Demand Response Dynamic Modelling of Load-Shifting Flexibility under Limited Consumer Observability

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Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.
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I thank Professor Pedro Carvalho for the interest shown in having me as his student and, more importantly, for not only suggesting me a subject so captivating and of upmost importance, but also for what he represented in my ever-increasing consideration and respect for science and academic research. I would also thank all my friends and colleagues for the moments we spent together that made this university period of my life so much enjoyable. Finally but supremely important, being the basis that allowed the possibility of me accomplishing this achievement, I thank my family for continuous support and teaching throughout my life.
Abstract

Demand flexibility and its responsiveness under price-based control is a major research field. Much attention has been payed to model demand elasticity. However, demand flexibility comes mostly as load shifting, and shifting dynamics have been neglected when modelling demand response.

In this study load shifting dynamics are modelled to simulate aggregate responses and analyze their predictability under time-varying prices. A particle hopping cellular automaton model is used to study demand response as a means to perform supply following.

With that model, simulation is used to analyze load-shifting main dynamics under indirect control strategies. An analysis on how demand responds to price signals and what implications dynamics may have on predictability and controllability, was conducted. Also, several properties and limitations of load-shifting w.r.t. target aggregate load or original load density distribution, were analyzed.

Keywords

Demand-Side management, demand response, load shifting, load-following, supply-following, supply dynamics, indirect control.
Resumo

A flexibilização dos consumos e o controlo dos mesmos com base em preços é uma importante área de investigação. Apesar da flexibilidade ser concretizada pelo adiamento de consumos discretos, a atenção tem sido muito focada na modelização da flexibilidade como uma elasticidade dos consumos aos preços, negligenciando as dinâmicas de adiamento na modelização da resposta dos consumidores.

Neste estudo, as dinâmicas de adiamento são modelizadas simulando a resposta agregada da procura e analisando a previsibilidade dessa perante variações pré-estabelecidas dos preços no tempo. A resposta agregada é obtida com recurso a modelos de resposta de consumidores individuais, programados como autômatos celulares de “particle hopping” para seguimento da oferta.

O modelo desenvolvido é utilizado para estudar as principais dinâmicas de resposta sob controlo indirecto, e as limitações que essas dinâmicas impõem à previsibilidade e controlabilidade da resposta. Os processos de adiamento foram estudados sob diferentes objectivos para a procura agregada e para populações de consumidores com diferentes densidades de consumo, numa tentativa de abstrair propriedades intrínsecas aos processos de adiamento de consumos.

Palavras Chave

Gestão de carga, resposta da carga, deslocamento de carga, seguimento de carga, seguimento da geração, dinâmicas de oferta, controlo indirecto.
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## Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>DR</td>
<td>Demand Response</td>
</tr>
<tr>
<td>DSM</td>
<td>Demand Side Management</td>
</tr>
<tr>
<td>w.r.t.</td>
<td>with respect to</td>
</tr>
<tr>
<td>CA</td>
<td>Cellular Automaton</td>
</tr>
<tr>
<td>ToU</td>
<td>Time of Use</td>
</tr>
<tr>
<td>ICT</td>
<td>Information and Communications Technology</td>
</tr>
<tr>
<td>HVAC</td>
<td>Heating, Ventilation, and Air Conditioning</td>
</tr>
<tr>
<td>LHS</td>
<td>Left-Hand Side</td>
</tr>
<tr>
<td>RHS</td>
<td>Right-Hand Side</td>
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<td>Electric Vehicles</td>
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Introduction

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1.1 Motivation

European Commission has recently established frameworks to reach targets for clean energy transition by 2030 and to reach climate neutrality by 2050 [1]. Reaching these targets requires novel solutions.

The current electric grid has a relatively simple architecture, where generation is largely predictable and deliberately dispatched. Demand is also well understood, predictable and passive. However, as penetration of renewable generation increases, together with the also increasing adoption of electric vehicles, this premise is changing at an impressive rate. Renewable Energies introduces variability in generation and with the increasing electrification of heat and transport, demand is predicted to increase considerably whilst becoming less predictable. These changes will contribute to challenges on peak-power supply and on what the reliability of electricity infrastructure is concerned. For this matter demand side management assumes a great importance [2–4].

Figure 1.1: Types of price- and incentive-based demand response by commitment timescale (from [4])

Figure 1.1 shows the load commitment timescale and when could demand response be useful depending on the way response is controlled. In what years/months ahead load commitment is concerned much has been studied and time-of-use prices exist for a long time as a way of decreasing peak demand. Conversely, day of dispatch/use or real time demand side management is a relatively recent subject, but with a lot of attention being given to it nowadays and already being implemented, particularly in the form of reserves or balancing markets. However, in a day ahead load commitment timescale, demand response could assume an important role on what could be called the change of paradigm from a “load following” to a “supply following” paradigm, as suggested in [5].

In the conventional supply-following paradigm generation is being dispatched to follow the current load and is seen as an ancillary service to pick up load ramp between scheduling steps and maintaining
the frequency of electricity within the required tolerance. With these changes taking place, the future will see demand being called on to follow supply. For this demand response programs more flexible demand, relying upon some form of energy storage, are going to assume a key role.

Loads that comply with sufficient elasticity requirement are so diverse that it will force utilities and other entities to deal with the future demand-side resource as a composite of different resource types, whose response characteristics they will have to segment into different classes in order to be able to predict individual class responses [6]. This creates the need for studying demand response main dynamics and their implications on load predictability and control, focusing mainly on its implementation.

1.2 Topic Overview

An impressive body of research has recently been made available in the topic of demand response. Many of the studies focus on specific applications as electric vehicles, water heaters and air-conditioners (HVAC). Plug-in hybrid electric vehicles have been studied as a means to regulate power in form of primary, secondary and tertiary frequency control and other ancillary services [7–10]. In [11] it is assumed that large groups of electrical loads can be controlled as a single entity to reduce their aggregate power demand and then, focuses on the dynamics of thermostatically controlled loads, modelling them in a way simple enough to be used in control design. With a similar objective, in [12] the dynamics of aggregate thermostatic loads are studied using a partial differential equation model aiming at designing nonlinear load control algorithms. Also, in [13], performance limits are established for probabilistic control schemes that coordinate populations of thermostatically controlled loads for the purpose of providing power system services, such as regulation and load following. The potential for Power-to-Heat/Cool applications to increase flexibility in the European electricity system is analysed in [14].

Different control strategies have been explored, from price based [15–18] to direct control for load following [19, 20] or price based demand response more focused on industrial applications with varying electricity prices based on Time-of-Use (ToU) rates [21].

These control strategies and others have been aimed at different objectives such as to manage real-Time energy imbalance [19], peak load reduction [15], increasing market efficiency, system reliability or even complement to generation and storage services.

Some papers propose centralized approaches that rely on information on load use that might be considered private or be shared under confidentiality to optimize customer responses. Other papers propose decentralized approaches that rely on load forecast to assess response elasticity with respect to varying price tariffs (e.g., [22]). Most papers focus on peak-load reduction only, which is an important issue but is not sufficient to support the increased integration of renewables in the future. Several papers deal with load-following but take demand response as bid-quantity market offers often ignoring
the dynamic specific limitations of load response (e.g., [23, 24]).

Under ideal direct control, response predictability is not an issue: within the intrinsic limitations of shifting, aggregate responses behave as expected. However, this type of control implies high investments in monitoring and control infrastructures. Yet, under indirect control enabled by time-varying prices or incentives, response predictability becomes very challenging. Without real-time auction mechanisms (such as those envisaged in peer-to-peer blockchain markets), indirect control based on prices introduces an uncertainty factor into the market clearing process, which comes from the underlying uncertainty in user availability to respond [25].

However valid and important past contributions to this field of research have been, they all lack a study of what are the intrinsic dynamics of the load-shifting process and what implications can these dynamics have on DR applicability.

1.3 Objectives

At the end of this thesis, it is expected to have advanced in several capabilities but also limitations intrinsic of the load-shifting process, when applied to supply-following as an ancillary service. Other studies have seen that one as to consider a set of technical obstacles to demand response implementation through load-shifting. It is intended to show other factors to take into account, that are intrinsic of the load-shifting process itself.

