Improving Emergency Medical Services Through Vehicle Location Optimization

Guilherme Costa Antunes Ferreira dos Santos
Department of Engineering and Management, Instituto Superior Técnico
guilherme.f.dos.santos@tecnico.ulisboa.pt

Abstract
Emergency Medical Services (EMS) are paramount in saving lives and their importance has long been recognized. Since each country organises EMS activities differently, there is no common standard for EMS, making them more challenging to model and optimize. Nevertheless, most EMS employ different vehicles to respond to emergencies with multiple priorities. Among many complex decisions, EMS planners must locate emergency vehicles, directly impacting the medical outcomes of the population. These decisions must account for an existing system already in operation. This feature has been mostly overlooked in previous EMS research. For this reason, a Multi-Objective Dynamic Mixed-Integer Programming model is developed to support decisions concerning the selection of stations, the allocation of vehicles to stations and the assignment of demand to vehicles. In general, the model considers multiple vehicles and call priorities, as well as micro and macro time periods, to determine how the existing system can be gradually improved. Three objectives are considered: coverage, cost and equity. Besides, a hybrid heuristic based on problem decomposition is proposed to streamline model solution for the first objective. Application of the model to the Portuguese EMS, SIEM, suggests that, under the current restrictions, only slight improvements are possible. The model is also explored to study fleet expansion and seasonal vehicle allocation decisions, aligned with current planning practices. Alternative scenarios show that, with the current resources, improvements up to 13.3% in coverage could be attained. Finally, computational tests of the heuristic prove it to be an effective solution procedure for large instances.

Keywords: Emergency Medical Services; Ambulance Location; Optimization; Hybrid Heuristic; Mixed-Integer Programming.

1. INTRODUCTION

Emergency Medical Services (EMS) systems are designed to save lives and are a vital component of pre-hospital medical care. In order to provide quality care, EMS planners must make multiple planning decisions regarding different resources, such as staff, the emergency fleet and emergency station locations.

The Portuguese integrated EMS system (SIEM) is no exception. The SIEM is managed by the National Institute for Medical Emergency (INEM). Its main goal is to provide pre-hospital medical care in continental Portugal, answering more than 1.300.000 calls per year, 90% of which require medical intervention. Despite having to manage such a challenging system, INEM planners must often rely on experience and intuition to make difficult planning decisions under uncertainty, budget constraints and multiple objectives. One of these decisions concerns the long-term positioning of emergency vehicles. Current vehicle positions have been gradually selected as the system was expanded, mostly guided by intuition and experience. Therefore, the implemented solution may not make the best use of available resources. In this context, the use of more sophisticated decision-support methods could lead to improved solutions and enhance the overall quality of care. Limitations of the current methods and need for a scientific approach were recognized in 2010 by the Portuguese Audit Office, who highlight that the SIEM comprises more vehicles than other EU countries with better performance (Tribunal de Contas, 2010).

This context motivates the present research, whose primary goal is to apply Operations Research (OR) techniques to assist INEM's vehicle planning processes and improve the SIEM's performance. Additional goals include characterizing EMS systems; providing a review of the wide literature of optimization models for EMS planning and contributing to the literature by modelling, formulating and implementing an optimization models in a real EMS context.

To meet these goals, the features of the case study lead to the development of an optimization model that takes into account the current system and a dynamic environment, in which the existing system is progressively adjusted. This approach contrasts with most approaches, which ignore the existing system. The model covers three decisions: selecting stations, allocating vehicles to these stations and defining responsibility areas. Additionally, it considers three objectives: coverage, cost and equity. The model is applied to INEM's case study, resulting in a set of recommendations.

Despite being inspired by INEM's case study, the model is applicable to different EMS systems, providing insights into the impact of alternative policies in the system's performance. Additionally, a hybrid heuristic is developed to streamline model solution for the first objective and its computational performance evaluated.

The remainder of this paper is organized as follows. Section 2 presents the case-study. Section 3 presents a literature review of the strategic EMS location literature, while section 4 presents the proposed optimization model and heuristic approach. Section 5 presents key results. Finally, section 6 concludes the paper.

2. CASE-STUDY

The SIEM and INEM

The INEM is the public entity responsible for managing the SIEM. The SIEM comprises several coordinated activities involved in providing pre-hospital medical care, including patient transportation, reception, referral and treatment at the receiving health unit. These activities are performed by multiple entities, namely INEM, firefighters, police officers, the Red Cross, doctors, nurses and hospitals.

There are several stages of SIEM's intervention. Firstly, an emergency is detected and reported by calling 112. Based on the triage procedure, one of four priority levels can be assigned to a call: P1 (emergent), P3 (urgent), P5 (non-urgent) and other priorities. SIEM's activity is driven mostly by P3 emergencies, which account for 70-75% of all calls, while 10-15% are P1 and 7-12% are P5. According to the call's priority, an appropriate vehicle is dispatched. Critical situations require Basic-Life Support (BLS) and Immediate/Advanced Life Support (ILS/ALS), while urgent situations require only BLS. If necessary, the patient is transported to a hospital, after which the vehicle and its crew are released.

The SIEM operates a multi-vehicle system. Some of these vehicles are owned and operated by INEM, while others are co-financed by INEM but operated by a partner. The basis of the system is a network of four types of BLS ambulances - AEM, PEM, RES and NINEM - complemented by motorcycles. To provide differentiated care, two ALS vehicles, VMER and Helicopters, as well as the Immediate Life Support (SIV) ambulance are used. Vehicles owned or operated by firefighters and the Red Cross, namely PEM, RES and NINEM, account for 75% of the emergency fleet.

Vehicle Location Planning

At INEM, vehicle location planning is under the responsibility of the *Gabinete de Planeamento e Controlo de Gestão* (GPCG).

A vehicle base is an infrastructure where an emergency vehicle is stationed while waiting for emergencies. For most vehicles, the Portuguese legislation establishes which facilities are potential base locations. In particular, VMERs and SIVs must be located at emergency departments of NHS hospitals, while PEMs are located in firefighters or the Red Cross facilities. It has been politically established that there should be one PEM in each municipality. AEMs and MEMs must be located in areas where certain types of emergency departments exist. Potential bases for these vehicles include hospitals, health centres, firefighters or police stations. As such INEM does not build dedicated stations.

