Dirac neutrinos in the 2HDM with maximally-restrictive Abelian symmetries

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The recent observation of neutrino oscillations provided undeniable evidence of physics beyond the Standard Model (SM), requiring neutrinos to be massive and leptons to mix. Motivated by this, we consider an extension of the SM with a second Higgs doublet (2HDM) and three right-handed (RH) neutrinos where lepton number is conserved and, thus, neutrinos are Dirac particles. We identify the most restrictive texture-zero combinations for the Dirac-neutrino and charged-lepton mass matrices that lead to masses and mixing parameters compatible with current neutrino oscillation data. We then systematically determine which of these combinations can be realised by Abelian continuous U(1) or discrete $\mathbb{Z}_N$ symmetries in the described framework. We find that only 5 of the 28 non-equivalent maximally-restricted lepton mass matrix pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ have a symmetry realization in the 2HDM. For these 5 cases, we explore leptonic CP violation and discuss the constraints arising from lepton universality in τ decays and rare lepton-flavour-violating (LFV) processes.
Keywords: Dirac neutrinos, 2HDM, flavour physics, Abelian symmetries, texture zeros

I. INTRODUCTION

Particle (or high-energy) physics is the study of fundamental particles and their interactions. It is based on the self-consistent and elegant Standard Model (SM) of particle physics [1–3], which shows a remarkable agreement with experiment in particle accelerators. However, we know today that this theory is incomplete, leaving several phenomena unexplained. For instance, the recent observation of neutrino oscillations [4, 5], implying that neutrinos are massive and that lepton mix, provided undeniable evidence of physics beyond the SM (BSM).

In the last decades, several experiments have been measuring most of the parameters involved in neutrino flavour oscillations with very good precision [6, 7]. In spite of this remarkable achievement, several important questions about neutrinos, such as whether these are Dirac or Majorana particles, remain unanswered. Unfortunately, this fundamental question cannot be addressed by neutrino oscillation experiments and current experimental data is compatible with both scenarios which should, therefore, be equally considered. On the other hand, one is always confronted with the problem of explaining the observed neutrino mass and mixing pattern. As discussed in the thesis, dimension-five operators in an effective theory realised at tree level by, for instance, a seesaw mechanism, can provide an explanation for the smallness of (Majorana or Dirac) neutrino masses. However, in general, these frameworks do not address the flavour problem (or puzzle) per se. Thus, one is compelled to consider sophisticated realisations of certain neutrino mass models in which flavour symmetries are considered.

One of the approaches is to explore the existence of vanishing elements (texture zeros) in the Yukawa and mass matrices which reflect the violation of a symmetry by a certain interaction [8–14]. The simplest of these symmetries are those based on continuous U(1) and discrete $\mathbb{Z}_N$ (i.e. Abelian) transformations. As shown in the thesis, in the SM extended with RH neutrinos, U(1) or $\mathbb{Z}_N$-motivated texture zeros are incompatible with data since, in general, they lead to massless charged leptons and/or vanishing lepton mixing angles (already excluded by data [6, 7]). This follows, in part, from the fact that all fermions in the SM couple to the same Higgs field. However, this is not the case in the two-Higgs-doublet model (2HDM), one of the simplest BSM scenarios, where two scalar (Higgs) doublets are considered instead of one (see Ref. [15] or the thesis for more details).

Motivated by the remarks above, we consider *restrictive* Abelian symmetries in the 2HDM extended with RH neutrino fields such that neutrinos are Dirac particles. We call these symmetries restrictive since the number of relevant flavour (Yukawa coupling) parameters in the resulting lepton sector is the same as the number of observables, i.e. nine (ten) in the case of two (three) massive Dirac neutrinos. Our goal is to determine if these maximally-restricted mass matrix textures can be generated by Abelian flavour symmetries in this model, while maintaining compatibility with charged-lepton masses, neutrino oscillation data and the experimental bounds on lepton universality in τ decays and rare lepton-flavour-violating (LFV) decays. The present work closely follows Ref. [16].

II. DIRAC NEUTRINOS IN THE 2HDM

As in the SM, Dirac neutrino masses can be generated in the 2HDM by adding RH neutrino singlet fields ν_R, which couple to the left-handed (LH) SM lepton doublets ℓ_L and the two Higgses $\Phi_a = \begin{pmatrix}\phi_a^+ & \phi_a^0\end{pmatrix}^T$, $a = 1, 2$, being ϕ_a^+ and ϕ_a^0 the charged and neutral components of the scalar doublets. In this framework, the lepton Yukawa interactions can be written as

$$\mathcal{L}_{\rm Y} = -\overline{\ell_L}\, \mathbf{Y}_a^\ell \Phi_a\, e_R - \overline{\ell_L}\, \mathbf{Y}_a^\nu \widetilde{\Phi}_a\, \nu_R + \text{H.c.}\,, \tag{1}$$

where a sum over a is implicit, e_R are the charged-lepton RH singlets, and $\widetilde{\Phi}_a = i\sigma_2\Phi_a^*$. The general 3×3 complex matrices $\mathbf{Y}_a^\ell$ and $\mathbf{Y}_a^\nu$ encode the charged-lepton and Dirac neutrino Yukawa interactions, respectively. In line with the discussion presented in the thesis, very small Yukawa couplings $\mathbf{Y}_a^\nu$ may originate from dimension-five operators:

$$-\mathcal{L}_5^D = \overline{\ell_L}\, \frac{\mathbf{Y}_a}{\Lambda} S\, \widetilde{\Phi}_a\, \nu_R + \text{H.c.}\,, \tag{2}$$

such that $\mathbf{Y}_a^\nu \equiv \mathbf{Y}_a v_S/\Lambda$ are sufficiently suppressed to generate sub-eV Dirac neutrino masses upon EWSB, i.e. when ϕ_a^0 acquire VEVs $\langle\phi_a^0\rangle \equiv v_a/\sqrt{2}$, $\tan\beta \equiv v_2/v_1$, $v^2 = v_1^2 + v_2^2$, with $v \simeq 246$ GeV. Thus, from now on we will consider that there is such a mechanism responsible for the smallness of $\mathbf{Y}_a^\nu$. The resulting charged-lepton and Dirac-neutrino mass matrices

$$\mathbf{M}_\ell = \sum_a \mathbf{Y}_a^\ell \frac{v_a}{\sqrt{2}}\,, \quad \mathbf{M}_\nu = \sum_a \mathbf{Y}_a^\nu \frac{v_a}{\sqrt{2}}\,, \tag{3}$$

can be diagonalized by a set of appropriate unitary matrices $\mathbf{U}_{L,R}^{\ell,\nu}$ so that

$$\begin{aligned} \mathbf{U}_L^{\ell\,\dagger}\mathbf{M}_\ell\mathbf{U}_R^\ell &= \mathbf{D}_\ell = \text{diag}(m_e, m_\mu, m_\tau)\,,\\ \mathbf{U}_L^{\nu\,\dagger}\mathbf{M}_\nu\mathbf{U}_R^\nu &= \mathbf{D}_\nu = \text{diag}(m_1, m_2, m_3)\,. \end{aligned} \tag{4}$$

where $m_{e,\mu,\tau}$ and $m_{1,2,3}$ denote the charged-lepton and neutrino masses, respectively, all being real and positive. To extract the LH rotation matrices, one diagonalizes the Hermitian matrices

$$\mathbf{H}_\ell = \mathbf{M}_\ell\mathbf{M}_\ell^\dagger\,, \quad \mathbf{H}_\nu = \mathbf{M}_\nu\mathbf{M}_\nu^\dagger\,, \tag{5}$$

in the following way:

$$\begin{aligned} \mathbf{U}_L^{\ell\,\dagger}\mathbf{H}_\ell\mathbf{U}_L^\ell &= \mathbf{D}_\ell^2 = \text{diag}(m_e^2, m_\mu^2, m_\tau^2)\,,\\ \mathbf{U}_L^{\nu\,\dagger}\mathbf{H}_\nu\mathbf{U}_L^\nu &= \mathbf{D}_\nu^2 = \text{diag}(m_1^2, m_2^2, m_3^2)\,. \end{aligned} \tag{6}$$

The unitary transformations $\mathbf{U}_L^{\ell,\nu}$ define the lepton mixing matrix $\mathbf{U}$ appearing in lepton charged-current interactions as

$$\mathbf{U} = \mathbf{U}_L^{\ell\,\dagger}\mathbf{U}_L^\nu\,. \tag{7}$$

In the case of massive Dirac neutrinos, $\mathbf{U}$ can be parametrized by three mixing angles θ_{ij} and a single CP-violating Dirac phase δ, such that [7]

$$\mathbf{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}\,, \tag{8}$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. Thus, the lepton sector is characterised by ten parameters: three charged-lepton and three neutrino masses, three mixing angles and one phase.

