Being Dory: Multicarrier System For Underwater Acoustic Communication

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Abstract

Underwater wireless communications is a challenging subject with many scientific and economic applications. The deployment of underwater equipment can be held back due to the lack of wireless communications, relying instead on cables to transmit information. Terrestrial radio-frequency systems are not a viable solution due to strong attenuation of the electromagnetic waves in seawater. The mainstream solution for underwater wireless communications is to use acoustic waves.

This paper aims to explore a matrix based modulation, Orthogonal Signal-Division Multiplexing, with flexible spectrum management and relatively simple equalization techniques. The matrix interpretation of the modulation allows a turbo like receiver to be implemented. The effectiveness of the modulation is studied here through simulation of underwater channels under different environmental conditions and link parametrizations. A MIMO extension is also developed to increase the spectral efficiency of the system.

Keywords: Underwater communications, Underwater Acoustic Channel, Orthogonal Signal-Division Multiplexing, Turbo Receiver, Multiple Input Multiple Output

1. Introduction

Wireless communications systems are the backbone of contemporary society. In terrestrial communications the use of radio-frequency systems has allowed for great strides in economical and social development. As ocean exploration activities increase there is a need for underwater wireless communications, to be used in different applications such as environmental sensors or underwater vehicles. In an underwater situation the terrestrial radio-frequency systems are not a viable solution, mainly due to electromagnetic waves not propagating over long distances. For the underwater medium the mainstream solution is to use acoustic waves, which can propagate for tens or hundreds of kilometres [8].

The underwater acoustic channel was initially thought to only allow for non-coherent detection schemes, as the channel would greatly impair the capabilities of phase coherent detection. As such the initial attempts at wireless communications relied on modulations that allowed for non-coherent detection, such as Frequency Shift Keying, and while these were successful in some applications they were not bandwidth efficient, making them ill-suited for high data rate applications. Further developments on the use of proper receiver structures proved that coherent modulations were viable [7].

2. Fundamental Concepts

The underwater communications channel is a challenging medium for reliable, high-rate wireless communications. This section briefly describes the main mechanisms which affect the channel and how this influences the communications systems.

There are a few models currently in use to describe the propagation of an acoustic wave [4] in the underwater scenario. We will use the ray propagation model, which is a reasonable approximation when the propagating wave has a high frequency, as seen in [8] and [2]. In this approximation the wave is represented as propagating along paths or rays through the ocean. In a homogeneous environment, the rays’ paths would follow a straight line radiating from the source, however, in the underwater scenario the water column can be very inhomogeneous leading to variable sound speed and causing ray bending, or refraction.

2.1. Underwater Acoustic Channel

The source of an acoustic system can be seen as radiating in multiple directions, represented as multiple rays leaving the source in different directions. As such, in addition to being refracted, a ray can be reflected if it reaches any of the channel boundaries. Accounting for refraction and reflection there are multiple paths which a ray can take from the source to the receiver. Then, different rays emanating from the same trans-
manner. This mechanism is responsible for intersymbol interference (ISI), where at any given instant a signal is affected by late arrivals of previous symbols. Development of receivers which can estimate the channel response and eliminate intersymbol interference has been one of the main focus of research, as this is recognized as the main distortion introduced by the underwater acoustic channel.

The propagating sound waves will suffer signal losses which decrease the available power at the receiver. The main loss mechanisms are spreading loss, absorption loss and scattering loss, as described in [6]. When there is relative motion between the source and the receiver or scattering surfaces, the propagation path length may change during a transmission, and the change in length will induce a corresponding scaling of the received signal time axis. That is, if the path increases with a rate of \( v \) [ms\(^{-1}\)] and a narrowband signal \( Re[s(t)e^{j\omega t}] \) is transmitted with \( c \) [ms\(^{-1}\)] sound speed, then there is a mean frequency shift \( \omega_d = \omega_0(1 - v/c) \) called Doppler shift of the signal. Residual fluctuations are called Doppler spread, and are on the order of 10 Hz for carrier frequencies in the kHz range and speeds under 10 ms\(^{-1}\). These Doppler effects are one of the main concerns when using a multicarrier communications system, as the frequency shift does not affect every carrier in the same manner and may lead to loss of orthogonality between carriers. This will require a specific equalizer structure.