A model for load-shifting main dynamics is proposed to study load-shifting being it under direct control or indirect control. From this model it is intended to derive relevant conclusion on the implications that load-shifting dynamics can have on supply-following implementations.

1.4 Thesis Outline

This dissertation is organized in five chapters. In the first chapter, the topic of the thesis is introduced by referring to other related studies as well as detailing the motivation behind it and the objectives that are to be accomplished.

In Chapter 2, a model for load-shifting main dynamics is introduced, as it is proposed in [26]. In this same chapter this model is well detailed and applied to several load-shifting applications enabled by direct control.

In Chapter 3, the same model introduced before is modified for an application to load-shifting enabled by indirect control. Also, two different indirect control strategies are proposed and compared. The study focuses, then, on some limitations introduced by indirect control. Finally, a model for the supply dynamics that are part of the indirect control strategies introduced, is derived. From this approximation
some insights on the effect of the supply into the process, are extracted.

In Chapter 4, there is a focus on relating previous conclusions to real life applications by making some generalizations regarding the load following target function and the original load density.

In the last Chapter, the thesis is concluded with a summarized overview of the results obtained throughout the dissertation emphasising the implication they can have on real life demand response applications enabled by load-shifting. Future work to be carried out in this area following this line of thought is also proposed.
Load Dynamic Model

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2.1 Modelling load-shifting

This study stands, primarily, on a model for load-shifting main dynamics, as proposed in [26]. Thus, it is relevant to describe this model into further detail, as a framework for an analysis on the provision of ancillary services, through demand response. The following sections replicate much of the background contents of [26] in an effort to describe the model into further detail.

Shiftable loads are referred to as interruptible, a subclass of curtailable loads that are perfectly elastic in time, as they rely upon perfect storage, but whose elasticity is confined to a time window limited ahead by a given time for load use. An example of such a load is an electric water heater whose time to heat the water is initially scheduled to a period, say \([s, s + \tau]\), that ends a few periods before the time instant \(t\) required to use the hot water, i.e., \(u > s + \tau\). The load “particle” that corresponds to the power consumption used during the storage period (for water heating) can be shifted ahead in time until \(t = u - \tau\) without anticipated discomfort.

Figure 2.1: Illustration of the time-constrained maximum flexibility of a time-shiftable incompressible load from a single user with \(\tau = 1\) under the assumptions of instantaneous load use (read the figure from top to down plots starting from the left column before going to the righ one).

In this study, the time window is set to its theoretical maximum assuming that (i) the load use is instantaneous and (ii) the original scheduling of storage is set to start immediately after every such use, anticipating the original storage period \([s, s + \tau]\) as much as possible and this way maximizing the room for shifting it ahead in time. Under such assumptions, the original load particles are flexible to be moved ahead as long as they do not overlap the scheduled times of use, \(u\).

Figure 2.1 illustrates the dynamics of load-shifting for a single user under the assumptions stated above. The figure shows four plots in consecutive time instances. In the right-hand side plot, the original schedule of load particles is shown. The schedule can be changed at \(t = 4\) by shifting the first particle ahead in time. The lower and left and side plots show the successive shifting of load particles: in the second and third plots the schedule at \(t = 5\) and \(t = 6\) is obtained after shifting the first particle once
and again; in the fourth plot, for $t = 7$, the first particle could no longer be shifted as there was another particle ahead, so, the second particle is moved ahead instead.

### 2.2 Particle Hopping

Particle hopping models have been used to simulate a variety of dynamic processes, including traffic-flow [27, 28]. The utilization of particle hopping models is proposed in [26], and presented here with more details, to simulate load-shifting dynamics.

#### 2.2.1 Model Description

Each individual load-shifting process is represented as a particle hopping process in a one dimensional lattice of cells (a string of cells), each cell representing a time period. In the lattice, cells are either occupied by exactly one load particle or are empty and shifting takes place by hopping between cells in one direction only (the time direction). Because particles can only be shifted ahead onto empty cells, the process of consecutive shifting can be seen as a travelling process with elastic collisions with particles ahead.

The population of load-users is then modelled as a two-dimensional lattice of cells, one being the time-stamp dimension of the schedule and the other being the user dimension. Users are considered homogeneous resources in the sense that their load particles have the same associated energy, i.e., the same power consumption in any period $\tau$. Figure 2.2 illustrates one such lattice for eight users and eighteen time periods. Note that the schedule depicted in the first row of the lattice (first user) corresponds to Figure 2.1 original schedule.

![Figure 2.2: Two-dimensional lattices of cells representing the schedule of storage periods for the population of load users: original schedule (left) and after exploring maximum flexibility (right). (Adapted from [26])](image)

Load-shifting flexibility is represented in the lattice by the empty positions ahead (in the time-stamp dimension) of each occupied position. The empty positions ahead of any occupied position can be used to regulate the population’s aggregate output (a summation in the user dimension). The empty positions
change every time an occupied position is moved ahead to control the population’s aggregate output. So, control over the aggregate output in each time $t$ is exercised by deciding which particles are to be moved ahead, and by doing so it determines in $t$ the configuration of the lattice in $t + 1$. The process is therefore dynamic. Both the number of particles available to be moved ahead in each time period $t$ and their flexibility depend on the number of particles one decided to move ahead in $t - 1$.

Some definitions are established to help understanding the model and the algorithm to be proposed. For simplicity of notation, it can be assumed that $\tau = 1$ and use indifferently periods, $k$, and time stamps, $t$. Let then the users be identified by $n = 1, \ldots, N$, where $N$ designates the population cardinality. Let $p(k)$ be the aggregate (normalized) output of the population in time $k$:

$$p(k) = \sum_{n=1}^{N} d_n(k), \quad k = 1, \ldots, T$$  \hspace{1cm} (2.1)

$$d_n(k) \in \{0, 1\} \quad n = 1, \ldots, N; \quad k = 1, \ldots, T$$  \hspace{1cm} (2.2)

where $d_n$ is the load demand of user $n$.

### 2.2.2 Generating Stochastic Lattices of a Given Load Density

Defining the original schedule of load particles is needed for simulation purposes and it is also convenient to further detail the concept of load density, $d$, in this study, as a key variable in the process. As it is proposed in [26], load homogeneity, as a simplification of the model, is understood not only as load particles being all similar, which after normalization led to $d_n(k) \in \{0, 1\}$. The concept of homogeneity, in this case, is extended from size to time-distribution. This imposes that load particles be distributed in time in the same expected way, given that it is assumed for load particles to be distributed randomly in time with the same stochastic process. This leads to a simple process that can be described as a Markov process with just two states, $\{0, 1\}$, as described in (2.2) and illustrated in Figure 2.3.

![Figure 2.3: State-diagram illustration representing possible transitions within the Markov process that generates the original schedule.](image)

To describe a Markov process, a transition matrix, $P = [p_{ij}]$ needs to be defined by setting the likelihoods of power staying in a given state or changing between states. Each entry of the matrix, $p_{ij}$ represents the likelihood of a load being in power state $i$ in one time period and transitioning to power
state \( j \) in the next time period. In order to avoid time-consecutive particles, the process has been defined by a transition matrix of the form:

\[
P = \begin{bmatrix}
1 - p_{01} & p_{01} \\
1 & 0
\end{bmatrix},
\]

(2.3)

where \( p_{01} \) represents, for each user, the probability of a change from an empty cell in \( k \) to an occupied cell in \( k + 1 \). To find the stationary probabilities \( \pi \) of the two states \( \{0, 1\} \), one has to solve the vector equation, \( \pi P = \pi \), whose solution is the following [29]:

\[
\pi = \begin{bmatrix}
\frac{1}{1 + p_{01}} \\
\frac{1}{1 - \frac{1}{1 + p_{01}}}
\end{bmatrix}.
\]

(2.4)

The stationary probabilities can then be used to parameterize the stochastic process, by defining the particle density, \( \hat{d} \equiv P_1 \) based on just one parameter, \( p_{01} \), as follows:

\[
\hat{d} = 1 - \frac{1}{1 + p_{01}}.
\]

(2.5)

Density, as defined, will be the sole parameter used to characterize load-use flexibility. Note, for example, that the lattice of Figure 2.2 has an average density \( \hat{d} = 0.22 \). The lattice represents a population of users, and its cardinality, \( N \), will naturally play a role in characterizing flexibility in absolute terms. Yet, if flexibility is used in relative terms, as the ability to change relative load (relative to its expected value), then the underlying flexibility of the population of users will depend on density alone, as it will be the case in the subsequent sections.