Global analyses of all available neutrino oscillation data constrain the parameters of the matrix $\mathbf{U}$ as shown in Table I [6, 17, 18], for both normal hierarchy (NH) ($m_1 < m_2 < m_3$) and inverted hierarchy (IH) ($m_3 < m_1 < m_2$) neutrino mass spectra. In the case of $m_1 = 0$ ($m_3 = 0$), m_2 and m_3 (m_1 and m_2) are fully determined by the mass-squared differences $\Delta m_{21}^2 = m_2^2 - m_1^2$ and $\Delta m_{31}^2 = m_3^2 - m_1^2$ (see thesis), so that the number of measurable quantities is reduced to nine. The cosmological constraint on the sum of the neutrino masses, $\sum_i m_i < 0.12$ eV at 95% CL [19], should also be taken into consideration. If one neutrino is massless, then $\sum_i m_i \simeq 0.059$ eV for NH and $\sum_i m_i \simeq 0.099$ eV for IH, thus being the cosmological limit automatically obeyed.

III. ABELIAN SYMMETRIES IN THE 2HDM WITH DIRAC NEUTRINOS

As shown in the thesis, Abelian-symmetry motivated texture zeros in $\mathbf{M}_\ell$ turn out to be incompatible with data, in the SM with RH neutrinos, as these lead to massless charged leptons or non-viable mixing angles. This is not the case in models with extra scalar fields like the 2HDM, where Abelian symmetries may lead to interesting constraints in the flavour sector [8, 9, 20, 21]. Thus, in this section we will go through some general aspects on this subject.

Denoting $\Phi \equiv (\Phi_1\ \Phi_2)^T$, and requiring that the full Lagrangian is invariant under the field transformations

$$\begin{aligned} \Phi \to \mathbf{S}_\Phi\Phi\,, &\quad \ell_L \to \mathbf{S}_\ell\ell_L\,,\\ e_R \to \mathbf{S}_e e_R\,, &\quad \nu_R \to \mathbf{S}_\nu\nu_R\,, \end{aligned} \tag{9}$$

where $\mathbf{S}_\Phi \in \text{U}(2)$ and $\{\mathbf{S}_\ell,\ \mathbf{S}_e,\ \mathbf{S}_\nu\} \in \text{U}(3)$, yields the following constraints on the Yukawa couplings

$$\mathbf{Y}_a^\ell = \mathbf{S}_\ell\mathbf{Y}_b^\ell\mathbf{S}_e^\dagger(\mathbf{S}_\Phi^\dagger)_{ba}\,, \quad \mathbf{Y}_a^\nu = \mathbf{S}_\ell\mathbf{Y}_b^\nu\mathbf{S}_\nu^\dagger(\mathbf{S}_\Phi^T)_{ba}\,, \tag{10}$$

Parameter	Best fit $\pm 1\sigma$	3σ range
θ_{12} ($^\circ$)	$34.5^{+1.2}_{-1.0}$	$31.5 \to 38.0$
θ_{23} ($^\circ$) [NH]	$47.7^{+1.2}_{-1.7}$	$41.8 \to 50.7$
θ_{23} ($^\circ$) [IH]	$47.9^{+1.0}_{-1.7}$	$42.3 \to 50.7$
θ_{13} ($^\circ$) [NH]	$8.45^{+0.16}_{-0.14}$	$8.0 \to 8.9$
θ_{13} ($^\circ$) [IH]	$8.53^{+0.14}_{-0.15}$	$8.1 \to 9.0$
δ ($^\circ$) [NH]	238^{+38}_{-27}	$157 \to 349$
δ ($^\circ$) [IH]	281^{+23}_{-27}	$202 \to 349$
Δm^2_{21} (10^{-5} eV^2)	$7.55^{+0.20}_{-0.16}$	$7.05 \to 8.14$
$\lvert\Delta m^2_{31}\rvert$ (10^{-3} eV^2) [NH]	2.50 ± 0.03	$2.41 \to 2.60$
$\lvert\Delta m^2_{31}\rvert$ (10^{-3} eV^2) [IH]	$2.42^{+0.03}_{-0.04}$	$2.31 \to 2.51$

TABLE I. Neutrino oscillation parameters obtained from the global analysis of Ref. [6] for NH and IH neutrino mass spectra (see also Refs. [17, 18]).

where a sum over $b = 1, 2$ is implicitly assumed. By performing basis transformations identical to those in Eq. (9), with the appropriate choices of unitary matrices $\mathbf{V} \in$ U(2) and $\{\mathbf{V}_\ell, \mathbf{V}_e, \mathbf{V}_\nu\} \in$ U(3), one can bring the matrices $\mathbf{S}$ into a diagonal form with [21]:

$$\begin{aligned}(\mathbf{S}_\Phi)_{ii} &= e^{i\theta_i}\,, \quad (\mathbf{S}_\ell)_{ii} = e^{i\alpha_i}\,,\\ (\mathbf{S}_e)_{ii} &= e^{i\beta_i}\,, \quad (\mathbf{S}_\nu)_{ii} = e^{i\gamma_i}\,,\end{aligned} \tag{11}$$

where θ_i, α_i, β_i and γ_i are continuous phases. Under the general transformation (11), the invariance condition (10) reads

$$(\mathbf{Y}^x_a)_{ij} = e^{i(\Theta^x_a)_{ij}} (\mathbf{Y}^x_a)_{ij}\,, \tag{12}$$

where $i, j = 1, 2, 3$ are flavour indices and $x = \ell, \nu$. The phase matrices Θ^x_a, which encode the transformation properties of each Yukawa interaction, are defined as

$$(\Theta^{\ell,\nu}_a)_{ij} = (\beta, \gamma)_j - \alpha_i + \theta_a\,. \tag{13}$$

These phases can be written in terms of charges $(\alpha', \beta', \gamma', \theta')$ and a parameter $\varphi \in [0,\ 2\pi[$ such that

$$(\Theta^{\ell,\nu}_a)_{ij} = [(\beta', \gamma')_j - \alpha'_i + \theta'_a]\ \varphi\,. \tag{14}$$

The particular case of $\varphi = 2\pi/N$, with $N = 2, 3, ...$, corresponds to a discrete $\mathbb{Z}_N$ symmetry. Alternatively, invariance of $(\mathbf{Y}^x_a)_{ij}$ under a U(1) symmetry implies $(\Theta^x_a)_{ij} = 0$ (mod 2π). It is also straightforward to conclude that, as a consequence of having $\theta_1 - \theta_2 \neq 0$ (mod 2π),[1] a non-zero entry in $\mathbf{Y}^x_1$ will automatically imply a zero entry in $\mathbf{Y}^x_2$, and vice versa. Moreover, as shown in the thesis, appropriate overall rephasings of $\mathbf{S}_{\Phi,\ell,e,\nu}$ allows us, without loss of generality, to set one of the Higgs and fermion transformation phases to zero. Here, we choose $\theta_1 = \alpha_1 = 0$.

With these redefinitions, Eq. (10) can be interpreted in terms of charge relations. The aforementioned U(1) charges determine the presence (or absence) of zero entries in the Yukawa and mass matrices $\mathbf{Y}^x_a$ and $\mathbf{M}_x$, defined in Eqs. (1) and (3), respectively. In particular, with (mod 2π) implied,

$$\begin{aligned}(\mathbf{M}_x)_{ij} = 0 &\Leftrightarrow (\Theta^x_1)_{ij} \neq 0 \ \wedge\ (\Theta^x_2)_{ij} \neq 0,\\ (\mathbf{M}_x)_{ij} \neq 0 &\Leftrightarrow (\Theta^x_1)_{ij} = 0 \ \vee\ (\Theta^x_2)_{ij} = 0\,.\end{aligned} \tag{15}$$

Vanishing elements in a mass matrix or Yukawa interaction matrix are usually dubbed as “texture zeros”. In this work, whenever a general matrix structure contains texture zeros, we will refer to it as a “texture”.

