Adaptive Dynamics and Structural Properties of Financial Markets

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Thesis to obtain the Master of Science Degree in

Information Systems and Computer Engineering

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October 2018
Acknowledgments

I would like to thank my parents, grandparents, uncle and my sister for their friendship, encouragement, example and caring over all these years, for always being there for me through thick and thin and without whom this project (and many others) would not be possible. I would also like to acknowledge my dissertation supervisors Prof. Francisco C. Santos, Prof. Jorge M. Pacheco and Prof. Fernando P. Santos for their insight, support and sharing of knowledge that has made this Thesis possible. Additionally, a special thank to FCT (Fundação para a Ciência e Tecnologia) for financial support and to João Ribeiro for the technical support and helpful coding insights. Moreover, to Prof. Andreia Sofia Teixeira and Prof. Alexandre P. Francisco for their work, support and feedback in the article "Capturing Financial Volatility Through Simple Network Measures".

Last but not least, to all my friends, colleagues and girlfriend that helped me grow as a person and were always there for me during the good and bad times in my life. Thank you.

To each and every one of you – Thank you.
Abstract

Modeling economic agents is quite often naively rooted in the maximization of a non-trivial and sometimes subjective utility function. Finding statistical evidences of whose economic features explain what in the financial world is most of the times, if not impossible, challenging. Nonetheless, in the recent years, developments in Evolutionary Game Theory (EGT) have endorsed those who believe markets are nothing but a giant and complex evolutionary game bounded by both behavioral and economic rules. In this thesis, we firstly develop an adaptive model with 2 strategies as Market participants such as Momentum and Fundamental Traders. Through their investment decisions, we measure how population collective behavior evolve in time, through the dynamics of peer-influence and changes in the Market. We show that even in a oversimplified population dynamic, Momentum Strategy dominate as spreads out to the entire population, under several different settings. Also, inequality and power-law distributions tend to constantly surge. Furthermore, measuring the inner characteristics of financial markets risks have been proven to be key at understanding what promotes financial instability and volatility. Advances in complex network analysis have shown the capability to characterize the specificities of networks. In the second part of this thesis, we present a price-correlation network model in which Standard & Poors’ members are nodes connected by edges corresponding to price-correlations over time. We use the degree and the frequency of specific motifs to evaluate if it is possible to identify volatility. Our results suggest the existence of a significant correlation between VIX and the degree of the network and the number of balanced positive triads. These results are shown to be robust to a wide range of time windows and correlations thresholds, suggesting that market instability can be inferred from simple topological features.

Keywords

Evolutionary Game Theory, Financial Markets, Adaptive Agents, Complex Networks, Social Balance
Resumo

A modelação de agentes econômicos é muitas vezes ingenuamente enraizada na maximização de uma função de utilidade não trivial e às vezes subjetiva. Encontrar evidências estatísticas de cujas características explicam o mundo financeiro é, na maioria das vezes, se não impossível, desafiador. No entanto, nos últimos anos, os desenvolvimentos na Teoria de Jogos Evolutiva confirmaram aqueles que acreditam que os mercados não são nada além de um gigantesco e complexo jogo evolutivo, limitado por regras comportamentais e econômicas. Nesta tese, primeiro desenvolvemos um modelo de EGT com duas estratégias opostas, participantes do mercado, como Momentum e Fundamental. Através de suas estratégias, vamos medir como o comportamento coletivo da população pode ser responsável por mudanças na dinâmica do mercado. Concluiu-se que, mesmo com uma dinâmica populacional simplificada, a desigualdade de rendimentos, tendem a surgir de forma constante. Adicionalmente, com os avanços obtidos em redes complexas, a medição das características intrínsecas das redes financeiros provou ser a chave para a compreensão daquilo que promove a instabilidade financeira. Aqui, apresentamos um modelo de correlação de preços, no qual os membros da Standard & Poor's 500 (SPX), são os nós ligados por eixos que correspondem à correlação de preços. Usando o grau e a frequência dos motivos específicos, baseados no balanço estrutural, conseguimos provar a existência de uma correlação significativa entre a volatilidade implícita do Index e o grau médio da rede e o número de tríades positivas equilibradas. Estes resultados, mostraram ser validamente robustos, para uma ampla gama de janelas temporais e limites de correlações, sugerindo que a instabilidade dos mercados pode ser inferida de simples características topológicas.

Palavras Chave

Teoria de Jogos Evolutiva, Mercados Financeiros, Agentes Adaptativos, Redes Complexas
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## Acronyms

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<tr>
<td>ABS</td>
<td>Adaptive Belief System</td>
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<td>CAPM</td>
<td>Capital Asset Pricing Model</td>
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<td>EMH</td>
<td>Efficient Market Hypothesis</td>
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0.1 Thesis General Overview

Without us even noticing, Financial Markets are present in all our daily lives. From our mortgage loan, labor contract, iPhone to our simple latte macchiato, all of its components were priced and traded somewhere in a Market. Nowadays, financial industry by itself moves more than any other economic sector, around 300 trillion dollars per year [4], and keeps growing at a 10% rate [5]. In order to move such amount of money, there are dozens of thousands of different economic agents, markets and products. Those so-called Markets are nothing but a giant amount of Market players exchanging cash-flows and goods between them, being therefore responsible for managing one of the most complex multi-agent systems humanity have ever created, permanently connecting every corner of the Globe [4,5].

Predicting the dynamics of such far complex multi-agent system has been one of the most wanted achievements in the Financial and Economics Academia (as well as in the Industry), not only to allow investors to protect their savings and investments against periods with large volatility, but mostly to profit from a risk-less model. Despite the huge literature on Market predictability, modeling so many agents (that can go from Mutual Funds, Wealth and Asset Management Funds, Brokers, Pensions Funds, Market Makers, Hedge Funds, Insurance Companies, High-Frequency Trading Funds, Proprietary Trading Firms, Investment and Commercial Banks to a simple small investor) and so many financial products (such as Stocks, Interest Rates, Future contracts, Commodities, Exchange Rates, Options, etc.) [4, 5] has been nearly impossible to prove and achieve.

One can say that within such a context, it is an open question if modeling such a diverse environment is possible. Many authors throughout time addressed this question and were motivation to the present work. One of them, Andrew Lo [6, 7], was key in order to change the reasoning when dealing with such problems. He said that it can only be done by moving from a strategy or price discovery to agent-based models. Combining world-accepted knowledge from Biology empowered by recent develops in Computer Science allow us to simulate a population with a financial reasoning and compare it to what happens in reality. In Richard Bookstaber’s “The End of Theory” book [8], he mentioned the fact that Financial Markets are ruled by entities that adapt to the current market or trading environmental conditions in order to maximize their "fitness" and avoid "extinction". Therefore, investors and Financial Professionals should also be ruled by the same properties as any other biological being no matter if the decision is to make a trading call or an animal searching for food.

Throughout decades of research, Economics and Finance academia built instruments to prove that markets are unpredictable yet perfect. Believing differently was showing society was doing something wrong. When the first contrariwise studies appeared, they were called either ”chaos and random propaganda” or proclaimed as research not rooted in free-market assumptions [6, 7]. Well as many others before the present work, where others see chaos movements, it should be seen order to be decoded, decipher and modeled. Despite being a complex reality that does not mean is wrong or malfunction-
In his seminal work, Andrew Lo [6, 7] said that Economic theory needs to move from physics and mathematical fields to meet once again with the rest of Social Sciences. Therefore, what it is going to be studied is not an “accident of nature” [6, 7] but the pure and mostly rational maximization of Market participants utility in a form of risk-adjusted profits. Within this unstoppable run, an army of investors move prices up or down driven by expectations on future market opportunities or expected informational advantages. This conveys that by the simple investors’ collective action (based on individual and biased expectations), Market prices “quickly” converge to an hypothetical steady-state and “eliminate the profit opportunities that first motivated their trades” [6, 7]. In short, instead of rejecting hypothesis, it is our belief that those should be combined to enrich our knowledge base.

In order to perform such a journey, the present work is split in two parts: firstly we develop a evolutionary model where investors, based on their strategy such as Momentum and Fundamental, invest on the largest country-wise Index: the Standard & Poor’s 500 (SPX). Having this population as research framework, we study the impact and the evolution of each strategy and its impact not only on investors performance but also global and general welfare. We may say that Momentum Strategy is a dominant strategy, relative to Fundamentalists, “Buy and Hold” and Random Investing (for the period in study, ie., 1927-2018). Moreover such environment is responsible to generate power-Law income distribution, where a few nodes concentrate most of wealth and the large majority under-performed and lost money with their investment actions. In the second Part1, we zoomed out to study the impact SPX constituents network have on the volatility of the Index itself. Through a timed-depended correlation network, we measure the SPX network average degree and its inner triads motifs, rooted in complex science and social structural balance, to acknowledge at what degree it can explain and replicate the volatility (instability) Index - the so-called Volatility Index (VIX). We demonstrated the powerful correlation between simple measures as the degree and the “fear Index”, as well as, the amount of positive correlated triads (whose nodes performance was up-warding for the period considered and the correlation among them also positive) and VIX.

"Chaos is just order to be deciphered" - José Saramago

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1Accepted as full paper at the The 7th International Conference on Complex Networks and Their Applications (December 11-13, 2018, Cambridge, UK) and inclusion in the proceedings to be published by Springer Verlag Lectures.
Part I

Evolutionary Games in Financial Markets
INTRODUCTION

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1.1 Motivation

Financial Industry is, by definition, always searching for better accuracy in market predictability. If someone can understand and replicate better what happens in real live-markets, then it has an edge that can be taken profit out of it [9]. In such environment, several thousand of agents compete to outperform its peers, by having not only more money, but also lower levels of risk per unit of profit made. This concept of competition in a population sounds quite familiar to those who study, since long time ago, Evolutionary Game Theory (EGT). The hypothesis underneath the present reasoning comes from the fact that a population with interchangeable behaviors in a Darwinian Evolutionary dynamic is as close to biology as to Financial Markets. Our model replication power, therefore, can be derived from the simple aggregation of agents with few properties or economic features [10]. As William A. Brock [11] asked in his study on the impact of different agents working together in the market, is it possible to constantly have asset prices reflecting all the economic fundamentals leaving no “free lunches” for investors? Or is there a chance that Market players expectations on the future deviate prices long enough allowing profitable opportunities to smart investors? Can we have bearish\(^1\) or bullish\(^2\) views that “cluster together” [11] and lead prices to spike or fall without any “rational evidence”?\(^3\)

Different Market expectations are indeed one of the motivation sources to the present study. Whenever we have agents making decision based on different activation functions, we are in a presence of a conflicts of interests [11,12], and contrarily views about future markets returns (having, therefore, opposite trading and investment decisions that leads ultimately, to opposite performance in the short-term). Having financial Markets framework in presence, agents are competing to have the strategy that can produce higher future expected risk-adjusted sustainable returns. In Farmer and Lo “Frontiers of finance: Evolution and efficient markets” [13] they pose a simple example: assuming 2 populations (from 1926 to 1976) where the first one can only buys the SPX Index and the second one is simply buying the U.S. Treasury bills (one of the safest securities in the world), the accumulated gain of investing 1 dollar is $1,370 and $14, respectively. However, just by letting agents to copy peers’ strategy and if perfect foresight was possible, the expected gain would be more than 2 million dollars. Therefore, an ecology of strategies seems to be relevant enough to study and measures its impact on individual and global performance.

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\(^1\) Pessimistic view of the Economy. Most of the times followed by a short-selling position in the Stock Market, meaning, betting that the market is about to fall;

\(^2\) Optimistic belief in the Market and Economy. Most of the times in line with a long position in securities underlying a upper movement in price in the recent future;
1.2 Objectives

Using SPX real historical data (from 1927 until 2018), we measure which of the main strategies is better, and which one is more likely of spreading across investors and Financial Professionals. Additionally, We investigate the disparity in income distribution of the agents in the population. Therefore, the investigation questions that the present problem can arise are the following:

1. Is there a Dominant strategy in Financial Markets?
2. As time goes by, Does the Dominant/dominated Strategies change over time?
3. Can Evolution and allowance to copy promote wealth distribution or it is promoting inequality in income distribution?

