Seismic Evaluation of Bonet Building of Sintra National Palace

Comparison of Two Different Approaches

Madalena Malcata

madalena.malcata@tecnico.ulisboa.pt

ABSTRACT

Complex masonry monuments represent an important part of the built cultural heritage and most of them are vulnerable to seismic actions. Their large scale, irregularity and heterogeneity, makes it challenging to characterize their structural behaviour.

This work addresses the state of conservation as well as the structural behaviour and seismic vulnerability of the most ancient body of the National Palace of Sintra, Portugal: the Bonet building. This body, a URM building, was built on top of Arabic foundations during the reign of King Dinis, around the year of 1281. Since then few alterations were made to the building. Due to an exemplary conservation of the Palace, crack patterns were identified only on the top floor of the Bonet building, an area that is not open for visitors and that has been recently submitted to restoration works.

In order to minimize the multiple uncertainties usually existing in complex masonry buildings, whether related to geometry or to masonry mechanical properties, a detailed structural survey was conducted using a laser-scanner and a drone and *in-situ* experimental tests were performed. Different *in-situ* experimental tests took place, semi-destructive and non-destructive, such as: samples collection, flat-jack tests, GPR (ground penetrating radar) campaign and ambient vibration tests. All these tests are important to the adequate characterization of building and to the calibration of the numerical model. The final values adopted for the mechanical properties of the rubble stone masonry are presented and can be used as reference for future works in ancient Portuguese monuments of the same period.

Afterwards, nonlinear static numerical analyses were performed in two different FEM software (3Muri and Abaqus), and comparisons and discussion of the results are made. The differences in modelling strategies and characterization of materials between the two software are considered with regard to their realism, computational effort, data availability and applicability to large scale structures. Efforts to calibrate and obtain the same behaviour of the building for the different software were made, involving geometry, boundary conditions and characterization of the material constitutive laws. One of the most difficult challenges when modelling complex structures is the definition of the boundary conditions. Therefore, the adjacent buildings were modelled to consider the interactions.

Keywords

Bonet Building, Sintra National Palace, Safety Verification, Seismic Evaluation, Damage Pattern, Finite Element Method, Modal Analyses, Nonlinear Static Analyses

1. Introduction

This work belongs to a research project with the scope of Seismically evaluate the National Palace of Sintra, promoted by Parques Sintra-Monte da Lua, S.A. (PSML) and coordinated by Instituto Superior Técnico, from University of Lisbon.

PSML is a public company, founded in 2000 due to the classification of the Cultural Landscape of Sintra as World Heritage, in 1995, by UNESCO, that has as primary mission to manage the important cultural and natural values.

The Palace is an agglomerate of buildings of different volumes and styles, added over time that turns out to be a beauty and harmonious unity, characterized by its iconic chimneys, as it can be seen in Figure 1.

Fig. 1: National Palace of Sintra. (Silva, 2002)*, on the left. On the right: image captured by the drone of the Instituto Superior Técnico, University of Lisbon team, involved in the project*

This study aims to analyse the Bonet Building, one of the oldest structures from the National Palace of Sintra, built over foundations of ancient Arab palace. At the behest of D. Dinis, whose long reign took place between 1261 and 1326, the Bonet Building was expanded, and the adjacent Palatine Chapel building was built from scratch, both identified in Figure 2.

Fig. 2: Bonet Building in National Palace of Sintra

Sintra is part of the Lisbon district, a zone of Continental Portugal considered to have a high seismic risk. Indeed, the seismic evaluation of Bonet building, including the identification of structural anomalies, vulnerability factors and, consequently, the eventual parts to be reinforced is an important work and is explained in this paper.

2. Characterization and Site Investigation of Bonet Building

This chapter aims to described the tests results, used to characterize the Bonet Building, performed before the beginning of this project by the IST team.

2.1. Laser Scanner

The geometric inspection plays a fundamental role in the characterization of the structure to be modelled, since the documentation regarding the National Palace of Sintra complemented with manual surveys does not have the necessary precision for the structural evaluation of the building. It is of the utmost importance to know the thickness and alignment of the structural walls, the dimensions of the openings and their railings, in order to correctly reproduce the behaviour of the structure.

Using a laser scanner, it is possible to obtain data with an accuracy of up to $1x10^{-6}$ m, thus constituting a suitable methodology for cases of complex, congested and difficult to access geometry (Laser Scanning, 2018). Data collection was aided by a drone so that it was possible to collect the full information from outside the palace.

The set of data obtained gives rise to a three-dimensional point cloud, later treated and used in Revit (Autodesk, 2019) software for the development of the BIM model and the collection of the necessary geometric information.

