to allocate a nurse to a shift, among the existent types of shift. Nurses should be allocated to shifts according to their availability, which can be limited by days allowed, types of shifts allowed or a certain shift on a certain day from the time horizon. Some nurses may still find themselves in integration - and not on an internship, in which case they are not counted on the roster - and need monitoring by another service nurse, whenever they are scheduled to work.
Limits are also imposed on consecutive working days, up to a legal limit of 6 days - shifts M, T, N - and consecutive rest days, set at 6 in HESE - shifts D; the number of shifts of type A is not subject to consecutiveness restrictions. In addition, there are certain sequences of shifts that are legally prohibited and that cannot exist on a work schedule, related to positive shift rotation and the requirement for a rest day after a night of work.
Regarding the balance of work distribution and rest on the roster, it is mandatory for a nurse to enjoy a weekend-day off every 4 weeks of work, so there may be a need to allocate a certain nurse to a certain number of Saturdays or Sundays Tt is preferable that a nurse enjoys a complete weekend of rest. The ideal limit for night shifts is also set at 2, for each work week, for each nurse.
Lastly, the preferences of each nurse for each type of shift each day should be included, according to their availability, so that their satisfaction is also taken into account; nurses can present negative preferences - not being allocated to a certain shift - or positive - preferring to be allocated to a specific shift. For each day, a nurse should not present more than a positive preference, for the types of existing shifts.

Standard Model Formulation The model developed under the specifications of the problem statement is presented.
Sets

- $n \in N$: available nurses
- $s \in S$: shift types (e.g.) {1=M, 2=T, 3=N, 4=D, 5=A})
- $d \in D$: planing horizon days

[2] As nurses do not work at the same time, the 30-minute overlap serves only for passing information from one shift to the next.

[3] Data relating to other services and hospitals contacted is taken into account during the development of the model

[4] Although there is a half hour overlap between work shifts, this time is allocated to the shift between nurses and not the practical overlap of available nurses.

- $e \in E$: specialities
- $w \in W$: planing horizon weeks

Subsets

- S^W : working shift types (e.g. $\{M, T, N\}$)
- $S_{s'}$: forbidden shift types after s'
- S_n^P : nurse n allowed working shift types
- D_n^P : nurse $n$allowed working days
- D_n^A : nurse n out days
- D^{SUN} : Sundays
- N_{ds}^{DS} : available nurses for rostering on day d shift $s \in S^W$
- N^L : nurses in integration
- N_l^N : nurse l chaperon
- N_e^E : nurses with given e speciality

Parameters and Weights

- μ^w,μ^r : maximum consecutive working and resting days
- ρ_n^{sat} ρ_n^{sun} : nurse n minimum Saturdays and Sundays off in the planing horizon
- ψ : weekly maximum working nights
- λ_s : $s \in S^W$ duration
- σ_n : nurse n ideal daily working hours
- η : number of holidays
- β : maximum deviation from η worked hours
- δ_s : $s \in S^W$ ideal allocated nurses
- q_s^+, q_s^- : under- and overstaffing allowed on shift $s \in S^W$
- π_e : same speciality ideal maximum nurses in each shift
- υ_{nds}^+, υ_{nds}^- : nurse n positive or negative preference fo shift s in day s
- ω^{sun}, ω^{sat} : weight of penalty variables for incomplete weekends off (working only on Saturday or on Sunday)
- ω^{nig} : weight of penalty variables for weekly overworked night shifts
- ω^{h+}, ω^{h-} : weight of penalty variables for under- and overtime
- ω^{n+}, ω^{n-} : weight of penalty variables for under- or overstaffing
- ω^e : weight of penalty variables for excess same speciality nurses in the same shift
- ω^{p+}, ω^{p-} : weight of penalty variables for non met positive and negative preferences

Decision Variables

- $x_{nds} = 0, 1$, equals 1 if nurse n is allocated to day d shift s, 0 otherwise
- Φ_n^{sun},Φ_n^{sat} : number of incomplete weekends off (working only on Saturday or on Sunday)
- Ψ_{nw}^{nig} : nurse n overworked night shifts in week w
- Λ_n^{h+},Λ_n^{h-} : nurse n under- and overtime
- Δ_{sd}^{n+},Δ_{sd}^{n-} : under- and overstaffing for shift s in day d
- Π_{eds}^e : excess same speciality nurses in the same shift s of day d
- Υ_n^{p+},Υ_n^{p-} : nurse n non met positive and negative preferences

Objective function

$$min: \sum_{n \in N}(\omega^{sun}\Phi_n^{sun} + \omega^{sat}\Phi_n^{sat}) + \sum_{n \in N}\sum_{w \in W}(\omega^{nig}\Psi_{nw}^{nig}) + \tag{1}$$

$$+ \sum_{n \in N}(\omega^{h+}\Lambda_n^{h+} + \omega^{h-}\Lambda_n^{h-}) +$$

$$+ \sum_{d \in D}\sum_{s \in S}(\omega^{n+}\Delta_{sd}^{n+} + \omega^{n-}\Delta_{sd}^{n-}) +$$

$$+ \sum_{e \in E}\sum_{d \in D}\sum_{s \in S}(\omega^e\Pi_{eds}^e) + \sum_{n \in N}(-\omega^{p+}\Upsilon_n^{p+} + \omega^{p-}\Upsilon_n^{p-})$$

Each summed term in the multi-objective function 1 corresponds to the minimization of a specific objective:

- incomplete weekends off;
- overworked night shifts in week;
- under- and overtime;
- under- or overstaffing;
- exces same speciality nurses in the same shift;
- non met positive and negative preferences.