3. Proposed solution

The proposed solution examined in this paper is based on orthogonal signal-division multiplexing (OSDM), as proposed in [3] and [5], also known as vector OFDM. The study of the behaviour and tuning of this modulation in the underwater acoustics medium, as well as, the development of a MIMO extension for the modulation, will be the major focus of this work. The description of the processing blocks in an OSDM link is mainly adapted from [5].

3.1. Transmitter

The structure for the proposed transmitter can be described as follows: First, the data block containing the information bits is partitioned into \( N - 1 \) blocks and encoded using a CRC error detecting code followed by a convolutional code. Then the data is randomly interleaved. Finally, the bit sequence is mapped onto a QPSK constellation. The mapping results in vectors which can be described as \( d_n = [d_{n,0}, d_{n,1}, \ldots, d_{n,M-1}] \) for all \( n = 1, 2, \ldots, N - 1 \). These vectors are then stacked to build a matrix for the OSDM modulation. In this step a pilot vector, known at the receiver, will be inserted resulting in the \( N \times M \) matrix \( D \). The OSDM modulation is performed and a cyclic prefix is added. Finally, this signal is converted to the carrier frequency and transmitted. In this text the signal will be treated as a baseband signal (i.e., complex envelope) for simplicity.

The OSDM Modulation can be though of as a middle ground between single carrier and OFDM, allowing for resource management, frequency domain equalization and better peak to average power ratio control. The modulation process is depicted in Figure 1. Taking the matrix \( D \) and changing the notation to simplify future representations results in

\[
D = \begin{bmatrix}
  d_0 & d_1 & \cdots & d_{M-1} \\
  d_M & d_{M+1} & \cdots & d_{2M-1} \\
  \vdots & \vdots & \ddots & \vdots \\
  d_{(N-1)M} & \cdots & \cdots & d_{NM-1}
\end{bmatrix}
\]

A point inverse discrete Fourier transform is performed columnwise on the matrix \( D \) resulting in a matrix \( S \). This can be described as

\[
s = (F_N^H \otimes I_M) d
\]

where \( F_N \) is the \( N \times N \) unitary DFT matrix, \( \otimes \) is the Kronecker product, \( I_M \) is the \( M \times M \) identity matrix, \( d \) is a vector with the concatenation of all \( d_n \) and \( s \) is a vector with the concatenation of all \( s_n \). Then the matrix \( S \) is read row-wise resulting in the baseband transmitted signal \( s = [s_0, s_1, \ldots, s_{M-1}, s_M, \ldots, s_{(N-1)M}, \ldots, s_{NM-1}]^T \) with \( K = NM \). The coding and modulation process weaves the information together; each transmitted symbol \( s_j \) is dependent on the column it belongs to due to the columnwise IDFT and the symbol \( d_j \) is dependent on the row due to the encoding.

3.2. Channel Description

Two assumptions are made regarding the channel: during one block the path amplitudes are constant and a common Doppler scale is shared among all paths. As such, a resampling method is used for initial Doppler compensation resulting in the received signal after cyclic prefix removal

\[
r_k = \sum_{l=0}^{L} h_t e^{j\theta_l} s_{k-l} + n_k, k = 0, \ldots, K - 1
\]

where \( n_k \) is the additive noise term, \( h_t \) is the channel impulse response, \( L \) is the length of the channel impulse response, and \( \theta_l = 2\pi k T_s \) stands for the phase corresponding to the post-resampling carrier frequency offset \( \epsilon \) with \( T_s \) the sampling period. It is also considered that \( \theta_l \) varies slowly.
The received baseband signal can be described as follows

\[ r = \tilde{\Theta} \tilde{H} (F_N^H \otimes I_M) d + n \]  

where \( \tilde{H} \) is the circulant channel matrix with first column equal to the channel impulse response (CIR) vector \( h \) appended by \( K - L - 1 \) zeros, \( \tilde{\Theta} = \text{diag}\{e^{j\theta_0}, e^{j\theta_1}, ..., e^{j\theta_{K-1}}\} \) is the time varying phase matrix and \( n \) is the noise term.