### 2.3 Direct Load Control

An analysis can be made on the shifting dynamics for ideal direct control over hidden shifting constraints, as enabled by future ICT and imposed by confidentiality of user load schedules, respectively. The analysis relies on a simple Cellular Automaton (CA) algorithm that simulates real-time control over the aggregate output of a population of shiftable loads.

Let the target output of the aggregate load in time period \( k \) be defined by \( p^*(k) \). The following algorithm summarizes the main steps of the CA controller implemented.
Algorithm 2.1: Cellular Automaton control algorithm

Set \( k = 0 \); \( n = 0 \);
for \( k = 1, \ldots, T \) do
  if \( p(k) > p^*(k) \) then
    if \( d_n(k) = 1 \land d_n(k + 1) = 0 \) then
      Make \( d_n(k) = 0; d_n(k + 1) = 1 \);
      Update \( p(k) \) with (2.1);
    else
      Break;
      Store \( p(k) \);
  end if
end for

The CA controller described is used to perform supply-following simulations under the assumptions stated above, for load particles distributed randomly in time with a Markov process parameterized with a given density, \( \hat{d} \).

The relative number of responses is referred to as shifting velocity, \( v \), i.e.:

\[
v(k) \equiv \sum_{n=1}^{N} v_n(k), \quad v_n \in \{0, 1\}, \quad n = 1, \ldots, N \tag{2.6}
\]

where \( v_n(k) \) is defined to be: unitary if a particle \( n \) is shifted in time \( k \); and zero, otherwise.

Assuming that \( p(k) \) is uniformly distributed around its expected value, the changes in aggregate load necessary to yield \( p^* \) in time \( k \), \( \Delta p(k) \), can be obtained by controlling particle shift responses, \( v_n \in \{0, 1\} \). The changes in aggregate load necessary to yield \( p^* \) in time \( k \), \( \Delta p(k) = p^*(k) - \bar{p} \) are obtained by controlling the population velocity, as

\[
\Delta p^*(k) = -v(k + 1) + v(k) \tag{2.7}
\]

Note that, as demonstrated in [26], since \( v \) is defined as the population shifting velocity, then, \( v(k + 1) - v(k) \) corresponds to a measure of the population shifting acceleration, decreasing \( \Delta p^* \) when positive and otherwise increasing it. The difference equation in (2.7) can be rewritten as an ordinary discrete-time linear state equation, as in (2.8):

\[
v(k + 1) \approx v(k) - \Delta p^*(k) \tag{2.8}
\]

and be solved as

\[
v(k) \approx v(0) - \sum_{k=0}^{K} \Delta p^*(k) \tag{2.9}
\]

The summation in (2.9) represents the population load cumulative changes; the equality in (2.9) denotes that such changes are limited only by the ability to evolve the velocity of the population, \( v(k) \).

Figure 2.4 illustrates the output obtained with a population of \( N = 500 \) users and the corresponding
lattice of user responsive actions for a triangular supply-following target function, \( p^* (k) \). Each figure (LHS and RHS) describe the experience in three different perspectives being the first (on top) the evolution in time of the normalized aggregate output that results from individual load responses, \( \hat{p}(k) \), and the last two, respectively, the Evolution of the number of responsive actions – population shifting velocity \( v(t) \) – necessary to undertake the changes in aggregate output and the positions of such responsive actions in the two dimensional lattice of cells – to identify responsive user positions, \( v_n \). It is important to note that the ramping response follows the target triangle wave function accurately and that supply-following is possible even when the aggregate response should be ramping-up. This is an important conclusion leading to the fact that load-shifting, as defined in this model, is a possibility for supply-following even without any consideration of bidirectional power flow.

![Graphs showing load-shifting aggregate response from direct control over a population of 500 users, by priority calling (left) or random calling (right). The target function is a triangular wave function with magnitude of 40% of the average load. The lattice has an average density of \( d = 0.167 \) load particles per period.](image)

**Figure 2.4:** load-shifting aggregate response from direct control over a population of 500 users, by priority calling (left) or random calling (right). The target function is a triangular wave function with magnitude of 40% of the average load. The lattice has an average density of \( d = 0.167 \) load particles per period.

Note that, in the algorithm described, there is an underlying prioritization of the users being called for shifting, given that the inner For-loop of the CA algorithm implicitly prioritizes user calls. A user that has shifted its schedule consumption in a given time period, \( k \), will be called to shift it again in the following time period, if the algorithm is used as it is.

If minor changes are made in the algorithm for it to call users randomly, regardless of their previous actions, a different aggregate response is obtained, as shown in the RHS plot of Figure 2.4. Aggregate response quality is considerably worse if users are called randomly, because, in fact, priorities favour flexibility and therefore improve accuracy. The necessary changes on velocity to successfully perform
supply-following are very difficult to obtain if one does not persist on moving ahead particles that are already on the move. The success in ramping relies on persistence and persistence is favoured by *prioritization*. 

3

From Direct to Indirect Control

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3.1 Real-Time Pricing

The desired prioritization effect on the way users are called for shifting their loads could be achieved by indirect control enabled by dynamic pricing, i.e., the use of economic incentives (or disincentives) to encourage or discourage voluntary changes in the end-user consumption pattern, as suggested in [12-23]. This is a solution that not only avoids intrusive control orders, but also relies upon a small communications effort, when compared to direct control solutions or some other indirect control solutions based on day-ahead price profiles.

3.1.1 Paid Incentives

A possibility to incentivize users to postpone their consumption would be to, actually, broadcast a price incentive to be paid to users, $\pi(k)$, for every time there is a response to a shifting call. In this case let us suppose each user bids a price $\pi_n$ to be rewarded for shifting load use $n$. To turn the direct CA controller presented before into an indirect price based controller of this type, it would be sufficient to overwrite the original CA condition,

$$\text{If } p(k) > p^*(k)$$

with the new condition,

$$\text{If } \pi_n > \pi^*(k)$$

guaranteeing for all $k = 1,\ldots,T$ that:

$$\sum_{n=1}^{N} d_n(k) = \hat{p} - p^*(k)$$

where $\hat{p}$ is the original expected aggregate load for a given density, $\hat{d}$, of particles in the lattice and $\pi^*(k)$ is the ideal price function to obtain the desired aggregate load, $p^*(k)$. The algorithm obtained is the following:

$$\sum_{n=1}^{N} d_n(k) = \hat{p} - p^*(k)$$

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Algorithm 3.1: Cellular Automaton Indirect Control Algorithm (incentive based)

Set $k = 0$ ; $n = 0$

for $k = 1, ..., T$ do
  if $\pi_n > \pi^*(k)$ then
    if $d_n(k) = 1 \land d_n(k + 1) = 0$ then
      Make $d_n(k) = 0$; $d_n(k + 1) = 1$
      Update $p(k)$ with (2.1);
    else
      Break;
  Store $p(k)$;

In the case of indirect control, the price $\pi^*(k)$ needs to be “learned” from experience as it is specific of the load changes evolution in time, $\hat{p} - p^*(k)$. Once learned, $\pi^*(k)$ can be seen as the best possible information one can have on the minimum incentive required to get the target response $p^*(k)$, and therefore be used to turn the direct CA controller into an indirect price based controller.