Constraints

$$\sum_{s \in S} x_{nds} = 1, \qquad \forall n \in N, d \in D \tag{2}$$

$$x_{nds} = 0, \qquad \forall n \in N, d \in D, s \notin S_n^P \tag{3}$$

$$x_{nds} = 0, \qquad \forall n \in N, d \notin D_n^P, s \in S^W \tag{4}$$

$$x_{nd,5} = 1, \qquad \forall n \in N, d \in D_n^A, s \in S \tag{5}$$

$$x_{nds} = 0, \qquad \forall n \notin N_{ds}^{DS}, d \in D, s \in S^W \tag{6}$$

$$x_{nds'} + x_{n,d+1,s} \leqslant 1, \qquad \forall n \in N, d \in D \backslash |D|, s' \in S, s \in S_{s'} \tag{7}$$

$$\sum_{i \in \{d,d+1,\ldots,d+\mu^w\}} \sum_{s \in S^W} x_{nis} \leqslant \mu^w, \tag{8}$$

$$\forall n \in N, d \in D \backslash \{|D|, |D|-1, \ldots |D|-\mu^w\}$$

$$\sum_{i \in \{d,d+1,\ldots,d+\mu^r\}} x_{ni,4} \leqslant \mu^r, \tag{9}$$

$$\forall n \in N, d \in D \backslash \{|D|, |D|-1, \ldots |D|-\mu^r\}$$

$$x_{nds} \geqslant x_{lds} \qquad l \in N^L, n \in N_l^N, d \in D, s \in S^W \tag{10}$$

$$\sum_{d \in D^{SUN}} x_{n,d-1,3} \geqslant \rho_n^{sat} \qquad \forall n \in N \tag{11}$$

$$\sum_{d \in D^{SUN}} x_{nd,3} \geqslant \rho_n^{sun} \qquad \forall n \in N \tag{12}$$

$$x_{nd,3} - x_{n,d-1,3} + \Phi_n^{sun} - \Phi_n^{sat} = 0 \qquad \forall n \in N, d \in D^{SUN} \tag{13}$$

$$\sum_{d \in \{7w-6,7w-5,\ldots,7w\}} x_{n,d,3} - \Psi_{nw}^{nig} \leqslant \psi, \qquad \forall n \in N, w \in W \tag{14}$$

$$\sum_{d \in D}\sum_{s \in S^W}(x_{nds} * \lambda_s) - \Lambda_n^{h+} + \Lambda_n^{h-} = \tag{15}$$

$$= \sigma_n * (|D| - 2 * |D^S| - |D_n^A| + \eta), \forall n \in N$$

$$\Lambda_n^{h+} \leqslant \beta \qquad \forall n \in N \tag{16}$$

$$\Lambda_n^{h-} \leqslant \beta \qquad \forall n \in N \tag{17}$$

$$\sum_{n \in N} x_{nds} - \Delta_{sd}^{n+} + \Delta_{sd}^{n-} = \delta_s \qquad \forall d \in D, s \in S^W \tag{18}$$

$$\Delta_{sd}^{n+} \leqslant q_s^+ \qquad \forall d \in D, s \in S^W \tag{19}$$

$$\Delta_{sd}^{n-} \leqslant q_s^- \qquad \forall d \in D, s \in S^W \tag{20}$$

$$\sum_{n \in N_e^E} x_{nds} - \Pi_{eds}^e \leqslant \pi_e \qquad \forall e \in E, d \in D, s \in S^W \tag{21}$$

$$\sum_{d \in D}\sum_{s \in S\{|S|\}} x_{nds} * \upsilon_{nds}^+ - \Upsilon_n^{p+} = 0 \qquad \forall n \in N \tag{22}$$

$$\sum_{d \in D}\sum_{s \in S\{|S|\}} x_{nds} * \upsilon_{nds}^- + \Upsilon_n^{p-} = 0 \qquad \forall n \in N \tag{23}$$

$$x_{nds} \in 0, 1 \qquad \forall n \in N, d \in D, s \in S \tag{24}$$

$$\Phi_n^{sat}, \Phi_n^{sun} \in \mathbb{N} \qquad \forall n \in N \tag{25}$$

$$\Psi_{nw}^{nig} \in \mathbb{N} \qquad \forall n \in N, w \in W \tag{26}$$

$$\Lambda_n^{h-}, \Lambda_n^{h-} \in \mathbb{Q}^+ \qquad \forall n \in N \tag{27}$$

$$\Delta_{sd}^{+n}, \Delta_{sd}^{-n} \in \mathbb{N} \qquad \forall d \in D, s \in S \tag{28}$$

$$\Pi_{esd} \in \mathbb{N} \qquad \forall e \in E, d \in D, s \in S \tag{29}$$

$$\Upsilon_n^{+p}, \Upsilon_n^{-p} \in \mathbb{N} \qquad \forall n \in N \tag{30}$$