### 3.3. Receiver

The receiver is based on the one proposed in [5], and can be described in two blocks: the demodulation block, where a coarse Doppler compensation and time alignment are performed, as well as the inverse of the modulation operation, outputting a signal where the channel impulse response and some residual phase noise are still present. The second block is an iterative OSDM detection block, in which the channel impulse response will be matched as well as the remaining phase noise.

The received signal is brought to baseband and a time alignment and coarse Doppler compensation are performed. The resulting samples \( r_k \) for all \( k = 1, 2, ..., K \) are the ones described in the previous chapter. These samples are written into a \( N \times M \) matrix \( R \) and an \( N \)-point DFT is performed columnwise resulting in a matrix \( X \). This OSDM demodulation process is illustrated in Figure 2.

The demodulation process can be mathematically described as

\[ x = (F_N \otimes I_M) r \]  

This description of the demodulation allows for some manipulation,

\[ x = GHd + n. \]  

where,

\[ G = (F_N \otimes I_M) \tilde{\Theta}(F_N^H \otimes I_M) \]

\[ H = (F_N \otimes I_M) \tilde{H}(F_N^H \otimes I_M), \]

the matrix \( G \) contains the Doppler information as a phase shift and the matrix \( H \) contains the channel information. The channel matrix \( H \) can also be described as

\[ H = \begin{bmatrix} H_0 & H_1 & \cdots & H_{N-1} \end{bmatrix} \]

where \( H_n \) are the \( M \times M \) channel submatrices corresponding to the \( n \)th symbol vector. These channel submatrices can be described as

\[ H_n = \Lambda_M^{nH} F_N^H \tilde{H}_n F_N \Lambda_M^n \]

where \( \Lambda_M^n = \text{diag}\{1, e^{-j\frac{2\pi}{M}}, ..., e^{-j(M-1)\frac{2\pi}{M}}\} \) is a frequency shifting submatrix, and \( \tilde{H}_n = \text{diag}\{H_n, H_{N+n}, ..., H_{(M-1)N+n}\} \) is the decimated frequency response submatrix with \( \tilde{H}_n = \sum_{l=0}^{M-1} h_le^{-j\frac{2\pi}{M} l}. \)

The matrix \( G \) can also be described using submatrices, that is,

\[ G = \begin{bmatrix} G_0 & G_{N-1} & \cdots & G_1 \\ G_1 & G_0 & \cdots & G_2 \\ \vdots & \vdots & \ddots & \vdots \\ G_{N-1} & G_{N-2} & \cdots & G_0 \end{bmatrix} \]

where \( G_i = \text{diag}\{g_i\} \) is the phase submatrix of the \( i \)th frequency sample with \( i = 0, \cdots, M - 1 \) and \( g_i = [g_i,0, g_{i+1}, \cdots, g_{i+M-1}] \) and each entry \( g_{i,m} = \frac{1}{M} \sum_{n=0}^{M-1} e^{j(\theta_{n,M+n} - \frac{2\pi i}{M})} \).

The previous description of the submatrices allows for the per vector treatment of the demodulated signal. Considering that the phase is time invariant and that \( \theta_k = 0 \) for all \( k \) then the per vector received symbol description is

\[ x_n = H_n d_n + z_n, n = 0, ..., N - 1. \]

For time variant channels there is interference among symbol vectors, resulting in

\[ x_n = G_0 H_n d_n + \sum_{i \neq 0} G_i H_{n-i} d_{n-i} + z_n, n = 0, ..., N - 1. \]

where the indices are taken modulo-\( N \) for notational simplicity.

The OSDM Symbol detection and decoding process consists of four components: a joint channel and phase estimator, an equalizer, a soft-input soft-output (SISO) demapper, and a decoder where the cyclic redundancy check is verified. These iterations will be tracked using a superscript index \( \beta \) and if a maximum number of iterations is reached without successful conclusion then the process will be terminated. While the process is not finished the decoding will allow for previous information to be fed back into the system.