The price could be "learned" from a simulation like the one on the LHS of Figure 2.4, and be reinforced by running it several times, understanding each run as a previous similar experience, and averaging the number of responses obtained. The average number of responses together with the bidding function would be sufficient to have an approximation of the price function $\pi^*(k)$, if it is to be used for the same population and the same target aggregate function $p^*(k)$, used for Direct Control in Section 2.3. By comparing Figure 3.1 with Figure 2.4 one may notice that the controlled magnitude output is much worse for the case of indirect control. Reasons are related with the fact that learning is incapable of dealing with randomness of particles distribution in time. This aspect will be further detailed in Section 3.1.3

Figure 3.2 illustrates a possible bidding function obtained for a population of $N = 500$ users when considering that the values for the price incentives were distributed randomly by the users. The function maps number of users into price incentives required for such users to be shifted ahead in time. Figure 3.2 also shows the price function, $\pi^*(k)$ obtained from the learning experience described, for the bidding function shown in the same figure. The price is shown as an incentive over a flat rate tariff. In this case the bidding function $b(n) : n \rightarrow \pi_n$ is linear and defined as:

$$\pi_n = \frac{\alpha n}{N}$$  \hspace{1cm} (3.2)

with $\alpha = 0.4$ and $N = 500$. 

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Figure 3.1: load-shifting aggregate response from indirect control (based on incentives) over a population of 500 users. The target function is a triangular wave function with magnitude of 40% of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period (cf Figure 2.4).

Figure 3.2: Possible bidding function resulting from a population of $N = 500$ users with incentive values distributed randomly (left) and price function, in time, obtained from a learning experience of ten periods for the previous triangular load function (right).
3.1.2 Price Discounts

Another possibility to incentivize users to postpone their consumption would be to make a discount on the tariff, instead of paying them to shift load use \( n \). In this case, it is assumed that each user bids a price discount \( \delta_n \), below which load use \( n \) would take place (be used). Proceeding in analogy with Section 3.1.1, to turn the direct CA controller presented before into an indirect price based controller based on price discounts, it would be sufficient to overwrite the original CA condition,

\[
\text{If } p(k) > p^*(k)
\]

with the new condition,

\[
\text{If } \delta_n < \delta^*(k)
\]

guaranteeing, like before, that (3.1) would be satisfied.

The algorithm obtained is now the following:

**Algorithm 3.2: Cellular Automaton Indirect Control Algorithm (discount based)**

1. Set \( k = 0 \); \( n = 0 \);
2. for \( k = 1, ..., T \) do
   1. if \( \delta_n < \delta^*(k) \) then
      1. if \( d_n(k) = 1 \) and \( d_n(k + 1) = 0 \) then
         1. Make \( d_n(k) = 0 \); \( d_n(k + 1) = 1 \);
         2. Update \( p(k) \) with (2.1);
      2. else
         1. Break;
   3. end if
3. end for
4. Store \( p(k) \);

Like before for the incentive, the price discount \( \delta^*(k) \) needs to be “learned” from experience, as it is specific of the load changes evolution, \( \hat{p} - p^*(k) \).

The price could also be "learned" from a simulation like the LHS process of Figure 2.2. The average number of responses together with the new bidding function would then be sufficient to have an approximation of the price discount function \( \delta^*(k) \), if it is to be used for the same population and the same target aggregate function \( p^*(k) \), as before. Figure 3.3 illustrates the output \( p(k) \) obtained with the same population of \( N = 500 \) users and the corresponding lattice user responsive actions for the same triangular supply-following target function, \( p^*(k) \).

Figure 3.4 illustrates a possible bidding function obtained from a population of \( N = 500 \) users. This bidding function shows that for a certain discount on price given, \( \delta_n \), the corresponding number of users on the abscissa axis is the number of users that are actually going to shift consumption given that discount on price, assuming that, for the maximum discount (100%), no load is shifted. Therefore,
Figure 3.3: load-shifting aggregate response from *indirect control* (based on discounts) over a population of 500 users. The target function is a triangular wave function with magnitude of 40% of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period (cf Figure 3.1).

Figure 3.4: Possible bidding function resulting from a population of $N = 500$ users with discount values distributed randomly (left) and price discount function, in time, obtained from a learning experience of ten periods for the previous triangular load function (right).
those are the users that are not going to shift its consumption due to the discount value not being high enough. Figure 3.4 also shows (in the RHS) the discount function, $\delta^*(k)$ obtained from the learning experience described. The price is shown as a discount on a flat rate tariff. In this case the bidding function $b(n) : n \to \delta_n$ is linear, and defined as:

$$\delta_n = \frac{\delta_{\text{max}}(N-n)}{N}$$

with $\delta_{\text{max}} = 1.0$ and $N = 500$.

### 3.1.3 Randomness and Population Size

Due to the underlying randomness of load use, a price function $\pi^*(k)$ or $\delta^*(k)$ obtained for a given population in a given horizon, $k = 1, ..., T$ results in supply-following mismatches if used in a different time period both for incentive or discount-based approaches.

Such mismatches are illustrated in Figure 3.1 and Figure 3.3 by depicting the aggregate output $p(k)$ and the corresponding lattice of user responsive actions obtained from indirect control over the same population (as used in Figure 2.4) with a price function that resulted from a learning experience of ten periods for the triangular load function.

If the learning process is run on a lattice of more users instead, mismatches are expected to become smaller. Figures 3.5 to 3.7 illustrate the response for increasing number of users, showing the effect it has on supply-following accuracy.

It is important to note that it could exist a practical, but relevant, obstacle regarding the incentive-based solution. As assumed before, the market operator is not aware of the individual consumer daily schedules (day-ahead load profiles). Thus, when paying a user to postpone its schedule in a given time period, actually that user is being paid not to consume instead of being paid to shift, given that it is not possible for the operator to know if consumption was planned to take place at that timing or not. The result is that, in each time period, $k$, instead of paying users that shift their consumption, the market operator would be paying every user that has not consumed. This would not have any implication on supply-following accuracy being only a matter of overall expense accountability for the operator.

By looking at Figures 3.5 to 3.7 it can be observed that incentive and discount-based approaches lead to different bidding curves and consequently different prices to be paid. However, in terms of dynamics regarding supply-following aggregate response and total number of response actions, the two approaches are very similar. Thus, it is possible to study process’s intrinsic dynamics, through any of the pricing strategies being discussed. For the subsequent sections, the load-shifting process is assumed to be enabled by indirect control from dynamic incentive pricing. Every conclusion taken would be, equally, valid for indirect control based on discounts.
Figure 3.5: load-shifting aggregate response from indirect control, based on incentives (left) over a population of 100 users and from indirect control, based on discounts (right) over a population of the same size. The target function is a triangular wave function with magnitude of 40% of the average load, for both simulations. The two lattices, both, have an average density of $\hat{d} = 0.167$ load particles per period.

Figure 3.6: load-shifting aggregate response from indirect control, based on incentives (left) over a population of 1,000 users and from indirect control, based on discounts (right) over a population of the same size. The target function is a triangular wave function with magnitude of 40% of the average load, for both simulations. The two lattices, both, have an average density of $d = 0.167$ load particles per period.
3.2 Price-Velocity Characteristic

The price evolution $\pi^*(k)$ can be combined with the evolution of the number of responses in time to synthesize the information obtained in the learning experience. The learning experience can be shown in a price-velocity characteristic built by eliminating time dependence from both evolutions.

The characteristic built from the learning experience with 5,000 users is depicted in Figure 3.8 assuming that the users’ population bidding function $b(n) : n \rightarrow \pi_n$ is linear, i.e., that $\pi_n = \alpha n/N$, as assumed before in Equation (3.2). The characteristic exhibits hysteresis caused by the supply dynamics introduced by the shifting process.

The effect of hysteresis on price and response time functions $\pi(k)$ and $v(k)$ is represented in Figure 3.9.