Equations 2 to 6 refer to restrictions on the allocation of certain days or shifts to nurses. Equation 7, 8 and 9 define allowed and forbidden sequences of working and resting times. Equation 10 defines the chaperoning constraint. Equations 11 to 14 define the constraints over worked weekends and nights. Equations 15 to 17 pertain to the worked hours and Equations 18 to 20 to the coverage constraints. Equation 21 involves the speciality nurses and Equations 22 and 23 count the number or violated preferences. Finally, Equations 24 to 30 define the variables domains.

Model Implementation and Results

The models has been implemented in test instances and real instances based on the case study presented. Two solution approaches were used: the Weighted Sum Method and the Lexicographical Method.

For the Weighted Sum Method, Equation 31 depicts the values for each of the weights associated with each variable, defined with the stakeholders using the swing weighing procedure.

$$min: \sum_{n\in N}(0.124\cdot\Phi_n^{sun}+0.115\cdot\Phi_n^{sat}) \tag{31}$$

$$+\sum_{n\in N}\sum_{w\in W}(0.142\cdot\Psi_{nw}^{nig})+\sum_{n\in N}(0.133\cdot\Lambda_n^{h+}+0.053\cdot\Lambda_n^{h-})+$$

$$+\sum_{d\in D}\sum_{s\in S}(0.017\cdot\Delta_{sd}^{n+}+0.177\cdot\Delta_{sd}^{n-})$$

$$+\sum_{e\in E}\sum_{d\in D}\sum_{s\in S}(0.062\cdot\Pi_{eds}^{e})+\sum_{n\in N}(-0.071\cdot\Upsilon_n^{p+}+0.106\cdot\Upsilon_n^{p-})$$

For the Lexicographical Method, the preferences defined are:

Table 1: Lexicographical Method Preferences

Objective Function	Preference
$min: \sum_{n\in N}(\Phi_n^{sun}+\Phi_n^{sat})$	3
$min: \sum_{n\in N}\sum_{w\in W}(\Psi_{nw}^{nig})$	5
$min: \sum_{n\in N}(\Lambda_n^{h+}+\Lambda_n^{h-})$	4
$min: \sum_{d\in D}\sum_{s\in S}(\Delta_{sd}^{n+}+\Delta_{sd}^{n-})$	6
$min: \sum_{e\in E}\sum_{d\in D}\sum_{s\in S}(\Pi_{eds}^{e})$	1
$min: \sum_{n\in N}(-\Upsilon_n^{p+}+\Upsilon_n^{p-})$	2

MED_1 and MED_1^{T1} This instances intend to mirror the *Medicina 1* service. The first does not include preferences.

Table 2: Sets and Subsets

N :	$\{1,2,...,23\}$
S :	$\{1,2,3,4,5\}$
D :	$\{1,2,...,28\}$
E :	$\{1\}$
W :	$\{1,2,3,4\}$
S^W :	$\{1,2,3\}$
$S_{s'}$:	$\{\{\varnothing\},\{1\},\{1,2,3\},\{\varnothing\},\{\varnothing\}\}$
D_n^A :	$\{\emptyset\}$
D^{SUN} :	$\{7,14,21,28\}$
N_{ds}^{DS} :	$\{\varnothing\}$
N^L :	$\{\varnothing\}$
N_l^N :	$\{\varnothing\}$
N_e^E :	$\{2,7,15\}$

Table 3: Parameters

μ^w :	6
μ^r :	6
ψ :	2
λ_s :	$\{8.5,8.5,8.5\}$
ω_n :	$\{7,7,...,7\}$
η :	0
β :	17.5
δ_s :	$\{7,4,3\}$
q_s^- :	$\{1,0,0\}$
π_e :	1

Besides these sets, subsets and parameters presented in Tables 2 and 3, others were defined for allowed working shifts and days, number of off Saturdays and Sundays and preferences. E nurses were not scheduled as their roster is the predefined. Because the model is unfeasible under the defined instances, a simplification took place, with the input from the stakeholders, according to Equations 32.

$$\sum_{n\in N}x_{nds}-\Delta_{sd}^{n+}+\Delta_{sd}^{n-}=\delta_{sd} \qquad \forall d\in D, s\in S^W \tag{32}$$

The results from this experiments are displayed in table 4. The best found values are denoted by green. The resulting rosters, approved by the stakeholders under small editions, are portrait in Figures 3 to 6.