The initial CIR estimate is obtained through a pilot vector, in the zeroth iteration of this process. By ignoring the residual Doppler effects the received signal can be considered time invariant, resulting in (11). Considering that the pilot vector is the \( n \)th vector then the CIR estimate can be computed from

\[ x_n = \Lambda_M^{nH} F_M^H D_n (\Gamma_n h) + z_n \]

where \( D_n = \text{diag}\{F_M \Lambda_M^0 d_0\} \) is the frequency domain symbol matrix and \( \Gamma_n \) is a \( M \times L + 1 \) matrix with entries
\[ |\Gamma_n|_{m,l} = e^{-j(2\pi/K)(mN+n)l}. \]
Solving for \( h \), and with \( \beta = 0 \), results in
\[ \hat{h}^{(0)} = (\Gamma_n^H\Gamma_n)^{-1}\Gamma_n^H D_n^{-1} F_M \Lambda_n^H x_n. \] (14)

From the CIR estimates the channel submatrices can be computed as follows
\[ \hat{H}_n^{(0)} = \text{diag}\{\Gamma_n^{(0)}\}, \]
\[ \hat{H}_n = \Lambda_M^H F_M \hat{H}_n^{(0)} F_M \Lambda_M. \] (16)

Finally, considering (11) and (16), the initial symbol estimates can be computed in a zero forcing sense as
\[ \hat{d}_n^{(0)} = (\hat{H}_n^{(0)})^{-1} x_n, \quad n = 1, \ldots, N - 1. \] (17)

Considering a MAP decoding strategy the demapping of the complex equalized symbols can provide the necessary information for the branch metric used in the BCJR decoding algorithm. Under the assumption that the equalized signal is described as \( d_n^{(0)} = \mu_n^{(0)} d_n + \xi_n^{(0)} \), where \( \xi_n^{(0)} \) is a Gaussian noise term with zero mean and variance \( \sigma_n^{(0)} \), then the branch metric, or extrinsic LLR, is computed for the \( q \)th iteration and the parameters \( \mu_n^{(0)} \) and \( \sigma_n^{(0)} \) are computed by
\[ \mu_n^{(0)} = \frac{1}{M} \sum_{m=0}^{M-1} d_n^{(0)} r_{n,m}^{(0)}, \]
\[ \sigma_n^{(1)} = \frac{1}{M} \sum_{m=0}^{M-1} |d_n^{(0)} - \mu_n^{(0)}|^2, \] (19)

where \( r_{n,m}^{(0)} \) is the hard decision symbol.

The extrinsic LLRs are the input of a BCJR decoder followed by a deinterleaver which outputs the a posteriori \( \text{LLR} \) \( L_n^{(0)}(c_{n,m}(q)) \) for each bit. From these a posteriori LLRs hard decisions are taken, the remaining bit decisions are checked using the CRC, the successfully decoded vectors are saved and the remaining will update their a priori \( \text{LLR} \) \( L_n^{(0)}(c_{n,m}(q)) = L_n^{(0)}(c_{n,m}(q)) \). For the erroneous vectors a SISO mapping using the LLR information can be done according to (20) at the top of the next page. The correct decoded vectors are hard-coded and together with the SISO mapped ones a matrix \( \hat{D} \) is built, the indexes of the valid CRC and invalid CRC vectors are written into sets \( N_V \) and \( N_R \) respectively, where
\[ N_V \cup N_R = \{0, 1, 2, \ldots, N-1\}. \]

From the best guess symbol matrix \( \hat{D} \) a new channel and phase estimate can be computed. Considering the received signal described by (12) and taking front-end resampling into account, the remaining Doppler effect is modelled as a time-varying phase slowly changing over one block. With the following two assumptions, the computation of the phase submatrices can be simplified. First, in the frequency domain it is reasonable to assume that the Doppler spread of the time varying phase is bounded, resulting in a reduction of the number of phase submatrices, that is,
\[ G_i = 0_{M \times M}, \quad I < i < N - I \] (21)

where \( I \) is the Doppler span parameter. Second, in the time-domain the slow varying nature of the phase means that it can be taken as constant over a \( J = M/M \) symbols, where \( J \) is the length and \( M \) is the number of the quasi-static subvectors. The phase submatrices are then described as
\[ G_i = \text{diag}\{g_i \} \otimes I_J, \quad -I \leq i \leq I \] (22)

with \( g_i = [g_{i,0}, g_{i,1}, \ldots, g_{i,M-1}] \).