Figure 3.10 illustrates the dependence the characteristic has on the value of the average density, $d$. Note that, despite the maximum price being different in every curve, the bidding function and the target load-aggregate output is the same. This shows the effect that the average density, $d$, has on the number of responses requirements and consequently on the price function, $\pi^*(k)$. Figure 3.9, also shows that it is always impossible to obtain the exact number of responses for a certain price, given by the bidding function. This can be explained by the fact that some of the users on the bidding function are constrained and because of that the price given corresponding to the number of calls must be higher.
than the desired number of shifting responses. The figure also illustrates, through the hysteresis, that flexibility is different if the number of responses is ramping up or down, proving the dynamic behaviour of the process. This is explained by the fact that, when ramping-up, as the supply increases, more particles are in movement (as more users are shifting) and because of that more users are willing to shift again for a lower increase on price. From a market perspective, indirect control introduces a dynamical effect into the market clearing process: when a consumer shifts a load particle ahead in a period, this same particle will probably become available to be shifted again in the next period for a similar price, this way expanding particles supply in the future period and consequently decreasing clearing prices for the same target variation of the aggregate load.

Figure 3.8: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process illustrated in Figure 3.7 for a linear bidding function with $\alpha = 0.4$. 

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Figure 3.9: Connection between price incentive vs. population velocity hysteresis characteristic (top left), velocity delayed from price in time (top right) and controlled output magnitude (bottom right). Trajectories obtained from experience with 5,000 users subject to the indirect price-based control process illustrated in Figure 3.10 for a linear bidding function with $\alpha = 0.4$. 

Illustrated in Figure 3.10 for a linear bidding function with $\alpha = 0.4$. The data is obtained from experience with 5,000 users subject to the indirect price-based control process.
Figure 3.10: Price incentive vs. population velocity. Trajectories obtained from experience with different density populations of 5,000 users subject to the indirect price based control process, with the same triangular target aggregate load function as in Figure 3.8, for a linear bidding function with $\alpha = 0.4$.

3.2.1 Modelling Supply Dynamics

Supply dynamics are found to be complex. However, for large density values the process can be approximated by an ordinary difference equation in time.

The step response of the system will be modelled as an exponential function, as seen in Figure 3.11. Thus, assuming superposition principle and a general bidding function $b(n) : n \rightarrow \pi_n$, the ramping-up process will be described as the sum of an increasing number of exponentials (illustrated in Figure 3.13), being the first one (represented in Figure 3.11), defined by:

$$v_1^k = [(2\hat{d} - 1)e^{-\frac{\hat{d}}{\pi_1} k} + (1 - \hat{d})] \cdot b^{-1}(\pi_{i=1}), \quad \text{for } k \geq 1 \quad (3.4)$$

where $\hat{d}$ is the original lattice’s particle density and $b^{-1}$ is the inverse of the price-quantity bidding function.

In the same way, the second exponential is defined as:

$$v_2^k = [(2\hat{d} - 1)e^{-\frac{\hat{d}}{\pi_1} (k-1)} + (1 - \hat{d})] \cdot b^{-1}(\pi_{i=2}), \quad \text{for } k \geq 2 \quad (3.5)$$

In conclusion, in every time instance, $k$, an exponential is added to the sum, with its correspondent
time delay and price step, \( \pi_i \). It is then possible to write a general expression for every exponential function that takes part on a general ramping up process:

\[
v^k_i = [(2\hat{d} - 1)e^{-\frac{d}{\hat{d} - \pi_i} (k-1)} + (1 - \hat{d})] \cdot b^{-1}(\pi_i), \quad \text{for } k \geq i
\]  \hspace{1cm} (3.6)

Since the objective in this section is to approximate the process by an ordinary difference equation, the first price step increase is written in terms of differences as:

\[
v^k_{i+1} - v^k_i = -v^k_i \cdot \frac{\hat{d}}{1 - \hat{d}} + (b^{-1}(\pi_i) - b^{-1}(\pi_{i-1})) \cdot \hat{d}, \quad i = 1
\]  \hspace{1cm} (3.7)

and, then, for every \( i \leq k \):

\[
v^k_{i+1} - v^k_i = -v^k_i \cdot \frac{\hat{d}}{1 - \hat{d}} + (b^{-1}(\pi_i) - b^{-1}(\pi_{i-1})) \cdot \hat{d}
\]  \hspace{1cm} (3.8)

Making use of this knowledge it is, finally, possible to write an expression, based on differences, that models the ramp-up process as a sum of exponentials, as previously supposed:

\[
v^{k+1} - v^k = \sum_{i=1}^{k} \left[ \left( -v^{k+i+1} \cdot \frac{\hat{d}}{1 - \hat{d}} \right) + (b^{-1}(\pi_i) - b^{-1}(\pi_{i-1})) \right] \cdot \hat{d}, \quad \forall k \leq i
\]  \hspace{1cm} (3.9)

Equation (3.9) can be simplified, knowing that, since the time constant is the same in every exponenti-
tial, then \( \sum_{i=1}^{k} (-v_{k-i}^{i+1}) = -v^k \), i.e., in a specific time, \( k \), the sum of the velocity values, \( v_i^k \) generated by every correspondent price step, \( \pi_i \) delayed in time and added before \( k \), is equal to the actual total velocity in \( k \), \( v^k \). Also, \( \sum_{i=1}^{k} [b^{-1}(\pi_i) - b^{-1}(\pi_{i-1})] = b^{-1} \left( \sum_{i=1}^{k} (\pi_i - \pi_{i-1}) \right) = b^{-1}(\pi^k - \pi^0) \). Then, for \( \pi^0 = 0 \), meaning the price starts at zero, velocity is given by:

\[
v_{k+1} = v^k \left( 1 - \frac{\hat{d}}{1 - \hat{d}} \right) + b^{-1}(\pi^k) \cdot \hat{d}, \quad \text{for } v_{k+1} > v^k
\]  

(3.10)

Equation (3.10) describes accurately the process when price steps are being added, i.e., when there is a price increase. However, it is reasonable to use the same equation in the whole domain where the response number is increasing, being the equation an approximation. This approximation is more accurate for higher average density values, \( \hat{d} \).

For the expression’s domain where the response number is decreasing, this expression isn’t satisfactory. This subprocess that corresponds to the deceleration of the load particles movement, cannot be described by the same expression since it now corresponds to the stopping of the previous velocity evolutions, \( v_{k} \). It is because the fact that the number of price velocity evolutions being stopped depends on the “size” of the price step decrease, that another expression must be written for modelling purposes.

If (3.10) is written with respect to price as in (3.11), it can be interpreted by separating it in two different terms. One that depends on velocity, \( v^k/\left(1 - \hat{d}\right) \) and other depending on changes in velocity, \( (v_{k+1} - v^k)/\left[\left(1 - \hat{d}\right)\hat{d}\right] \). The first term can be interpreted as the particles already in movement that keep moving unless they collide with another particle ahead with a probability of \( \left(1 - \hat{d}\right) \), whereas the second term corresponds to the process of getting new particles into the movement by searching in the set of particles not yet called for shifting. This is achieved with a probability of \( \left(1 - \hat{d}\right)\hat{d} \).

\[
b^{-1}(\pi^k) = \frac{v^k}{1 - \hat{d}} + \frac{v_{k+1} - v^k}{\hat{d}}
\]

(3.11)

Assuming that, for the case where the number of responses is decreasing, only the first term applies and that the process of stopping particles to move is linear, an expression can be obtained, in analogy to (3.11), as:

\[
b^{-1}(\pi^k) \approx \frac{v^k}{1 - \hat{d}}
\]

(3.12)

which corresponds, w.r.t velocity, to:

\[
v_{k+1} \approx b^{-1}(\pi_{k+1}) \left(1 - \hat{d}\right), \quad \text{for } v_{k+1} < v^k
\]

(3.13)

Finally a system of equations, that can, approximately, model the shifting process exhibiting hysteresis caused by the supply dynamics, can be written as:
\[ v^{k+1} \approx \begin{cases} v^k \left(1 - \frac{\hat{d}}{1 - d}\right) + b^{-1}(\pi^k) \cdot \hat{d} & \text{if } v^{k+1} > v^k \\ b^{-1}(\pi^{k+1}) (1 - \hat{d}) & \text{if } v^{k+1} < v^k \end{cases} \]

(3.14)

The equation can then be used to generalize the learning process to other densities and other load target functions. Figure 3.12 illustrates the approximate price-response characteristic, as modelled by (3.14), for different average load densities, $\hat{d}$.