Vehicle location planning at INEM comprises two processes: long-term and short-term planning. Long-term planning concerns choosing stations and assigning vehicles to those stations for everyday operation. This is an incremental process: whenever a new vehicle is to be added to the fleet, the GPCG helps the board decide where it should be positioned. As such, location planning is not approached holistically, and already positioned vehicles are not changed.

In order to choose an area for a vehicle, the GPCG evaluates demographics and emergency history, as well as existing vehicles. After gauging this balance, the GPCG uses experience to choose where the vehicle is more beneficial. Subsequently, a base within that area is selected. In practice, this is not an easy task, due to the unavailability of suitable locations (according to legislation). Furthermore, to support these decisions, the GPCG uses three indicators: Rescue Time (time between dispatch and arrival); External Dependency (fraction of calls answered by outside vehicles) and Dispatch Time (time between triage and dispatching). Currently, the target of 15 minutes of Rescue Time in urban areas is only met 75% of the time.

Problem Scope Definition

The proposed model is to be applied on P1 and P3 emergencies, and the corresponding response vehicles: SIV, VMER, AEM, PEM, RES and NINEM. Furthermore, Lisbon and Setúbal have been selected as case-study areas since they are challenging urban areas in what concerns emergency vehicle planning. On the one hand, Lisbon is the capital and the mostly populated municipality in the country. On the other hand, Setúbal relies mostly on PEM ambulances and presents a high external dependency and has never been explored in previous studies of the same nature.

3. LITERATURE REVIEW

Location decisions arise both in the public and private sectors and are usually long-term investment decisions with lasting impacts (Daskin, 1995). As such, building location models has been a topic of Operations Research (OR) for several years. Since solving real-sized instances can be challenging, multiple solution approaches have also been proposed (Owen and Daskin, 1998).

This review focuses on discrete models for strategic and tactical planning, thus excluding operational decisions and continuous models.

Early Static Covering Models

Covering models focus on demand coverage: a demand node is covered if it can be reached within the coverage radius. The first covering model is the Location Set Covering Problem (LSCP) (Toregas *et al.*, 1971), which determines the minimum number and location of facilities that guarantee coverage of all nodes. Later, Church and ReVelle (1974) introduce the Maximum Covering Location Problem (MCLP), which maximizes coverage by a limited number of facilities. Both these models were very influential, but they rely on strong assumptions. Therefore, researchers have been extending these formulations to get more accurate representations of EMS systems.

Vehicle Unavailability

When a vehicle is dispatched, its designated regions are no longer covered. To deal with this issue, three alternatives have been explored.

Multiple coverage models seek to ensure that more than one vehicle is capable of covering a demand node. The most influential is the Double Standard Model (DSM), which maximizes back-up coverage, ensuring basic coverage of a fraction of demand and full coverage within a larger radius (Gendreau, Laporte and Semet, 1997).

Probabilistic models, which focus on the vehicles' busy fraction. One alternative is to use a system-wide busy fraction (Daskin, 1983). By dividing the region in subareas, area-specific busy fractions can be calculated (ReVelle and Hogan, 1988). Closer to reality, server-specific busy fractions require treating this parameter as endogenous (Goldberg and Paz, 1991). To compute the busy fraction, historical data, Simulation (Davis, 1981) and the Hypercube Queueing Model (HQM) (Larson, 1974) can be used. Models also differ on the way they explore the busy fraction. Reliability models establish that a node is covered if one vehicle is available within the coverage radius with a given probability (ReVelle and Hogan, 1988). Expected coverage models maximize the expected population receiving care within the coverage radius (Daskin, 1983). Hybrid models mix both approaches.

Another alternative are **capacitated models,** which limit the demand that each vehicle can cover (Pirkul and Schilling, 1991).

Demand and Travel Time

There are several methods for modelling EMS demand. Models using Queueing Theory usually assume that demand follows a Poisson process (Marianov and Serra, 1998). Demand can also be treated as a random variable (Beraldi, Bruni and Conforti, 2004), or demand scenarios can be used, usually within a Stochastic Programming framework (Nickel, Reuter-Oppermann and Saldanha-da-Gama, 2016).

Similar approaches can be used to model travel time: scenarios were used by Berman, Hajizadeh and Krass (2013), while Marianov and ReVelle (1996) model travel times as normal variables. A different approach is to use the probability that a vehicle can reach an area within the coverage radius to determine expected coverage (Goldberg and Paz, 1991).

Time-Dependent Models

Since the performance of EMS can change throughout the day, many researchers have transformed classical formulations into time-dependent models, including MEXCLP (Repede and Bernardo, 1994; Van Den Berg and Aardal, 2015), PLSCP (Rajagopalan, Saydam and Xiao, 2008), MALP2 (Cheu, Lei and Aldouri, 2010), DSM (Schmid and Doerner, 2010) and BACOP1 (Başar, Çatay and Ünlüyurt, 2011).

However, fewer authors account for the relocation cost of vehicles (Van Den Berg and Aardal, 2015). Also, studies contemplating multiple periods in long-term planning are rarer in the EMS literature.

Multiple vehicles and call priorities

Even though early models consider only one vehicle type, most real EMS use multi-tiered systems. Early multiple vehicle models include the TEAM, FLEET (Schilling *et al.*, 1979) and the FAST (Revelle and Snyder, 1995).

Some models use variable coverage radius for different call priorities (Liu *et al.*, 2016), while other approaches to model differentiated call priorities use Queueing Theory (Silva and Serra, 2008; Davoudpour, Mortaz and Hosseinijou, 2014). Although locating a heterogeneous emergency fleet has been addressed, most existing models are designed specifically for the application at hand. A general framework would be desirable.

Alternative Performance Measures

Many models consider a single coverage objective. However, considering other performance measures into location models may be beneficial (Knight, Harper and Smith, 2012).

The original notion of coverage has three limitations: either a facility totally covers a demand node or not; the coverage radius is fixed; coverage depends only on the closest facility (Berman, Drezner and Krass, 2010b). To overcome these limitations, gradual coverage models were introduced, (Berman and Krass, 2002), the coverage radius can be treated as an endogenous parameter (Berman *et al.*, 2009) and cooperation can be modelled as facilities sending a "signal" which decays with distance (Berman, Drezner and Krass, 2010a).

Furthermore, the assumption of coverage models is that short response times improve the outcomes of patients. Recently, researchers have modelled this objective explicitly by introducing survival objectives (Erkut, Erdogan and Ingolfsson, 2008; Knight, Harper and Smith, 2012).