Table 4: Output parameters for MED_1 and MED_1^{T1}

	MED_1		MED_1^{T1}	
Output parameter	MSP	ML	MSP	ML
Computing Time	0.91 s	6.71 s	1.03 s	7.44 s
Φ_n^{sun}	0	0	0	0
Φ_n^{sat}	0	0	0	0
Ψ_{nw}^{nig}	0	0	0	0
Λ_n^{h+}	22.5	76	21.5	72
Λ_n^{h-}	64	24	80	20
Δ_n^{n+}	2	0	0	0
Δ_n^{n-}	13	0	13	0
Π_{eds}^{e}	0	0	0	0
Υ_n^{p+}	47	48	0	0
Υ_n^{p-}	0	0	0	0
2 nights worked sequences	28	21	20	23
# nurses with 1 full weekend	16	17	16	17
# nurses with 2 full weekend	4	2	4	2
# nurses with 3 full weekend	0	1	0	1
# nurses with 4 full weekend	3	3	3	3

The developed model is able to incorporate nurses preferences in the roster within an acceptable computing time. The found solutions are considered by the stakeholders as acceptable under minor changes.

Conclusions

Nursing is considered an essential activity for the provision of healthcare, and its correct organization guarantees

Enfermeiros	Dia																												Totais					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Manhã	Tarde	Noite	Descanso	Descanso S+D	Horas
1	D	M	D	M	N	D	D	D	N	D	D	D	M	M	M	M	T	D	M	N	N	D	T	N	D	T	T	T	7	5	5	11	2	144.5
2	T	N	D	D	N	D	D	D	M	T	D	D	N	N	N	N	D	M	D	D	D	T	N	D	M	T	T	T	3	6	7	12	4	136.0
3	M	N	D	D	M	M	M	N	D	M	D	M	T	T	D	M	D	M	D	D	D	N	N	D	T	D	M	M	10	3	4	11	2	144.5
4	T	N	D	D	M	M	M	D	T	T	T	D	M	M	M	D	T	T	D	D	D	N	D	M	D	M	D	D	8	6	2	12	4	136.0
5	T	T	T	D	D	M	M	M	M	D	M	T	N	N	D	D	D	N	D	M	M	M	D	M	M	N	D	D	10	4	4	10	2	153.0
6	D	M	D	M	D	M	M	M	D	M	M	M	M	M	M	T	D	D	M	M	M	D	N	D	T	D	D	D	14	2	1	11	2	144.5
7	M	M	M	M	M	N	N	D	N	N	D	D	T	T	T	D	D	D	M	N	N	D	D	D	N	D	D	D	6	3	7	12	2	136.0
8	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
9	N	D	T	T	N	D	D	M	D	D	T	T	D	D	D	D	M	M	T	D	D	M	T	D	M	M	N	N	6	6	4	12	6	136.0
10	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
11	T	D	T	N	D	T	T	T	D	T	D	T	D	D	D	M	M	M	T	T	T	D	T	D	D	D	T	T	3	13	1	11	2	144.5
12	M	M	N	N	D	D	D	D	M	M	M	D	D	D	M	T	T	T	N	D	D	M	M	M	D	N	D	D	9	3	4	12	8	136.0
13	D	D	D	T	D	T	T	D	T	N	D	M	T	T	D	D	N	D	N	D	D	M	M	T	N	D	M	M	5	7	4	12	2	136.0
14	D	D	D	T	T	T	T	N	D	D	T	D	M	M	T	N	N	D	M	M	M	T	D	T	D	N	D	D	5	8	4	11	2	144.5
15	D	T	D	D	T	T	T	T	D	D	N	N	D	D	M	M	M	T	D	T	T	D	T	D	D	M	M	M	6	9	2	11	2	144.5
16	D	T	T	D	T	D	D	N	D	N	D	D	M	M	T	N	D	N	D	M	M	M	M	N	D	D	M	M	8	4	5	11	2	144.5
17	N	D	N	D	M	D	D	T	T	T	D	M	T	T	D	D	D	D	T	T	T	D	M	D	M	M	M	M	7	8	2	11	2	144.5
18	M	T	D	M	T	N	N	D	D	M	D	M	N	N	N	D	M	D	N	D	D	D	D	M	M	D	N	N	7	2	8	11	2	144.5
19	N	D	M	T	D	M	M	D	M	M	T	N	D	D	D	M	N	D	D	M	M	D	D	N	D	M	M	M	11	2	4	11	2	144.5
20	D	M	N	D	D	M	M	T	D	D	M	N	D	D	D	D	M	M	T	T	T	T	D	T	T	D	M	M	8	7	2	11	2	144.5
21	D	D	M	N	D	M	M	M	T	D	N	D	M	M	T	T	D	D	M	M	M	T	D	M	N	D	D	D	10	4	3	11	2	144.5
22	D	D	M	D	M	N	N	D	N	D	M	T	D	D	N	D	T	N	D	M	M	D	M	D	T	T	N	N	6	4	7	11	2	144.5
23	M	D	M	M	D	D	D	M	M	D	N	D	M	M	D	T	D	T	D	N	N	N	D	T	D	T	T	T	7	6	4	11	2	144.5
Total M	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7						
Total T	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4						
Total N	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3						
Total D	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9						