Figure 3.12: Price incentive vs. population velocity. Trajectories obtained with (3.14) for different densities, $\hat{d}$, and linear bidding function with $\alpha = 0.4$ (c.f Figure 3.10).

Figure 3.12 in comparison to Figure 3.10 shows that, (3.15) and (3.14), despite not being an exact expression (as postulated before) to describe the process, could be seen as a reasonable approximation, especially for higher densities. In fact the expression’s domain where velocity is decreasing is not linear nor the domain where velocity is increasing and decreasing prices is described by the same expression as for increasing prices.

3.2.2 Insights over Hysteresis Effect on Price

Figure 3.9 shows that the price saturates when velocity is increasing (between points $A$ and $B$). This is even more visible when average density, $\hat{d}$ is lower, as it is seen in Figure 3.10. This means that, in that interval, even with constant price, the velocity, or number of shifting responses, increases, which is something not easily expected. It is even possible to have the velocity increasing with the price
decreasing as it is seen on the curves with lower densities on Figure 3.10.

This effect can be, intuitively, explained by making a distinction between velocity (number of shifting responses) and the number of calls, as the number of users called to shift its consumption. It is trivial to conclude, from previous sections, that the number of calls is always lower than velocity. That is, as a certain number of users is called for shifting, some of them will be constrained since the instant load density in the following time instance will be different from zero and therefore, collisions will always occur. As velocity increases meaning that more particles are in the movement, consecutively being shifted, the instant density in the following time instant will be higher, as particles accumulate, which results in an increase on the number of responses (velocity) w.r.t the number of calls.

![Figure 3.13: Price step summation resulting in exponential responses (illustration). The curves in black solid lines represent the price function whereas the curves in green dot lines represent velocity’s response for each price step increase.](image)

The effect being discussed could also be explained from the dependence it has on the time constant in (3.6) (remind that the time constant is equal for every exponential being summed, and that it depends on the value of the average density, $\hat{d}$). Figure 3.13 exemplifies a process in which the price function, $\pi^k$, is given by an increase of two steps and symmetric decrease. Note that, in the figure, two exponentials with the same time constant are depicted, showing the exponential evolution of the velocities specific to each price step, as defined in (3.6). It is important to note that for the decreasing price steps, velocity does not respond in a similar way to the ones corresponding to an increase on price. For decreasing price steps, the particles whose price is above the new one, would stop abruptly, whereas the particles below this price would continue accelerating, as Figure 3.13 illustrates. This explains the hysteresis phenomena, as, when there is a price step decrease, depending on the time constant given by the original value for the average density, $\hat{d}$, the still accelerating particles can contribute to an overall increasing velocity, despite the decrease on price.
3.2.3 Dynamical Effects on Learning

Equation (3.14) highlights the fact that predictability cannot be yield without information on population velocity. It makes clear that the relationship between aggregate output changes, \( \Delta p^* \), and price, \( \pi^* \), depend on a state-variable - velocity. If one uses (3.14) together with (2.7) to express aggregate output changes on price, one may write:

\[
\Delta p^k \approx \begin{cases} 
\frac{\hat{d}}{1 - \hat{d}} \cdot v^k - \hat{d} \cdot b^{-1}(\pi^{k+1}) & \Leftarrow \Delta p^*(k) < 0 \\
v^k - (1 - \hat{d}) \cdot b^{-1}(\pi^{k+1}) & \Leftarrow \text{otherwise} 
\end{cases}
\]  

(3.15)

which quantifies the role of velocity in price-based demand response, for a given bidding function \( b(n) \).

Load uses cannot be assumed homogeneous, so (3.15) cannot be used directly to predict load changes induced by price. Load densities will vary from use to use, and uses will not be perfectly elastic. This fact will force utilities and other load serving entities to deal with the future demand-side resources as a composite of different resource types, whose response characteristics they will have to segment into different classes in order to be able to predict individual class responses could be supported by machine learning techniques.

Standard machine learning techniques could be used to classify resource types and to parameterize response models for each resource class, and could do that with bidding information and data on both price (input) and resulting aggregate load (output) \([13,23–27,30,31]\). However, as it was shown in (3.15), that cannot be done without data on velocity as well.

![Machine Learning](image.png)

**Figure 3.14:** Learning the aggregate response from load use shifting implies modelling velocity and price altogether as input data.

Without velocity data, learning is impossible: given a shifting price and a bidding function for the population of users, one cannot predict the changes in aggregate load without knowing the number of loads under shifting – this results from hysteresis and is quantified in (3.15). The standard machine learning approaches make use of data from input \( \pi^*(k) \) and output \( \Delta p^*(k) \) to “discover” a predicting model. It is clear that the standard is insufficient because of the shifting dynamics and propose that future learning processes need to take velocity as an input. Figure 3.14 illustrates the machine learning approach as envisioned.
4 Dynamic Properties and Limitations

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4.1 Target Aggregate Load Dependencies

In the previous chapter, difficulties regarding the supply dynamics’ learning process were postulated. In this section, the model will be used to derive other properties w.r.t to variables kept constant before, namely the shape of the target function or its magnitude and frequency. The bidding function, \( b(n) \), however, is assumed linear as before: \( \pi_n = \frac{\alpha n}{N} \) with \( \alpha = 0.4 \) and \( N = 5,000 \).

4.1.1 Magnitude and Frequency

Magnitude, \( m \), as referred to in previous chapters, is defined as a normalized quantity:

\[
m \equiv \max_t \left\{ \frac{p^*(k) - \hat{d} \cdot N}{\hat{d} \cdot N} \right\}
\]

so that the ramping rate, defined as the slope of \( p^*(k) \) be given by \( m \times f \).

![Figure 4.1: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process, with triangular target aggregate load functions with varying magnitude \( m \), constant frequency \( f = 0.04 \), load density \( \hat{d} = 0.16(6) \) and for a linear bidding function with \( \alpha = 0.4 \).

Figure 4.1 shows the effect of magnitude changes on price-velocity characteristic and consequently on supply-following accuracy. As expected, the maximum velocity increases with magnitude, and so does the process’s hysteresis “width”, i.e, the slope difference between ramping-up and ramping-down trajectories. This could bring extra difficulties into the predictability and controllability exercise, as magnitude increases. Also, note that, as magnitude reaches the value of \( m = 0.7 \), the price has already peaked at its maximum (\( \pi_{\text{max}} = 40\% \)), meaning that no other users can be called to fulfil the needed number of shifting responses. In this situation, it can be said that supply-following can no longer be
The comparison between Figure 4.1 and Figure 4.2 shows the goodness of the approximate model derived in Chapter 3. In fact, as the figure comparison shows, the accuracy of the model is not significantly affected by changes in magnitude.

Despite what one might be led to conclude, frequency changes have the opposite effect of magnitude’s. That is because when frequency increases, supply-following becomes less demanding from velocity needs (see Figure 4.3). In contrast to magnitude changes’ effect on price-velocity, frequency changes do not have a significant impact on hysteresis cycle "width", meaning that predictability would not be affected directly by frequency changes.

Regarding the accuracy of the model derived in Chapter 3, changes in frequency, are not so well modelled as changes in magnitude (compare Figure 4.3 with Figure 4.4). In fact, as frequency increases the model’s accuracy tends to decrease. The effect that frequency changes have on the velocity’s downward trajectory (lower slope) is significant enough for deviating it from linearity. Since this is the part of the model’s domain were the most significant approximations were made, it is then explainable that accuracy is affected.

Prior to this chapter, in what supply-following accuracy is concerned, one might be led to believe that a constant slope of the triangular wave function would result in constant accuracy degree. Yet, taking into account the described magnitude and frequency effect on price-velocity characteristic, together with Figure 4.5, it is clear that this is not the case. Again, it is seen in a Figure 4.5 that depending on the values for magnitude and frequency, supply-following can become inaccurate, as an excessive maximum
Figure 4.3: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process, with triangular target aggregate load functions with varying frequency $f$, constant magnitude $m = 0.4$, load density $\hat{d} = 0.16(\ell)$ and for a linear bidding function with $\alpha = 0.4$ (c.f Figure 4.1)

Indeed, as it had been conjectured before, there is a limitation on velocity that is intrinsic of the load-shifting process. This limitation is derived and well detailed in [26], yet some important details are worth mentioning here.