Moreover, authors have acknowledged its importance and formulated different equity objectives (Drezner, Drezner and Guyse, 2009; McLay and Mayorga, 2010; Chanta *et al.*, 2011).

Finally, recognizing that EMS planners may wish to consider multiple objectives, several multi-objective models have been proposed, studying: costs and demand satisfaction (Zhang and Jiang, 2014); efficiency and equity (Chanta, Mayorga and McLay, 2014); and coverage and server workload (Alsalloum and Rand, 2006).

Strategic, tactical and operational Models

Although the distinction between strategic, tactical and operational issues is common, these decisions are interrelated. Therefore, some authors have attempted to combine them in a single model (Davoudpour, Mortaz and Hosseinijou, 2014), while other employ one model for the dispatching policy and another for the location decision (Chong, Henderson and Lewis, 2016). Alternatively, Stochastic Programming can be used (Sung and Lee, 2018).

Literature Review Conclusion

It is possible to conclude that, although a variety of methods exist to model accurately different components of EMS systems, the literature is still lacking a simple general framework considering all these perspectives. Additionally, most models fail to account for the existing system.

4. MODEL FORMULATION

Problem Statement

The proposed model seeks to assist the location EMS stations on a region and assignment of emergency vehicles to these stations. Conversely, main areas of responsibility of each station/vehicle pair are also derived. The goal is to formulate a strategic plan considering both present and future consequences, as well as the varying needs of the population. The problem can be summarized as:

Given:

1. A set of customers which generate emergency requests with different priorities;
2. A region of interest represented by a network;
3. Sites for potential and existing emergency stations, some of which can be changed (selectable), alongside their capacities;
4. Different types of emergency vehicles, some which can be relocated (selectable), capable of providing different care levels;
5. A fixed planning horizon and working shifts;
6. The initial state of the system;
7. Each customer's demand;
8. Travel, service times, coverage probabilities;
9. The allowed number of vehicle relocations, opened and closed facilities;
10. Initial, operating and closing costs;

Determine:

1. The configuration of emergency stations on different periods;
2. The allocation of emergency vehicles among these stations, specifying which vehicles are relocated or added/removed from the fleet;
3. Areas of responsibility of each vehicle/station;

In order to:

1. Maximize the expected coverage;
2. Minimize total costs;
3. Maximize equity.

Relationship to the Literature

The main contributions of the model are:

- The formulation of a tri-objective model capturing the main concerns – coverage, equity and cost – that drive EMS location decisions;
- Considering the possibility of having several types of emergency requests and vehicles which may be dispatched together to a call;
- Allowing strategic and tactical relocations;
- Considering the existing EMS system.

Mathematical Formulation

The notation required for the model is presented in Table 1, while model parameters and decision-variables are introduced in Table 2.

Table 1 - Model notation.

Notation	Description
Sets	
$s \in S$	Working shifts
$t \in T$	Periods in the planning horizon; $t = 0$ is the beginning of the planning horizon
$\|T\|$	Number of planning periods
$d \in D$	Demand points
$p \in P$	Emergency priorities
$v \in V$	Vehicle types
$l \in L$	Care levels
$f \in F$	Emergency station locations
Subsets	
$f \in F^{exi}$	Existing station locations
$f \in F^{new}$	Potential new station locations
$f \in F^{sel}$	Selectable station locations
$v \in V^{sel}$	Selectable vehicles
Indexed Sets	
$v \in V^{f}$	Vehicles that can be located at station f
$v \in V^{l}$	Vehicles capable of providing care level l
$f \in F^{v}$	Stations where vehicles v may be located
$l \in L^{p}$	Care levels l required by a call of type p

Table 2 - Parameters and decision-variables.

Notation	Description	Notation	Description
Parameters			
$OpeningCost_f^t$	Cost of opening station f at the beginning of period t	ε	Minimum fraction of coverage that for a vehicle to be considered as actively cooperating to serve the node
$ClosingCost_f^t$:	Cost of closing a station at site f at the beginning of period t	$StationCap_f^t$	Maximum number of vehicles that can be housed at station f during period t
$CapacityCost_f^t$:	Average cost per vehicle of type v of operating station f during period t	$VeicAva_v^t$	Number of vehicles v available during period t
$OperatingCost_v^{ts}$	Average cost of operating a vehicle of type v during shift s on period t	$MinVeic_{fv}$	Minimum number of vehicles of type v that must be located at station f
$AssignmentCost_{dp}^{ts}$	Average cost of providing care level l to a call of priority p from node d with a vehicle of type v from station f during period t	$InitialVeic_{fv}$	Number of vehicles of type v at station f at the beginning of the planning horizon
Dem_{dp}^{ts}	Requests of priority p from demand node d during on shift s of period t	$MaxVeicShift_v^{ts}$	Number of vehicles v that are available to during shift s of planning period t
$ServiceTime_{dpl}^{ts}$	Average service time to provide care level l to a priority p call from demand node d on shift s of period t	$ShiftLength^s$	Number of time units in shift s
$TravelTime_{fvd}^{ts}$	Travel time for a vehicle of type v from station f to node d on shift s of period t	W_{pl}^1, W_t^2, W_s^3	Weight of covering an emergency of priority p with care l ; during period t and working shift s
γ_{vpl}	1, if a vehicle of type v can provide care level l to call of priority p; 0, otherwise	τ_f	Minimum amount of time that a station at f must remain in operation once opened
$\emptyset_{fvdpl}^{ts}$	Probability that a vehicle of type v departing from a station at site f can cover at care level l a call of priority p from demand node d on shift s of planning period t	$MaxStations^t$, $MaxOpen^t$, $MaxClosed^t$	Maximum number of stations that can be operated/opened/closed during period t
θ	Maximum travel time in the system	i^t	Inflation rate on period t
δ_{fd}	1, if station f can be assigned to calls at node d; 0, otherwise	$Days^t$	Number of days in planning period t
Decision Variables			
$y_f^t \in [0;1]$	1, if an emergency station is operating at site f during planning period t; 0, otherwise.	$ATT_{dpl}^{ts} \in \mathbb{R}_0^+$:	average travel time for care level l to calls p from node d during period t and shift s
$x_{fv}^t \in \mathbb{N}_0$	number of vehicles of type v assigned to a station at site f during planning period t	$MinATT_{dp}^{ts} \in \mathbb{R}_0^+$	average travel time for the first responding vehicle to calls of type p from node d during period t and shift s
$sh_{fv}^{ts} \in \mathbb{N}_0$	number of vehicles of type v assigned to a station at site f during planning period t that are active on working shift s	$E_{fv}^{t+}/E_{fv}^{t-} \in \mathbb{N}_0$:	number of vehicles of type v added to/removed from facility f at the beginning of time period t
$a_{dplfv}^{ts} \in [0;1]$	proportion of demand of priority p from node d for care level l allocated to vehicles of type v at station f during period t and shift s	$H_v^{t+}/H_v^{t-} \in \mathbb{N}_0$	total number of vehicles of type v added to/removed from the fleet at the beginning of period t
$closed_f^t \in [0;1]$, $opened_f^t \in [0;1]$	1, if a station at f is closed/opened at the beginning of planning period t; 0, otherwise.	$b_{fv}^t \in [0;1]$	1, if vehicles of type v are added to site f at the beginning of period t; 0, otherwise.
$w_{dplfv}^{ts} \in [0;1]$	1, if vehicles v at station f are assigned to calls of type p from node d at care level l during shift s and period t	$c_v^t \in [0;1]$	1, if vehicles of type v are added the fleet at the beginning of period t; 0, otherwise.
$\overline{ATT} \in \mathbb{R}_0^+$:	maximum average travel time for the first responding vehicle over the entire region and planning periods	$\beta_{dpl}^{ts} \in [0;1]$	1, if care level l has the shortest average response time for calls of type p from node d during planning period t and working shift s