2.2. Ground Penetrating Radar

GPR (Ground Penetrating Radar) is a non-destructive method, whose test aims to characterize the masonry, detecting types of materials and anomalies (voids,

cracks, water), using antennas of different frequencies (Ponte & Bento, 2019).

The main conclusions drawn indicate that “the majority of the walls are composed of two interior-filled masonry leaves, and usually the outer leaf is approximately 0.30m thick” (Ponte & Bento, 2019). Furthermore, the walls around the rock mass were built directly against it, with no empty space in the middle.

2.3. Flat-jack tests

Flat-jack tests, considered to be semi-destructive *in-situ* tests, aim to characterize the mechanical capacity of structural masonry walls.

Since the Palace is a built heritage, the difficulty of locating the experimental tests adds to the fact that they involve the removal of the plaster in a window of approximately 1x1 m^2 and 25 cm cuts perpendicular to the façade, where the flat-jack fits. Two locations in the Bonet building were defined, one on the north façade and the other on the south.

For each inspection window, single tests were carried out, with only one flatjack, to determine the local state of stress of the masonry; followed by double tests, using two flat-jacks, to determine the deformability and strength characteristics of the masonry.

The results of the single test indicate that the Bonet structure is poorly compressed in the tested areas, although the results may be unreliable due to their punctual character and the physical characteristics of the walls, which present high thicknesses.

From the double tests the stress-strain curves are obtained through which the stiffness, maximum stress and Poisson's ratio are determined. Young’s Modulus values are high, justified by the boundary conditions of the walls examined, which are built against the bedrock (the only possible locations on the building to perform these tests). Therefore, the parameter was taking into account as an upper limit value during the numerical model calibration.

2.4. Samples Collection

It should be noted that, parallel to the flat-jack tests, samples were also collected in the form of cores (another semi-destructive test), which main objective is to characterize the material and its constituents. Note that, since masonry is a heterogeneous material, the samples only represent the material locally.

From this test it was possible to conclude that the state of degradation of the masonry in the Bonet building is quite high, being qualitatively one of the worst of the entire Palace.

2.5. Ambient vibration tests

Ambient vibration testing is a non-destructive *in-situ* methodology suitable for historic and heritage buildings. By measuring the structure vibrations subjected to external environmental inputs (vibration caused by traffic, wind, etc.), it is possible to identify the modal dynamic characterization parameters, with the calibration of the numerical model as objective.

The processing of the obtained data was performed with the ARTeMIS (Structural Vibration Solutions (SVIBS), 2015) software and allowed to obtain the fundamental frequencies and the vibration modes of the structure . The Enhanced Frequency Domain Decomposition method was used and the modes were determined by selecting peaks in the response spectral density functions for the set of tests (Milosevic & Bento, 2015). Results can be consulted in Table 1.

3. Numerical Modal

The Bonet Building was modelled in parallel in two software, 3Muri (S.T.A. DATA., 2018) and Abaqus (Simulia, 2014a). It was ensured that the structure geometry is identical in both models, even with the required simplifications, and consistent with the data collected by Laser Scanner.

A Revit model was created (Figure 3) prior to the realization of the Laser Scanner survey, therefore without a verified geometry (Querido, 2018). It is possible to export the Revit geometry and to import it

into Abaqus, saving time and manual effort. Thus, the Revit model has been fully updated with the Laser Scanner's point cloud.

BIM models are very detailed, with all the architectural details, and cannot be used directly in FE software. As explained in Castellazzi et all (2015) "in order to achieve the expected FE model, several manual corrections are needed to guarantee the mesh compatibility, to avoid mesh local distortions or small elements and to model complex architectural objects". With that in mind, some geometric simplifications were adopted, namely the uniformity of thicknesses in walls of variable thickness.

The geometric structure was divided into cells in order to assign different properties to different areas of the global structure. However, this division process is complex, may be less rigorous and creates distorted elements in the finite element mesh.

For the Abaqus model, consisting of solid elements, a 10-node tetrahedral finite element mesh (Pereira, 2000) with a maximum size of 700mm was defined, Figure 4, thus ensuring that for each thickness there are always three nodes.