Taking this into account, a soft interference cancellation and phase compensation can be performed to facilitate future estimates, this can be done as follows,
\[ x_n^{(q)} = \left( \gamma_0^{(q)} \right)^{-1} \left( x_n - \sum_{0 < |i| \leq I} G_i H_{n-i} d_{n-i} \right). \] (24)

The channel impulse response and phase matrices can now be jointly estimated by solving
\[ \min_{h, g_i} = \sum_{n=0}^{N-1} \|x_n^{(q)} - \sum_{i=0}^{I} G_i H_{n-i} d_{n-i} \|^2, \] (25)

where \( d_n \in \hat{D} \). This optimization problem is bilinear and thus non-convex. A popular (suboptimal) heuristic to tackle this is to alternate between the computation of the \( h \) and \( g_i \), keeping the other one fixed. Each alternating step is an easily solvable least-squares problem. The process ends when there is no significant decline in the cost function or a maximum number of iterations is reached.

For the CIR estimate the received signal after soft interference cancellation and phase compensation is described by
\[ x_n^{(q)} = H_n d_n + z_n^{(q)} \] (26)
\[ = \Lambda_M^H F_M D_n (\Gamma_n h) + z_n. \] (27)

Solving the previous equation with respect to \( (\Gamma_n h) \) results in
\[ D_n^{-1} F_M \Lambda_M^H x_n^{(q)} = (\Gamma_n h) + v_n \] (28)

where \( v_n = D_n^{-1} F_M \Lambda_M^H z_n \). By defining \( y_n = D_n^{-1} F_M \Lambda_M^H x_n^{(q)} \), \( \Gamma = \left[ \Gamma_0^T, \Gamma_1^T, \ldots, \Gamma_{N-1}^T \right]^T \), \( y = [y_0^T, y_1^T, \ldots, y_{N-1}^T]^T \) and \( v = [v_0^T, v_1^T, \ldots, v_{N-1}^T]^T \) then it follows that
\[ y = \Gamma h + v. \] (29)
The CIR estimate can be calculated by
\[ \hat{h}^{(r+1)} = (\Gamma^H \Gamma)^{-1} \Gamma^H y. \] (30)

Given the CIR estimate, we can substitute \( \hat{H}_n^{(r+1)} \) into (23) to obtain
\[ x_n = \sum_{i=-I}^{I} G_i \hat{H}_{n-i}^{(r+1)} d_{n-i} + n_n. \] (31)

Extracting the \( m \)th subvector, that is, premultiplying the previous equation by \( E_m = [I_m | m: m+J-1: \ldots] \), which represents the \( J \times M \) identity submatrix from row \( m \) to row \( m+J-1 \), we obtain
\[ \tilde{x}_{n,m} = \sum_{i=-I}^{I} g_i,m E_m \hat{H}_{n-i}^{(r+1)} d_{n-i} + z_{n,m} \] (32)

where \( z_{n,m} = E_m n_n \). By defining \( \tilde{g}_{m} = [g_{-I,m}, \ldots, g_{0,m}, g_{1,m}]^T \) and \( B_{n,m}^{(r+1)} = E_m [\hat{H}_{n-i}^{(r+1)} d_{n-i}, \ldots, \hat{H}_{n+J}^{(r+1)} d_{n+J}] \) for \( m = 0, \ldots, M-1 \), then (32) can be written as
\[ \tilde{x}_{n,m} = B_{n,m}^{(r+1)} \tilde{g}_{m} + z_{n,m}. \] (33)