The mathematical formulation of the model is presented next:

Subject to:

$$Z_1 = \max \sum_{p\in P}\sum_{l\in L} W_{pl}^1 \times \sum_{t\in T} W_t^2 \times \sum_{s\in S} W_s^3 \times \sum_{d\in D}\sum_{f\in F}\sum_{v\in V^f} a_{dplfv}^{ts} \times Dem_{dp}^{ts} \times \emptyset_{fvdpl}^{ts} \quad \textbf{(1)}$$

$$\begin{aligned} Z_2 = \min & \sum_{t\in T\setminus\{0\}} \frac{1}{(1+i^t)^t} \times \left(\sum_{f\in(F^{exi}\cap F^{sel})} Closed_f^t \times ClosingCost_f^t + \sum_{f\in(F^{new}\cap F^{sel})} Opened_f^t \times OpeningCost_f^t \right) \\ & + \sum_{t\in T}\frac{1}{(1+i^t)^t} \times \sum_{f\in F}\sum_{v\in V^f} x_{fv}^t \times CapacityCost_{fv}^t + \sum_{s\in S} sh_{fv}^{ts} \times OperatingCost_v^{ts} \\ & + \sum_{t\in T}\frac{1}{(1+i^t)^t} \times \sum_{d\in D}\sum_{p\in P}\sum_{l\in L^p}\sum_{s\in S} a_{dplfv}^{ts} \times Dem_{dp}^{ts} \times AssignmentCost_{dplfv}^{ts} \end{aligned} \quad \textbf{(2)}$$

$$Z_3 = \min \overline{ATT} \quad \textbf{(3)}$$

$$sh_{fv}^{ts} \le x_{fv}^t, \quad \forall f \in F, v \in V^f, t \in T, s \in S \quad \textbf{(4)}$$

$$\sum_{f\in F^v} x_{fv}^t \le VeicAva_v^t, \quad \forall v \in V, t \in T \quad \textbf{(5)}$$

$$x_{fv}^t \ge MinVeic_{fv}, \quad \forall f \in F, v \in V^f, t \in T\setminus\{0\} \quad \textbf{(6)}$$

$$\sum_{v\in V^f} x_{fv}^t \le StationCap_f^t \times y_f^t, \quad \forall v \in V, t \in T \quad \textbf{(7)}$$

$$\sum_{v\in V^f} x_{fv}^t \ge y_f^t, \quad \forall f \in F, t \in T \quad \textbf{(8)}$$

$$\sum_{f\in F} sh_{fv}^{ts} \le MaxVeicShift_v^{ts}, \quad \forall v \in V, t \in T, s \in S \quad \textbf{(9)}$$

$$x_{fv}^0 = InitialVeic_{fv}, \quad \forall f \in F^{exi}, v \in V^f \quad \textbf{(10)}$$

$$y_f^0 = 1, \quad \forall f \in F^{exi} \quad \textbf{(11)}$$

$$x_{fv}^t \ge x_{fv}^{t-1}, \quad \forall v \in \overline{V^{sel}}, f \in F^v, t \in T\setminus\{0\} \quad \textbf{(12)}$$

$$y_f^t \ge y_f^{t-1}, \quad \forall f \in \overline{F^{sel}}, t \in T\setminus\{0\} \quad \textbf{(13)}$$

$$y_f^t - y_f^{t-1} = opened_f^t - closed_f^t, \quad \forall f \in F, t \in T\setminus\{0\} \quad \textbf{(14)}$$

$$opened_f^t + closed_f^t \le 1, \quad \forall f \in F, t \in T\setminus\{0\} \quad \textbf{(15)}$$

$$x_{fv}^t - x_{fv}^{(t-1)} = E_{fv}^{t+} - E_{fv}^{t-}, \quad \forall f \in F, v \in V^f, t \in T\setminus\{0\} \quad \textbf{(16)}$$

$$\sum_{f\in F^v}\left(x_{fv}^t - x_{fv}^{(t-1)}\right) = H_v^{t+} - H_v^{t-}, \quad \forall v \in V, t \in T\setminus\{0\} \quad \textbf{(17)}$$

$$E_{fv}^{t+} \leq b_{fv}^{t} \times StationCap_f^t , \quad \forall f \in F, v \in V^f, t \in T\backslash\{0\} \quad \textbf{(18)}$$

$$E_{fv}^{t-} \leq (1 - b_{fv}^{t}) \times StationCap_f^{t-1} , \quad \forall f \in F, v \in V^f, t \in T\backslash\{0,1\} \quad \textbf{(19)}$$

$$E_{fv}^{1-} \leq (1 - b_{fv}^{1}) \times InitialVeic_{fv} , \quad \forall f \in F, v \in V^f \quad \textbf{(20)}$$