Fig. 3: Revit Model, Bonet Building, North Façade

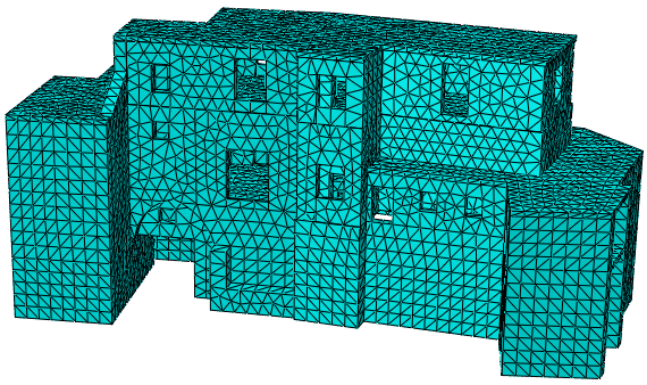


Fig. 4: Abaqus Model with Finite Element Mesh, Bonet Building, North Facade

In 3Muri, the model is defined by macro elements, piers, spandrels and rigid nodes.

Through the point cloud obtained by the Laser Scanner, the use and coating of the floors of each room were studied, as well as the existence of a false ceiling. Thus, the necessary information was collected to define the linear properties of the pavements, including roofs, and the loads acting according to the seismic combination.

Parts of adjacent buildings were also modelled to account for the interaction and impact they have on the overall behaviour of the Bonet Building.

3.1. Calibration

It is described the important process of calibration of the numerical model, in Abaqus and in 3Muri, and the analysis results.

At this stage, the boundary conditions of the structural walls in contact with the bedrock were also defined to understand how constraint the displacements and rotations are.

The Young's modulus (E) of the masonry (Bonet Building top floor - cracked and other floors) was defined during the numerical model calibration with the results processed in ARTeMIS. The final calibrated values are shown in Table 3.

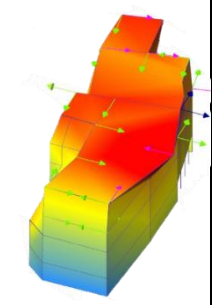
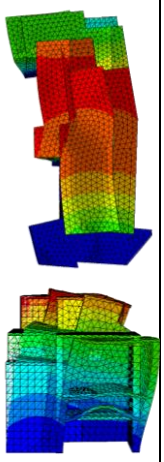
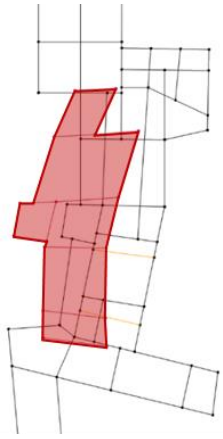
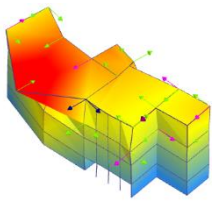
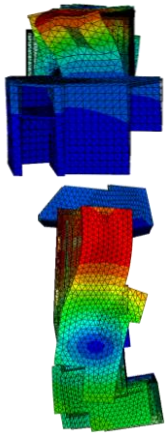
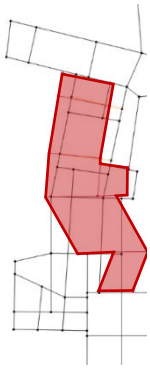
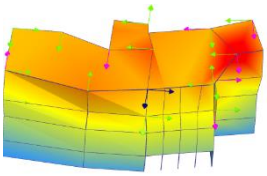
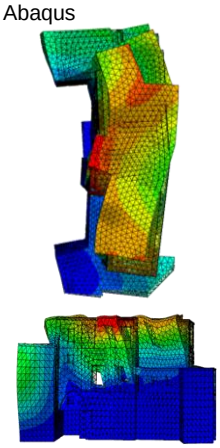
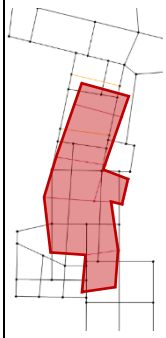
Table 1 shows the vibration modes obtained for the first three models, and the relative error between numerical models and the ambient vibration test, satisfactory in all cases.

Tab. 1: Vibration Modes, Relative Errors of Abaqus and 3Muri Modes in comparison to ARTeMis

	Model ARTeMIS	**Model Abaqus**		**Model 3Muri**	
Mode:	**f (Hz)**	**f (Hz)**	**Error (%)**	**f (Hz)**	**Error (%)**
1st Y	5.00	5.05	0.96%	4.98	-0.40%
2nd Y	5.18	6.17	19.11%	5.55	7.14%
1st X	8.20	7.27	-11.34%	7.03	-14.27%

Table 2 shows the deformed shape of the vibration modes for the three models, and it is possible to conclude once again the close similarity between them.

Tab. 2: Deformed Shapes of Vibration Modes

1st Vibration Mode - Y (transverse direction)		
ARTeMIS	Abaqus	3Muri
		
2nd Vibration Mode – Y (transverse direction)		
ARTeMIS	Abaqus	3Muri
		
1st Vibration Mode – X (longitudinal direction)		
ARTeMIS	Abaqus	3Muri
		

4. Nonlinear Static Analyses

To verify the seismic safety of the structure, nonlinear static analyses were performed.