MED_1

Figure 3: Generated Roster for MED_1 with the Weighed Sum Method

Enfermeiros	Dia																												Totais					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Manhã	Tarde	Noite	Descanso	Descanso S+D	Horas
1	D	N	D	M	D	T	T	D	D	D	D	D	M	M	M	M	D	T	T	T	T	D	M	T	T	T	D	D	6	9	1	12	2	136.0
2	D	D	M	D	D	T	T	N	N	D	M	D	M	M	N	D	M	D	D	M	M	T	N	D	T	T	D	D	7	5	4	12	2	136.0
3	D	N	D	D	D	M	M	T	T	D	T	N	D	D	D	D	M	T	T	N	N	D	M	D	T	D	T	T	4	8	4	12	2	136.0
4	D	T	T	D	T	D	D	M	T	D	N	D	T	T	D	M	N	D	D	M	M	T	T	D	D	M	N	N	5	8	4	11	2	144.5
5	M	D	M	D	N	D	D	D	M	M	T	D	M	M	M	M	D	T	T	T	T	D	T	D	D	D	N	N	8	6	3	11	2	144.5
6	T	D	T	T	T	N	N	N	D	N	D	T	D	D	M	T	T	D	M	D	D	D	T	D	D	M	N	N	3	8	6	11	4	144.5
7	D	M	D	T	T	N	N	D	M	D	D	M	N	N	D	M	N	N	D	D	D	N	D	M	M	M	D	D	7	2	7	12	4	136.0
8	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
9	D	D	M	M	D	D	D	D	M	D	M	M	T	T	D	N	N	D	M	T	T	D	T	T	D	T	T	T	6	9	2	11	2	144.5
10	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
11	M	D	D	D	T	T	T	D	M	T	N	D	M	M	M	N	D	M	M	M	M	M	D	D	D	N	D	D	10	4	3	11	2	144.5
12	M	N	D	M	M	D	D	T	T	N	D	N	D	D	T	D	M	M	N	D	D	M	M	M	M	N	D	D	9	3	5	11	8	144.5
13	D	M	D	T	N	D	D	D	M	M	N	D	M	M	D	M	M	M	D	N	N	D	D	N	N	D	M	M	10	1	6	11	2	144.5
14	N	D	N	D	M	M	M	M	D	T	T	D	N	N	D	D	T	N	D	D	D	T	D	M	N	D	T	T	5	6	6	11	2	144.5
15	D	T	N	D	M	M	M	M	D	M	D	T	D	D	T	T	D	D	T	N	N	D	D	T	D	D	T	T	5	8	3	12	2	136.0
16	M	M	M	M	D	D	D	D	D	T	D	M	T	T	T	D	M	D	M	M	M	M	D	T	D	D	M	M	12	5	0	11	2	144.5
17	N	D	D	D	M	M	M	M	N	N	D	D	M	M	M	D	D	M	N	D	D	T	N	N	D	D	M	M	10	1	6	11	2	144.5
18	N	D	D	T	D	T	T	D	D	D	M	M	M	M	N	N	D	M	D	D	D	N	D	M	M	M	M	M	10	3	4	11	2	144.5
19	T	T	N	N	D	M	M	T	N	D	M	M	D	D	D	T	D	D	D	T	T	D	D	M	M	D	M	M	8	6	3	11	2	144.5
20	M	M	T	N	D	M	M	M	D	M	D	D	T	T	T	D	D	D	M	M	M	D	M	D	T	T	D	D	10	6	1	11	2	144.5
21	T	T	T	D	M	M	M	D	T	T	T	T	N	N	D	D	D	N	D	M	M	N	D	D	D	N	D	D	5	7	5	11	2	144.5
22	D	M	M	N	N	D	D	T	D	M	M	T	D	D	D	T	T	D	N	D	D	M	M	N	N	D	M	M	8	4	5	11	6	144.5
23	T	D	D	M	D	N	N	N	D	D	D	N	D	D	N	D	T	T	D	M	M	M	N	D	M	M	M	M	8	3	6	11	2	144.5
Total M	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7						
Total T	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4						
Total N	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3						
Total D	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9						