$$H_v^{t+} \leq c_v^t \times VeicAva_v^t , \quad \forall f \in F, v \in V^f, t \in T\backslash\{0\} \quad \textbf{(21)}$$

$$H_v^{t-} \leq (1 - c_v^t) \times VeicAva_f^{t-1} , \quad \forall f \in F, v \in V^f, t \in T\backslash\{0,1\} \quad \textbf{(22)}$$

$$H_v^{1-} \leq (1 - c_v^1) \times \sum_{f \in F^v} InitialVeic_{fv} , \quad \forall f \in F, v \in V^f \quad \textbf{(23)}$$

$$\sum_{b=t}^{t+\tau_f^t - 1} y_f^b \geq opened_f^t \times \tau_f^t , \quad \forall f \in F, t \in T\backslash\{0\} \quad \textbf{(24)}$$

$$\sum_{f \in F} opened_f^t \leq MaxOpen^t , \quad \forall t \in T\backslash\{0\} \quad \textbf{(25)}$$

$$\sum_{f \in F} closed_f^t \leq MaxClosed^t , \quad \forall t \in T\backslash\{0\} \quad \textbf{(26)}$$

$$\sum_{f \in F} y_f^t \leq MaxStations^t , \quad \forall t \in T\backslash\{0\} \quad \textbf{(27)}$$

$$\left(\sum_{f \in F} E_{fv}^{t+} \right) - H_v^{t+} \leq MaxReal_v^t , \quad \forall v \in V, t \in T \quad \textbf{(28)}$$

$$\sum_{f \in F} \sum_{v \in V^f} a_{dplfv}^{ts} \leq 1, \quad \forall d \in D, p \in P, l \in L^p, s \in S, t \in T \quad \textbf{(29)}$$

$$\sum_{l \in L^p} a_{dplfv}^{ts} \leq sh_{fv}^{ts}, \quad \forall d \in D, p \in P, f \in F, v \in V, s \in S, t \in T \quad \textbf{(30)}$$

$$a_{dplfv}^{ts} \leq \gamma_{vpl} \times \delta_{fd}, \quad \forall \in D, p \in P, l \in L, t\, f \in F, v \in V, s \in S, t \in T \quad \textbf{(31)}$$

$$w_{dplfv}^{ts} \geq a_{dplfv}^{ts} , \quad \forall \in D, p \in P, l \in L, t\, f \in F, v \in V, s \in S, t \in T \quad \textbf{(32)}$$

$$\varepsilon \times w_{dplfv}^{ts} \leq a_{dplfv}^{ts} , \quad \forall \in D, p \in P, l \in L, t\, f \in F, v \in V, s \in S, t \in T \quad \textbf{(33)}$$

$$\sum_{d \in D} \sum_{p \in P} \sum_{l \in L} Dem_{dp}^{ts} \times a_{dplfv}^{ts} \times \left(TravelTime_{fvd}^{ts} + ServiceTime_{dpl}^{ts} \right) \leq \rho^{max} \times sh_{fv}^{ts} \times ShiftLength^s , \quad \forall f \in F, v \in V^f, s \in S, t \in T \quad \textbf{(36)}$$

$$\sum_{f \in F} \sum_{v \in V^f} \sum_{l \in L^p} w_{dplfv}^{ts} \geq N , \quad \forall d \in D, p \in P, s \in S, t \in T \quad \textbf{(37)}$$

$$ATT_{dpl}^{ts} = \left(\sum_{f \in F} \sum_{v \in V^f} a_{dplfv}^{ts} \times TravelTime_{fvd}^{ts} \right) + \left(1 - \sum_{f \in F} \sum_{v \in V^f} a_{dplfv}^{ts} \right) \times \theta, \quad \forall d \in D, p \in P, l \in L^p, s \in S, t \in T \quad \textbf{(38)}$$

$$MinATT_{dp}^{ts} \geq ATT_{dpl}^{ts} - \theta \times \beta_{dpl}^{ts}, \quad \forall d \in D, p \in P, l \in L^p, s \in S, t \in T \quad \textbf{(39)}$$

$$\sum_{l \in L^p} \beta_{dpl}^{ts} = |L^p| - 1 , \forall d \in D, p \in P, l \in L^p, s \in S, t \in T \quad \textbf{(40)}$$

$$\overline{ATT} \geq MinATT_{dp}^{ts} , \quad \forall d \in D, p \in P, s \in S, t \in T \quad \textbf{(41)}$$

Objective (1) maximizes the expected coverage (Z_1), weighted by W_{pl}^1, W_t^2 and W_s^3. Total coverage is the amount of demand allocated to each server ($a_{dplfv}^{ts} \times Dem_{dp}^{ts}$) times its coverage probability $\emptyset_{fvdpl}^{ts}$. Additionally, the model minimizes the total cost of the system (Z_2). Finally, the third objective (Z_3) promotes equity, by minimizing the maximum average travel time of the first care level, across all shifts and periods.

Constraints (4) state that the number of vehicles of type v on station f available during shift s and period t must be smaller than the number of vehicles of that type allocated to that station. Constraints (5) limit the number of vehicles of each type deployed during each period to the number of available vehicles. Constraints (6) impose a minimum number of vehicles to be allocated to certain stations at all times. These constraints are mostly related with legal requirements. Constraints (7) limit the number of vehicles assigned to a station to its capacity. Constraints (8) force at least one vehicle to be assigned to each open station. Constraints (9) limit the number of vehicles in operation at each shift. Constraints (10) define the number of vehicles allocated to existing stations, while constraints (11) set $y = 1$ for all existing stations.

Constraints (12) and (13) state that, for non-selectable vehicles, assigned vehicles cannot be removed and that non-selectable facilities, once opened, must remain in operation. Constraints (14) and (15) keep track of which stations are opened and closed at each period and constraints (16) and (17) keep track of the vehicles added and removed from each station and the overall fleet. Since there are infinite possibilities for the differences $E_{fv}^{t+} - E_{fv}^{t-}$ and $H_v^{t+} - H_v^{t-}$, it must be ensured that only one of the E_{fv}^t and one of the H_v^t variables are positive. This is accomplished in constraints (18) to (23).