4.1. Nonlinear Properties of Masonry

The nonlinear properties of masonry were defined coherently for both programs.

The compressive yield strength (f_c) of masonry is proportional to the calibrated Young's modulus, according to (MIT, 2009). The value of shear strength (T_0) was similarly calculated.

3Muri makes use of the Turnsek-Cacovic theory, which indicates that tensile yield stress (f_t) is given by 3/2 of the shear strength value. These values were then calculated and adopted in Abaqus, thus ensuring greater coherence between both numerical models.

In Abaqus, it is necessary to define the behaviour of the material, that is, the evolution of stresses for both compression and tensile regimes.

The Concrete Damage Plasticity Model (CDPM) is a model designed to define the plastic properties of concrete or quasi-brittle materials developed by (Lee & Fenves, 1998). This model uses the concepts of damaged elastic isotropic material in conjunction with the compressive and tensile plastic stress of isotropic material to represent the non-elastic behavior of masonry, as explored in ABAQUS Theory Manual (Simulia, 2014b). Therefore, it is advised to use for masonry materials, despite being developed for concrete.

The plastic model assumes that the two main breaking mechanisms are tensile diagonal cracking and compressive crushing. The evolution of the yield surface is controlled by the tensile and compressive strain, considering the different types of behavior related to the failure mechanisms under load, as explained in ABAQUS Theory Manual (Simulia, 2014b).

The CDPM suggests the constitutive laws of the material for traction and compression, respectively, as shown in Figure 5.

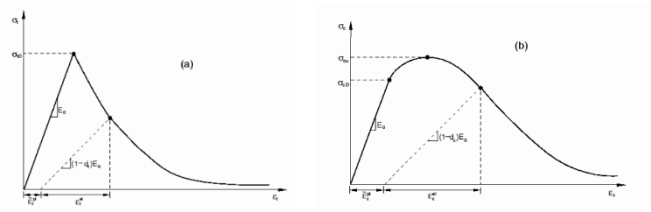


Fig. 5: Constitutive Laws of the Masonry for tensile and compression regime

The simplified generic constitutive models that were adopted, according to (Pozzato, 2018), are represented in Figure 6.

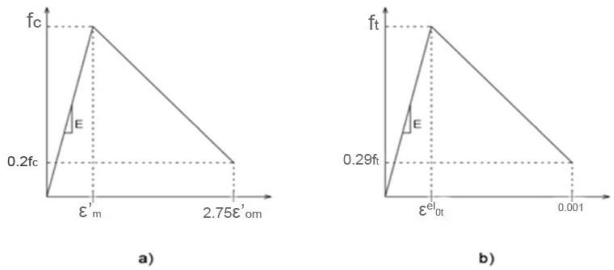


Fig. 6: Simplified Constitutive Laws of CDPM

The final data can be found in Table 3.

Tab. 3: Masonry Properties

	E (GPa)	G (GPa)	f_t (MPa)	f_c (MPa)	w (kN/m³)
Disorganized irregular stone masonry					
MIT (2009)	0.69 – 1.05	0.23 – 0.35	0.03 – 0.048	1.0 – 1.8	19
Bonet Building (Last Floor) Cracked	0.40	0.13	0.018	0.62	18
Bonet Building	0.80	0.26	0.036	1.24	18
Rough block masonry with external leaf of limited thickness and internal core					
MIT (2009)	1.02 – 1.44	0.34 – 0.48	0.053 – 0.077	2.0 – 3.0	20
Chapel	1.20	0.40	0.063	2.43	18
Brasões Room	1.44	0.48	0.080	3.00	18

Damage variables for the compression and tension of each material are defined, responsible for the loss of stiffness and resistance of the material that occurs upon entering the plastic regime, increasing progressively with the stress decay.

The damage variables are fundamental for the behaviour of the material to represent the real behaviour since the damaged masonry presents significant stiffness reductions, which have to be considered in the analysis.

A linear progression was adopted since the beginning of plastic regime, whose damage is null, to the maximum strain of decay, which was assigned the maximum damage value of 0.9.

To define the CDP model in Abaqus, the values of the following parameters are assumed as being the typical ones for masonry, as explained (Degli *et al.*, 2018): 1) the dilation angle is equal to 10°; 2) the flow potential eccentricity is set equal to 0.1; 3) the ratio of initial biaxial compressive yield stress to initial uniaxial compressive yield stress is equal to 1.16; 4) the ratio of the second stress invariant on the tensile meridian to that on the compressive meridian is set equal to 0.666; 5) the viscosity parameter, which defines visco-plastic regularization, is assumed equal to 0.002.