On a final note, it is worth comparing the lepton and quark sectors in the framework of the 2HDM with RH neutrinos, as the Yukawa interactions for quarks are the same as in Eq. (1) after replacing ℓ_L, e_R, ν_R by q_L, d_R, u_R (see thesis). Obviously, $\mathbf{Y}^{\ell,\nu}_a$ must be replaced by $\mathbf{Y}^{d,u}_a$. Textures for quarks can, in principle, be implemented in the same way as for leptons by imposing Abelian symmetries as the ones discussed above, being the main difference in the fact that all six quarks must be massive, in contrast with leptons, for which a massless neutrino is allowed by current experimental data. As explained in the thesis, just as for leptons, Abelian-symmetry motivated textures zeros for quarks are viable in the SM.

IV. MAXIMALLY-RESTRICTIVE TEXTURES FOR LEPTONS

In Ref. [22], all possible textures for $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$ were identified and grouped into equivalence classes, considering both Majorana and Dirac massive neutrinos. For charged leptons, two textures $\mathbf{M}_\ell$ and $\mathbf{M}'_\ell$ are equivalent if they can be transformed onto each other by performing permutations of the ℓ_L and e_R fields, i.e. if

$$\mathbf{M}'_\ell = \mathbf{P}^\dagger_\ell \mathbf{M}_\ell \mathbf{P}_e\,, \tag{16}$$

where $\mathbf{P}_{\ell,e}$ can be any two matrices of the 3-dimensional representation of the S_3 permutation group (see thesis). Said otherwise, two $\mathbf{M}_\ell$ textures are equivalent when they are equal up to permutations of rows and columns.

In the case of Dirac neutrinos, two textures $\mathbf{M}_\nu$ and $\mathbf{M}'_\nu$ are considered equivalent if

$$\mathbf{M}'_\nu = \mathbf{M}_\nu \mathbf{P}_\nu\,, \tag{17}$$

where $\mathbf{P}_\nu$ is also a permutation matrix. Thus, two neutrino mass matrix textures are equivalent if they can be transformed onto each other by column permutations.

We shall combine the above $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$ classes into all possible $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ pairs, keeping only one representative texture of each $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$ equivalence class. Pairs

[1] Otherwise one recovers the SM.

leading to the same leptonic mixing matrix $\mathbf{U}$ are equivalent and, thus, redundant. This is the case when the mass matrices can be related by

$$\mathbf{M}'_\ell = \mathbf{P}^\dagger_\ell \mathbf{M}_\ell \mathbf{P}_e \,, \quad \mathbf{M}'_\nu = \mathbf{P}^\dagger_\ell \mathbf{M}_\nu \mathbf{P}_\nu \,, \tag{18}$$

for any two texture pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ and $(\mathbf{M}'_\ell, \mathbf{M}'_\nu)$. Notice that, in order to leave $\mathbf{U}$ invariant, $\mathbf{P}_\ell$ must be the same in both transformations. Therefore, two texture pairs are equivalent if they can be obtained from each other through column and row permutations, being the row permutation identical for both mass matrices in the pair. This is why in Eq. (17) only column permutations are considered, avoiding the possibility of excluding relevant cases. The outlined procedure aims at eliminating redundant cases that reproduce the same mass and mixing parameters.

Since in this work we are interested in the most predictive $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$, it is crucial to introduce the concept of maximally-restrictive textures [12, 22]:

A texture pair $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ is said to be maximally restrictive if the predicted values for the lepton masses, mixing angles and CP phase are compatible with the experimental data, and the addition of one more texture zero in either $\mathbf{M}_\ell$ or $\mathbf{M}_\nu$ makes the pair incompatible with data.

Essentially, these are the pairs with least parameters, which are viable when confronted with observations.

$\mathbf{M}_\ell$	$\mathbf{M}_\nu$								
3^ℓ_2	7^ν_1	7^ν_3							
4^ℓ_1	6^ν_1	6^ν_3	6^ν_4	6^ν_5	6^ν_6				
4^ℓ_2	6^ν_1	6^ν_2	6^ν_3	6^ν_7	6^ν_8				
4^ℓ_3	6^ν_1	6^ν_2	6^ν_3	6^ν_4	6^ν_5	6^ν_6	6^ν_7	6^ν_8	6^ν_9
5^ℓ_1	5^ν_1	5^ν_4	5^ν_5	5^ν_6	5^ν_8				
6^ℓ_1	4^ν_1	4^ν_{17}							

TABLE II. Maximally-restrictive pairs of leptonic mass matrix textures, consistent with both NH and IH neutrino mass spectra at 1σ CL, except for the texture pair $(6^\ell_1, 4^\nu_{17})$, found to be consistent with experimental data only at 3σ and for a NH mass spectrum.

$$3^\ell_2: \begin{pmatrix} 0 & \times & \times \\ 0 & \times & \times \\ \times & 0 & \times \end{pmatrix} \qquad 4^\ell_1: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & \times \end{pmatrix}$$

$$4^\ell_2: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & 0 & \times \end{pmatrix} \qquad 4^\ell_3: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

$$5^\ell_1: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix} \qquad 6^\ell_1: \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & 0 \end{pmatrix}$$

TABLE III. Representative textures of the $\mathbf{M}_\ell$ equivalence classes appearing in Table II.

A. Compatibility with neutrino oscillation data

In order to identify the maximally-restrictive texture pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ we shall perform an analysis similar to the one of Refs. [12, 22], considering the updated neutrino oscillation parameters and including the current ranges for the Dirac phase δ (see Table I). We require compatibility at 3σ confidence level (CL) and perform a standard χ^2-analysis with the function

$$\chi^2(x) = \sum_i \frac{\left[\mathcal{P}_i(x) - \overline{\mathcal{O}}_i\right]^2}{\sigma_i^2} \,, \tag{19}$$

where x denotes the matrix elements of $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$, $\mathcal{P}_i(x)$ is the model prediction for the observable $\mathcal{O}_i$, $\overline{\mathcal{O}}_i$ is the corresponding best-fit value, and σ_i denotes its 1σ error.

In our search for viable pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$, we require the charged-lepton masses to be at their central values [7], so that the χ^2-function is minimised only with respect to the six neutrino observables $\mathcal{O}_i$ (the two neutrino mass-squared differences $\Delta m^2_{21,31}$, the three mixing angles θ_{ij} and the Dirac phase δ) following the numerical method presented in Refs. [11, 12]. If the deviation of each neutrino observable from its experimental value is at most 3σ at the χ^2 minimum for a given $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ pair, the corresponding lepton textures are said to be compatible with data. In such cases, we test compatibility at the 1σ as well.

Our results show that the maximally-restrictive pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ compatible with data are those presented in Table II, where the labelling follows the notation of Ref. [22].[2] A representative texture of each equivalence class is presented in Tables III and IV for $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$, respectively. All pairs in Table II were found to be consistent with neutrino oscillation data at 1σ, for both NH and IH, except for the pair $(6^\ell_1, 4^\nu_{17})$ which is consistent with data only at 3σ CL and for a NH neutrino mass spectrum. With the exception of texture 4^ν_{17}, any representative of $\mathbf{M}_\nu$ given in Table IV features a massless neutrino, since it contains a full column of zeros.

We emphasise that these maximally-restrictive texture pairs cannot be implemented in the SM by imposing Abelian symmetries [20]. Hence, in the next section we will address the question of whether (or which of) the texture pairs in Table II can be implemented in the 2HDM with Abelian flavour symmetries.

[2] Note that our matrices $\mathbf{M}_\nu$ correspond to $\mathbf{M}^\dagger_\nu$ in Ref. [22], since in this reference the RH neutrino fields appear on the left in the Dirac neutrino mass term.