Equation (4.2) describes a relation for the maximum velocity depending on magnitude and frequency,$$

\text{velocity leads the price to hit its ceiling.}

Indeed, as it had been conjectured before, there is a limitation on velocity that is intrinsic of the load-shifting process. This limitation is derived and well detailed in [26], yet some important details are worth mentioning here.

Equation (4.2) describes a relation for the maximum velocity depending on magnitude and frequency,
Figure 4.5: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process, with triangular target aggregate load functions with varying magnitude $m$ and frequency $f$, keeping a constant $m \times f$ relation. Load density is $\hat{d} = 0.16(6)$ and the bidding function is linear with $\alpha = 0.4$ as demonstrated in [26].

\[
\left( \frac{m}{f} \right)^* = 4\rho \left( 1 - \hat{d} \right) \tag{4.2}
\]

for a maximum velocity $v^*$ given by:

\[
v^* \approx m/(4f) \tag{4.3}
\]

From (4.2), after manipulation, (4.4) is derived as an expression giving the maximum magnitude $m^*$ for constant frequency $f$.

\[
m^* \left( \hat{d} \right) \approx m_0 e^{-\hat{d}} \tag{4.4}
\]

Note that, despite not being trivial to guess, in an accurate supply-following process for an aggregate output set to follow a triangle wave function, it is a constant $m/f$ relation that leads to the certainty that supply-following will keep its accuracy, and not a constant slope given by $m \times f$. By looking at Figure 4.6, it is evident that maximum velocity is the same for a constant $m/f$ ratio.

This result, despite being specific of triangular wave supply following functions, corroborates the previously findings for the effect magnitude and frequency have on supply-following accuracy. For other target functions, the relations in (4.2) and in (4.4) do not hold anymore. However, for any target function there is a maximum velocity, corresponding to the maximum number of shifting responses one can ob-
Figure 4.6: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process, with triangular target aggregate load functions with varying magnitude $m$ and frequency $f$, keeping a constant $m/f$ relation. Load density is $d = 0.16(6)$ and the bidding function is linear with $\alpha = 0.4$.

Figure 4.7 illustrates the output $p(k)$, together with velocity $v(k)$ obtained with a population of $N = 5,000$ users and the corresponding lattice user responsive actions for a sinusoidal load target function, $p^*(k)$.

Figure 4.8 depicts the price-velocity characteristic correspondent to the experience illustrated in Figure 4.7. Since every variable was kept constant except for the target load function shape, both Figure 3.10 and Figure 4.8 can be compared. From this comparison, it becomes evident that sinusoidal supply-following is more demanding than triangular supply-following. It requires both higher maximum velocity and higher maximum price.

Figure 4.9 illustrates the output $p(k)$, together with velocity $v(k)$ obtained with a population of $N = 5,000$ users and the corresponding lattice user responsive actions for a square wave target function, $p^*(k)$.

4.1.2 Target Load Shapes

Conclusions derived in the previous chapter stand on the relevant example of a triangular target load function, as being representative of the process ramping capabilities, in analogy with what is done for generation side control. However, it is also relevant to take considerations on the process dynamics for other types of target functions.

41
Figure 4.7: load-shifting aggregate response from indirect control over a population of 5,000 users. The target function is a sinusoidal wave function with magnitude of 40% of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period (cf Figure 3.7).

Figure 4.8: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process, with a sinusoidal target aggregate load function with amplitude of 40% of the average load, for a linear bidding function with $\alpha = 0.4$. The lattice has an average density of $\hat{d} = 0.167$ load particles per period.

Figure 4.10 depicts the price-velocity characteristic correspondent to the experience illustrated in Figure 4.9. From this comparison, it becomes evident that a square wave supply-following is more
Figure 4.9: load-shifting aggregate response from indirect control over a population of 5,000 users. The target function is a square wave function with magnitude of 40% of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period (cf Figure 3.7 and Figure 4.7).

Figure 4.10: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price based control process, with a square target aggregate load function with amplitude of 40% of the average load, for a linear bidding function with $\alpha = 0.4$. The lattice has an average density of $\hat{d} = 0.167$ load particles per period.

demanding than both the triangular and the sinusoidal functions to a point that, for this magnitude (40%), inaccuracy is not only due to randomness and size effect but also due to the fact that the price reaches
its maximum. In this case, to properly compare the price-velocity characteristic with the previous ones, magnitude should be reduced.

Figure 4.11: Load-shifting aggregate response from indirect control over a population of 5,000 users. The target function is a square wave function with magnitude of 20% of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period (cf Figure 3.7, Figure 4.7 and Figure 4.9).

Figure 4.12: Price incentive vs. population velocity. Trajectories obtained from experience with 5,000 users subject to the indirect price-based control process, with a square target aggregate load function with amplitude of 20% of the average load, for a linear bidding function with $\alpha = 0.4$. The lattice has an average density of $\hat{d} = 0.167$ load particles per period.
Figure 4.11 illustrates the output $p(k)$ together with velocity $v(k)$ obtained with a population of $N = 5,000$ users and the corresponding lattice user responsive actions for a square wave target function, $p^*(k)$ with a reduced magnitude of 20%. In this case it becomes clear that the price does not reach its previously defined limit of 40%. However, supply-following is still inaccurate due to randomness and population size effects that become more visible for the square wave target function.

Figure 4.13 illustrates an experience similar to the one in Figure 4.11 except for the number of users that is increased to $N = 50,000$. This figure shows that, for the square load function, the required number of users to create the output accurately is much larger.

Figure 4.13: load-shifting aggregate response from indirect control over a population of 50,000 users. The target function is a square wave function with magnitude of 20% of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period (cf Figure 4.11).

Note that, as the step duration increases, the more demanding it is to obtain an accurate output function. Contrary to what might be thought, it is more difficult for the load to follow a constant target, different from its average value, than it is for it to ramp-up or ramp-down frequently. This could be explained by the result reached in the previous section regarding frequency impact on supply-following accuracy. As frequency is reduced, maximum velocity approaches its limit, and therefore the closer supply-following is to become inaccurate.

Figure 4.14 shows the output $p(k)$ for an attempt of "smoothing" the original load density to a constant
target function with a value 15% higher than the average load. The result is a ramping velocity $v$. The output function is accurate until velocity reaches its maximum value. At that point, the population is unable to keep following the target function $p^*(k)$. It should also be noted that, for this target function, the role of randomness and size assumes is so important that a simulation for a population of $N = 100,000$ had to be carried out to obtain a reasonable accuracy on the target function $p(k)$.

This set of experiences not only shows the effect the target aggregate load function has on the price-velocity characteristic, having a direct influence on the demand for shifting responses, but also illustrates the idea of existence of a maximum velocity above which an accurate output function $p(k)$ is not attainable.

### 4.1.3 Process Termination

In view of the dynamic characteristic of the process, where velocity assumes such a major part, terminating the control process is not an insignificant matter.

Remember (2.9) and note that, for an abrupt variation in velocity $v$, as it represents the integral of the changes in aggregate load $\Delta p$, this changes, being its derivative, would tend to be very large.
Figure 4.15 shows the transient resulting from an abrupt interruption on price incentives and, hence, on velocity. As this figure shows, bringing price incentives to zero, results in a considerable overshoot on load aggregate response, of approximately 200% of the average load, in this case.

Yet, as shown in Figure 4.16, there is a possibility to bring the load aggregate output to its average value without any undesired overshoot. This possibility is to keep the price constant at the value it takes when the process is interrupted. However, this could result in price incentives being paid indefinitely, as Figure 4.17 shows.

An alternative way of terminating the process, bringing load aggregate output to its average value, would be to stop the incentives as velocity crosses zero. At that moment no particle is being shifted and an interruption would not imply any significant transient on controlled magnitude (as seen in Figure 4.18 and Figure 4.19).