4.2. Capacity Curves

The control points determined by the 3Muri sensitivity analyzes were adopted in both models to ensure a better comparison between the results. Control points are defined on the top floor of the alignment where the first walls collapses.

Figure 7 and 8 shows the capacity curves obtained by the pushover analyzes of the two models, highlighting the similarity and proximity of the stiffness between each analysis. This similarity corroborates the correct calibration between both models.

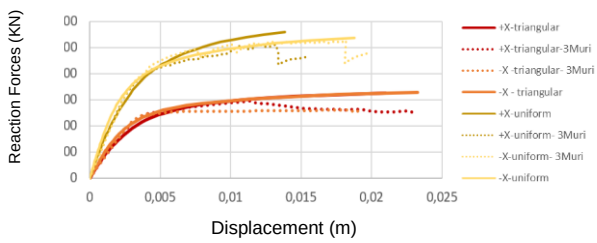


Fig. 7: Capacity Curves, X direction, Abaqus and 3Muri

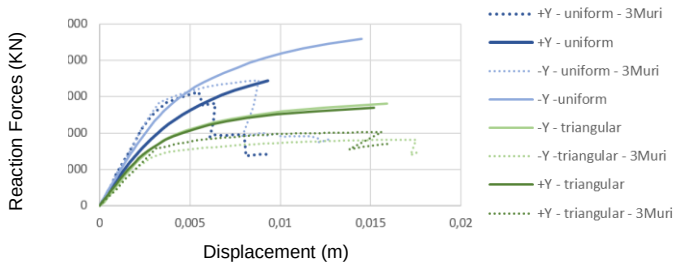


Fig. 8: Capacity Curves, Y direction, Abaqus and 3Muri

The Abaqus model has the ability to take much more into account the spread of nonlinear entry in some of the most vulnerable areas of the model, while the remaining are still in linear elasticity. In contrast, in 3Muri nonlinear input occurs for the entire model simultaneously.

To overcome this difficulty, it was decided to consider in 3Muri that the Young's modulus of all masonry materials is equal to 2/3 of the original value. In this way, the propagation of nonlinear behaviour in the macro-element is considered until reaching the ultimate values of the forces (shear force and bending moment).

The graphs obtained by 3Muri are compared before the reduction of the elastic moduli and after for the X direction and positive direction, as it can be seen in Figure 9. As might be expected, an initial stiffness reduction of the global model is observed, proportional to the reduction of the Young's Module.

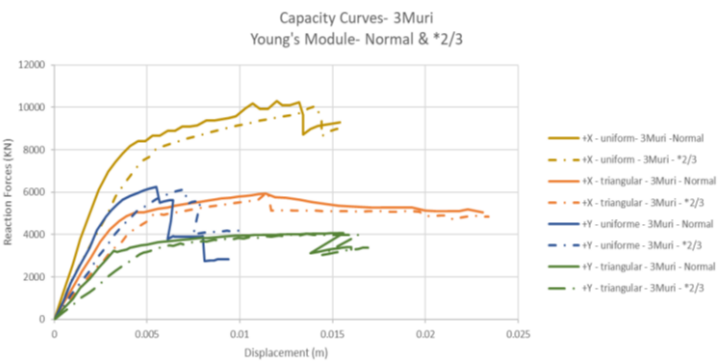


Fig. 9: Capacity curves obtained in 3Muri, with and without reduction of the Young's Module

4.3. Ultimate displacement definition

The CDPM implies that at each time increment the applied load is also higher and, regardless of the loss of material strength derived from the damage variables, the total strength of the structure does not decrease.

Since one of the main limitations of Abaqus is the difficulty in determining the ultimate displacement, there is a need to define a criterion for determining it.

3Muri assumes the last displacement to be what occurs first between: 1) the reduction of 20% of the base shear force regarding the maximum strength of the structure; 2) the occurrence of a collapse mechanism.

On the other hand, the Italian norm indicates that the ratio between the ultimate displacement value and the yield displacement, where the structure enters the nonlinear plastic regime of the bi-linear curve, must be between 3 and 6.

Since one of the main objectives of the Abaqus model study is to compare the results with the 3Muri model, in addition to performing the seismic evaluation, it was used the ultimate displacement calculated in 3Muri in the corresponding curves in Abaqus.

Subsequently, the ultimate displacement are verified using the Italian Standard (NTC, 2008) criterion and, if necessary, their correction.