1.3 Structure

The structure of the present Part will be split as follows: Chapter 2 describes the state of the art in Evolutionary Game Theory, Behavioral Finance and Network Science dynamics as well as related work in economically driven Multi-Agents systems such as the experiment we are conducting in this study. Then, Chapter 3 addresses our proposal and methods employed, namely, the necessary analytic components in order to build a population that fits into the financial reality. Then, it is computed several simulations using real time data. Finally, in Chapter 4, it is presented the first results and further questions that those results brought, and Chapter 5 is where we debate our findings and the necessary next steps to be made in the presented area of study.
2 RELATED WORK

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Designing economic simulations in a multi-agent system is rooted in several different fields and areas of study. This multidisciplinary comes up naturally as we are dealing with a system made by different types of players when it comes to invest (field of Traditional Finance), where those players may not be the most rational ones since they are driven by behavioral human deceptions, such as biased preferences and personal choices (Behavioral Economics area).

Furthermore, Market players are making investment decisions in several marketplaces being more connected to those who are geographically closed, industry related, Wealth proximity (well-known as "league of gentleman"), among several other relevant features, as nicely described by Hellmich [14]. Therefore, it is also needed to report the related work in Complex Network Science to better describe the impact of such ties in Market prices and its fluctuations. Finally, all investors are thirsty not only to outperform their peers but mostly the overall market. Thus, they are intensively seeking to know other people thoughts about the current market environment and consequent investment actions in order to profit out of this marginal information. So, it is undeniable the need to reflect the state of art in Game Theory (or Evolutionary Game Theory if we are speaking about populations throughout time) to fully capture the essence of economic relations and the Financial Markets [14].

2.1 Questioning the Efficient Market Hypothesis

In the center of Economic Theory regarding Asset Prices is the Efficient Market Hypothesis (EMH). Firstly proposed by Eugene Fama et al. in 1969, EMH is the concept of having markets that quickly react and adjust to new information [15]. By this it is meant that efficient Markets are consistently "reflecting all the available information" [16], where their players are constantly processing the information rationally, ignoring the irrelevant one and avoiding systematic errors and deviations from the fundamental value of the Economy. This concept of "fundamental valuation" is the currently fair value of all assets in the economy and the present value of future expected income from current asset allocation. In other words, it measures how much the economy worths.

Since its origin, EMH began to be tested in the majority of markets and different environments and it has as many critics as supporters. A good survey related with this topic was written by Meredith Beechey, David Gruen and James Vickery [17]. On the top of critics of EMH is the definition of information availability and fundamental value of the Economy. On one hand, information is not costless to obtain [17]. Thus, Market players need to perform research and develop methods and tools to better capture what is happening in the market. Grossman and Stiglitz said that even if, by absurd, all the information was constantly reflected in asset prices, there would not be any incentive to obtain it, being price fluctuations close to a random walk which is far from being accurate [18].

After the important remarks obtained with Grossman and Stiglitz's research, Jensen proposed to
move from this strictly EMH to a weaker one (yet economically more realistic), whereas prices reflect the newest information at to the point where the gain (or the marginal benefit) on acting on the researched information does not "exceed the marginal costs of collecting it" [17, 19].

On the other hand, there is no such thing as a fundamental value of a Company, Market or Economy. Several models have being proposed, tested and rejected being difficult to spot a clear winner [17]. Nonetheless, few years later, Fama et al defended their hypothesis arguing that not only there is no alternative if EMH is rejected, since an economy can only work if investors have accurate information to reallocate optimally the available resources, but also not having a clear model to define the economy does not mean it is a chaotic unpredictable random walk [20].

It was not until the beginning of the XXI century that Capital Asset Pricing Model (CAPM) benchmarks previously stated started to be profoundly questioned by both psychological and economic researchers. Even though the criticisms were rooted in human behavior and known deceptions, it had an enormous impact in Finance and Economics. It is good to remember that in this same period, Daniel, Hirshleifer and Subrahmanyam presented statistical proofs that something greater than the Adam Smith "invisible hand" was moving the markets to odd "corners" [21]. Abnormal fluctuations in prices, spikes in volatility, long-term reversals\(^1\), short-term momentum\(^2\) and event-based returns were responsible to outperform what the EMH would predict. Therefore, a smart investor could exploit such deviations from the fundamental value. Moreover, in 2000 the awarded Nobel-Prize Robert Shiller published his "Irrational Exuberance" book, showing persistent deviations from a "normal path" in Stocks [22].

Despite lacking some more statistical evidences [23], a behavioral approach is getting widespread throughout all academic studies and research. This Behavioristic view of Financial Markets is rooted at rejecting a perfectly rational population of investors and limiting the power of arbitrage. The greatest contribution on the stated issues came from Kahneman and Tversky [24] whose impact in the economic theory is still today far from being accomplished. These psychologists’ earlier research helped to conclude that even in a mix population of fully rational investors with noisy traders\(^3\), the latter ones can be responsible for consistently deviating asset prices (causing for instance bubble cycles), being the first ones powerless to arbitrage\(^4\) those incomprehensible price fluctuations. Furthermore (and perhaps more important) their research made on choices prove that humans are at the most partially rational due to their beliefs and preferences [24]. These systematic biased and "bounded rationality" (concept firstly proposed by Nobel-prize-winning economist Herbert Simon) drive prices to a permanently quasi-random dynamic process due to the unstable and not universal investors’ expectations.

These two Economic approaches were from the very beginning completely separated, whereas the

\(^1\) Positive short-term auto-correlation between stock returns [9];
\(^2\) Well-known as "Overreaction", it is a negative auto-correlation between short-term returns and longer lagged ones [9];
\(^3\) Investors that somehow do not follow any perceived economic rule, being responsible to add "noise" or irrationality into daily prices.
\(^4\) Power to Arbitrage is comprehended as the power of smart investors to price correctly asset prices, driving irrational traders out of market;
Traditional Economics field is continuously blaming Behaviorists from not having models to capture reality while Behavioristic Academia is accusing economists from being “in love” with binary economic agents that are far from being accurate at describing Human Nature [6].

The main responsible of connecting these two contrarily worlds was, with no doubt, Andrew W. Lo. Within his paper on “The Adaptive Markets Hypothesis”, which can be described as the study of “Market Efficiency from an Evolutionary Perspective” [6], Lo launched a disruptive view of Markets based on “Socio-Biology” and “Evolutionary Dynamics”. His approach to Finance is heavily influenced by the principles of competition and natural selection between investors, market sentiment or investment strategies. By connecting the dots of multiple diverse areas of study, he developed a framework that allows economic agents to be modeled as a social entity (rather than a binary “Homo Economicus” [14]) with behavioral characteristics while operating in a constantly changing system far less homogeneous that what its predecessors believed [6]. For the present work, Lo’s research is by far a central piece. We used throughout this study, his main concept that “individuals make choices based on past experience and their “best guess” as to what might be optimal, and they learn by receiving positive or negative reinforcement from the outcomes (...)” [6]. Therefore, as long as the economy does not change, the present heuristic will converge eventually to “the most rational and optimal” decision. Yet, if something changes, individuals will need to learn from others at the expense of their fitness. That it is why it is important to study the link between financial realities and evolutionary dynamics.

2.2 Economics within Evolutionary Game Theory

2.2.1 Game Theory

After reaching this stage, it seems reasonable to do a step-back to specify how economic reality can be described under the laws of Game Theory (GT). Game Theory is the study on how outcomes change when agents are interacting with each other and making strategic decisions based on that. The first breakthroughs on GT were proposed by Von Neumann and Morgenstern [25]. Actually few years later, John Nash’s discoveries (one of the fathers of Game Theory) were deeply used in all sorts of economic studies and well-known dilemmas. His proposal was the idea of an equilibrium (which is a core concept in Economics) where none of the players have any incentive to change their strategy, regarding the co-player decisions [26]. The introduction of an equilibrium is crucial due to the fact that if investors have no gain in changing their strategy towards an asset price, it means that the market fully reflect, at that specific time, all the available information and stock prices’ converged to its fair valuation [6].

Nash Equilibrium is mostly used as a tool to predict social interactions’ outcome. Having, as reference, a 2-player 2-strategy cooperative-defection framework, where cooperation means being altruistic and engaging in a donation and defection is the choice of not to play. As it is clearly, seen the best
outcome to the overall society is when both cooperate. However, the best possible outcome to one player is when he defects, waiting to the opponent to cooperate. By doing so, this player will be able to receiving a benefit without paying any additional cost. More important it is to mention that after choosing defection, a player does not have any incentive to change his strategy, whatsoever other’s strategy (See Table below). Having each player maximizing his utility or payoff relative to other people decisions, both will end up defecting, being mutual defection a so-called Nash Equilibrium.

<table>
<thead>
<tr>
<th>Table 2.1: Payoffs of Prisoner’s Dilemma</th>
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<tbody>
<tr>
<td><strong>Cooperate</strong></td>
</tr>
<tr>
<td><strong>Cooperate</strong></td>
</tr>
<tr>
<td><strong>Defect</strong></td>
</tr>
</tbody>
</table>

Knowing that reality it is slightly different from a 2-by-2 scenario, Game Theory needed to be reconfigured to move from pair interaction to group interaction. Games with this extension are also called N-Person Games. Those are most of the times a generalization of Prisoner’s Dilemma with more than two persons. Nonetheless, in the prior framework we are neither considering what are the changes if both players would have played a series of games (instead of a one-off decision) nor the impact of a particular outcome on his/her future decision. This likelihood of different dynamics constraint to fitness and population characteristics are crucial to the “survival” of an individual throughout time.

### 2.2.2 Evolutionary Game Theory

The combination of payoffs and fitness allow us to empower and enlarge the previous model and its applications. By doing so, agents with greater payoffs (trough time) are perceived as more fit, leading to a higher level of reproduction (in the eyes of a biologist) or higher level of being copied by his fellows (in social sciences). Therefore, adding these new features brings us to Evolutionary Game Theory (EGT) where it is possible to evaluate evolution of agents throughout time [27]. A key aspect is the fact that EGT focuses more on the dynamics of the strategic change rather than on the properties of strategy equilibria and avoids a set of strong assumptions on rationality [10]. Therefore, the goal moved from predicting how agents behave when interacting with others like them, to the study of the evolutionary dynamics regarding how agents’ behavior changes through time.

Nonetheless, by considering a model that is played by in large populations of arbitrary compositions, there is the need to describe some necessary steps and notation. Let $e = (e_1, e_2, ..., e_n)$ be the set of all possible strategies to use, while $x$ is a vector representing the portion of the population using a given strategy, $x = (x_1, x_2, ..., x_n)$. Thus, we are interested to acknowledge the evolution of vector $x$, while individuals are making decisions regarding strategies.

Designing evolutionary dynamics within EGT is slightly different from modeling pure rational agents, that fully know the payoff structure of the game and choose actions following heuristics such as best
response. In EGT evolution is driven by the best individuals being copied more often than the rest of the population. Likewise, being copied more often can be directly translated by the increase of the fraction of individuals that share the most fit strategies. Mathematically speaking, this process can be processed through a replicator equation. This replicator tells us which fraction of the population (using a given strategy) will increase/decrease proportionally to the adaptivity provided. Therefore, strategies that yield some additional advantage to the population will spread faster as they are more copied over time. The replicator equation can be as the following:

\[ \dot{x}_i = x_i [(Ax)_i - x.Ax] \]  \tag{2.1} 

The presented formula can be simplified, since we are dealing with only two different strategies \( n = 2 \).

If M and F stand for momentum and fundamental populations, respectively, the system of differential equations in (5) become equivalent to:

\[ \dot{x}_i = x(1 - x)(f_m - f_f) \]  \tag{2.2} 

where \( x \) is the portion of momentum traders in the system and \( 1 - x \) the fraction of fundamental investors. Additionally, \( f_m \) and \( f_f \) are the fitness of momentum and fundamental strategies. Nonetheless, the equation above has some restrictions. Firstly, it only has a single solution given an initial fraction of \( x \) strategy and the distribution of strategies in the population. Secondly, it assumes a system with an infinitely large number of agents where all can interact with each other (in other words the replicator equation is configured for a well-mixed and fully connected network with infinitum amount of individuals). Due to the need of considering finite populations within scale-free networks, the next section will address the introduction of stochastic effects.