Constraints (24) state that, once a station is opened, it must remain open for the following τ_f periods. Constraints (25) and (26) place a limit on the amount of stations that can be opened/closed at each period, while constraints (27) limit the total number of stations. Constraints (28) limit on the relocations of each vehicle type. Constraints (29) ensure that it is not possible to assign more than the existing demand. Constraints (30) prevent any station/vehicle pair from being responsible for providing more care levels to the same call than the number of existing vehicles. Constraints (31) only allow requests to be assigned to a vehicle of type v if that vehicle is capable of providing care l to a priority p call and vehicles at station f are allowed to respond to calls at node d. Constraints (32) assign the value to variables w_{dplfv}^{ts}, while constraints (33) force a station to cover a minimum amount of demand, if it cooperates to serve a node. Constraints (36) set an upper bound on the unavailability probability of each vehicle, while the minimum number of vehicles that must share the responsibility of an emergency request is set by constraints (37).

Lastly, constraints (38) calculate the average travel time for each emergency request at each time period. Unassigned requests are penalized to a maximum value of travel time. Constraints (39) and (40) calculate the minimum average travel time across all care levels. Finally, constraints (41) assign the largest of these to $\overline{ATT}$.

Heuristic Approach

The proposed model includes three levels of decision-making: selection of facilities, allocation of vehicles and demand assignment. Although making these decisions simultaneously is expected to yield better solutions, it is possible to perform them sequentially. This observation motivates the heuristic, which consists of decomposing the model in two sub-models and iterate between them to get good solutions. The first sub-model (SP1) decides the location of stations and allocation of vehicles by maximizing potential coverage, being defined as:

SP1:

$$Z_1{}' = \max \sum_{p\in P}\sum_{l\in L} W_{pl}^1 \times \sum_{t\in T} W_t^2 \times \sum_{s\in S} W_s^3 \times \sum_{d\in D}\sum_{f\in F}\sum_{v\in V f} sh_{fv}^{ts} \times Dem_{dp}^{ts} \times \emptyset_{fvdpl}^{ts} \quad \textbf{(42)}$$

Subject to (4) to (28).

While the second sub-model assigns demand to vehicles, considering the original objective function and remaining constraints:

SP2:

$$Z_1 = \max \sum_{p\in P}\sum_{l\in L} W_{pl}^1 \times \sum_{t\in T} W_t^2 \times \sum_{s\in S} W_s^3 \times \sum_{d\in D}\sum_{f\in F}\sum_{v\in V f} a_{dplfv}^{ts} \times Dem_{dp}^{ts} \times \emptyset_{fvdpl}^{ts} \quad \textbf{(43)}$$

Subject to (29) to (33) and (36) to (37).

The proposed heuristic is described in Table 3.

Table 3 - Pseudo-code of the proposed heuristic.

Algorithm Hybrid Heuristic

Initialize: $BFS \leftarrow 0,\ iter \leftarrow 1, cons \leftarrow 0$
Set $N,\ \varepsilon,$
While ($iter \leq Maximum\ iterations$) ***and*** (cons $\leq Maximum\ iterations\ without\ improvement$) ***do:***
    Solve *$SP1$ and retrieve optimal decision-variable sh^{iter}*
    Solve *$SP2$ and retrieve optimal solution value Z^**
    If ($Z^* \geq BFS$) ***do:***
        Set $BFS \leftarrow Z^*$
        Set $cons \leftarrow 0$
    Else:
        Set $cons \leftarrow cons + 1$
    For Each ($sh_{ts}^{fv^{iter}}$) ***do***:
        Compute $rank_{fv}^{ts\ iter} \leftarrow \sum_{p\in P}\sum_{l\in L} W_{pl}^1 \times \sum_{t\in T} W_t^2 \times \sum_{s\in S} W_s^3 \times \sum_{d\in D}\sum_{f\in F}\sum_{v\in V f} sh_{fv}^{ts} \times Dem_{dp}^{ts} \times \emptyset_{fvdpl}^{ts} - \sum_{p\in P}\sum_{l\in L} W_{pl}^1 \times \sum_{t\in T} W_t^2 \times \sum_{s\in S} W_s^3 \times \sum_{d\in D}\sum_{f\in F}\sum_{v\in V f} a_{dplfv}^{ts} \times Dem_{dp}^{ts} \times \emptyset_{fvdpl}^{ts}$
    Sort $sh_{fv}^{ts\ iter}$ according to $rank_{fv}^{ts\ iter}$
    For *the first N $sh_{fv}^{ts\ iter}$* ***do***
        Add *constraint* $sh_{fv}^{ts} \leq \max(1,\ sh_{fv}^{ts\ iter} - 1)$
    Update $N \leftarrow round\left(\frac{N}{\varepsilon}\right)$
    Set $iter \leftarrow iter + 1$

5. CASE-STUDY RESULTS

This section presents the application of the model to INEM's case study. The results of experiments are presented, as well as computational results of the optimization model and the heuristic. The model is implemented in IBM ILOG CPLEX Optimization Studio 12.8.0.0, using Optimization Programming Language (OPL). A Lexicographic approach is used to handle multiple objectives. For this purpose, objectives are ranked in decreasing order of importance: Z1, Z2 and Z3.

The heuristic approach and the lexicographic method are implemented in OPL script (IBM Knowledge Center, 2019a). Experiments are conducted on a 2.40 GHz Intel Core i7-4700MQ processor and 12.0 GB of RAM laptop running Windows 10. To validate the model, two toy instances are analysed, and extreme conditions are introduced by varying several parameters and testing the model's response to such changes. In all experiments, the model behaved as expected.

Preliminary Scalability Analysis

In order to assess the scalability of the model, several instances of different sizes are analysed.

The results are presented in Table 4.

Table 4 - Results of the scalability analysis.

	Run 0			
Instance	**Coverage** (Z1)	**Gap** (%)	**BB Time (s)**	**Total Time (s)**
L0.0	1691.43	0.47%	188.7	237.3
L1.0	1647.43	0.44%	874.2	971.4
L2.0	1652.12	0.18%	8821.2	9191.1
S0.0	323.47	0.00%	5.5	22.4
S1.0	322.41	0.00%	18.1	51.2
S2.0	322.81	0.00%	179.4	247.6
	Run 1		**Run 2**	
Instance	**Cost** (Z2)	**Gap** (%)	**Equity** (Z3)	**Gap** (%)
L0.0	60423.46	1.4%	15.12	0.0%
L1.0	65131.90	4.2%	15.48	0.0%
L2.0	65934.69	4.9%	15.86	0.0%
S0.0	15157.96	3.2%	16.34	0.0%
S1.0	13038.88	4.2%	17.48	0.0%
S2.0	13986.25	4.6%	16.69	0.0%

It can be concluded that the computational effort increases exponentially with the instance size, mostly due to the second and third runs of the Lexicographic method since the additional constraints related with the previous objectives render the model much more difficult. The results also suggest that the cluster partition only slightly influences the resulting objective function values.