By stacking all the \( m \)th subvectors from \( x_n \), that is by defining the following matrices
\[ x_m = \begin{bmatrix} \tilde{x}_{0,m}^T, \tilde{x}_{1,m}^T, \ldots, \tilde{x}_{N-1,m}^T \end{bmatrix}^T, \]
\[ z_m = \begin{bmatrix} z_{0,m}^T, z_{1,m}^T, \ldots, z_{N-1,m}^T \end{bmatrix}^T \]
and \( B_m^{(r+1)} = \begin{bmatrix} B_{0,m}^{(r+1)T}, B_{1,m}^{(r+1)T}, \ldots, B_{N-1,m}^{(r+1)T} \end{bmatrix}^T \) resulting in
\[ \tilde{x}_m = B_m^{(r+1)} \tilde{g}_m + z_m. \] (34)

Solving for \( \tilde{g}_m \) yields
\[ \tilde{g}_m^{(r+1)} = (B_m^{(r+1)} H_m^{(r+1)})^{-1} B_m^{(r+1)} H_m^{(r+1)} \tilde{x}_m. \] (35)

After solving for all \( m = 0, 1, \ldots, M-1 \) and defining \( \tilde{g}_m^{(r+1)} = [g_{-I,m}^{(r+1)T}, \ldots, g_{0,m}^{(r+1)T}, g_{1,m}^{(r+1)T}]^T \) and \( \tilde{g}_m^{(r+1)} = [g_{0,m}^{(r+1)T}, g_{1,m}^{(r+1)T}, \ldots, g_{M-1,m}^{(r+1)T}]^T \) then
\[ \tilde{g}^{(r+1)} = P_{M,2I+1} \tilde{g}^{(r+1)}, \] (36)

where \( P_{M,2I+1} \) is a permutation matrix defined by
\[ P_{M,2I+1} = \begin{bmatrix} I_M & e_{M,2I+1}^T(0) \\ I_M & e_{M,2I+1}^T(1) \\ \vdots & \vdots \\ I_M & e_{M,2I+1}^T(2I) \end{bmatrix}. \] (37)

The optimization ends, then the process of equalization, SIC, demapping and decoding can begin with the new estimates.

### 3.4. MIMO Extension

Consider a situation with \( N_t \) transmitters and \( N_r \) receivers, where each transmitter is independent of the remaining ones and the transmitter operations described before apply.

Regarding the OSDM modulation step, there is only one difference to the single transmitter situation: in MIMO each transmitter should transmit a pilot symbol vector. For the duration of the pilot symbol the remaining transmitters must be silent, which can be accomplished by changing the symbol matrix \( D \). Each transmitter will be allocated a subcarrier for its pilot vector and the remaining ones will not use that subcarrier, filling it with null symbols instead.

The baseband signal to be transmitted for each transmitter can be expressed as \( s = (F_N^H \otimes I_M)(d) \) for \( \varsigma = 1, 2, \ldots, N_t \), where \( d \) is the \( K \times N \) length vector containing all the symbols to be transmitted, including the pilot vector and null symbols. By stacking all transmitter signals then the multiple transmitter setup can be described as
\[ (\tilde{M}s) = (I_{N_t} \otimes F_N^H \otimes I_M)(\tilde{M}d) \quad (38) \]
where \( (\tilde{M}d) = [d_1, d_2, \ldots, d_{N_t}]^T \) is a \( N_t K \times 1 \) vector containing the symbols for each transmitter.

Considering a time-invariant channel with no post-resampling phase offset, then a system transmitting \( N_t \) OSDM symbols simultaneously can be described as follows for the \( \rho \)th receiver
\[ \rho r_k = \sum_{\varsigma=1}^{N_t} \sum_{l=0}^{L} (\varsigma r_b)^{l} (\varsigma s_{k-l}) + (\rho n_k) \quad (39) \]
for \( k = 0, 1, \ldots, K \). For each pair \( (\varsigma, \rho) \) with \( \varsigma = 1, 2, \ldots, N_t \) and \( \rho = 1, 2, \ldots, N_r \), there is a CIR vector \( (\varsigma \rho h) \). These vectors can be used to build \( K \times K \) circular channel matrices \( (\varsigma \rho H) \) where the first column is the respective CIR appended by \( K-L-1 \) zeros. Taking these circular channel matrices \( N_t K \times 1 \) received signal vector \((\tilde{M}s) = [r_{k1}, r_{k2}, \ldots, r_{kN_r}]^T \) can be expressed as
\[ (\tilde{M}r) = \hat{H}(\tilde{M}s) \quad (40) \]
is the \(N_K \times N_K\) MIMO channel matrix containing circulant channel matrices.