### 2.2.3 Stochastic Effects in Evolution

Moving from an infinite population to a computer EGT simulation based on a finite population forces the introduction of a stochastic process. With infinity populations we know, deterministically, the dynamic of the population (using the replicator equations). Nonetheless, for finite populations it is needed to consider the probability of an agent moving from one strategy to the other.

In order to implement such a method, it is needed to generate a "pairwise comparison rule" [28]. With this rule it will be determined the probability of an individual changing his strategy, based on his fitness and after observing another agent’s characteristics. Likewise in many other studies [28, 29], in this work it is proposed to use Fermi function as a pairwise comparison rule:

\[ P(x \text{ imitates } y) = \frac{1}{1 + e^{(f_x - f_y)}} \]  \tag{2.3} 


As seen in the equation above, when using this update rule, agent \( x \) imitates agent \( y \) with a probability proportional to the gap between agent's fitness and the one he is comparing to \( (f_x - f_y) \).

Note that if both have the same fitness, the imitation probability is 0.5, implying that fitness does not influence the decision making process of changing agent's strategy. Moreover, \( \beta \) parameter represents the strength or intensity of population selection. For lower values, imitation will be close to happen randomly, whereas for larger \( \beta \), imitation will strongly depend on the differences between fitness [29].

### 2.2.4 Financial Evolutionary Game Theory

There is a profound research to be mentioned regarding Economic EGT applications, with a special attention to Daniel Friedman works [30,31]. Dedicated to prove that Evolutionary Stable Strategies (ESS) could be a concept useful in economics, he presented several studies to demonstrate his point of view.

A population in a ESS is the situation where not only there is no incentive to change strategy, but mostly that even if a small population of "mutants" invade with an abnormal behavior, they will eventually disappear due to lack of fitness and natural selection.

Despite being rooted in Biology, as aforementioned when we are dealing with economic realities, it is undeniable the need to move from this "mathematical" genetic mechanism to a more complex learning and copy mechanism where social interactions and a broader imitation processes variety [30] can exists.

In Daniel's study, it was developed a population whose actions were a product of a strategy profile set. To define those strategies he mentioned several examples with a growing level of complexity. If, for instance, in a population of Buyers and Sellers where the latter ones can cheat or be honest about what they are selling and the first ones be naive or suspicious, each one of the elements of the populations desire to know who they are dealing with to take advantage out of it (cheating if far less attractive to sellers when they are dealing with more suspicious buyers). With this setting and a couple of differential equations the only stable solution is where both populations are fairly mixed [30].

Nonetheless, in the prior studies presented, several non-real assumptions were made. In the Financial Industry, market players are very diverse in their actions, beliefs and risk preferences. Moreover, investors tend to "accurately" perceive other people's fitness (having a preferential attachment to those who are recently wealthier), and finally individuals connections are way more persistent (as an example, Friedman used random matching in both of his studies [30, 31]), not only due to shared strategies, but also to geographical constraints, wealth, among several others.

Knowing the diversity of trading investment profiles, additional studies were performed to better picture whose profiles are more determinant and how they evolve throughout time. William A. Brock et al. [11], under the notion of a theoretical Adaptive Belief System (ABS) [12], developed an artificial market with several competitive (and different) buyers and sellers that can randomly interchange their beliefs.

---

5Concept brought by biologists to ADN and Cell Studies. As for this work it is understood as a simple change of opinion;
and actions based on two different asset classes (one relatively risk-free and one close to a stock market index, thus riskier). These works shown that as we add more “intensity of adaptation” or diversity of beliefs in those market types, price deviations or fluctuations appear to be more persistent relative to fundamental benchmarks and responsible for generating excess volatility [11]. Another relevant discovery was the fact that model accuracy to a real-live market vanishes as they introduce more players into the simulation. However, Diks and van der Weide [32] heavily criticize their approach because the distribution of “available” strategies are dependent of price fluctuations and price dynamics. Since agents based their preferences and choices on past performance, belief distribution changes with realized gains/losses. This is done intuitively as new price information arrives, investors make a re-evaluation of the strategies and update their decision-making process [32].

Nevertheless, as aforementioned, updating information anytime something change is costly. Few Market players have the capability of reacting to every event in the Stock Market. The reasons for the stated come from quite different sources: from Economic to population and behavioral constraints. On one hand, some players (as Mutual Funds and Trackers) are by construction constraint to follow strict rules regarding investment. As Chan pointed out on his survey about the main characteristics of major Market players, those who are constraint tend to broadly cluster their investment strategy to the overall market performance, without taking quite extreme positions and preferentially favor higher past winners [4, 33]. Therefore, even if the presented strategy is not the optimal under a given time horizon, several investors are “stubborn” fundamentalists or trend followers [10].

On the other hand, as we are going to see in the next section, the topology and the relations between the several moving pieces matter to fully replicate real-live markets. Individuals tend to copy relatively closed neighbors when they perceive an improvement in his expected future fitness [14]. Moreover, it is of capital importance to realize the network inner relationships that happen in real-life markets and whose are the individuals (or nodes) with a greater access to information, the importance of initial neighbors and starting day, lack of adaptability to the quick changes in the environment, behavioral biases towards supporting bubbles or crashes, among several other.

2.3 Population of Investors in Complex Multi-agent Connected Networks

2.3.1 Complex Network Science

Despite being relatively new, Network Science has proved to explain and characterize multiple real-life examples. Theorized by Erdos and Renyi, when they studied Random Graphs and its stochastic properties, it was only in the late 90’s that the first systematic studies were developed [34]. Modern
Complex Network Analysis was founded on D. J. Watts, S. H. Strogatz, A. L. Barabási and R. Albert research and is used from Biology and Computer Science to Social Sciences [7, 35, 36].

A network or a graph is simply a diagram of sets of nodes connected through links. For instance, "WWW is a network of web documents linked by URL's" or a "society is a network of individuals linked by family, friendship or professional ties" [36]. Modern techniques and growing computational power were important to understand scale-free networks composed of millions of interconnected parts.

Until this stage, most studies presented so far assume that all agents can interact freely with any other individual. Yet, well-mixed populations, where there is an equal probability of interacting with any element of the network are rarely found in real examples. Furthermore, the well mixed assumption ignores the fact that social clustered interactions, where most people interact with only a limited number of people. In short, real-life Populations are highly structured. Therefore, it is possible to study these kind of populations with the help of network science and graphs, where nodes are individuals that can only interact with those who are linked to.

2.3.2 Networks driven by Economic Agents

Well-ahead of the idea of combining Network Science with Economics, most economic thinkers were already somehow linking economic agents. Perhaps the biggest example of such was after the 2008-Crisis where most prominent Market participants alert to the need of saving Global Institutions because "they were too big too fail". Griffin’s Testimony to the House Committee [37] was one of the first mentions to the concept of "too interconnected to fail". Actually, based on such a wave of questions regarding the reliability of Financial Institutions, we performed a study on companies’ interconnection (that can be seen in Appendix 1) and discussed if current tools are able to handle distress periods, crashes and excess volatility. Despite being a recently new research topic, some studies deserve a place in the present project. One of the most interesting works with Networks and Finance, was done in 2012 by Leonidov and Rumyantsev [38]. Within their work, we can see the roots of combining agent-based models in a financial network. They define a network with all the Russian banks and its interdependences regarding credit, where the in or outgoing links define the relationships of being a net borrower or lender, respectively. They conclude that with only an additional 4% increase in their capital requirements, more than 8 banks would automatically collapse. Nonetheless, they are assuming static ties. Babus [39], however, studied the impact of dynamic relations in OTC markets. In the Over-the-counter (OTC) context the inner relationships are crucial for investors and banking institutions. Without a good network they cannot find the best prices and deals. Combining these two studies it is easy to conclude that the structure in each investors are in matters when it comes to make an investment decision.

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6 capital requirements is the amount of money banks need to have to face stress periods, shortage of credit or excess of liquidity demand

7 OTC = Over-the-counter markets where there is any stock exchange being investors responsible to find buyers and sellers
3.1 Investors Decision Making Process

In order to understand how Markets behave, it is crucial to define how agents react to changes in the environment. Based on the Chan's book [40], a trading strategy is a pre-set of methods to make money out of the fluctuations of the market. Defining investment strategies is challenging since the disparity in types of trading profiles. Yet, a strategy can only be profitable if it is either mean-reverting or following a trend. If we are in the presence of a random market whose prices move without any specific order, trading will be pointless and investment strategies will perform no better than a toss of a coin. Therefore, Market participants can be grouped in 2 different profiles: Momentum and Fundamental Investors.

Before diving into the explanation of such strategies, it is important to stress that Market Agents are free to either buy or sell any asset, without a specific order, meaning, short-selling is allowed. In practice, short-selling is a naked sell\(^1\) where investors are selling an asset they still do not have. In short, short-sellers are betting the price of the asset is about to fall and they will buy it back cheaper later on. Moreover, the usage of daily prices seems limited considering Financial Market complexity. Nonetheless, it is the belief of this proposal that investors tend to rely on prices to make investment decisions. In Karl Marx's Capital [41], it is mentioned as a major criticism of Capitalism the fact that 2 products with the same price are perceived as similar despite the differences in sources of value (like the intensity or type of labor used). Therefore, even if some Market players use complicated methods and models to produce their investment “activation function”, those tools tend to be dependent and mostly a function of past prices.

3.1.1 Strategy Based On Past Performance

When it comes to designing any investment strategy, it is fundamental to define a reference point. Without having a clear fundamental value of the economy, today’s fair valuation can only be priced relative to past performance and to future expectations. Therefore, investors compare today’s prices to a given look back period in time. This oversimplification of a moving average is the same as to say that the cheapness or expensiveness of a given asset is constraint to what it has been doing in the past. So, despite the strategy used, investors have a specific look back period, \(\kappa\), that they used at performing investment decisions. For instance, if a given investor has a \(\kappa = 25\), then his decision of buying or selling in the Stock Market will be derived from the relation between today’s price and the price of in \(\kappa\), meaning, \(P_t - P_{t-25}\). Therefore, Investment trading decisions can be summarized as two contrariwise boolean activation’s functions, such as, \(b(P_t, P_{t-\kappa})\) and \(s(P_t, P_{t-\kappa})\), for buying and selling, respectively.

\(^1\)Naked Sell happens when an investor wants to sell the asset before acquiring it. This can be done by borrowing from someone else that own the asset accepting a small fee in exchange;
3.1.1.A Momentum Investment Strategy

Market Agents that use momentum strategies are those in favor of “surfing the trend waves”. Using their own indicator ($\kappa$), they are willing to buy if prices are trending up relative to historical prices, and sell if they are already falling. Their reasoning comes from believing that the aggregate market knows better where the value is. Thus, a single investor can only follow Market tendency and buy/sell what is fashionable. Consequently, Momentum investors are responsible for increase bubble cycles if prices are rising and augment panic cycles in the Stock Market when prices are going down.

\[
b(P_t, P_{t-\kappa}) = \begin{cases} 
  1 & \text{if } P_t > P_{t-\kappa} \\
  0 & \text{if otherwise} 
\end{cases} \\
(3.1)
\]

\[
s(P_t, P_{t-\kappa}) = \begin{cases} 
  0 & \text{if } P_t > P_{t-\kappa} \\
  1 & \text{if otherwise} 
\end{cases} \\
(3.2)
\]

3.1.1.B Fundamental Investment Strategy

Contrariwise to Momentum, Fundamental investors have an opinion on Market Valuation. They think the economy has an intrinsic value and it is more robust than the “irrational” short-term ups-and-downs. Therefore, they follow a trend reversal strategy. Fundamental traders invest on the logic of capturing the local minimum/maximum, believing that if a given stock is trending up will eventually converge to a broader moving average which is its fair value. So, using the same reasoning as above, with an indicator ($\kappa$), they are willing to sell if prices are trending up relative to historical prices, and buy if they are already falling. Agents following such a strategy are those that are betting against big swings in prices and trust in price stability over time.

\[
b(P_t, P_{t-\kappa}) = \begin{cases} 
  0 & \text{if } P_t > P_{t-\kappa} \\
  1 & \text{if otherwise} 
\end{cases} \\
(3.3)
\]

\[
s(P_t, P_{t-\kappa}) = \begin{cases} 
  1 & \text{if } P_t > P_{t-\kappa} \\
  0 & \text{if otherwise} 
\end{cases} \\
(3.4)
\]