$4_1^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$4_{17}^\nu : \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$	$5_1^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & \times & \times \end{pmatrix}$
$5_4^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ 0 & \times & \times \end{pmatrix}$	$5_5^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$5_6^\nu : \begin{pmatrix} 0 & \times & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}$
$5_8^\nu : \begin{pmatrix} 0 & \times & \times \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$	$6_1^\nu : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$6_2^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & 0 \\ 0 & \times & \times \end{pmatrix}$
$6_3^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$	$6_4^\nu : \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}$	$6_5^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$
$6_6^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ 0 & 0 & 0 \end{pmatrix}$	$6_7^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ 0 & \times & 0 \end{pmatrix}$	$6_8^\nu : \begin{pmatrix} 0 & \times & \times \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$
$6_9^\nu : \begin{pmatrix} 0 & \times & \times \\ 0 & 0 & \times \\ 0 & 0 & 0 \end{pmatrix}$	$7_1^\nu : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$	$7_3^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}$

TABLE IV. Representative textures of the $\mathbf{M}_\nu$ equivalence classes appearing in Table II.

V. ABELIAN-SYMMETRY REALIZATION OF COMPATIBLE TEXTURES

Having identified the maximally-restrictive texture pairs that are compatible with data, next we aim at ascertaining the pairs in Table II that can be realised in the 2HDM by imposing continuous or discrete Abelian symmetries. At the same time, we wish to identify the corresponding transformation properties of the various fields according to Eq. (9). Keeping this in mind, two methods shall be employed, namely the canonical and SNF methods, which are detailed in the thesis. See also Refs. [21, 23] and Ref. [24], respectively.

We first apply the canonical method to find which of the $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ pairs given in Table II cannot be implemented with Abelian flavour symmetries in the 2HDM. From this analysis, detailed in the thesis, we conclude that 23 of the 28 maximally-restrictive pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ appearing in Table II cannot be realised through Abelian symmetries in the present framework. We then determine the decompositions into Yukawa matrices $\mathbf{Y}^{\ell,\nu}_{1,2}$, of the textures comprising the 5 remaining pairs $(4_3^\ell, 6^\nu_{1,3,7,9})$ and $(5_1^\ell, 5_8^\nu)$, which can be realised by an Abelian symmetry in the 2HDM. Here, we take advantage of the results of Ref. [23] which are based on the canonical method. We then generate all the possible resulting pairs of $(4_3^\ell, 6^\nu_{1,3,7,9})$ and $(5_1^\ell, 5_8^\nu)$ decompositions and apply the SNF method to determine the minimal rephasing symmetry under which the allowed Yukawa interactions are invariant, for each case. Relying again on the canonical method, we check whether this rephasing symmetry can reproduce all the required texture zeros in $\mathbf{Y}^{\ell,\nu}_{1,2}$, i.e. can realise the respective mass matrix texture pair decomposition. We find that the 5 mass matrix pairs have only one decomposition which can be implemented, the Yukawa interaction ordering is for $4^\ell_{3,\mathrm{I}}, 5^\ell_{1,\mathrm{II}}$ $a_i(b_i)$ belong to $\mathbf{Y}_1^\ell(\mathbf{Y}_2^\ell)$, for $6^\nu_{1,\mathrm{I}}, 6^\nu_{3,\mathrm{I}}$ $y(x_i)$ belong to $\mathbf{Y}_1^\nu(\mathbf{Y}_2^\nu)$ and for $6^\nu_{7,\mathrm{I}}, 6^\nu_{9,\mathrm{I}}, 5^\nu_{8,\mathrm{I}}$ $x_i(y_i)$ belong to $\mathbf{Y}_1^\ell(\mathbf{Y}_2^\ell)$ (see Table V). For these 5 decompositions the texture structure can be achieved by imposing a single U(1) symmetry (see Table VI).

$\mathbf{M}_\ell$	$\mathbf{M}_\nu$
$4^\ell_{3,\mathrm{I}} : \begin{pmatrix} 0 & 0 & a_1 \\ 0 & a_2 & b_1 \\ a_3 & b_2 & 0 \end{pmatrix}$	$6^\nu_{1,\mathrm{I}} : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_2 \\ 0 & x_1 & ye^{i\alpha} \end{pmatrix}$
	$6^\nu_{3,\mathrm{I}} : \begin{pmatrix} 0 & 0 & x_1 \\ 0 & 0 & ye^{i\alpha} \\ 0 & x_2 & 0 \end{pmatrix}$
	$6^\nu_{7,\mathrm{I}} : \begin{pmatrix} 0 & 0 & x_2 \\ 0 & ye^{i\alpha} & 0 \\ 0 & x_1 & 0 \end{pmatrix}$
	$6^\nu_{9,\mathrm{I}} : \begin{pmatrix} 0 & x_1 & ye^{i\alpha} \\ 0 & 0 & x_2 \\ 0 & 0 & 0 \end{pmatrix}$
$5^\ell_{1,\mathrm{II}} : \begin{pmatrix} 0 & 0 & a_1 \\ 0 & b_1 & 0 \\ a_2 & 0 & b_2 \end{pmatrix}$	$5^\nu_{8,\mathrm{I}} : \begin{pmatrix} 0 & y_1 & x_1 \\ 0 & 0 & y_2 \\ 0 & x_2e^{i\alpha} & 0 \end{pmatrix}$

TABLE V. Parameter conventions for the maximally restrictive pairs $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ which are simultaneously compatible with charged-lepton and neutrino data and realisable in the 2HDM framework through a U(1) flavour symmetry. We have used the field-rephasing freedom to eliminate all unphysical phases, placing the only irremovable phase α in $\mathbf{M}_\nu$.

$(\mathbf{M}_\ell, \mathbf{M}_\nu)$	U(1): $(\alpha_1, \alpha_2, \alpha_3)$	$(\beta_1, \beta_2, \beta_3)$	$(\gamma_1, \gamma_2, \gamma_3)$
$(4^\ell_{3,\mathrm{I}}, 6^\nu_{1,\mathrm{I}})$	$(0, \theta, 2\theta)$	$(2\theta, \theta, 0)$	$(\eta, 3\theta, 2\theta)$
$(4^\ell_{3,\mathrm{I}}, 6^\nu_{3,\mathrm{I}})$	$(0, \theta, 2\theta)$	$(2\theta, \theta, 0)$	$(\eta, 3\theta, \theta)$
$(4^\ell_{3,\mathrm{I}}, 6^\nu_{7,\mathrm{I}})$	$(0, \theta, 2\theta)$	$(2\theta, \theta, 0)$	$(\eta, 2\theta, 0)$
$(4^\ell_{3,\mathrm{I}}, 6^\nu_{9,\mathrm{I}})$	$(0, \theta, 2\theta)$	$(2\theta, \theta, 0)$	$(\eta, 0, \theta)$
$(5^\ell_{1,\mathrm{II}}, 5^\nu_{8,\mathrm{I}})$	$(0, -\theta, \theta)$	$(\theta, -2\theta, 0)$	$(\zeta, \theta, 0)$

TABLE VI. Implementation of the texture pairs from Table V in a 2HDM. With $\theta \in \mathbb{R}$, $\eta \neq \{0,\ \theta,\ 2\theta,\ 3\theta\}$ and $\zeta \neq \{-\theta,\ 0,\ \theta,\ 2\theta\}$. The continuous phases $(\alpha_i, \beta_i, \gamma_i)$ correspond to Eq. (13) with $(\theta_1, \theta_2) = (0, \theta)$.

VI. PHENOMENOLOGY

In the following, we focus on the phenomenology of the texture pairs in Table V in what concerns leptonic CP violation [25], rare LFV decays and lepton universality.

A. Leptonic CP violation

Although, in general, all elements of the Yukawa matrices $\mathbf{Y}_a^{\ell,\nu}$ in Eq. (1) are complex, some phases have no physical significance and, thus, can be removed by rephasing the fermion fields as $\psi_j \to e^{i\varphi_j}\psi_j$. For the pairs of matrices in Table V, all elements of $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$ (or $\mathbf{Y}_{1,2}^{\ell,\nu}$) can be made real and positive, except one. The single phase α which remains will be necessarily correlated with the Dirac CP-violating phase δ in Eq. (8). Our convention for the position of α is given in Table V, where all parameters a_i, b_i, x_i and y_i are real and positive.