MED_1^{T1}

Figure 4: Generated Roster for MED_1 with the Lexicographical Method

Enfermeiros	Dia																												Totais					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Manhã	Tarde	Noite	Descanso	Descanso S+D	Horas
1	N	D	D	M	D	D	D	D	T	T	D	T	N	N	N	D	M	T	D	T	T	N	D	D	N	D	M	M	4	6	6	12	2	136.0
2	M	M	M	M	T	T	T	D	M	D	M	T	N	N	D	D	D	D	D	M	M	D	D	T	D	N	D	D	8	5	3	12	2	136.0
3	M	D	M	D	M	M	M	D	M	D	T	D	N	N	D	T	N	D	D	M	M	M	N	D	D	N	D	D	9	2	5	12	2	136.0
4	N	D	M	D	D	M	M	D	D	D	D	M	D	D	M	T	N	D	M	M	M	M	N	D	N	D	M	M	11	1	4	12	2	136.0
5	D	D	D	M	T	N	N	D	D	M	M	M	D	D	D	M	D	D	M	M	M	T	D	M	M	D	N	N	10	2	4	12	2	136.0
6	D	M	D	D	T	T	T	N	D	D	D	M	M	M	D	D	M	D	M	T	T	T	D	M	T	T	D	D	7	8	1	12	2	136.0
7	T	D	D	D	M	M	M	N	D	M	N	D	M	M	T	D	M	T	N	D	D	D	D	D	D	T	T	T	7	6	3	12	2	136.0
8	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
9	D	T	D	N	N	D	D	N	D	M	M	T	D	D	D	T	D	M	M	N	N	D	D	N	D	M	M	M	7	3	6	12	4	136.0
10	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
11	D	N	D	N	D	T	T	T	D	D	D	D	M	M	T	T	T	N	D	T	T	N	N	D	T	D	D	D	2	9	5	12	2	136.0
12	M	D	M	T	N	D	D	M	N	N	D	M	D	D	N	D	D	M	N	D	D	M	M	M	N	N	D	D	8	1	7	12	8	136.0
13	T	N	N	D	D	D	D	T	D	N	D	D	M	M	D	D	M	D	T	T	T	D	M	T	T	D	T	T	4	9	3	12	2	136.0
14	T	D	N	D	M	M	M	D	D	M	M	D	M	M	D	M	D	T	T	D	D	D	D	D	M	M	N	N	10	3	3	12	2	136.0
15	D	D	T	T	N	D	D	T	T	D	D	M	T	T	D	M	T	D	T	D	D	D	M	M	M	M	M	M	8	8	1	11	4	144.5
16	D	N	D	N	D	T	T	D	T	D	T	N	D	D	M	M	T	N	D	M	M	D	M	N	D	D	T	T	5	7	5	11	2	144.5
17	D	T	T	D	M	M	M	M	T	T	T	T	T	T	D	D	D	N	D	D	D	D	D	T	D	D	M	M	6	9	1	12	2	136.0
18	N	D	T	D	T	D	D	M	M	T	T	D	T	T	D	D	D	M	M	D	D	N	D	D	M	T	T	T	5	9	2	12	4	136.0
19	T	T	D	M	D	D	D	D	D	T	D	N	D	D	N	N	D	T	D	M	M	M	D	M	T	T	N	N	5	6	5	12	4	136.0
20	D	M	D	T	D	N	N	D	M	N	D	D	M	M	M	N	D	M	N	D	D	T	T	D	D	D	M	M	8	3	5	12	2	136.0
21	D	T	T	D	D	M	M	M	N	D	N	D	M	M	M	D	D	M	D	N	N	D	T	T	D	M	D	D	8	4	4	12	2	136.0
22	M	M	N	D	D	M	M	T	N	D	D	N	D	D	T	D	T	D	T	N	N	D	T	N	D	D	M	M	6	5	6	11	2	144.5
23	D	D	D	T	D	N	N	D	M	D	N	D	T	T	T	N	N	D	D	M	M	T	T	D	M	M	D	D	5	6	5	12	2	136.0
Total M	6	6	6	6	6	7	7	6	7	6	6	7	7	7	6	6	6	7	7	7	7	6	6	7	7	7	7	7						
Total T	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4						
Total N	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3						
Total D	10	10	10	10	10	9	9	10	9	10	10	9	9	9	10	10	10	9	9	9	9	10	10	9	9	9	9	9						