Finally, we also considered three additional strategies: "Random Buy and Sell", "Random Strategy" and "Buy and Hold" strategy. As it says the name, a "Random Buy and Sell" strategy is the one where an investor $i$ has the same probability of buying and selling and any investment time horizon ($\kappa$). On the other hand, an investor following a "Random Strategy" is indifferent between following any of the two previously stated strategies. Thus, with the same probability chooses a strategy and his corresponding action. Lastly, a "Buy and Hold" strategy is where an investor buys an asset and holds it until the end. Thus, it only does an initial buy and a final sell, holding a given asset throughout the time.
3.2 Our Proposal

In the present work, we proposed a sequential dynamic model composed in three stages. Focusing on an arbitrary agent or investor \( i \), these stages can be described as follows:

1. Investor \( i \) based on his strategy, takes an action of buying or selling a given product (in the present study it is used as asset to be exchangeable, the S&P 500 Index\(^2\));

2. Based on his trading operations, a daily positive or negative income flow is added on his fitness, based on the following equation:

\[
 f_t = f_{t-1} \times \text{Return}(t);
\]

\[
 \text{Return}(t) = 1 + \log \left( \frac{P_t}{P_{t-1}} \right) \times \text{Strategy}(t-1);
\]

3. After the market is closed for the day, Investor \( i \) has the possibility of copying a random neighbor if the gap in fitness is large enough (using a \( \beta \) of 1):

\[
 P(i \text{ imitates } j) = \frac{1}{1 + e(f_i - f_j)};
\]

3.2.1 AKKA: Programming concurrent Agents

The present model was implemented in JAVA language (Version 8). In order to fully create separate agents, we used AKKA toolkit. AKKA is designed for the development of applications that are willing to run in concurrency, and especially, to generate/create agents that fully run in parallel (taking advantage of threads) [42]. For that it uses as milestone a general concept of an “actor”. AKKA’s actors are nothing but runnable entities where each one of them is running in a thread. This is really important to distinguish from using simple JAVA objects, when those just “follow the script”, while agents have a choice (“Objects do it for free, agents do it because they want to”) [43]. Our main class, denominated “Agent”, extends Actor JAVA class. Since they are running separately, Actors in AKKA communicate through messages. For instance, using our model we have several agents (or investors) communicating between each other and sending messages between them and the Market, as they would be doing in the real live by sending their buying or selling order into the Stock Exchange.

The combination of those two main features (agents that can run in parallel and the possibility of exchanging message in real time), led us to the conclusion that the inclusion of AKKA into our study, will not only empower our modeling accuracy as well as improving our efficiency in terms of time and resource consumption [42].

\(^2\)Despite the fact of being unrealistic having a population of investors focused on buying/selling one single product, it can be understandable as a study that it is only interested in capturing investors’ trading decision on a given index either through buying any Index constituent or an Exchange-Traded Fund that aggregates Market members.
3.2.1.A Market Model-Base

**Figure 3.1:** Market Model Structure

**interaction:**

- **Send**: *Open Notification*, *Close Notification*;

- **Receive**: *Trade Request*, *Reward Request*, *Price Request*, *Table Request*;

**Market States:**

- **Idle** or Initial State: While all the model components are created, being launched for the first time afterwards;

- **Open**: Market notifies all investors that it is open. During this state, it receives *Price Requests* (where investors ask for current and historical prices) and *Trade Requests* (Investors send their Buy/Sell orders to the Exchange). After receiving investors orders, Market state changes to closed.

- **Closed**: Market sends a notification to all its participants that it is no longer opened. Thus, investors now can ask their daily performance (though a *Reward Request*) and if agents are interested in changing their strategy they can request a general picture of the Market (through a *Request Table*). After all investors acknowledged their daily profits and losses, a new trading day starts by changing Market State to Open.
3.2.1.B Market Trading Model

Having a trading system efficient and capable of quickly match orders and allow all the participants to execute their trading and investment decisions can take many formats. Several prior studies and researchers discussed the impact of different auctions and types of exchanges in order to promote more liquidity, higher transacted quantity and lower prices [4,9,44–48]. In the current project we are assuming an exchange without any intermediary and/or broker, being every investor responsible to send their order to the market. Thus, what matters is the amount of buyers versus the sellers, assuming the price is given by the exchange.

3.2.1.C Investors Model-Base

Figure 3.2: Agent Design Model

interaction:

- **Send**: Trade Request, Reward Request, Price Request, Table Request;
- **Receive**: Open Notification, Close Notification;

Agent States:

- **Awaiting Market Opening**: In the present state, the investor is waiting Market opening notification. After receiving it, it sends a Price Request and it changes to Awaiting Prices;
• **Awaiting Prices**: Investor is on hold, waiting for the Prices he ask to the Market. After receiving an answer, the agent defines its investment trading decision (based on its own Strategy) and sends to the Exchange/Market its order (through a *Trade Request*). After completed, it changes to *Awaiting Market Closing*.

• **Awaiting Market Closing**: In this state, investors are waiting the closing bell notification from the Market. At receiving it, they make a *Reward Request* to acknowledge their daily performance in order to update their fitness. After completing this stage, it changes to *Awaiting Reward*;

• **Awaiting Reward**: Awaiting for his daily performance that it will be used for his fitness update function. After receiving it, through reconsider function, the investor decides if it is better to copy someone else strategy (by asking it to the market, through a *Table Request* and moving to the *Awaiting table* state) or if he is good to start trading again (moving back to the state *Awaiting Market Opening*);

• **Awaiting Table**: After deciding to reconsider its strategy, investor is now waiting the answer to the *Table Request*. After receiving it, he picks an investor (following Pareto rulers and followers already explained in prior sections) and takes the decision if he desires to copy other investor’s strategy. After that, he return to the *Awaiting Market Opening*;

3.2.2 **K-Strategy and N-Person Game in Large Populations**

One of the most important concepts in these kinds of models is the rule of action dependency, meaning, what an agent does affects the overall game. Nonetheless, player’s impact on the game is dependent of his/her degree of magnitude, like fitness, public knowledge, money, external recognition, among several others. Therefore, by design, as more agents are in the game the lower is the impact of a single agent. In the limit, if in a given game there would be an infinity number of players, each agents’ impact would be approximately zero. Following the same reasoning, in our experiment it is assumed as assumption that no matter how many agents our model has or how rich (higher fitness) an investor is, it is impossible to move World Market Prices.

Academia still lack for an answer of how Market players accurately access and process information to be fully integrated in stock prices. Yet, this happens not because investors are incapable of doing an accurate analysis but mostly due to differences in expectations, time restrictions and behavior biases. As briefly explained, expectations have a central role in economics. Current prices are biased to investors’ expectation on future economic and financial developments. For instance, Apple Inc. stock price is not only dependent on current iPhone sales or the number of streamed musics on iTunes, but also on investors expectations regarding future sales, market share, future developments in the competitors, etc.
Thus, such expectation analysis is severely dependent of each investors opinion and valuations. Therefore, even in a simple buy/sell World, different expectations make investors with very similar investment strategies differ on the Market side in a given asset. Since it is the purpose of this study capturing this crucial feature, it is going to be used as knowledge base studies made under games in a EGT World with different strategies. Our hypothesis is that just by changing the period investors are past-dependent (namely having several different strategy $\kappa$) the model gets closer to a real-life Market, which is in the first place our goal and objective. By K-Strategy is meaning an investor $i$ with $\kappa = 25$ will take his decisions based upon the price of the asset 25 trading days ago, whereas a strategy with $\kappa = 500$

### 3.2.3 Bloomberg Database

In order to populate the aforementioned strategies, it is needed real time data. In the present work, we are using throughout the entire study (described in detailed in the previous sections) one of the most important Index Stock Exchanges in the World: S&P 500 SPX (the aggregation of the 500 largest companies in the United States) from Bloomberg database\(^3\). Considering real life data is vital in financial modeling. All Market agents derive their decisions based on prices’ past performance as well as their wealth is constraint to their trading decisions on those prices. Using as an example a investor that is now following a momentum strategy, his fitness (or wealth) changes every day according to the correctness of his trading decisions. If he is buying a stock and then the Market crashed, his fitness is about to collapse.

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\(^3\)which is on the most accurate database when we are dealing with financial data
RESULTS

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4.1 Homogeneous Populations

Consider a population of $10^4$ similar individuals, whose initial fitness $^{1}$ is set to 100 and all sharing the same strategy. Within such simulation framework, coping each other strategy does not have any impact, since all agents have the same profile. Nonetheless, it is important to acknowledge which strategy generates the highest payoffs in the period under study. This analysis offers a baseline reference to the study of heterogeneous cases.

As summarized in Table A.1 and shown in Figure 4.1, we can argue some of the strategies are considerably better relative to its peers, individually speaking. One can say that this may indicate that if we mix these strategies into a well-mixed population, dominant strategies must appear. Momentum Strategy seems to constantly outperform a fundamental one. This confirms the well-documented Carhart's Momentum factor $^{49}$ proving that following a momentum strategy yields superior risk-free returns, all else equal. Nonetheless, it is important to refer the fact that the performance of a strategy is not constant throughout time. For instance, an investor $i$ in the period of 1998-2008 that blindly followed a Fundamental strategy would have higher payoffs than any other strategy, for the same period. This results can

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$^{1}$It can be understood as the monetary disposition that an agent has to invest in a given period of time.
be easily confirmed by the data, since it was the period of the "dot.com" bubble and the Financial Crisis in 2008. Finally, it is interesting that, relative to a simple "Buy and Hold" investment reasoning, most strategies not only under-performed but also lost money in most of the sample periods, as described in Figure 4.1.

4.2 Semi-Heterogeneous Populations

In the present section, we study the impact of population dynamics and evolution. If a strategy is far better than any other, it should spread and be increasingly adopted by the agents in the population. Again, we are considering a population of $10^4$ similar individuals, but assuming that 50% of them were initially following one strategy and the rest another one. For instance 50% were Fundamental investors while the rest were momentum traders. We are now considering a population with a semi-heterogeneous profile assuming that agents can only adopt one out of two strategies and individuals with the same strategy are equal. Additionally, every agent is allowed to pick in a given time-stamp any random investor to (re)evaluate his investment strategy\(^2\). Therefore, the number of interactions (and possibilities of changing strategy) is equal to the number of periods of each simulation. In the case of the entire sample, the number of possibilities is above the $2.3 \times 10^3$ daily iterations. Within such context, we measured the impact of the under and out-performance of a strategy relative to the other. Moreover, one can evaluate how fitness distribution is across time and test how the population evolves.

4.2.1 Momentum Strategy Dominance

Given the confirmed fact that, individually, Momentum outperform most strategies throughout time, we tested whether that would be valid if agents can copy other investors' investment decisions, leading to a convergence to a fully Momentum population. Thus, we run a set of 100 simulations for 3 different initial settings: Fundamental versus Momentum, Random Buy/Sell versus Momentum and finally, Buy an Hold versus Momentum. Yet, it is important to mention that we are assuming fully-committed agents that live forever and the number of players is fixed. Thus, we are neglecting some real-life aspects of appearance and disappearance of market players related to their wealth/fitness.

4.2.1.A Fundamental Strategy Versus Momentum Strategy ($\kappa = 50$)

As it can be seen in Figure 4.2, after running no more than $10^3$ iterations, Fundamental investors are completely dominated by Momentum investors. The results seem to be in line with both Figure 4.1, Table A.1 and Carhart's Momentum factor [49], given the discrepancies in performance. Furthermore,

\(^2\)It is assumed a complete graph where all nodes are linked between each other.
after confirming momentum dominance, we were also interested in understanding the behaviour of fitness/wealth or income distributions were, on average, for the simulations made. As displayed in Figures 4.3 and 4.4, where we have the cumulative log-log income distribution, we can argue that we are in the presence of a power-law distribution or Pareto Law income inequality, where a few wealthy nodes accumulate most of the earnings from the market whereas the large majority has relatively under-performed, for the period in study.