Given that for all $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ pairs there are nine real parameters in total, four of them remain undefined after ensuring a mass spectrum that reproduces the observed charged-lepton masses $m_{e,\mu,\tau}$ and neutrino mass-squared differences $\Delta m^2_{21,31}$ (since one neutrino is massless, the two Dirac neutrino masses can be written in terms of $\Delta m^2_{21,31}$). Focusing on the $(5^\ell_{1,\mathrm{II}}, 5^\nu_{8,\mathrm{I}})$ case, which we will show is the most interesting one, we are left with one (three) free parameters in $5^\ell_{1,\mathrm{II}}$ $(5^\nu_{8,\mathrm{I}})$. For the texture $5^\ell_{1,\mathrm{II}}$ we choose to write a_1, $b_{1,2}$ in terms of the free a_2 and $m_{e,\mu,\tau}$. As for $5^\nu_{8,\mathrm{I}}$, we express x_1, y_1 in terms of the also free x_2, y_2 and $\Delta m^2_{21,31}$. These defining relations, and equivalent ones for $4^\ell_{3,\mathrm{I}}$ and $6^\nu_{i,\mathrm{I}}$, can be found in the thesis. For these texture pairs we then perform a scan of the free parameters in their validity ranges and, for each input set, $\mathbf{H}_\ell$ and $\mathbf{H}_\nu$ are computed considering their definition as in Eq. (5). After diagonalizing these two matrices, $\mathbf{U}$ gets determined by Eq. (7), and θ_{ij} and δ appearing in Eq. (8) can be extracted in terms of the parameters of $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$. Demanding agreement with the 3σ ranges given in Table I, we plot δ as a function of α. Our results are illustrated in Fig. 1, where both the NH and IH mass spectra were considered. Identical plots for the $(4^\ell_{3,\mathrm{I}}, 6^\nu_{i,\mathrm{I}})$ pairs can be found in the thesis. The first aspect to remark is that, in the matrix 5^ℓ_1, one of the charged-lepton states is decoupled. Notice that the Hermitian matrix $\mathbf{H}_\ell = \mathbf{M}_\ell \mathbf{M}_\ell^\dagger$ and its diagonalizing unitary transformation $\mathbf{U}_L^\ell$ is given by

$$\mathbf{H}_\ell = \begin{pmatrix} a_1^2 & 0 & a_1 b_2 \\ 0 & b_1^2 & 0 \\ a_1 b_2 & 0 & a_2^2 + b_2^2 \end{pmatrix}, \; \mathbf{U}'_L = \begin{pmatrix} c_L & 0 & s_L \\ 0 & 1 & 0 \\ -s_L & 0 & c_L \end{pmatrix}, \quad (20)$$

where $c_L \equiv \cos\theta_L$ and $s_L \equiv \sin\theta_L$. As shown in the thesis, $\tan(2\theta_L)$ can be written in terms of a_2 and m_{ℓ_2,ℓ_3}, which correspond to the masses of the two non-decoupled charged-lepton states. Thus, depending on which charged lepton ℓ_1 is identified as decoupled, three different cases must be considered: $\ell_1 = e, \mu, \tau \;\to\; 5^\ell_1 \equiv 5_1^{e,\mu,\tau}$. This explains the notation used in the plots of Fig. 1. Taking into account that the unitary matrix $\mathbf{U}_L^\ell$ must be such that Eq. (6) is verified with the correct charged-lepton mass ordering, we have

$$5^e_1 : \mathbf{U}^\ell_L = \mathbf{U}'_L \mathcal{P}_{12}\,, \quad (21)$$

$$5^\mu_1 : \mathbf{U}^\ell_L = \mathbf{U}'_L\,,\; 5^\tau_1 :\; \mathbf{U}^\ell_L = \mathbf{U}'_L \mathcal{P}_{23}\,. \quad (22)$$

The unitary matrix which diagonalizes $\mathbf{H}_\nu$ as in Eq. (6), when $\mathbf{M}_\nu$ is of the type 5^ν_8, can be found in the thesis for both NH and IH. Taking it into account, we have shown, after some algebra, that the charged-lepton rotation set by θ_L given in Eq. (20) is crucial to obtain compatibility with the measured neutrino mixing angles. Namely, considering $\mathbf{U} = \mathbf{U}^\nu_L$, we find that it is not possible to achieve, regardless of the charged-lepton which is decoupled, θ_{23} in the 3σ range of Table I. Furthermore, neglecting the mixing coming from $\mathbf{U}^\ell_L$ would also lead to $\delta = 0$ since α could be removed by rephasing the LH and RH charged-lepton fields. Moreover, it can be shown that the Jarlskog invariant $\mathcal{J}_{CP}$ [26], which signals Dirac-type CP violation, obeys

$$\mathcal{J}_{CP} = \mathrm{Im}[\mathbf{U}_{11}\mathbf{U}_{22}\mathbf{U}^*_{12}\mathbf{U}^*_{21}] \propto \sin(2\theta_L)\sin\alpha\,, \quad (23)$$

confirming the fact that, for CP violation to occur in the lepton sector, $\theta_L \neq n\pi/2$ and $\alpha \neq n\pi$ (n is integer) must hold. Further detail regarding this analysis can be found in the thesis.

We now focus on the $(5^e_1, 5^\nu_8)$ pair and obtain relations among the parameters in $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$ (a_2, x_2, y_2 and α) and the three mixing angles θ_{ij} and the CP phase δ. From Eqs. (7), (20), (21), together with the definition of $\mathbf{U}^\nu_L$ for $5^\nu_{8,I}$, the lepton mixing matrix $\mathbf{U}$ is computed and the mixing angles and the phase δ are extracted. Notice that, for the case $(5^e_1, 5^\nu_8)$, one has $\mathbf{U}_{1j} = (\mathbf{U}^\nu_L)_{2j}$. Therefore, given the parametrization (8), x_2 and y_2 in $\mathbf{M}_\nu$ depend only on θ_{12} and θ_{13} through the relations

$$\begin{aligned} \mathrm{NH}: x_2^2 &= \frac{\Delta m^2_{21} c^2_{12}(r c^2_{13} s^2_{12} + s^2_{13})}{r s^2_{12}(r - 2s^2_{13} - r c^2_{13} s^2_{12}) + s^2_{13}}\,, \\ y_2^2 &= \Delta m^2_{31}(s^2_{13} + r c^2_{13} s^2_{12})\,, \\ \mathrm{IH}: x_2^2 &= \frac{\Delta m^2_{31}(1+r)(1+r s^2_{12}) s^2_{13}}{r s^2_{12}(r - 2s^2_{13} - r c^2_{13} s^2_{12}) + s^2_{13}}\,, \\ y_2^2 &= \Delta m^2_{31} c^2_{13}(1 + r s^2_{12})\,. \end{aligned} \quad (24)$$

It now remains to express θ_L (or a_2) appearing in Eq. (20) and the phase α in terms of the measurable neutrino parameters. Including the charged-lepton corrections to the mixing (i.e. $\mathbf{U} = \mathbf{U}^{\ell\,\dagger}_L \mathbf{U}^\nu_L$), we can write $\tan^2\theta_{23}$ in terms of $\theta_L, \alpha, \theta_{12}, \theta_{13}$ and, subsequently, determine θ_L through the approximate relation

$$\tan\theta_L \simeq \cot\theta_{23} \mp \frac{r c_\alpha \sin(2\theta_{12})}{2 s_{13} s^2_{23}}\,, \quad (25)$$

where the $-$ $(+)$ sign corresponds to the NH (IH) case and $r \equiv \Delta m^2_{21}/\Delta m^2_{31} \simeq 0.03$, according to the data given in Table I. The above relation provides a very good approximation for the behaviour of the charged-lepton mixing angle θ_L in terms of θ_{ij}, r and α. Considering the