4.4. Safety Verification – N2 Method

Safety is verified using the N2 method proposed in EC8-1 (CEN, 2009). The method intends to compare the ultimate displacement with the target displacement: the displacement of the structure when subjected to a seismic action, determined through the intersection of the resistant capacity curve, with the response spectrum of the seismic action. This comparison is made after the transformation of the multiple degrees of freedom system into one equivalent degree of freedom system using the calculated transformation factor for each direction and the bilinearization of the pushover curves.

Seismic action is defined according to EC8-1 (CEN, 2009), complemented by the National Annex, from response spectra, for each ultimate limit state and for distant and near earthquake.

The bilinear capacity curve is composed of a first section, a constant slope, until it reaches the yield force and correspondent displacement. The second section, which begins with the entry into plastic regime, has zero slope and extends to the intersection with the ultimate displacement (Cattari & Lagomarsino, 2006).

As opposed to 3Muri, Abaqus does not automatically obtain the transformation coefficient, so this factor has been manually calculated for each direction, which is a demanding process for complex structures such as the Bonet Building. The degrees of freedom were considered as the complete floors of the structure that are not in contact with the bedrock. The mass associated with each degree of freedom was calculated considering the displacements of each floor associated with the vibration modes.

The target displacement depends on the intersection of the bilinear curve with the seismic action response spectrum previously defined for each limit state in acceleration-displacement form.

Safety is verified when the ratio between ultimate displacement and target displacement is greater than 1. Figures 10 and 11 show the safety verifications for the type 1 earthquake, the most demanding one, for the analyses carried out at Abaqus and 3Muri, for the most demanding limit state that, for historic buildings, is near collapse.

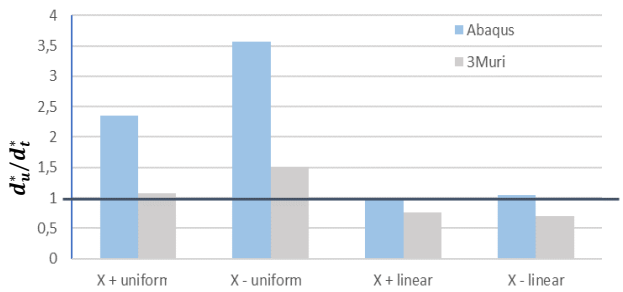


Fig. 10: Safety Verification, Ultimate Limit State, X direction, Type 1 Seism

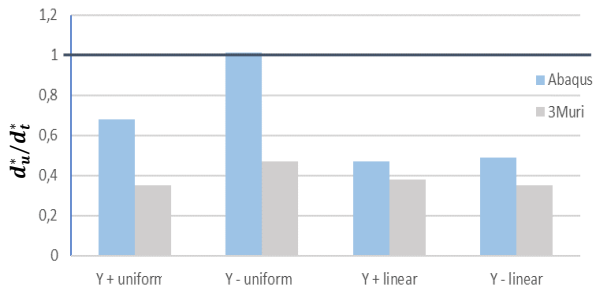


Fig. 11: Safety Verification, Ultimate Limit State, Y direction, Type 1 Seism

It is then possible to state that safety is not verified in any direction for the Bonet Building, with direction Y being the most vulnerable.

4.5. Damage Pattern

Finite element analysis in the Abaqus model allows us to study the local evolution of the damage in detail, which can be increased by mesh refinement.

This program allows the observation of the damage through the separate interpretation between the damage caused by tension or compression.

On the other hand, 3Muri provides the damage pattern of the structure by characterizing the type of damage to occur in each macro element.

Damage pattern comparisons were performed for all pushover analyzes performed between both programs.

Figure 12 to 14 shows the damage pattern for the most demanding analysis: + Y, pseudo-triangular distribution.

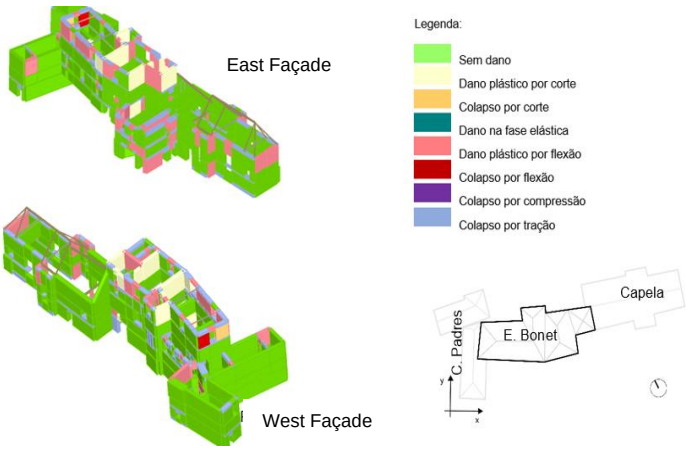


Fig. 12: Damage Pattern, 3Muri (+Y, pseudo-triangular distribution)