The presented Figures 4.3 and 4.4 allow us to make some comments about the inner characteristics of our financial model simulations. In the first place, however incipient, allowing individuals to copy other people strategy changes the population dynamics. We can argue that whenever the model introduces evolution, income distribution changes. Secondly, comparing both Figures 4.3 and 4.4, we can see power-law tend to smooth as agents can copy someone else way of investing. For the same period, adding the presented feature, it empowers equality on investors fitness. Therefore, one can say that evolution tries to mitigate the “rich gets richer” effect, where flowing most of the profits from the markets to the ones that are wealthiest. This “evolution effect” can be somehow explained by the simple fact that, by allowing less fit agents to copy others’ strategy, the likelihood of also performing better increases.

Figure 4.2: Full Momentum Strategy Dominance (With an initial condition of 50/50 Fundamental and Momentum Strategy, \( \kappa = 50 \))
4.2.1.B Random Buy/Sell Versus Momentum Strategy ($\kappa = 50$)

Financial professionals tend to constantly emphasize the fact that active managing is always safer and more profitable than leaving our investment decision into inexperienced hands. Thus, in order to test it, we run a set of simulations where half of the population was initially Momentum, whereas the other half is performing a Random "Buy/Sell" trading decision.

The results are shown in Figure 4.5. Momentum is outperforming a simple random investing strategy (dominates most of the times). However, it takes longer time to dominate this strategy than dominating a Fundamental population (as detailed above). On the other hand, in close to 15% of the trials, it fails to achieve a total dissemination of the strategy (despite being relatively greater the percentage of investors being Momentum) and in 4% of the runs gets dominated by the random population. Yet, it might be argued that the presented results are an additional proof that a consistent strategy can do better than accepting financial time series follow a random walk and prediction is, as a consequence, fairly difficult. Nonetheless, to understand if active managing of financial assets is more profitable than accepting Market swings, we need to test it against a passive strategy, as a "Buy and Hold" described in detailed in the next section.

4.2.1.C Buy and Hold Versus Momentum Strategy ($\kappa = 50$)

As we tried to briefly explain in the related work and state of the art, academia and the industry show results proving and disproving the likelihood of beating the Market. One can say that beating the market is performing constantly above the average gain of a Buy and Hold Strategy, where an investor simply
accepts what the market is doing. Within the context of our over-simplified model, we wanted to test if our strategy winner (Momentum) was indeed capable of dominating by spreading into a population of "holders". From Figure 4.6, it can be said that Momentum still dominates more than 75% of the times the Buy and Hold Strategy, despite being dominated almost 14% of the runs and being relatively lower in the number of players whenever none of the strategies were able to spread out completely (which being, in this case, the strategy with only 44% of the investors). Moreover, taking a careful look into the chart, we can also point the fact that due to our sample data, Momentum is far better at surfing crisis relative to an agent that it is holding an asset despite the market characteristics (which is exactly the beginning of our sample, with 1928 Market Crash). However, if this strategy cannot spread out to every agent, as time goes by Momentum Strategy loses its "momentum" or edge and "Buyers and Holders" become more popular among their peers. The results here presented go somehow in line with those who believe Markets are more predictable than what economists defend, and some strategies can outperform the market and beat it. This is quite important because it questions the so-called "Efficient Market Hypothesis" whereby share prices reflect all the available information and consistent risk-adjusted superior return generation is, if not impossible, a question of luck.

Figure 4.5: Momentum Strategy dominates 86% of times, Random Buy/Sell dominates in 4% of the runs and in the 10% left population follows on average more a momentum than a random strategy (58.5%)
Figure 4.6: Momentum Strategy dominates 75.86% of times, Buy Hold dominates in 10.34% of the runs and in the 13.8% left population follows, on average, more a buy and hold strategy than Momentum (44.05%)

Figure 4.7: Log-Log Income Distribution over a Population with a 50/50 Momentum and Buy and Hold Strategy without evolution (allowance to copy others). Each line represents a decade starting in 1927 and finishing in 2018

Figure 4.8: Log-Log Income Distribution over a Population with a 50/50 Momentum and Buy and Hold Strategy with evolution (every agent is allowed to copy). Each line represents a decade starting in 1927 and finishing in 2018
Furthermore, when plotting the differences between population dynamics with and without allowing investors to copy someone else strategy, we confirm once more that evolution brings a population more similar (in terms of fitness distribution), allowing some players with relatively worse strategies to change their losing strategy and outperform from what it would have happened if they were isolated. On one hand, these arguments can be somehow hard for “Darwinist people” where evolution is just the “survival of the fittest”, whereas in this case results suggest that evolution leads to an improvement also of those who were not the fittest. On the other hand, at the expense of the stated, we can also see that the richest agents earn more than without the possibility of copy. One can say that the factor that took place was that some of the investors changed their strategy to local maximum rather than keeping the strategy that was best for the entire period. For instance, during the 80’s, Momentum Strategy was consistently under-performing close to one third of what a Buy and hold was doing. (See Figure 4.1 and Table A.1). Thus, the best investment policy is, not only picking the winning strategy for the entire period, but also changing it accordingly to the environment in presence. Therefore, this can only be done by allowing strategy copy and this was responsible to empower some agents’ performance, as seen in Figure 4.8.

4.3 Heterogeneous Populations

Adding the possibility of having many different investment profiles is moving a step closer to the real-live markets. Firstly because, bearing in mind the amount of different types of players that Financial Markets can have, it is challenging to accept that they all share the same reasoning when it comes to investing. Secondly, being difficult to access the investment time horizon an investor has, sampling a larger set and leaving the work in hand of evolution, is, if not accurately what happens in reality, the closest guess we can find. As in the previous contexts, we considered a population of $10^4$ individuals, but giving to each one of them a “unique investment profile” ($\kappa$), ranging from 1 to 500 trading days. The chosen range was thought to include a large spectrum of investors, since High-frequency traders to the most conservative funds. Therefore, instead of having a population with only two ways of looking into the environment and act, now we have 1000 different strategies, where half of them follow a Momentum mindset of trading, while the rest is Fundamental investors.

4.3.1 Momentum Strategy Dominance

Having a population with a larger set of strategies, it is not only interesting to understand which (if any) of the strategies dominates but also which is the prevailing and most profitable investment time horizon ($\kappa$).
4.3.1.A Using Real Data

In the first place, as predicted and pictured in Figure 4.9, Fundamental investors are getting dominated as time goes by if they are allowed to copy Momentum traders. Far more interesting, is perhaps, the persistent initial variability in strategies and the greater amount of time that a Momentum Strategy need to dominate relative to the previous stage where all agents shared the same profile ($\kappa = 50$). This fact may indicate that the existence of a diverse ecology of investment look-back periods give some strength to Fundamental population, even though not enough to dominate Momentum.

Figure 4.9: Full Momentum Strategy Dominance (With an initial condition of 50/50 Fundamental and Momentum Strategy, with investment look-back period ($\kappa$) ranging from 1 to 500 trading days)

Secondly, it is interesting to see that evolution (allowance to copy relative to full-committed agents) is now not so different comparing to previous examples, as displayed in Figure 4.10 and Figure 4.11. Income distribution, whenever we have a more diverse set of strategies, does not change qualitatively when evolution takes place, i.e., the "copy effect" seems to averages out.

4.3.1.B Strategy Updating based on Investment Profile

Some of the results presented seem, to some degree, unrealistic when confronted with financial markets reality. On the one hand, there is a percentage of investors that are most times stubborn and not willing to change their investment position. On the other hand, it is not reasonable to compare the rate of
strategy change between an investor that only looks to yesterday’s asset price to make a trading call, relative to a conservative investor that looks to a broader period. Thus, we simulated what would be the impact of creating a population with heterogeneous and random investment profiles (different look-back periods $\kappa$ ranging from 1 single day to 500 trading days). Having said that, within such environment, it is possible to study the impact of having agents with different update rates. We assume that those with lower $\kappa$ will be faster at looking to their neighbors to re-evaluate their strategy, following the equation:

$$
Prob(Reconsider) = \begin{cases} 
1, & \text{if } Day \mod \kappa = 0, \\
\frac{Day}{\kappa}, & \text{Otherwise.} 
\end{cases}
$$

(4.1)

Thus, despite its own strategy, while an agent with look-back period ($\kappa = 1$) will re-evaluate its trading and investment decision every day, another and different investor with a ($\kappa = 100$) will be more reluctant to change with small daily variations (because he is willing to wait for the big fundamental moves). Yet, despite the changes presented, population dynamics seems to be quite resilient and it converges, as time goes by, to a fully Momentum Population. However, one can say that after adding the present feature of delayed updates, convergence is harder. This may indicate that Fundamental strategy benefit from delayed updates, avoiding to quickly copy its neighbours. In terms of income distributions, we do not see fundamental differences when delayed strategy change is implemented (Figure 4.14) relative to daily copy (Figure 4.11). Nonetheless, as in the previous examples, the power-law exponent (alpha) ranges from the delayed version are different from the ones without the possibility of copy investors’ strategies (moving from [2.88; 6.44] to [3.32; 4.15], as seen in Figure 4.13 and Figure 4.14).
Figure 4.12: Full Momentum Strategy Dominance (With an initial condition of 50/50 Fundamental and Momentum Strategy, with investment look-back period ($\kappa$) ranging from 1 to 500 trading days)

Figure 4.13: Log-Log Income Distribution Without Evolution and Strategy Copy (With an initial condition of 50/50 Fundamental and Momentum, with investment look-back period ($\kappa$) ranging from 1 to 500 days; every agent is allowed to copy). Each line represents a decade starting in 1927 to 2018.

Figure 4.14: Log-Log Income Distribution under Evolution and Strategy Copy (With an initial condition of 50/50 Fundamental and Momentum, with investment look-back period ($\kappa$) ranging from 1 to 500 days; every agent is allowed to copy). Each line represents a decade starting in 1927 to 2018.
4.3.1.C Using Periodic Data

The usage of real life examples is, in our view, a good approach to get closer to what a real financial agent would have won or lost if he was participating in the live Financial Market environment. Nonetheless, we find relevant to measure what would have been different in the dominant strategies and income distribution if the environment was structurally different. This allows us to isolate the effect of having an upward trend in the financial data (that could benefit some trend-following strategies, like Momentum).

For this purpose, we build a periodic time series (shown in Figure 4.15), with period equal to $40\pi$, which is approximately 125 trading days, described in the equation below:

$$Price_t = 100 + 50 \times \cos(0.1 \times t) \times \sin(0.05 \times t)$$ (4.2)

![Figure 4.15: Periodic price time series (period = $40\pi$, which is approximately 125 trading days)](image)

Considering the periodic time series generated, the results are now different. In the first place, we see the disappearance of Momentum Strategy as the winner and consistently dominant above all the others. In the present case, as displayed in Figure 4.16, both Momentum and Fundamental dominate in a close to 50% of the times. Having this function to determine Market prices, the strategy that, by chance, gets more trendy in the beginning of each simulation, appears to dominate in few iterations. Furthermore, income distributions are no longer what we have seen in previous examples (See Figure 4.17 and Figure 4.18). Within the present time series, power-law style distribution where inequality is the main factor, seemed to disappear in both cases (with and without allowance to copy a peer Strategy) and not changing with the introduction of allowance to copy another's strategy.
Figure 4.16: Momentum Strategy dominates 41% of times, Fundamental Strategy dominate in 44% of the runs.

Figure 4.17: Log-Log Income Distribution Without Evolution and Strategy Copy (With an initial condition of 50/50 Fundamental and Momentum, with investment look-back period (κ) ranging from 1 to 500 days; every agent is allowed to copy). Each line represents a decade starting in 1927 and finishing in 2018.