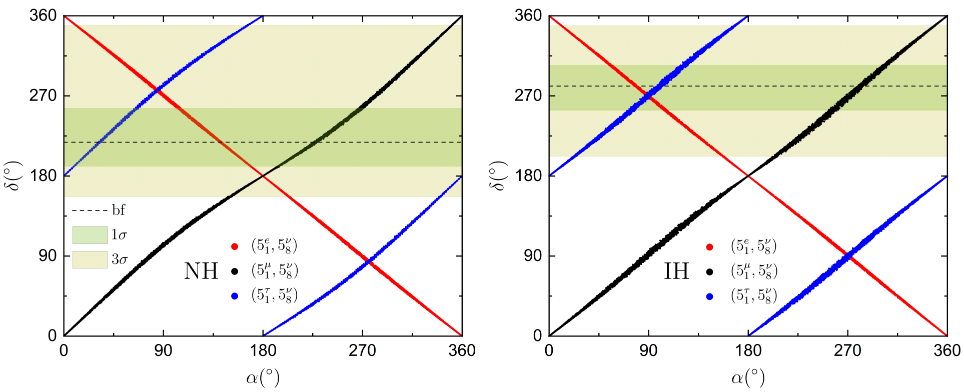

FIG. 1. Correlation between the phase α appearing in $\mathbf{M}_\nu$ and the Dirac CP-violating phase δ of the lepton mixing matrix $\mathbf{U}$ for the $(5_1^{e,\mu,\tau}, 5_8^\nu)$ texture pairs. The results shown in the left (right) panel correspond to a NH (IH) neutrino mass spectrum. In all points, the mixing angles θ_{ij} lie within the 3σ ranges given in Table I.

definitions of $\tan(2\theta_L)$ and a_1^2, $b_{1,2}^2$ in terms of a_2 (see the thesis), together with Eq. (25), one can determine the parameters $a_{1,2}$ and $b_{1,2}$ in terms of m_e, m_μ, m_τ and θ_{23}, namely,

$$\begin{aligned} a_2^2 &\simeq \frac{2m_\mu^2 m_\tau^2}{m_\mu^2 + m_\tau^2 \pm (m_\tau^2 - m_\mu^2)\cos(2\theta_{23})}\,, \\ a_1^2 &\simeq \frac{1}{2}\left[m_\mu^2 + m_\tau^2 \pm (m_\tau^2 - m_\mu^2)\cos(2\theta_{23})\right]\,, \\ b_1^2 &= m_e^2\,,\; b_2^2 \simeq \frac{(m_\tau^2 - m_\mu^2)^2 \sin^2(2\theta_{23})}{2\left[m_\mu^2 + m_\tau^2 \pm (m_\tau^2 - m_\mu^2)\cos(2\theta_{23})\right]}\,. \end{aligned} \tag{26}$$

As shown in the thesis, in order to relate δ with α, we notice that $\arg(\mathbf{U}_{23})$ and $\arg(\mathbf{U}_{33})$ can be written, approximately, in terms of $\theta_L, \alpha, \theta_{12}, \theta_{13}$, for both NH and IH, in such a way that implies

$$\arg(\mathbf{U}_{23}) \simeq \arg(\mathbf{U}_{33}) \simeq -\alpha\,, \tag{27}$$

from which, after performing some rephasing transformations to bring $\mathbf{U}$ to the form given in Eq. (8), we obtain

$$\delta = \arg(\mathbf{U}_{23}) \simeq -\alpha\,, \tag{28}$$

confirming the results plotted in Fig. 1. Following the same procedure for the 5_1^μ (5_1^τ) we obtain $\delta \simeq \alpha$ ($\delta \simeq \pi + \alpha$), which also agrees with the numerical output shown in Fig. 1. In conclusion, all parameters in the mass matrices $\mathbf{M}_\ell$ and $\mathbf{M}_\nu$ can be determined in terms of the charged-lepton and neutrino masses and mixing angles through Eqs. (24)-(26) and (28).

B. Lepton universality and rare LFV decays

In the 2HDM, Yukawa interactions encode flavour-changing scalar processes which may induce FCNCs at the tree and loop levels. Therefore, the viable maximally-restrictive textures previously obtained (cf. Table V) should be confronted with the current experimental bounds on such processes. Namely, the constraints on universality in purely leptonic decays, as well as the lepton-flavour-violating decays $\ell_\alpha^- \to \ell_\beta^- \ell_\gamma^+ \ell_\delta^-$, and $\ell_\alpha \to \ell_\beta \gamma$, should be considered.[3] In order to discuss physical processes, we must precisely define the framework which is considered. As discussed in the thesis, since we are considering scenarios with a U(1) symmetry under which one of the Higgs doublets is charged, there is no CP violation in the scalar potential and, thus, no mixing between CP-even and CP-odd scalars. It is convenient to rotate (Φ_1, Φ_2) to the Higgs basis (H_1, H_2) such that one can write [28]

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\,G^+ \\ v + H^0 + i\,G^0 \end{pmatrix},\; H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\,H^+ \\ R + i\,I \end{pmatrix}, \tag{29}$$

with $\langle H_2 \rangle = 0$. The neutral (charged) Goldstone boson is denoted G^0 (G^+), while I is the U(1) Goldstone boson, which is massless in the exact U(1) symmetric limit. In order to avoid this massless particle, a soft U(1) symmetry breaking term of the type $m_{12}^2 \Phi_1^\dagger \Phi_2$ can be included in the scalar potential originating a mass $m_I^2 \propto m_{12}^2$ for the decoupled CP-odd scalar. We will assume there is no mixing between R and H^0 (i.e. the alignment limit), such that these can be identified with the heavy (H) and light (h) neutral CP-even mass states of the 2HDM, respectively. We also consider that H^0 corresponds to the observed Higgs boson [29, 30] and thus $m_{H^0} \equiv m_h \simeq 125$ GeV. As no mixing between the neutral scalars is allowed, m_R, m_I and the mass of the charged scalar, $m_{H^\pm}$, can be taken to be independent parameters (cf. thesis for more detail).

In the conditions described above, the relevant scalar-fermion interactions can be read off from the Lagrangian (1) which, after appropriate transformations,

[3] In Refs. [12, 27], the implications of these processes have been analysed in alternative 2HDM realisations.

takes the form

$$-\mathcal{L} = \frac{1}{v}\overline{e_L}\,\left[\mathbf{D}_\ell(v+h) - \mathbf{N}_e R - i\mathbf{N}_e I\right]\, e_R - \frac{\sqrt{2}}{v}\overline{\nu_L}\,\mathbf{U}^\dagger \mathbf{N}_e\, e_R\, H^+ + \text{H.c.}\,, \tag{30}$$

with the FCNC couplings

$$\mathbf{N}_e = \mathbf{U}_L^{\ell\dagger}\mathbf{N}_e^0\mathbf{U}_R^\ell\,, \quad \mathbf{N}_e^0 = \frac{v}{\sqrt{2}}(\mathbf{Y}_1^\ell s_\beta - \mathbf{Y}_2^\ell c_\beta)\,, \tag{31}$$

where $s_\beta, c_\beta = \sin\beta, \cos\beta$. In Eq. (30) all fields are mass eigenstates.

Lepton universality tests aim at probing the SM prediction that all leptons couple with the same strength to the charged weak current. A relevant quantity to test universality in purely leptonic τ decays is

$$\left|\frac{g_\mu}{g_e}\right|^2 \equiv \frac{\text{Br}\,(\tau\to\mu\nu\bar{\nu})}{\text{Br}\,(\tau\to e\nu\bar{\nu})}\frac{f(x_{e\tau}^2)}{f(x_{\mu\tau}^2)}\,, \tag{32}$$

where $x_{\alpha\beta} \equiv m_\alpha/m_\beta$ and $f(x)$ is a phase space function. The explicit form of $f(x)$ and the decay width $\Gamma(\ell_\alpha \to \ell_\beta\,\nu\,\bar{\nu})$, in the presence of scalar and vector interactions, can be found in Ref. [27] and the thesis. It is important, however, to notice that this decay width depends on coefficients $g^S_{RR,\alpha\beta}$, defined through

$$\left|g^S_{RR,\alpha\beta}\right|^2 \equiv \sum_{i,j=1}^{3} |\mathbf{U}_{\alpha i}|^2|\mathbf{U}_{\beta j}|^2|g_{i\alpha j\beta}|^2\,. \tag{33}$$

Current experimental constraints yield [31]

$$|g_\mu/g_e| - 1 = 0.0019 \pm 0.0014\,, \tag{34}$$

and require, at 95% CL [7],

$$\left|g^S_{RR,\alpha\beta}\right| < \overbrace{0.035}^{\alpha\beta=\mu e},\ \overbrace{0.70}^{\alpha\beta=\tau e},\ \overbrace{0.72}^{\alpha\beta=\tau\mu}\ . \tag{35}$$