MED_1^{T1}

Figure 5: Generated Roster for MED_1^{T1} with the Weighted Sum Method

Enfermeiros	Dia																												Totais					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	Manhã	Tarde	Noite	Descanso	Descanso S+D	Horas
1	D	M	D	M	D	D	D	N	N	D	D	D	M	M	M	T	T	T	T	T	N	D	D	M	D	D	T	T	6	7	3	12	2	136.0
2	T	D	T	N	N	D	D	M	D	T	T	D	M	M	D	D	M	T	D	N	N	D	D	D	M	D	N	N	5	5	6	12	2	136.0
3	D	M	D	D	T	N	N	N	N	D	T	T	D	D	D	D	M	N	D	T	T	T	T	D	T	D	M	M	4	8	5	11	2	144.5
4	T	D	N	D	D	M	M	D	T	T	N	D	T	T	N	D	T	D	N	D	D	T	D	T	N	N	D	D	2	8	6	12	4	136.0
5	M	D	D	D	M	N	N	D	T	D	D	T	T	T	T	D	D	D	M	D	D	M	T	D	T	T	T	T	4	10	2	12	2	136.0
6	D	D	M	D	D	M	M	M	D	N	D	D	M	M	M	M	D	M	T	D	D	N	D	T	T	D	M	M	11	3	2	12	2	136.0
7	M	T	D	D	M	N	N	D	M	D	N	N	D	D	M	M	N	D	D	M	M	T	T	N	D	N	D	D	7	3	7	11	4	144.5
8	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
9	D	N	D	T	N	D	D	M	D	D	D	T	D	D	D	D	N	N	D	M	M	M	M	T	T	T	N	N	5	5	6	12	4	136.0
10	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	M	M	M	M	M	D	D	20	0	0	8	8	170.0
11	N	D	M	D	M	M	M	T	D	D	T	N	D	D	D	M	M	D	M	M	T	D	D	D	M	D	M	M	11	3	2	12	2	136.0
12	M	N	D	M	T	D	D	M	M	T	T	T	D	D	N	N	D	D	M	D	D	M	N	N	D	T	D	D	6	5	5	12	8	136.0
13	T	D	D	T	N	D	D	D	D	D	M	M	T	T	T	D	M	D	D	N	N	D	M	N	N	D	M	M	6	5	5	12	2	136.0
14	M	D	D	D	D	T	T	T	D	M	D	D	N	N	D	M	M	M	N	D	D	D	M	M	M	D	N	N	8	3	5	12	2	136.0
15	D	D	D	T	D	M	M	N	D	N	D	M	T	T	T	T	D	M	T	T	T	N	D	D	D	M	D	D	5	8	3	12	2	136.0
16	N	D	N	D	M	D	D	T	D	M	M	D	M	M	D	D	T	T	N	D	D	D	D	M	M	M	T	T	8	5	3	12	4	136.0
17	D	M	T	N	D	M	M	M	M	M	M	D	D	D	D	D	D	D	D	N	N	N	N	D	D	M	M	M	10	1	5	12	2	136.0
18	N	N	D	M	T	T	T	D	M	D	M	D	N	N	D	N	N	D	D	D	D	D	N	D	D	M	M	M	6	3	7	12	2	136.0
19	D	M	M	M	D	M	M	M	N	N	D	M	M	M	N	N	D	M	D	M	M	D	D	D	D	N	D	D	12	0	5	11	2	144.5
20	D	T	T	N	D	T	T	D	T	D	D	N	D	D	M	D	T	N	D	M	M	M	D	T	D	T	T	T	4	10	3	11	2	144.5
21	D	T	N	D	D	M	M	D	M	M	D	M	M	M	T	T	D	D	T	T	T	D	M	M	N	D	D	D	9	6	2	11	2	144.5
22	T	T	T	T	T	T	T	D	D	D	N	D	M	M	M	M	D	D	M	M	M	D	D	M	D	D	D	D	8	7	1	12	2	136.0
23	M	D	M	D	D	D	D	T	T	T	D	D	N	N	D	T	D	T	D	M	M	T	T	D	D	M	M	M	7	7	2	12	2	136.0
Total M	7	6	6	6	6	7	7	8	7	6	6	6	7	7	7	7	7	6	6	7	6	6	6	7	6	7	7	7						
Total T	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4						
Total N	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3						
Total D	9	10	10	10	10	9	9	8	9	10	10	10	9	9	9	9	9	10	10	9	9	10	10	9	10	9	9	9						

MED_1

Figure 6: Generated Roster for MED_1^{T1} with the Lexicographical Method

a quality service. Nurse rostering is a decision made at the organizational level of hospitals, specifically for each nursing service, carried out by the Head Nurses who must periodically decide on the allocation of available nurses to work and rest shifts.
In Portugal, the surrounding macro environment, the legal provisions in force and general discontent by the professional class, add complexity to the rostering; on one hand, in some cases, the number of nurses needed is higher than those available (example of the Med 1 and Orto services, addressed) and on the other hand, nurses have limitations in the availability of days and work shifts, specifically in what concerns respect to working afternoons, nights and weekends.
Rostering at HESE, done manually, is hampered by the lack of professionals and legal restrictions in relation to the availability of nurses, particularly the group of those who have restrictions associated with Parental rights. The implementation of automatic rostering brings improvements in terms of efficiency of generated schedules and corresponding to nurses' preferences. In the case study, the Head Nurses incorporate preferences manually, making it impossible to incorporate all orders in a timely manner, which increases the generation time of the schedule, while decreasing the quality and satisfaction of nurses.
The achievement of the objectives is presented in the form of a mathematical model in Multi-Objective Mixed Integer Linear Programming, which retained the main characteristics of the case study and can be applied to instances with acyclic schedules and without work by teams.
The analysis of the results obtained in the model runs allows us to infer that the legal provisions are not fully complied with for the services analysed in the case study. The adaptation made to the model generates viable rosters. Subject to minimal manual changes these rosters could be implemented in the case study. The entire process of implementation and validation of the model was monitored by the *stakeholders* contributing to its acceptance and in keeping with the reality experienced at (HESE) and which mirrors the Portuguese reality.
The nurse scheduling problem presented is classified as $ASBCN|RN|PLM$, according to the classification $\alpha|\beta|\gamma$ of [7] for the classification of scheduling problems for nurses. The contribution of this work to the literature is its implementation in real instances and the inclusion of the *stakeholders* in the decision-making process.
The abundance of existing work on the Nurse Rostering Problem mirrors the multitude of problems encountered in scheduling nursing services. In the case of HESE there are services with cyclical schedules and teamwork which are not supported by the approach developed. The incorporation of work teams in the model, with acyclic schedules could be incorporated by extending the restriction associated with nurses in integration (Equation 10) to cases where there are nurses who should be scheduled together.
Another proposal for future work is related to the analysis of the demand and supply of the services of Med 1 and Orto which is in line with the proposal of the Nurse-Director to have floating nurses that according to the needs hospital staff would be allocated to different services. This possibility implies the simultaneous scheduling of several nursing services, so the model could be extended in order to incorporate services with greater needs. This extension would imply the resolution of the model for instances with dimensions much larger than those analysed, which would lead to an exponential increase in computational times (derived from the classification of the problem as NP-*hard*), so its resolution would possibly imply the application of heuristic algorithms.
Both suggestions presented were considered by the *stakeholders* as valid research paths and that provide value to the hospital.
Finally, the development of a Graphical User Interface is one of the fundamental aspects for the acceptance and use of the developed method. In this dissertation the development of this tool was not addressed, however it would be

desirable that in collaboration with the hospital's computer services, a user interface was developed, which could be integrated with the *software* currently available, the *Sisqual ® Operations Cockpit*.