Figure 4.18: Log-Log Income Distribution under Evolution and Strategy Copy (With an initial condition of 50/50 Fundamental and Momentum, with investment look-back period (κ) ranging from 1 to 500 days; every agent is allowed to copy). Each line represents a decade starting in 1927 and finishing in 2018.
4.3.2 The impact of Look-Back Investment Period ($\kappa$)

Most of what was discussed so far relied on the fact that Momentum Strategy is dominant in most environments (keeping in mind that those results are biased towards the presented time series). Despite the introduction of evolution, other strategies cannot perform and spread as effective as Momentum. However, as we discussed earlier, financial population is quite diverse in strategies and profiles. Our intent to use different investment look-back periods and delayed update function had as main goal capture this diversity in real-life markets. Having said that, in this section we try to understand how some strategies keep resisting in the population and there is no fully conversion into the fittest one. In Figure 4.19 and Figure 4.20 is represented the income distribution over the several tested investment look-back periods for Momentum and Fundamental Strategies (we did not plot for the remaining strategies since they do not depend on past observations ($\kappa$) to making a trading/investment decisions). On one hand, as displayed, Fundamental Strategy performs poorly relative to Momentum for every look-back period. One the other hand, for both strategies it seems that, all else equal, there are some investment look-back periods better than others. For instance, if two agents are not allowed to copy and the first is committed to a Fundamental strategy of $\kappa = 100$, whereas the latter has a Fundamental strategy of $\kappa = 250$, the first will outperform the second by having a higher fitness (Figure 4.20). On the contrary, having for the period in study a Momentum Strategy of $\kappa = 250$ it would generate more than the double in profits than any other Fundamental Strategy. Nonetheless, one can say that the most profitable investment look-back period is 1 trading day. Thus, buying if the Market was trending up in the previous day and selling it if yesterday’s performance was downwards, earns the most of the Market. This means that the simple strategy of “short-term equivalent retaliation” (response in the same fashion as the last interaction) wins above all others. Searching for the reasons of the obtained results is fundamental for future research. At the first glance and looking to Figure 4.19 we may say that if SPX price time series was a periodic function, it would have a longer and a short-term economic cycle, which it would be the double of the best found $\kappa$, of 250 and 1 trading days, respectively. Despite the stated, we think it is important to stress the fact that we are not considering the local winners as time goes by. We are just looking to the overall macro view of the entire sample.

Moreover, we still did not measure the impact on the overall wealth of allowing investors to copy and the impact of evolution. In Table 4.1 it is the wealth of a mixed population versus one with only Momentum investors and other with Fundamental individuals. This is relevant to check how populations benefit from copying someone else strategy. In the first half of the table we can see, once more, the disparity between Momentum/Fundamental Strategies but mostly we see the impact of a mixed population. Despite being reasonably better off for Fundamental investors to be in a mixed environment, Momentum investors that temporarily changed to this losing strategy, will be worse off in the end of the simulation. Between 1947-1977 and from 2007 until nowadays, the mixed population lost value compared with a
Figure 4.19: Income Distribution based on Momentum Strategy and Investment Look-Back Period

Figure 4.20: Income Distribution based on Fundamental Strategy and Investment Look-Back Period
Table 4.1: Summary Table of Wealth Distribution for mixed and non-mixed Population of Fundamental and Momentum Strategies (All Values are, an average from 100 runs, in Dollars). The investment look-back period considered ranged from 1-500.

<table>
<thead>
<tr>
<th>Period</th>
<th>Total Wealth</th>
<th>Average Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Momentum</td>
<td>Fundamental</td>
</tr>
<tr>
<td>1927-1937</td>
<td>82,823</td>
<td>21,077</td>
</tr>
<tr>
<td>1937-1947</td>
<td>60,273</td>
<td>33,490</td>
</tr>
<tr>
<td>1947-1957</td>
<td>71,984</td>
<td>29,964</td>
</tr>
<tr>
<td>1957-1967</td>
<td>51,679</td>
<td>42,679</td>
</tr>
<tr>
<td>1967-1977</td>
<td>58,216</td>
<td>35,959</td>
</tr>
<tr>
<td>1977-1987</td>
<td>56,844</td>
<td>35,986</td>
</tr>
<tr>
<td>1987-1997</td>
<td>51,757</td>
<td>40,739</td>
</tr>
<tr>
<td>1997-2007</td>
<td>44,507</td>
<td>42,574</td>
</tr>
<tr>
<td>2007-2018</td>
<td>47,102</td>
<td>42,581</td>
</tr>
</tbody>
</table>

scenario where evolution and copy was forbidden and the population of Momentum and Fundamental were separated in a 50/50 composition. This seems to be in line with what was discussed earlier about the "survival of the fittest" and dominant strategies spreading out in all cases. In the process of evolution, we find that there is a loss on the overall population wealth. Lastly, it is important to mention the fact that within the mixed population context, only two of the periods considered were profitable for the average investor. This can be seen on the second half of Table 4.1 where all the values below 100 represent a lost in wealth (since all agents start with an initial fitness or income of 100). The reason is quite easy to understand: whereas an investor copies the losing strategy it erases his future likelihood of making more income in the future.

Finally, one can say that, within the presented results and information, it could be extrapolated what would be the best combination of look-back investment periods ($\kappa$) for the sample in study. In Figure 4.21, and in more detail in the Appendix (Table A.2), we studied the stability of performance of each look-back investment period ($\kappa$), when combined with a Momentum Strategy. Thus, we measured not only the average performance of each single strategy, but also their standard deviation and quartiles (Table A.2). On one hand, the first striking evidence is the fact that a given investor, by only changing his strategy $\kappa$, would under or outperform relative to a similar population of agents. The average fitness range goes from [56.51; 1,006.37], meaning for some $\kappa$, the investor will lose money while on other he will make 100 times its initial investment, for the considered period.

On the other hand, as displayed in Figure 4.21 and following the equation below, if we compute the ratio of average fitness with its standard deviation, we have a simple measure of how safe a given look-back investment period ($\kappa$) is:

\[
\text{Risk Reward Ratio} = \frac{\text{Fitness Average}}{\text{Fitness Standard Deviation}}
\]  

The presented indicator can acknowledge those $\kappa$ that perform extremely higher relative to its peers,
but in some other moments this performance trend will decay, having a highly volatile fitness. In Figure 4.21 we can check that despite being the lowest values of $\kappa$ the best performers in terms of fitness, due to their volatility in performance throughout time, those investment look-back periods have a low ratio in terms of risk-reward. Yet, Strategies with $\kappa$ between 275 and 300 are the most balanced ones relative to the amount of fitness an investor gets per unit of risk.

Perhaps, considering the reasoning developed, one can argue that a population with a strategy composed by lower $\kappa$ will have higher fitness ranges, whereas a higher investment look-back period Strategy will be more constant. Therefore, we can extrapolate that an investor $i$ that copies more often wealthy investors (without taking into account the risk-reward of the strategy he intends to copy), will have a greater likelihood of larger spikes in fitness. Thus, this feature brings an all new area of study, which is the impact of investors network in their performance. The individual’s set of ties or connections, impacts on his future performance. The population a given agent can be connected not only impacts on his performance, but also on the variation of his wealth throughout time. In the case of our model, agents are only looking to copy someone (randomly) better in terms of fitness. Future studies need to study the impact of network structure in investors relations and its impact on fitness and risk-reward.

**Figure 4.21:** Ratio between each investment look-back period fitness and its standard deviation throughout the entire sample period (1927-2018).
5

CONCLUSION

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5.1 Summary of contributions ........................................... 47
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In the present work, we took the challenge of contributing to the many efforts that try to provide an explanation for the complexity of Financial Markets. Through this project we conclude that there are strategies that outperform others and income inequality based on investment decision tend to constantly appear when using real US data. Despite being just a starting point, the present preliminary results provide a promising starting point regarding the power of evolutionary game theory to understand and characterize financial population dynamics.

5.1 Summary of contributions

In view of the results obtained, we delineate here what we believe are lines in what concerns future studies. It is our view that, in the best of our abilities, those will be important guiding lines for future work, discussed in the section ahead.

5.1.1 Wisdom of the Crowds or Momentum

Despite being counter-intuitive, the present work gives strength to the argument that following the Market trends is not only profitable but also the best simple guess of how to outperform the Market. Therefore, following the "Wisdom of the Crowds" is the best strategy. To some extend, these conclusions put in question those believing in the EMH and highlights the benefits of using of Carhart's Momentum factor [49] into Asset allocation Models and Stock picking analysis.

5.1.2 Investment look-back Period matters: "Short-Term Equivalent Retaliation"

Besides using a Momentum Strategy, how far we look back to make an investment decision is crucial to overpass the results of a fully-committed and blind Momentum. As it appears in our results, for the sample period in study, one can say SPX has a short-term period of 2 trading day and a longer one around 420 trading days. Nonetheless, It is the short-term figure where we find the best results showing that having as activation or action function a simple "tit-for-tat" momentum strategy ("Buy if it went up, Sell it otherwise"), is the best response. In our view, this is a quite remarkable conclusion, since these experiments are rooted in EGT and the results seem to be in line with Anatol Rapoport findings in the Prisoner's dilemma and its good performance in Axelrod's Tournament [50, 51].

5.1.3 "My network empowers me!"

Despite the several findings and conclusions that can be taken from the present work, the prevalence of inequality that we observe, even in a population constituted by similar strategic profiles, has key importance. These differences can only be understood and rooted in timing of change and network
Ties. With timing of change we mean the time-stamp in which an investor \( i \) changes his trading strategy (either through copy a given neighbor or just by following his own strategy). Having the luck/capability to change an investment decision in the right time is key to outperform the rest of the community. However, an investor \( i \) surrounded by a group of “bad traders” (a relatively lower market prediction accuracy despite its fitness/actual performance) would eventually end up worse off than an investor \( j \) whose neighbors are, on average, making better decision calls when it comes to invest. Despite the concept of “trading network” was not used in the present work, we found that even in a scenario where every investor can be connected to the entire population, those to whom I connect during the simulation are responsible to define my performance. Additionally, not only copying is important to define fitness performance but also its prospect of future growth and variance, over time. Thus, those investors who were lucky enough to pick an agent with a Momentum Strategy with low investment look-back period, will outperform considerably all others, at the expense of higher wealth variance throughout time. On the other hand, by changing to some peers’ strategy based on longer-term investment look-back period would decrease its potential gains, but empower wealth stability, over the sample period.

5.2 Future Work

The questions raised by the present work are far greater than the work itself. Despite the multiple areas of focus that can be raised, it is our belief there are some topics that deserve future and careful study:

5.2.1 My actions impact on Market Prices

One can say that we are only in a presence of an EGT environment whenever agents can change, with their actions, the game they are part of. Therefore, for future reference, we should study the impact that our modeled agents bring to the real prices, or even, how different it is the time series produced by such population of oversimplified investment reasoning relative to real-live prices.

5.2.2 “The Economy, Stupid!”

One can say that one of the most striking evidences from the current work is the confirmation that Momentum Strategy performs quite above most strategies and during some persisting periods is even outperforming the Market, meaning, doing better than following a “Buy and Hold” strategy. There may be several reasons that can lead to this conclusion: \( i \) The behavioral effect of following the wisdom of the crowds; \( ii \) The belief that any system, after reaching a point of low friction, will move steadily and not in cycles; \( iii \) Accepting that it is not possible to predict future trends in the market and following the current one is the only way out;
Nonetheless, as James Carville said “the economy, stupid” is perhaps the responsible for creating such trends. Without taking into account macroeconomic data and just looking to financial markets prices, we are forgetting the role of interest rates, fiscal and monetary policies, advances in technologies, earnings and company results that eventually lead to sustainable trends that are not irrational but fundamentally sustained by the economy. Therefore, future works need to address more carefully the impact of such macroeconomic variables.

5.2.3 The impact of complex networks

As we tried to show in this present contribution, networks have a central role in the studied reality. Firstly, even with a homogeneous population, the simple fact of copying investor $i$ and not investor $j$, can be decisive at explaining higher fitness relative to its peers. Studying the impact of different networks is, with no doubt, the way to go in order to fully measure the impact evolution can have and which one of the networks can produce higher or lower degrees of inequality. Moreover, it would be interesting to understand what would be the impact of a single wealthy agent in the history of relations of a given investor $i$ and what needs to happen in order to this investor $i$ become wealthy as well. Finally, following the reasoning of a cooperator-defector game, it would be relevant to quantify how many agents are needed to start an upward or downward movement in financial Markets that can spread out and contaminate the entire network.

5.2.4 Companies Impacting on Investors Decisions

Another source of vital information when it comes to explain financial markets realities is the impact of corporate actions. Such events, as earnings releases, dividend and stock splits, mergers and acquisitions impact on how investors perceive value and risk, changing their actions accordingly. Therefore, in further analysis (such as the one detailed in Part II), we need to understand the connections between firms, how they move and what they represent in terms of market size to better acknowledge the impact of such environment in a study like the one here presented.