Lepton-flavour violating decays $\ell_\alpha^- \to \ell_\beta^-\,\ell_\gamma^+\,\ell_\delta^-$ are another source of experimental constraint for our models. In the present scenario, these are mediated by the neutral scalars R and I, at tree level. The branching ratio (BR) for such processes depends on coefficients $g_{LL}^{\alpha\beta,\gamma\delta}, g_{RR}^{\alpha\beta,\gamma\delta}, g_{LR}^{\alpha\beta,\gamma\delta}, g_{RL}^{\alpha\beta,\gamma\delta}$, which obey

$$\begin{aligned} g_{LL}^{\alpha\beta,\gamma\delta}\,, g_{RR}^{\alpha\beta,\gamma\delta} &\propto \left(\frac{1}{m_R^2} - \frac{1}{m_I^2}\right)\,,\\ g_{RL}^{\alpha\beta,\gamma\delta}\,, g_{LR}^{\alpha\beta,\gamma\delta} &\propto \left(\frac{1}{m_R^2} + \frac{1}{m_I^2}\right)\,. \end{aligned} \tag{36}$$

The explicit dependence of the BR on these coefficients can be found in Ref. [27] and the thesis. We consider the μ and τ total decay widths [7]

$$\begin{aligned} \Gamma_\mu &\simeq 3.0\times 10^{-19}\ \text{GeV}\,,\\ \Gamma_\tau &\simeq 2.3\times 10^{-12}\ \text{GeV}\,. \end{aligned} \tag{37}$$

and the current experimental upper limits on the branching ratios of the 3-body LFV decays [7]

$$\begin{aligned} &\text{Br}(\tau^-\to e^-e^+e^-) < 2.7\times 10^{-8}\,,\\ &\text{Br}(\tau^-\to \mu^-\mu^+\mu^-) < 2.1\times 10^{-8}\,,\\ &\text{Br}(\tau^-\to e^-\mu^+e^-) < 1.5\times 10^{-8}\,,\\ &\text{Br}(\tau^-\to e^-e^+\mu^-) < 1.8\times 10^{-8}\,,\\ &\text{Br}(\tau^-\to \mu^-e^+\mu^-) < 1.7\times 10^{-8}\,,\\ &\text{Br}(\tau^-\to \mu^-\mu^+e^-) < 2.7\times 10^{-8}\,,\\ &\text{Br}(\mu^-\to e^-e^+e^-) < 1.0\times 10^{-12}\,, \end{aligned} \tag{38}$$

at 90% CL.

Finally, radiative lepton-flavour violating process $\ell_\alpha \to \ell_\beta\,\gamma$ should also be considered. Neglecting contributions proportional to the neutrino masses and sub-leading terms in $m_\ell^2/m_{R,I}^2$, the corresponding decay width is given, up to one-loop level, by [27]

$$\Gamma(\ell_\alpha \to \ell_\beta\,\gamma) = \frac{\alpha_e\, m_\alpha^5 G_\text{F}^2}{128\pi^4}\left(|\mathcal{A}_L|^2 + |\mathcal{A}_R|^2\right)\,, \tag{39}$$

where $\alpha_e = e^2/(4\pi)$ and the explicit form of the amplitudes $\mathcal{A}_{L,R}$, in the present framework, can be found in Ref. [27] and the thesis. However, it is worth mentioning that, just as in the case of $\Gamma(\ell_\alpha^- \to \ell_\beta^-\,\ell_\gamma^+\,\ell_\delta^-)$, $\mathcal{A}_{L,R}$ depends partly on terms $\propto 1/m_R^2 - 1/m_I^2$. Current experimental upper bounds, at 90% CL, are [7]

$$\begin{aligned} &\text{Br}(\mu\to e\gamma) < 4.2\times 10^{-13},\\ &\text{Br}(\tau\to e\gamma) < 3.3\times 10^{-8},\\ &\text{Br}(\tau\to \mu\gamma) < 4.4\times 10^{-8}. \end{aligned} \tag{40}$$

We now aim at studying the compatibility of the texture pairs given in Table V with the constraints discussed above. Simultaneously to the analysis performed in the previous section, we randomly vary $\tan\beta$ in the range 0.01 to 100 (these values ensure that the Yukawa couplings are always $\lesssim 1$), the charged-Higgs scalar masses $m_{H^\pm} \gtrsim 80$ GeV [32], and the neutral scalar masses $m_{R,I} \gtrsim 100$ GeV [7]. We limit our search to cases where the $m_{H^\pm} \lesssim 1$ TeV and $m_{R,I} \lesssim 10$ TeV. For each input parameter set compatible with neutrino data, we compute $|g_\mu/g_e| - 1$, $g^S_{RR,\alpha\beta}$, and the BRs of all LFV 3-body and radiative charged-lepton decays. In all cases, we keep only those points obeying Eqs. (35), (38) and (40). As for $|g_\mu/g_e|$, we demand $|g_\mu/g_e| - 1 \geq 10^{-4}$, keeping in mind the result (34).

In Fig. 2 we show the results for the $(5_1^e, 5_8^\nu)$ texture pair. We find that the deviation from universality is in agreement with Eq. (34) for $80\,\text{GeV} \lesssim m_{H^\pm} \lesssim 400$ GeV and $\tan\beta \lesssim 0.03$ or $\tan\beta \gtrsim 20$, for both NH and IH. Notice that for large (small) $\tan\beta$ the Yukawa couplings in $\mathbf{Y}_1^\ell$ ($\mathbf{Y}_2^\ell$) are enhanced, leading to an enhancement of $|g_\mu/g_e| - 1$. These results were also obtained for the $(5_1^{\mu,\tau}, 5_8^\nu)$ cases which are disfavoured by the $|g_\mu/g_e| - 1$ constraint (34) (indicated by the horizontal grey bands

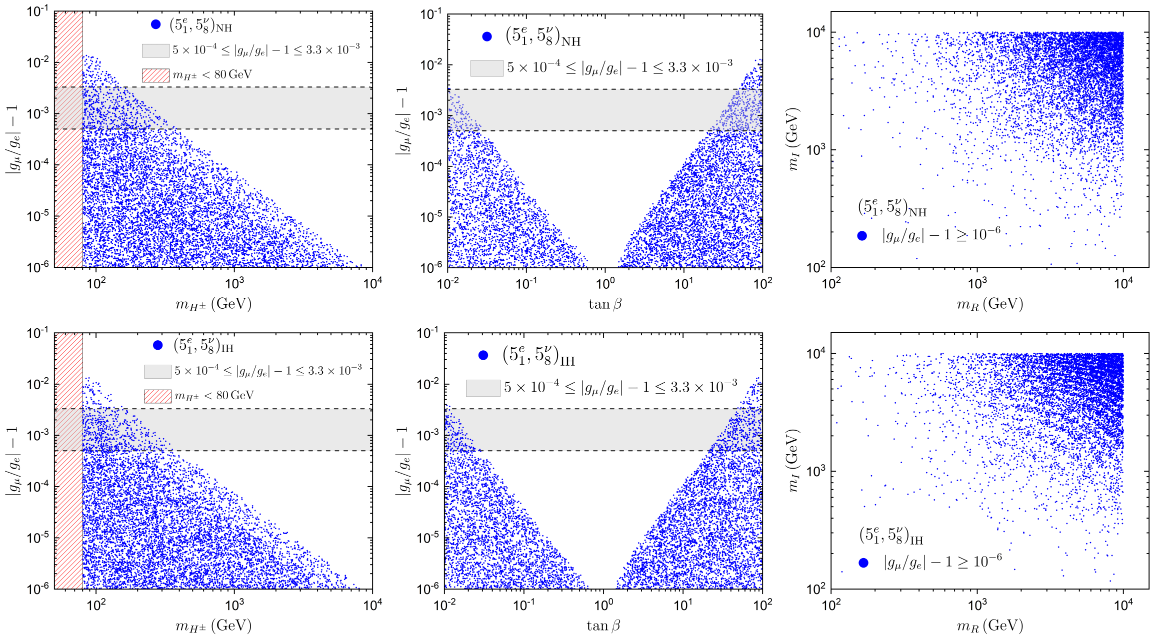

FIG. 2. Results for the $(5_1^e, 5_8^\nu)$ texture pair. In the left (middle) columns we plot $|g_\mu/g_e| - 1$ as a function of $m_{H^\pm}$ $(\tan\beta)$. The horizontal grey bands correspond to the constraint (34). In the right column, the same points as in the corresponding $|g_\mu/g_e| - 1$ plots are shown in the (m_R, m_I)-plane. All points obey the constraints (38) and (40), and the mixing angles θ_{ij} lie within the 3σ ranges given in Table I for NH and IH.

in the plots). On the other hand, the $|g_\mu/g_e| - 1 \ge 10^{-4}$ bound which was considered may be less important then the well-determined constraints on the $\left|g^S_{RR,\alpha\beta}\right|$ coefficients, in (35). As such, relaxing such a strict constraint would render the $(5_1^{\mu,\tau}, 5_8^\nu)$ cases equally viable to the $(5_1^e, 5_8^\nu)$ one.