Once again from, this example, it becomes reinforced the necessity of having information on velocity data in order to be able to control load-shifting. Without this information, it is not possible to adequately terminate the process.
**Figure 4.16:** load-shifting aggregate transient from an interruption. *Indirect* control over a population of 5,000 users. The target function is a triangle wave function with magnitude of 20% of the average load. The lattice has an average density of \( \hat{d} = 0.167 \) load particles per period.

**Figure 4.17:** Price function \( \pi^*(k) \), used in the experience depicted in fig. 4.16.
4.2 Dependence on Original Load Density

It was seen in previous sections that not only the particle hopping model proposed initially applies for every type of target load function but also does the model based on a difference equation derived in
Chapter 3, and that considerations regarding capabilities and limitations of the process also hold. However, every simulation stood under the assumption of an original load schedule with randomly distributed in time load particles generated with the same stochastic process. It is known, however, that consumption tends to be more concentrated in some periods of the day. Because of that, an analysis in which the original load density is not constant, would be of great importance to generalize derived conclusions in this study to real applications of load-shifting.

Figure 4.20: load-shifting aggregate response from indirect control over a population of 50,000 users whose loads are distributed to create a triangular wave function (with a magnitude of 40%). The target function is a constant function with the value of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period.

Figure 4.20 illustrates an experience where the original load density corresponds to a triangular wave function, whereas the target aggregate load $\Delta p^*$ corresponds to a constant function with the value of the load average density.

The original density is generated from a simulation like the one in Figure 4.7, as it outputs a population of users whose loads are distributed as a triangular wave function. Note that, because of that, the lower plot in Figure 4.20 shows a portion of particles that cannot be moved. This, however, does not represent any inaccuracy regarding supply-following capabilities but rather an issue related to the way the original density is generated. Also, the initial inaccuracy, which lasts until time period $k \approx 15$, is due to the fact that the initial difference between original density and the load target output is positive. In that
case, velocity should decrease but, as velocity starts in zero, supply-following is not possible until this difference becomes negative.

From Figure 4.20, it is clear that the process works in a similar way to the one where the process starts with a uniform original load density for a target function of a triangular wave form. From the comparison between Figure 4.20 and Figure 4.7 it can be observed that the output velocity function in both processes is very similar. This easily suggests that conclusions derived before, for a uniform original load density, also apply for any original load density.

![Figure 4.21: load-shifting aggregate response from indirect control over a population of 50,000 users whose loads are distributed to create a sinusoidal wave function (with a magnitude of 40%). The target function is a constant function with the value of the average load. The lattice has an average density of $\hat{d} = 0.167$ load particles per period.](image)

Figure 4.21 illustrates an experience where the original load density corresponds to a sinusoidal wave function. The target aggregate load $\Delta p^*$ corresponds, again, to a constant function with the value of the load average density.

It is also clear, from this figure, that the process of flattening a sinusoidal wave requires an evolution of velocity similar to the one in the process of obtaining a sinusoidal wave from a uniformly distributed original load distribution. From the comparison between Figure 4.21 and Figure 4.7 note that, not only the velocity evolution in both figures has the same shape, but also that the maximum velocity is approximately the same.
In Section 4.1.2 it was concluded that a sinusoidal wave function as a target function is more demanding in terms of velocity than a triangular wave. It is clear that it also holds, in an analogous way, for the comparison between the original triangular distribution and the sinusoidal distribution.

Figure 4.22: load-shifting aggregate response from *indirect* control over a population of 50,000 users whose loads are distributed to create a square wave function (with a magnitude of 20%). The target function is a constant function with the value of the average load. The lattice has an average density of \( \hat{d} = 0.167 \) load particles per period.

Figure 4.22 illustrates an experience where the original load density corresponds to a square wave function. The target aggregate load \( \Delta p^* \) corresponds, again, to a constant function with the value of the load average density.

Figure 4.22 also suggests the fact that conclusions derived in previous sections do not depend on the original load density. Again, from the comparison between Figure 4.22 and Figure 4.13 it is clear the similarity between the velocity evolution in both processes. Also, for the square wave function the number of users under the load-shifting process had to be increased to obtain an accurate output, as it was in Section 4.1.2.

In fact, the model could be generalized by replacing the target aggregate load function in the algorithm (for direct control),

\[
\text{If } p(k) > p^*(k)
\]
with the new condition,

\[
\text{If } \delta p^*(k) > 0
\]

for \( \delta p^*(k) = p(k) - p^*(k) \).

The same could be done for indirect control based on price incentives, this way turning the algorithm into one that does not depend on the original load density and aggregate load target function separately, but rather on the difference between the two function's densities.
Conclusion

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5.1 Overview

With this dissertation it was intended to characterize demand response main dynamics, mainly enabled by load-shifting as a means to supply-following. It was also intended to develop insights on what should be considered on demand response possible household applications.

By studying some of the literature available, related to DSM, it was concluded that it lacks information on DR intrinsic dynamics and the importance it might have on controllability and predictability of a users’ population. From that necessity, in [26] a model for load-shifting under direct control was proposed and from it, considerations on DR intrinsic limitations were taken. Also in this model, the particle density concept was introduced has the way of characterizing demand flexibility in absolute terms. The first step on this dissertation was to go through this model and analyse how it could be modified in order to study the case for indirect control.

In Chapter 3, from the modification of the model proposed in [26] for indirect control, some preliminary conclusions were drawn. It was concluded that, whether it being under a control strategy based on paid incentives or price discounts, the output load function obtained is of an acceptable accuracy. However, due to the underlying randomness of load use, supply-following accuracy is found to be negatively affected by the indirect control itself, especially for low size user populations. This means that for indirect control to be implemented, the population concerned should be bigger than a certain number of users.

The concept of velocity, defined as the number of shifting responses, was projected against price to show that the price-velocity characteristic exhibits hysteresis when following periodic load targets. Being this characteristic of great importance, as it might impact the way we see the predictability of a population’s response to a given price evolution, it was found relevant to derive a model for it. The existence of hysteresis provides evidence that elasticity is not sufficient to model aggregate response. Aggregate response dynamics and their dependence on prices were then expressed analytically on velocity to generalize such dependencies. The role of velocity in price-formation was discussed to emphasize that response output cannot be predicted without keeping track on velocity no matter how large the population of users might be.

Chapter 4 was aimed at generalizing previous conclusions and presenting a set of properties w.r.t. target aggregate load dependencies. We have seen that, for periodic load targets, the magnitude of the target function can only be increased until a certain point beyond which supply-following accuracy is no longer upheld. Conversely to magnitude, this limit takes place has the target function’s frequency decreases. Magnitude has also a role in what predictability is concerned as when it is increased the hysteresis’ cycle width increases that way possibly increasing predictability. Also, we have gone through the impact that the target load function shape has on predictability and controllability difficulty and also on supply-following accuracy. Important to note is the difficulty behind following a constant target, with an average value different from the initial average. The issue of terminating the process was addressed,
having been shown the importance of terminating at the point where velocity crosses zero to avoid unintended transients. The independence of the process from the original load density was shown has a way of generalizing previous conclusions derived.

5.2 Future Work

In this dissertation, several advances on demand response main dynamics were made. However, throughout this study several assumptions were kept, such as the homogeneity of particles’ density, the homogeneity of resources, in the sense that particles have the same associated energy, and the use of the same type of linear bidding. Also, along this study, the model relates strictly to load-shifting as the way DR is enabled.

In the future it would be beneficial to study different types of loads and to develop suitable dynamic models of their response to prices variation, considering the intrinsic dynamics studied here. Also, the behaviour of active consumers, micro-producers, electric vehicles (EV), could be studied with focus on modelling response flexibility of such loads as a composite of individual flexible uses, each with its intrinsic constraints of use in time. An adaptation of the model described in this study could, also, be used to represent charge reduction actions (load stretching) and evolve the model to combine both shifting and stretching for composites of heterogeneous density uses.

Hopefully, this study can lead the way for more diverse and specific approaches also taking in consideration the process’s main dynamics.
Bibliography