References

[1] Esra Agca Aktunc and Elif Tekin. Nurse Scheduling with Shift Preferences in a Surgical Suite Using Goal Programming. *Industrial Engineering in the Industry 4.0 Era – Lecture Notes in Management and Industrial Engineering*, 2018.

[2] Ivo Blochliger. Modeling staff scheduling problems. A tutorial. *European Journal of Operational Research*, 158:533–542, 2004.

[3] E. Burke, P. De Causmaecker, G.V. Berghe, and H. Landeghem. The State of the Art of Nurse Rostering. *Journal of Scheduling*, 7:441–499, 2004.

[4] E. K. Burke, T. Curtois, and G. Qu, R. ando Vanden Berghe. A scatter search approach for the nurse rostering problem. *Journal of the Operational Research Society*, 61:1667–1679, 2010.

[5] Conselho de Administração do HESE. Plano de Atividades & Orçamento. Technical report, HESE, Évora, 2017.

[6] G.B. Dantzig. A Comment on Edie's "Traffic Delays at Toll Booths". *Journal of the Operations Research Society of America*, 2(3):339–341, 1954.

[7] Patrick De Causmaecker and Greet Vanden Berghe. A categorisation of nurse rostering problems. *Journal of Scheduling*, 14(1):3–16, 2011.

[8] C.L. Edie. Traffic Delays at Toll Booths. *Journal of the Operations Research Society of America*, 2(2):107–138, 1954.

[9] Enf. Director José Chora. Entrevista com o Enfermeiro Director do HESE. Comunicação Pessoal, 2018. Dados de Novembro de 2018.

[10] Zeineb Fourati, Badreddine Jerbi, and Hichem Kammoun. Planning and modeling nurse timetabling at an intensive care unit in a tunisian university hospital. 2016.

[11] R.L. Graham, E.L. Lawler, J.K. Lenstra, and A.H.G. Rinnooy Kan. Scheduling Nursing Personnel According to Nursing Preference, A Mathematical Programming Approach. *Annals of Discrete Mathematics*, 5:287–326, 1976.

[12] Salim Haddadi. Three-phase method for Nurse Rostering. *Internation Journal of Management Science and Engineering Management*, 2018.

[13] Zhenyuan Liu, Zaisheng Liua, Zhipeng Zhua, Yindong Shena, and Junwu Dongc. Simulated annealing for a multi-level nurse rostering problem in hemodialysis service. *Applied Soft Computing*, 64:148–160, 2018.

[14] Alireza Moselm, Somayeh Zamirinejad, Fatemeh Ghezeli, and Arash Akaberi. The impact of night shift on working memory performance: A pilot study. *International Journal of Humanities and Cultural Studies*, 5:1132–1138, 2015.

[15] T. Osogami. Classification of Various Neighborhood Operations for the Nurse Scheduling Problem Classification of Various Neighborhood Operations for the Nurse Scheduling Problem (Extended Abstract). (May), 2010.

[16] Y.A. Ozcan. *Quantitative Methods in Health Care Management: Techniques and Applications*. Jossey-Bass Public Health. Wiley, 2009.

[17] Erfan Rahimian, Kerem Akartunalıa, and John Levine. A hybrid Integer Programming and Variable Neighbourhood Search algorithm to solve Nurse Rostering Problems. *European Journal of Operational Research*, 528:411–423, 2017.

[18] M. Rajeswari, J. Amudhavel, Sujatha Pothula, , and P. Dhavachelvan1. Directed Bee Colony Optimization Algorithm to Solve the Nurse Rostering Problem. *Computational Intelligence and Neuroscience*, 2017.

[19] República Portuguesa. Artigo 64.º, Constituição da Républica Portuguesa, VII Revisão Constitucional [2005], 1976.

[20] Seyda Topaloglu and Hasan Selim. Nurse scheduling using fuzzy modeling approach. *Fuzzy Sets and Systems*, 161:1543–1563, 2010.

[21] Benno Woskowski. The Use of Auction in Nurse Rostering. *Game Theory in Management Accounting: Implementing Incentives and Fairness*, pages 193–208, 2018.

[22] Anthony Wren. Scheduling, Timetabling and Rostering - A Special Relationship? In *Selected Papers from the First International Conference on Practice and Theory of Automated Timetabling*, pages 46–75, London, UK, UK, 1996. Springer-Verlag.

[23] Simone Zanda, Paola Zuddasb, and Carla Seatzua. Long term nurse scheduling via a decision support system based on linear integer programming: A case study at the University Hospital in Cagliari. *Computers & Industrial Engineering*, 126:337–347, 2018.