Similar results were also obtained for the $(4_3^\ell, 6_k^\nu)$ texture pairs given in Table V. These are partly identical to that of the $(5_1^e, 5_8^\nu)$ case i.e., in general, the deviation from universality is in agreement with Eq. (34) for a small and a large $\tan\beta$ region and small values of $m_{H^\pm}$, for both NH and IH. The main difference between the results for the $(4_3^\ell, 6_k^\nu)$ pairs and the ones in Fig. 2 lies in the (m_R, m_I) plots. While for the $(5_1^\ell, 5_8^\nu)$ texture pair all constraints from the LFV decays are verified for non-correlated $m_{R,I}$ masses, for the texture sets $(4_3^\ell, 6_k^\nu)$ a mass tuning $m_R/m_I \simeq 1$ is needed to pass these constraints. This fact becomes easy to understand once we notice that the matrix $\mathbf{N}_e$, defined in Eq. (31), has texture zeros in the 5_1^ℓ case and is full for 4_3^ℓ (see thesis). Ultimately, for a certain value of $\tan\beta$, the non-zero entries could be expressed in terms of the charged-lepton and neutrino masses and lepton mixing angles, as illustrated for the case of the 5_1^e texture discussed in the previous section.

Taking into account the dependence of the BRs for the $\ell_\alpha^- \to \ell_\beta^- \, \ell_\gamma^+ \, \ell_\delta^-$ and $\ell_\alpha \to \ell_\beta \, \gamma$ decays on the FCNC couplings $(\mathbf{N}_e)_{\alpha\beta}$, we can conclude that most of these channels are forbidden at the one loop level for the 5_1^ℓ textures (see Table VII). This is due to the coupling structure imposed by the U(1) flavour symmetry which, in the case of charged leptons, only allows mixing between two flavours. Thus, for 5_1^ℓ the constraints from LFV decays are respected without requiring any special relation among the scalar masses $m_{R,I}$, as can be seen from the plots in Fig. 2. The natural suppression of LFV decays does not however occur when $\mathbf{M}_\ell \sim 4_3^\ell$. As said, in these cases the couplings $\mathbf{N}_e$ do not exhibit any decoupling behaviour and, thus, the decay rates are not naturally suppressed. In the particular case of $\mu \to e\gamma$, the terms enhanced by m_τ/m_μ are potentially large and the experimental bound on that decay is respected only when there is a cancellation between the two terms proportional to m_τ/m_μ in $\mathcal{A}_R$, i.e. when $m_R \simeq m_I$. Notice that, in the 5_1^ℓ case, those terms were absent since $(\mathbf{N}_e)_{\mu\tau}(\mathbf{N}_e)_{\tau e} = 0$. For illustration, we show in Fig. 3 the dependence of $\mathrm{Br}(\mu \to e\gamma)$ on the mass ratio m_I/m_R for the texture pair $(4_3^\ell, 6_7^\nu)$, which confirms the fact that quasi-degenerate $m_{R,I}$ masses are required to respect the MEG $\mu \to e\gamma$ bound.

Decay	5_1^e	5_1^μ	5_1^τ
$\ell_\alpha \to \ell_\beta\gamma$	(τ,μ)	(τ,e)	(μ,e)
$\ell_\alpha^- \to \ell_\beta^- \, \ell_\gamma^+ \, \ell_\delta^-$	$(\tau,\mu\mu\mu)$	(τ,eee)	(μ,eee)
	$(\tau,ee\mu)$	$(\tau,\mu\mu e)$	

TABLE VII. Allowed $\ell_\alpha \to \ell_\beta\gamma$ and $\ell_\alpha \to \ell_\beta\gamma$ for $5_1^{e,\mu,\tau}$, indicated in each case by particle flavour indices (α,β) and $(\alpha,\beta\gamma\delta)$.

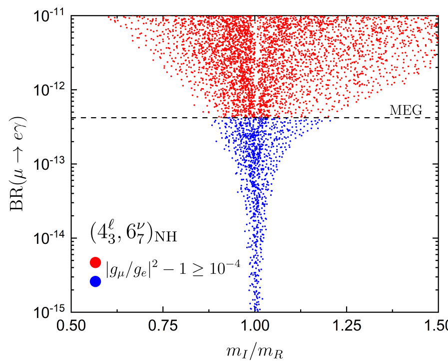

FIG. 3. Dependence of $\mathrm{Br}(\mu \to e\gamma)$ on m_I/m_R for the $(4_3^\ell, 6_7^\nu)$ texture pair. For the other $(4_3^\ell, 6_k^\nu)$ pairs in Table V the results are similar. In all points the mixing angles θ_{ij} lie within the 3σ ranges of Table I and $|g_\mu/g_e| - 1 \geq 10^{-4}$. The red points are excluded by the $\mu \to e\gamma$ MEG bound in (37).

VII. CONCLUDING REMARKS

We have analysed massive Dirac neutrinos in an extension of the SM known as the 2HDM, focusing on lepton mass matrices with zero entries originating from Abelian flavour symmetries. Special emphasis was given to the restrictiveness of these symmetries, in the sense that the minimal number of flavour parameters required to describe charged-lepton masses and neutrino oscillation data is achieved. We concluded that, in the SM extended with RH neutrinos, U(1) or $\mathbb{Z}_N$-motivated texture zeros are incompatible with data since they lead to massless charged-leptons and/or vanishing lepton mixing angles (excluded by the data). This motivated us to explore extensions of the SM such as the 2HDM. We demonstrated that this model serves as a minimal scenario for Abelian-symmetry motivated texture zeros in lepton mass matrices, while maintaining compatibility with observed lepton masses and mixing parameters. We find that, in this framework, it is even possible to implement, with a U(1) flavour symmetry, maximally-restricted $(\mathbf{M}_\ell, \mathbf{M}_\nu)$ pairs where the number of relevant flavour parameters in the lepton sector is the same as the number of observables.

For these specific pairs, we explore leptonic CP violation and show that a relation between the only complex phase α in the Yukawa interactions and the Dirac CP-violating phase δ in $\mathbf{U}$ can be obtained, both numerically and analytically. To further validate these texture pairs, we have confronted them with the constraints arising from lepton universality in τ decays as well as two and three-body rare LFV decays. Despite imposing a strict bound on the deviation from universality, we find that the pair $(5_1^e, 5_8^\nu)$ is consistent with these constraints due to the particular Yukawa interactions allowed by the U(1) flavour symmetry which lead to a natural suppression of specific LFV channels. If the strict universality bound was relaxed all pairs analysed would be viable.

A natural continuation of this thesis is the extension of our work to the quark sector. Just as in the case of leptons, in the SM, Abelian-symmetry motivated texture zeros in quark mass matrices are not compatible with observations of their masses and mixing. As such, a similar analysis to the one performed here, for quarks in the 2HDM, is due. In this case, besides compatibility with the observed quark masses and mixing, more severe constraints have to be checked, such as those coming from universality tests in τ and meson semileptonic decays, and from $B \to X_s\gamma$ and meson $\mu^+\mu^-$ decays. This analysis on the quark sector is under preparation [33], as well as the generalisation to the case of seesaw-generated Majorana neutrino masses.

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