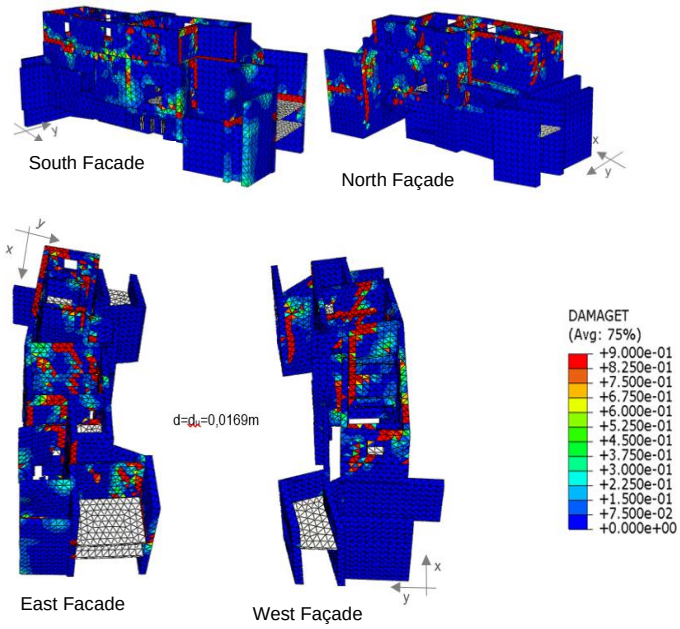


Fig. 13: Damage Pattern, Tensile Variable, Abaqus (+Y, pseudo-triangular distribution)

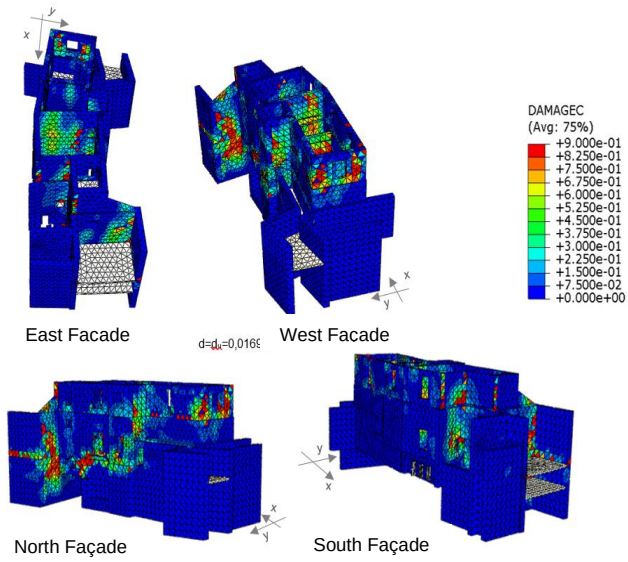


Fig. 14: Damage Pattern, Compressive Variable, Abaqus (+Y, pseudo-triangular distribution)

Analysing the damage pattern given by the 3Muri in Fig 12, it is known that the collapse occurs in the west side of last floor wall, by flexural failure mechanism in the posterior pier, according to the loading direction, and by shear failure mechanism in the anterior pier.

The upper level spandrels feature mostly tensile failure. Some last floor walls positioned according to X direction present flexural damage, while those positioned according to Y direction present shear damage. The lower floors are undamaged.

Analysing the damage parameter in Abaqus (Fig. 13), according to the tension variable, it can be concluded that exists a maximum damage concentration in the spandrel of the wall where the collapse occurs, which may correspond to the tensile collapse also identified in the 3Muri.

Tensile damage is also present in the piers but the anterior macro-element is not sufficiently damaged to indicate collapse by shear.

On the other hand, compression damage (Fig. 14) is present throughout the wall in question, but more significant at the pier where flexural collapse occurs.

Maximum tensile damage is present in the upper spandrels of the walls of the last floor room, in both façades, which has the tie rods, as well as in the lower spandrels of the façades, and may be associated with collapse by shear.

On the Y direction walls where 3Muri identifies shear damage, Abaqus has also presented tensile damage, particularly along the macro-element diagonal, verifying the similarity of results between both programs.

The damage pattern for compression in Abaqus reveals an accumulation on most Y direction structural walls, resulting in bending damage of the crushing mechanism. Damage is further reduced on the south façade due to the loading direction.

5. Conclusions

First of all, it is worth noting the difficulty of geometric construction of such a complex model as the Bonet building in Abaqus. Importing the structure directly from a BIM model is necessary, however with some care to be taken in advance. The BIM model has to be architecturally simplified and built to the point so as not to create distorted finite elements when generating the mesh. On the other hand, 3Muri is a program for building modeling and analysis, resulting in ease of construction and alteration of the geometric characteristics of the structure.