5.2.5 Market Prediction and Alpha generation

Producing models and research in Economics and Finance is cursed to be biased towards prediction. If a model has the likelihood of mimic, at a certain degree, what happens in reality, it is mostly doomed to be used at anticipating Markets moves and getting profits out of a risk-less strategy. Within this work, we choose not to focus on prediction or measuring at what extend our model can mimic today’s Markets. However, in our view testing model predictability is the natural step to acknowledge if we were able to capture Financial Markets.
5.2.6 Moving to micro and macro-approach

In order to grab such enormous area of study not only we believe that it is needed to include more markets in the study (and the inclusion of companies valuations as described previously), but also their inner relations between the several important features Global Financial Markets have nowadays. Specially, most economic relations are time dependent and are structured. Further Studies need to be done to combine population dynamics in an environment where they can buy several assets and those assets are correlated between each other and part of a global and connected economy.

5.2.7 Exogenous Action Validation

Capturing financial markets within a model can only be made knowing that market are connected and what happens in a certain part of the globe can, as most certainly will, affect the entire economic ecosystem. Therefore, the introduction of exogenous knowledge that it is known global investors tend to look and act based upon that is crucial to capturing shifts in the inner nature of the environment in study. Adding economic activity, financial reports and Twitter news into the decision making process of investing in a given market is far more realistic, leading to situations where for the same market value perceptions are different and so are the actions investor make.

5.2.8 The impact of Behavioral Economics

We argued earlier that agents tend to behave in a similar manner regarding asset prices. Nonetheless, it is undeniable there is still a path to go in order to achieve a live trading framework. So far we assume simple agents that do not influence that overall market trend and that reason based on previous prices. In the future, it may be relevant to test more sophisticated learning algorithms and eventually include in agents’ decision making forecast capabilities and the possibility to differentiate their investment strategy (e.g., momentum or fundamental) depending on particular assets’ characteristics. For that, it will be fundamental to consider the capitalization of different companies in isolation, rather than using an average index (as we have been doing so far). Also, agents may reason about future trends based on the particular way that different company capitalization’s interrelate. The introduction of behavioral biased, widely studied by Kanhneman and Tversky [24], it would make investor, for instance, over-invest whenever they are having a positive performance (despite the current market conditions) or under-invest or panic even if it is the best time/opportunity to buy cheap. While implementing such sophisticated agents is out of the scope of this thesis, in the next part we give a first step into the study of how SPX members are connected, and how this network of companies is responsible for changes in the aggregated Index.
Part II

Capturing Financial Volatility Through Simple Network Measures
Contents

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In recent years, it is undeniable how interconnected financial markets are becoming [6, 38, 52–54]. Modern economic agents and their institutions can operate globally in an interdependent and connected market. Proof is the most recent financial crises, in which we observed the growing importance of accounting for systematic risk, where a single financial institution can affect the global market and all its agents [55–62]. As a result, financial systems are a natural playground for network science. It is both convenient and insightful to describe the various connections between financial assets and institutions in the form of a graph. In Allen et. al [53], it is detailed the vital role of network connections in the interbank market. Banks are exposed to their peers, both by holding crossed positions (mutual exposure in their balance sheets) as well as sharing similar market portfolio, assets and liabilities (as creditors and depositors) [38, 53, 54]. Those so-called markets represent market players exchanging cash-flows and goods between them, being responsible for managing one of the most complex multi-agent systems humanity have ever created, permanently connecting every corner of the globe [4, 5].

6.1 Motivation

In this context, it remains to a large extent an open question how one can extrapolate from network properties to commonly used measures to detect risk [37–39, 55–63]. At a firm level, as detailed by Onnela et al. [64], it is possible to assess firms’ performance, looking at their stock price time series. It is believed that firms’ valuation is based on all the available information [9, 16]. This concept of “pricing” is understood as the current fair value of all company assets and the present value of future expected income flows from the current asset allocation. In other words, it measures how much the firm is worth. Nonetheless, these companies are not isolated. They are interacting with one another by exchanging cash-flows, products, clients, among others [7, 54, 64]. Despite the fact that is not straightforward to measure the nature of all this inner relations, stock price variations are the closest to public information it can be used to study this complex system [4, 64].

Some prior studies tried to capture financial stability (or risk), looking to the inner characteristics of the network. Battiston and Caldarelli et al. seminal work gave us the insights of how it is possible to detect systemic risk looking at interbank network, in the context of asset-liabilities relations [55–62]. They studied the impact of several types of networks (from random to scale-free) and sources of information (the degree at which nodes can accurately measure the risks of his peers), to access the likelihood of a bankruptcy cascade effect in a distress financial period [55, 58–61]. Moreover, these authors proposed some network measures, like the debt rank [56, 57], to better capture the importance of each single node in a financial network in a crisis context. Boss et al. [54] presented some encouraging results in the Austrian Interbank Market proving that network metrics can explain structural changes in the environment as in many other scientific fields. In Onnela et al. [64, 65], important steps were made when
it was introduced the time dependence, calling for a detailed study of the co-movement between network characteristics and an external validation. However, by having the opportunity of using US financial data, namely the companies that are part of the most relevant Stock Market Index (S&P 500), it is possible to check if the network metrics mimic the volatility and financial systematic risk, by comparison to the “Fear Index”, the so-called “VIX Index”. This index measures the implied market risk and accounts for the short-term implied-volatility derived from all the S&P stocks and its options and structured products [66]. Despite the difficulty at fully characterize VIX Index pricing (due to the complexity of its calculations and the amount of different products the computation uses), it measures the instability of the S&P Market and it is widely used by the industry. Thus, if by using simple network measures it is possible to capture the same daily movements, we may argue that complex science can give a quicker and easy to understand answer about financial instability.

6.2 Objectives and Contributions

In this work, we aim to study the financial systematic risk through complex networks (simple) measures, resorting to both weighted and signed networks representations. Using as framework real US data, we build a network from price correlations among companies and measure the likelihood of capturing volatility shifts looking at the time variation in: i) the network (weighted) average degree (or strength) and in ii) particular three-node motifs (triads). On one hand, the average degree is the simplest measure capturing the level of interdependence among companies, and pictures a global and macro view of how price variations affect network connectedness. On the other hand, because structural balance theory [1,2] has already been used to study financial networks [3], we are interested in the identification of specific motifs to perceive network shifts and their impact on network volatility. This way we are able to analyze the volatility of a network with a global network measure and with specific local patterns. In both cases, we find a statistical significance when it comes to explain and replicate what happen to the VIX Index in a given period of time.

6.3 Structure

This second Part is structured in the following manner: In Chapter 7, we define model architecture and define the process from extracting raw market data to a complex network. Thus, in Chapter 8 we present the major findings and conclusions that can be extrapolate from both macro-view of the network and the triads analysis. Finally, in Chapter 9 we define the next steps within the model and summarize the main contributions from the presented work.
7

MODEL

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7.1 Using Node Degree and Motifs to Analyze Financial Networks

7.1.1 Financial Markets Structure

Let us now build our financial networks based on correlation matrices, as introduced before. We consider a price time series from 1992-2018, whose source is Bloomberg database\(^1\), for a security set of 500 companies. Let us define \(P_i(\tau)\) as the closing price of stock \(i\) at time \(\tau\) and the daily logarithmic return of stock \(i\) as:

\[
r_i(\tau) = \ln[P_i(\tau)] - \ln[P_i(\tau - 1)]
\]

(7.1)

Then, by defining a time grouping window of bulk size \(T\), one obtains a cube of correlations, where \(\rho(x, y)_{[t,t+T]}\) is the returns correlation between firm \(x\) and firm \(y\) between \(t\) and \(t + T\) (giving \(n\) data points), defined as:

\[
\rho = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2(y_i - \bar{y})^2}}
\]

(7.2)

Having built the correlation cube, two independent studies were made. First, a weighted time-varying network was built, whose nodes were the constituents of the S&P 500 Index and their links were the correlations throughout time. By this it is meant that whenever two firms have a correlations different from zero (and ranging between -1 and 1), they will share an edge whose value is their correlation. As an example, if firm A has a price correlation in a given set of days (lets say 0.75) with firm B, the network will display two independent nodes (A and B) whose edge between them has a linkage (during this time-frame) of 0.75. From this stage, in order to capture network connectedness, we compute the time-dependent network average degree [67] as the average of the sum of the weights attached to each node (see below).

In the second study we build a signed network, in which every pair of nodes that share a correlation above a given threshold \(X\) will have a positive sign, whereas a negative sign is defined whenever the correlation value is below \(-X\). The links with values between \(-X\) and \(X\) were not considered to avoid adding noise or spurious edge relation for very low correlations.

7.1.2 Relation between VIX and Average Degree

Given its simplicity, we chose the network average degree to obtain a global view of the network, being easy to acknowledge the strength of each node with its peers. Since that, in this case, we are dealing with a weighted network, for simplicity we extend the notion of degree of a node, to its weighted degree or strength, i.e., the sum of the relative weights of all its links [57]. Using the correlation matrix as the

\(^1\)Which is one of the most accurate database when we are dealing with financial data. The dataset has, on average, 252 days per year during 26 years
network adjacency matrix \( A \) (the network is relatively close to a fully-connected graph), summing row-wise (neglecting self-correlations) gives us the (weighted) degree \( z_{i,t+T} \) of a node \( i \), in the time frame \( t+T \), as displayed in Equation 2.

\[
z_{i,t+T} = \sum_{j=1}^{N} A(i,j)_{[t,t+T]} - 1 \tag{7.3}
\]

By averaging over all nodes degree, we obtain the network average degree, \( < z > \), for a given time window, \([t+T]\). The degree of a node can be seen as a measure of the impact of a single node in the network. Nodes with higher degree can easily spread a good or a bad economic movement, being potentially responsible for generating instability in the network. Moreover, higher values of the average degree will mean that companies are increasingly connected and interdependent, while lower values reflect independent firms’ price movements. Here we try to correlate the time-evolution of the network average degree with the financial volatility, which can be independently assessed through the VIX index.

### 7.1.3 Relation between VIX, Motifs and Structural Balance

Signed networks have been used in the past to study financial portfolios (see, e.g., [3]). One of the main observations is that usually a portfolio presents high values of structural balance, being rare to have unbalanced relations. This concept of balance and unbalanced networks come from the structural balance theory [1, 2]. A given network is considered to be balanced if all the cycles are balanced, otherwise it is considered unbalanced. A cycle is balanced if the product of the signs of its edges is positive (see Figure 7.1). One can evaluate the “degree of balance” of a signed network as the ratio of the number of positive cycles to the total number of cycles. Let \( G \) be a signed graph, \( c(G) \) be the number of cycles of \( G \), \( c_+(G) \) be the number of positive cycles of \( G \), and \( b(G) \) be the degree of balance of \( G \). Then:

\[
b(G) = \frac{c_+(G)}{c(G)} \tag{7.4}
\]

In this work we use this measure to evaluate structural balance with cycles of size 3, following the same approach as in [3].

Let \( G = (V, E) \) be an undirected and signed network, with \( n = |V| \) vertices (individuals) and \( m = |E| \) edges (ties), and with edges labels \( w = \{-1, 1\} \) between two assets \((a,b)\): \( w(a,b) = w(b,a) = 1 \), if it is a positive correlation, \( w(a,b) = w(b,a) = -1 \) if it is a negative correlation. To calculate the degree of balance of a network we first use gtriesScanner [68]\(^2\) to obtain all triads of the network. Then, for each triad, we calculate the product of its signs and in the end we obtain the degree of balance.

\(^2\) http://www.dcc.fc.up.pt/gtries/
Figure 7.1: Triads considered balanced and unbalanced by the structural balance theory of Harary [1–3].

As was already observed by Harary [3] financial networks tend to have high values of structural balance. We are able to observe the same in our datasets, see Figure 8.4. Given this, and because when trying to relate to the VIX we want to extract the most discriminatory network characteristics effect, we removed the unbalanced triads, focusing only on those that are balanced. Additionally, since in our edge definition it was chosen firms’ prices correlations we could not differentiate triads whose components were trending up versus those that were jointly falling apart, we added into each node a positive or negative sign respectively to their performance in the time horizon in study, looking for the frequency of specific motifs in the form of Figure 7.2.