In light of these results, the remaining experiences are conducted using the L0 and S2 instances. Additionally, given that proving optimality requires significant computational time, a relative gap of 0.5% is used when maximizing coverage, while a gap of 5% is applied when optimizing the remaining objectives. Additionally, a time lime of 24 hours is set for each run.

Current System Performance

A heat map of emergencies in Lisbon and Setúbal suggests a higher emergency density in the centres of both cities, where population density is higher due to residents, commuters and tourists. Simultaneously, more stations are located in these areas. Nevertheless, the north of Lisbon and a large part of Setúbal are apparently less covered.

Regarding Setúbal, most emergencies are concentrated on the city centre and, as such, vehicle activity patterns are more balanced. Workload is approximately distributed. External dependency is high (close to 15%). Regarding Lisbon, external vehicles operate mainly on the northern part of the city. Additionally, vehicles with higher dispatching fractions are located closer to the city centre and the eastern part of the city, suggesting that additional vehicles in these regions could be beneficial. NINEMs present the highest dispatch fractions, which is surprising given that INEM favours own vehicles and PEMs. Some AEMs present low dispatch fractions, which suggests that their positions can be improved. External dependency is significant (10%).

Inputting the current solution into the model (Table 5), it is concluded that the current system should be able to cover around 83% and 96% of the calls, respectively.

Table 5 - Estimated current system performance.

Instance	**Coverage** (Z1)	**Gap (%)**	**Cost** (Z2)	**Equity** (Z3)
L0.1	1686.14	0.10%	66494.66	17.86
S2.1	320.43	0.00%	17119.53	18.32

Base-Line Optimization

In order to seek improvements for the current system complying with all the restrictions imposed by INEM, the legislation and SIEM partners, the base-line optimization instances are used again. The results are presented in Figure 1.

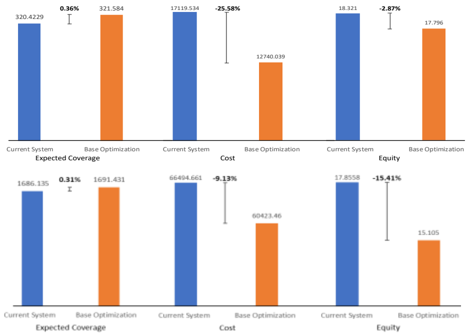

Figure 1 - Base line results (Top: Lisbon; Bottom: Setúbal).

The results highlight that, under the current restrictions, the present performance can only be slightly improved in what concerns coverage. This is mainly because all stations are classified as non-selectable, meaning that they cannot be relocated. Since the maximum number of stations in most months is also fixed to the current number of stations (20 in Lisbon and 5 in Setúbal), the underlying structure of the emergency system network cannot be modified. Analysing the resulting solutions, it is possible to verify that only limited changes are possible: seasonal stations are suggested during the summer and selected AEMs are progressively relocated to improve the system's performance.

Although the improvement in coverage is modest, the reduction in cost is more significative: 9.13% for Lisbon and 25.58% for Setúbal. The same result is valid for the equity objective, where there is a reduction in the maximum average travel time in the system of 15.41% in Lisbon and 2.87% in Setúbal. It is possible to conclude that, although the current restrictions are considerably tight and do not allow many modifications of the system, attractive improvements can still be attained in what concerns the system cost and the equity of service, together with a modest improvement in the SIEM's ability to cover emergencies. Therefore, it becomes important to study management policies which may help improve the system's performance.

Seasonal PEMs

Currently, INEM deploys seasonal PEMs during the summer. In order to assess the impact of this policy, an additional type of vehicle is included in the model (Seasonal PEMs, SPEM). The impact of this vehicle is evaluated by fixing the remaining vehicles at their original stations. The model suggests locating Setúbal's SPEM in *BV Setúbal – Azeitão*, while in Lisbon it should be located at *BV Cabo Ruivo*. The results suggest that seasonal PEMs improve the system's coverage by 0.20% in Setúbal and 1.02% in Lisbon and the equity indicator by 15.4.% and 0.48%, respectively.

The results indicate that, in Setúbal, SPEMs are more effective during the summer months, while, on the other hand, in Lisbon they would be more advantageous during the first months of the year, namely February, March and April.

Fleet Expansion

In line with the current planning practice at INEM, the impact of adding additional units of each type to the current fleet is analysed in Figure 2.

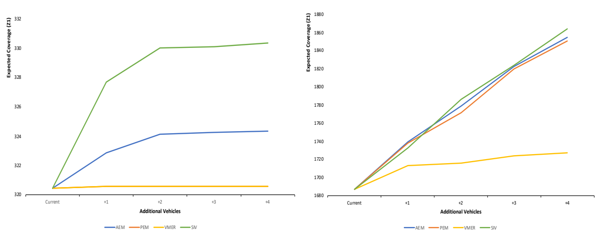

Figure 2 - Fleet expansion results (Left: Lisbon; Right: Setúbal).

If the fleet is to be expanded in one unit, then one AEM should be purchased for Lisbon and one SIV for Setúbal. However, two SIVs are more beneficial in Lisbon than two AEMs. Generally, expanding the fleet through the purchase of AEMs seems to be more advantageous than through PEMs. Additionally, with the current limitations in stations for VMERs, increasing these vehicles is not significantly attractive.

Furthermore, considering level 2 stations provides only marginal benefit when compared to level 1 stations. However, the benefit of level 2 stations seems to increase as the number of vehicles added to fleet also increases. Therefore, if more vehicles are to be added, then considering level 2 stations is important, provided that the location of these additional vehicles is planned in advance.

Alternative Operating Scenarios

Moreover, four alternative scenarios are tested: the effect of legislation concerning minimum vehicles at hospitals; a free vehicle relocation scenario; a green-field scenario and a fully flexible system. The results are presented in Figure 1.