It is positively concluded that the results obtained between the two programs are similar. Note that Abaqus presents a high graphical ability to observe the results. In modal analysis, it is possible to rigorously study the deformed structure for the various vibration modes. The same conclusion can be drawn when interpreting the damage pattern, since we observe the damage evolution for each finite element node, while 3Muri presents the damage evolution for each macro element, naming only the most significant mechanism.

However, 3Muri is a more suitable program for performing pushover analysis and subsequent security verifications. The program proposes for each analysis the ultimate displacement and automatically calculates the transformation coefficient of the models from MDOF to SDOF. These two points are of high difficulty in Abaqus, involving a complex calculation to perform, especially for complex buildings.

6. Bibliography

Autodesk. (2019). *Autodesk, Revit 2019, versão 2019*.

Castellazzi, G., D'Altri, A. M., Bitelli, G., Selvaggi, I., & Lambertini, A. (2015). From laser scanning to finite element analysis of complex buildings by using a semi-automatic procedure. *Sensors (Switzerland)*, *15*(8), 18360–18380. https://doi.org/10.3390/s150818360

Cattari, S., & Lagomarsino, S. (2006). Non Linear Analysis of Mixed Masonry and Reinforced Concrete Buildings. *First European Conference on Earthquake Engineering and Seismology*, (September), 3–8.

CEN. (2009). NP EN 1998-1: Eurocódigo 8 – Projecto de Estruturas para resistência aos sismos. Parte 1: Regras gerais, acções sísmicas e regras para edifícios. Bruxelas: Instituto Português da Qualidade. European Committee for Standardization. *Instituto Português Da Qualidade*.

Degli, S., Maria, A., Ottonelli, D., Castellazzi, G., Cattari, S., Miranda, S. De, & Lagomarsino, S. (2018). Seismic assessment of interacting structural units in complex historic masonry constructions by nonlinear static analyses. *Computers and Structures*. https://doi.org/10.1016/j.compstruc.2018.12.001

Laser Scanning. (2018). No Title. Retrieved May 30, 2019, from www.laser-scanning.co.uk

Lee, J., & Fenves, G. L. (1998). Plastic-Damage Model for Cyclic Loading of Concrete Structures. *Journal of Engineering Mechanics*, *124*(8), 892–900. https://doi.org/10.1061/(ASCE)0733-9399(1998)124:8(892)

Milosevic, J., & Bento, R. (2015). *Software for Operational Modal Analysis Artemis Version 3.0. Relatório ICIST DTC nº 12/2015.*

MIT. (2009). *Circolare n. 617 del 2 febbraio 2009. Istruzioni per l'Applicazione Nuove Norme Tecniche Costruzioni di cui al Decreto Ministeriale 14 gennaio 2008. (em Italiano)*.

NTC. (2008). *Norme Tecniche per la Construzioni. Decreto Ministeriale 17/01/2018, Official Gazette. Roma (em Italiano).*

Pereira, O. J. B. a. (2000). Análise De Estruturas II: Introdução Ao Método Dos Elementos Finitos Na Análise De Problemas Planos De Elasticidade. *DECivil, Instituto Superior Técnico*.

Ponte, M., & Bento, R. (2019). Caracterização de Paredes de Alvenaria no Contexto da Avaliação Sísmica: o Caso de Estudo do Palácio Nacional de Sintra. *Sísmica*, (11º Congresso Nacional de Sismologia e Engenharia Sísmica).

Pozzato, M. C. (2018). *Sintra National Palace: Dynamic and Seismic Characterization of the Chimneys Body. Dissertação de Mestrado, Instituto Superior Técnico.* https://doi.org/10.1590/s1809-98232013000400007

Querido, J. (2018). *O Edifício Bonet do Palácio Nacional de Sintra: Caracterização e avaliação do desempenho estrutural. Dissertação de Mestrado*

em Engenharia Civil, Instituto Superior Técnico.

S.T.A. DATA. (2018). *S.T.A. DATA., 3Muri, v.11.4.0*.

Silva, J. C. V. da. (2002). *O Palácio Nacional de Sintra* (1º; SCALA, Ed.). Londres.

Simulia. (2014a). *ABAQUS, Complete Solutions for Realistic Simulation, versão 16.14-1*.

Simulia. (2014b). *ABAQUS Theory Manual. Versão 6.6.*

Structural Vibration Solutions (SVIBS). (2015). *Structural Vibration Solutions, ARTeMIS Modal Pro, versão 4.0*.