Figure 7.2: Balanced triads with signs on the nodes. Those signs correspond to the performance of the firms represented by the nodes. A positive sign (+) is inputed whenever for the given time-frame node value has gone up, whereas a negative sign (−) is given when a given node lost value for the same period.

Therefore, as we calculate the product of the signs of each triad, we also gather the frequency of each different motif. The goal is to analyze if some specific motif can relate to VIX, always maintaining the notion of structural balance in the signs of the edges and using the signs of the nodes as supplementary information. Within such context, it is now possible to split those balanced triads with a positive impact on the network from those whose performance was poorly, relative to its peers.
RESULTS AND EVALUATION

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8.1 Testing Volatility on Network Properties

To answer our baseline question, which is the study of the interplay between network metrics and degree of risk or volatility, we considered the data from the S&P constituents to input in the network previously explained, as well as the VIX Index, as our exogenous and independent variable.

First, we performed one set of 1000 runs to acknowledge the sensitivity of our variables in the predictability of the average degree (summary of the results can be found in Table 8.1). Those variables – size of the bulk, $T$ and the correlation cut-off threshold – are relevant to perceive if this macro view of the network can capture short or long term trends in volatility as well as if nodes with lower correlation (being responsible for lower impacts in the network) add value in the model predictability. As it is possible to picture from Table 8.1, the degree of a node matters at mimic financial instability. Only 8% of the tested variables were not able to accomplish at least 50% of replication, being 59.29% of the simulations statistically significant, show as proof that the network average degree was correct more time stamps than wrong (rejecting, with 95% confidence interval, the hypothesis that the measurements were different from a toss of a coin).

Moreover, it is relevant to add that even the lowest tier ranked nodes are important to increase model replication power. This can be inferred from the replication levels in Table 8.1 regarding the correlation cut-off threshold: as the model gets pickier at choosing the strongest relations between nodes, the model loses its capability of following the volatility index trend.

Also, Figure 8.1 enlightens a very important remark: as the volatility index gets higher, network average degree tends to spike. When the fear in financial markets grows, firms specific price variations tend to be forgotten and prices move all in a similar manner. Thus, we move towards a highly connected financial structure (we can argue that the networks shrink) when instability and volatility are present, and
get less connected and “relaxed” in the presence of a low systemic risk.

8.2 Network Micro-Structure Analysis

In our second analysis, we measure the correlation between the fraction of different motifs/triads and the volatility index VIX throughout time. We see that these time-evolving networks portray high levels of balance (Figure 8.4), as has been previously found [3]. Given the observed levels of social balance, balanced triads tend to overweight the unbalanced ones, being the latter even in lower number in the presence of large volatility swings, meaning that whenever Market instability is at its highest, financial networks tend to be mostly connected and balanced. This outcome leads us to perform, within the same reasoning already developed and explained with the degree of network, a set of 50 runs to test if the likelihood of balance and the different motifs replicate the uncertainty of the market. From Table 8.2, we conclude that only the correlation between triads with three positive edges and the VIX index is statistically significant and Index replication consistently higher than 50%, (as displayed in Table 8.3).

Due to fact that only triads of fully positive edges seem to matter when mimicking the “fear Index"
we split those triads whose nodes were positive in a given time-frame, from the ones that were negative in the same period of time. In Figure 8.3 we show that looking to the node performance (where performance is the price variation for the time window considered), we may gain some additional explanatory power. In Table 8.2 we show the p-values from the sensitivity analysis run within the present context. As a matter of fact, we obtain significantly higher results using nodes performance for the considered period. Without having a correlation cut-off threshold and a grouping size-window of 5 trading days, triads with positive edges and positive performance of their nodes, VIX index is replicated at a rate of 70%.

The present results seem to be in line with our empirical reasoning that whenever there are times where all firms are moving together, is not because of their fundamental or intrinsic value, but mostly because of something exogenous in the financial world that spreads out into the entire network, despite its positive or negative impact. Our results suggest that when the VIX Index tend to spike, the number of

<table>
<thead>
<tr>
<th>Number Of Days</th>
<th>Correlation Cut-off Threshold</th>
<th>Statistical Significance (z-scores)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.00</td>
<td>-1.179</td>
</tr>
<tr>
<td>5</td>
<td>0.40</td>
<td>-1.179</td>
</tr>
<tr>
<td>15</td>
<td>0.80</td>
<td>-2.038</td>
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<tr>
<td>15</td>
<td>0.80</td>
<td>-2.038</td>
</tr>
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<td>15</td>
<td>0.80</td>
<td>-128.727</td>
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<tr>
<td>30</td>
<td>0.80</td>
<td>-0.768</td>
</tr>
<tr>
<td>30</td>
<td>0.80</td>
<td>-0.768</td>
</tr>
</tbody>
</table>

Table 8.2: Sensitivity Analysis: Z-scores from Statistical Tests (Green values are Statistical Significant with 95% confidence).
### Table 8.3: Summary of runs accuracies

(number of times the variation in VIX Index was correctly replicated by the tested type of triads (in Percentage points)).

<table>
<thead>
<tr>
<th>Number Of Days</th>
<th>Correlation Cut-off Threshold</th>
<th>Accuracy Versus VIX Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unbalanced (0 Pos Edges)</td>
<td>Balanced (1 Pos Edges)</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>48.38%</td>
</tr>
<tr>
<td></td>
<td>Bal(1 Pos Edges)</td>
<td>48.51%</td>
</tr>
<tr>
<td></td>
<td>Unbalanced (2 Pos Edges)</td>
<td>Balanced (3 Pos Edges)</td>
</tr>
<tr>
<td>5</td>
<td>0.40</td>
<td>48.46%</td>
</tr>
<tr>
<td></td>
<td>Bal(1 Pos Edge) 2/3 Pos Nodes</td>
<td>51.84%</td>
</tr>
<tr>
<td></td>
<td>Unbalanced (3 Pos Edges)</td>
<td>Balanced (3 Pos Edges)</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>48.71%</td>
</tr>
<tr>
<td></td>
<td>Bal(1 Pos Edge) 3/3 Pos Nodes</td>
<td>51.84%</td>
</tr>
<tr>
<td>10</td>
<td>0.00</td>
<td>43.99%</td>
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<tr>
<td></td>
<td>Bal(1 Pos Edges)</td>
<td>51.91%</td>
</tr>
<tr>
<td></td>
<td>Unbalanced (2 Pos Edges)</td>
<td>Balanced (3 Pos Edges)</td>
</tr>
<tr>
<td>10</td>
<td>0.40</td>
<td>44.44%</td>
</tr>
<tr>
<td></td>
<td>Bal(1 Pos Edge) 2/3 Pos Nodes</td>
<td>42.49%</td>
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<td></td>
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<td>15</td>
<td>0.00</td>
<td>48.05%</td>
</tr>
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<td>50.00%</td>
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<td></td>
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</tr>
<tr>
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<td>0.80</td>
<td>42.12%</td>
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<td></td>
<td>Bal(1 Pos Edge) 3/3 Pos Nodes</td>
<td>46.10%</td>
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<td>47.75%</td>
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<tr>
<td>30</td>
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<td>42.12%</td>
</tr>
<tr>
<td></td>
<td>Bal(1 Pos Edge) 3/3 Pos Nodes</td>
<td>46.10%</td>
</tr>
</tbody>
</table>

Triads with three positive edges and nodes tend to reduce, increasing otherwise, as shown in Figure 8.3.
CONCLUSION

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9.1 Summary of Contributions

In the present work, we propose two new approaches to strengthen the connection between network science and financial markets. We show that there is a close relationship between financial network characteristics and its performance. Without taking into account VIX complex pricing of volatility, network inner characteristics seem to emerge in the same fashion and magnitude. At a macro and global level, simple measures, as the average degree (or average strength), can help to replicate the short-term implied-volatility of the financial market network. As the network average degree rises, the VIX Index tends to produce a similar movement showing that in times of crisis firms get more connected and investors tend to bear their market valuation on the global macro data, leaving apart the firm intrinsic value. Reversely, when instability is lower, nodes tend to move more independently, meaning that nodes’ strength is weaker, leading a less connected network. On the other hand, despite its greater complexity and time constraints, motifs are also a viable source of mimic power, giving the combination of balanced triads with knowing the performance of its constituents an out-performance considerably higher relative to the other attempts. Preliminary results indicate that the combination of multiple network features may further increase our understanding and predictive power. Work along these lines is in progress.

9.2 Future Work

Further studies must be pursued for a better understanding of the financial world. In our view, there is the need to study whose firms are more often in the motifs that accurately replicate the VIX index and also to acknowledge the impact and stability of such ties throughout time. Each of these subgraphs, define a particular kind of interactions between vertices, reflecting a meso-scale pattern that will later lead to global financial observables. In particular, it is relevant to identify or engineer subgraphs that are more resilient and stable than the rest of the financial network, creating a portfolio with a lower likelihood of suffering in times of crisis or crashes. As future work, it seems relevant to understand if those measures are still reliable at anticipating and predicting future swings in market instability, and what are the main contributors of financial volatility and their relation with size, sector, cumulative performance, among others. Secondly, correlation networks may be spurious, i.e. induced by other variables not included in the analysis. Therefore, a deeper study is required to partial out the effect of network structure. Furthermore, producing similar studies in other markets, regions of the globe or joining them together, might be enlightening at detailing the origins of global movements or those that, despite its characteristics, easily spread throughout the financial market network.
Acknowledgments

This work was partly supported by national funds through Universidade de Lisboa and FCT – Fundação para a Ciência e Tecnologia, under projects SFRH/BD/129072/2017, PTDC/EEI-SII/5081/2014, PTDC/-MAT/STA/3358/2014, and UID/CEC/50021/2013.

We are grateful to Bruno Gonçalves for comments.
Bibliography


[29] F. P. Santos, “Evolution of fairness under n-person ultimatum games.”


Simulation Summary Dashboards
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<th>Period</th>
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<th>Random Buy/Sell*</th>
<th>Random Strategy (k = 50)</th>
<th>Full Momentum (k = 25)</th>
<th>Full Fundamental (k = 25)</th>
<th>Full Momentum (k = 50)</th>
<th>Full Fundamental (k = 50)</th>
<th>Full Momentum (k = 500)</th>
<th>Full Fundamental (k = 500)</th>
<th>Copy Strategy (Random Buy/Sell &amp; Mom k=50)**</th>
<th>Copy Strategy (Mom &amp; Fund k = 50)**</th>
<th>Copy Strategy (multiple k's)**</th>
<th>Partial Copy for Strategy and k (multiple k's)**</th>
<th>Full Strategy Copy (multiple k's)**</th>
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<td>2161.29 (83.94)</td>
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<td>76.30 (84.64)</td>
<td>412.91 (84.49)</td>
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<td>1531.63 (83.35)</td>
<td>1460.12 (93.20)</td>
<td>2106.82 (87.29)</td>
<td>2424.58 (85.19)</td>
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<td>127.96 (84.41)</td>
<td>76.30 (84.64)</td>
<td>412.91 (84.49)</td>
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<td>2207.96 (15.59)</td>
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<td>1948-1958</td>
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<td>197.79 (84.21)</td>
<td>127.96 (84.41)</td>
<td>76.30 (84.64)</td>
<td>412.91 (84.49)</td>
<td>412.91 (84.49)</td>
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<td>1531.63 (83.35)</td>
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<td>2424.58 (85.19)</td>
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<td>1958-1968</td>
<td>127.96 (84.41)</td>
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<td>412.91 (84.49)</td>
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<td>127.96 (84.41)</td>
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<td>127.96 (84.41)</td>
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<td>127.96 (84.41)</td>
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**Table A.1:** In the beginning of each period, every strategy has 100$ to invest in the S&P Index. Results are the accumulated earnings (including initial investment) from investing in the stated strategies. Thus, values below 100 mean strategies lost value throughout the time-frame, whereas above 100 the strategy was profitable (with an annual inflation correction of 2%). For the last 4 row each strategy has the maximum profit obtained (by a single investor) and the population average wealth in the simulations (in parenthesis). * Results from 1000 simulations; ** Results from 100 Simulations.
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Table A.2: Fitness descriptive statistics for a Momentum strategy with different investment Look-Back Periods (κ). All values are in US dollars.