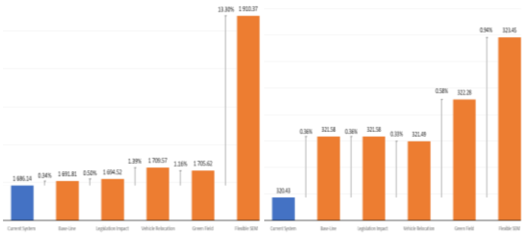

Figure 1 - Comparison of the results from the alternative operating scenarios (Left: Lisbon; Right: Setúbal).

Concerning the first experiment, the results suggest that the impact of legislative restrictions in isolation is small, mainly because the vehicles which are regulated by legislation – VMERs – are crewed by doctors from NHS hospitals and, as such, can only be located at the very sights in which the legislation requires them to be. On the other hand, results of the vehicle relocation scenario suggest that the performance on both regions could be improved with the current vehicles and stations. However, these improvements are modest, especially if the high number of required relocations is considered. Therefore, all in all, this strategy seems to be inefficient. Concerning the green-field scenario, only modest improvements are attained once again. It is interesting to note that the proposed distribution of emergency stations in both Lisbon and Setúbal is more scattered than the original solution. Finally, the fully flexible EMS is, as expected, the scenario which yields a more significant improvement. For Lisbon, using the same emergency vehicles and number of emergency stations, an improvement of 13.1% can be achieved. This improvement is higher than what would be obtained by adding four SIVs. In Setúbal, a 0.94% improvement is possible which, in turn, is less attractive than the addition of a single SIV, which leads to improvements of 2.66%. For both cities, the improvement in a flexible system is 3 to 10 times higher than allowing only vehicles to be relocated or scratching off the existing stations.

Sensitivity Analysis

Given the inherent uncertainty of several parameters, a sensitivity analysis is carried out by introducing a ±10% variation (for continuous parameters) and ±1 (for discrete parameters) in selected parameters.

The results lead to the conclusion that the parameters which have a higher impact in coverage are demand and coverage probabilities, followed by N. Remaining parameters – Travel and Service Time, ρ^{max} and ε – have little influence in the value of coverage. Concerning the second objective, the category which generates a higher effect in total cost is capacity cost, followed by the operating cost and, finally, assignment cost. Finally, travel time has a significant effect on the equity objective, while θ (maximum travel time) presents a smaller variation because, in the studied instance, most demand is assigned to vehicles, thus few requests are penalized at the maximum system travel time. Again, ρ^{max} and ε do not appear to significantly impact this objective.

Heuristic Approach

The performance of the proposed heuristic is compared to CPLEX. For this purpose, the instances used in the scalability analysis are applied. Since the heuristic is only applicable to the first objective, only the first run is considered.

A grid search is conducted to tune the two heuristic parameters (N and ε). It is concluded that, after a certain value of N, the heuristic solution remains unchanged regardless of ε. Furthermore, greater values of N require also higher values of ε to obtain improved results.

The results show that the heuristic provides good solutions with relative gaps never greater than 2.10% after five iterations. Even with just one iteration, the solutions are good (gaps under 2.32%), especially considering that the computational times are reduced by up to 95.54%. When further iterations are allowed, the solution is further improved, but only slightly. This may suggest that more effective cuts may exist.

For small instances, the gains in computational time are small for few iterations. For five iterations, the heuristic takes more time than CPLEX and produces worse results. Therefore, it can be concluded that the heuristic should not be applied for small instances, for which CPLEX is more efficient and effective. On the other hand, for larger instances, the computational gains of the heuristic are attractive. The heuristic outperforms CPLEX after just two iterations for some instances, reducing the computational time by 39.04% and 89.98%, respectively. Additionally, in the larger instances, the heuristic yields a worse solution (gap of 0.26% compared to 0.18% of CPLEX) but time is reduced by 84.41%.

Furthermore, after a given number of iterations (approx. 10), the heuristic procedure stalls. That is, after a significant number of cuts are added to SP1, the new cuts become redundant and the heuristic becomes stuck in the same solution. This phase is reached more quickly if N is larger, given that more cuts are added at the beginning of the optimization. Although this may help reduce the computational burden of the heuristic, it could be interesting to explore diversification strategies.

Overall, the heuristic is an effective solution method for large instances, but not appropriate for smaller instances.

6. CONCLUSIONS

EMS are highly complex systems which are designed to save lives. In this dissertation, SIEM is studied. Supported on an extensive literature review and case-study description, a Multi-Objective Dynamic MIP model is proposed. Its main goal is to assist the gradual reconfiguration of an existing EMS system over time by determining the configuration of emergency stations, the allocation of emergency vehicles, and the areas of responsibility of each vehicle. Three objectives are considered in the model: coverage, cost and equity. Besides, a hybrid heuristic based on decomposing the problem is proposed to streamline model solution for the first objective.

The model is applied to Lisbon and Setúbal, using a Lexicographic approach. The results suggest that unless INEM is able to lift some restrictions, only modest improvements can be expected.

The results also show that the seasonal vehicles slightly improve the system performance. Furthermore, in what concerns the deployment months for the seasonal PEMs, it is concluded that Lisbon would benefit more if such vehicles were deployed during the winter months, while for Setúbal the current deployment months seem appropriate. Concerning fleet expansion, in Lisbon, the addition of one AEM is the most promising choice. However, for more than two vehicles, the addition of SIVs is recommended. Similarly, for Setúbal, the addition of SIVs is recommended. Tests also show that considering a broader set of facilities as potential base stations does not provide a significant benefit unless a large expansion is expected. Furthermore, it is concluded that the impact of legislation is small, and that performance can be greatly improved with existing resources provided that both stations and vehicles are allowed to be relocated.

Besides overcoming limitations regarding data collection, suggestions for future work include: a) exploring alternative methods to determine the upper bound on vehicle workload and the minimum number of vehicles which must share a call; b) studying the impact of the chosen reliability level and other vehicle availability parameters; c) include demand stochasticity via scenarios or chance constraints; d) seeking alternative equity measures; e) accounting for the impact of external vehicles and emergency requests in the model; e) reviewing cost categories included in the model. Furthermore, concerning the solution approach, the heuristic can be improved by a) seeking diversification strategies and alternative cuts to polish the solution; b) the development of heuristics designed to handle the remaining objectives; c) study alternative hybrid heuristics, exploring other decomposition possibilities.

Finally, according to the literature, it is suggested that the model is validated against a simulation model, to identify opportunities for improvement.

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