The MIMO receiver for OSDM is analogous to the single transmitter case where the initial demodulation, CIR estimate, and equalization have some nuances. The ensuing steps for demapping and decoding are the same as the single transmitter situation. For initial time alignment and coarse Doppler compensation it is assumed that the transmitters are perfectly synchronous and the geometric distribution of the system is such that there is no significant difference in propagation times from each transmitter. On a MIMO situation the received signal is an overlap of the transmitted signals. As such the OSDM demodulation can be expressed as

\[
(\hat{M}x) = (I_{N_t} \otimes F_N \otimes I_M)(\hat{M}r),
\]

(42)

From (42) we can obtain a received signal description analogous to the single transmitter situation as follows,

\[
(\hat{M}x) = H(\hat{M}d) + (\hat{M}n)
\]

(43)

where

\[
H = (I_{N_t} \otimes F_N \otimes I_M)\hat{H}(I_{N_t} \otimes F_N^H \otimes I_M),
\]

(44)

\[
= \begin{bmatrix}
(1,1)\hat{H} & (2,1)\hat{H} & \cdots & (N_{t,1})\hat{H} \\
(1,2)\hat{H} & (2,2)\hat{H} & \cdots & (N_{t,2})\hat{H} \\
\vdots & \vdots & \ddots & \vdots \\
(1,N_{t})\hat{H} & (2,N_{t})\hat{H} & \cdots & (N_{t,N_{t}})\hat{H}
\end{bmatrix}
\]

(45)

and each entry \((\hat{^c}^p H) = (F_N \otimes I_M)(\hat{^c}^p \hat{H})(F_N^H \otimes I_M)\)

is identical to the matrix described in (8), allowing a similar approach. This results in a per vector received signal that can be expressed as

\[
(\hat{M}x_n) = (H_n)(\hat{M}d_n) + (\hat{M}n_n)
\]

(46)

where

\[
(H_n) = \begin{bmatrix}
(1,1)H_n & (2,1)H_n & \cdots & (N_{t,1})H_n \\
(1,2)H_n & (2,2)H_n & \cdots & (N_{t,2})H_n \\
\vdots & \vdots & \ddots & \vdots \\
(1,N_{t})H_n & (2,N_{t})H_n & \cdots & (N_{t,N_{t}})H_n
\end{bmatrix}
\]

(47)

is a \(N_t \times M \times N_t\) matrix, \((\hat{M}d_n)\) is a \(N_t \times M \times 1\) vector containing the \(n\)th vectors from the \(N_t\) transmitters, and \((\hat{M}x_n)\) is the \(N_t \times M \times 1\) vector containing the data corresponding to the \(n\)th vector from all receivers. Assuming that the pilots were properly placed, then at each receiver it is possible to do the initial channel estimate as (14) resulting in \(N_t \times N_t\) CIR estimates. For each CIR the corresponding \((\hat{^c}^p \hat{H})\) can be computed as described in (15) and (16), then the \(\hat{H}\) matrix can be built and used to compute the symbol estimates. In a zero-forcing sense we obtain

\[
(\hat{M}x_n) = \left(\left(H_n\right)^H\left(H_n\right)^{-1}\left(H_n\right)^H\right)(\hat{M}x_n)
\]

(48)

From this initial estimation demapping and decoding are performed independently for each transmitter, as coding and mapping are independent. The initial channel estimates and decoding might not be sufficient, in which case a refined channel estimate can be computed.

The description of the channel given by (43) has a limitation, as there is no direct reference to Doppler due to imperfect initial compensation. Considering that for every transmitter-receiver there is an associated Doppler shift the following MIMO extension of (6) may be written

\[
(\hat{M}x_n) = \sum_{i=1}^{I} G_i \circ H_{n-i}(\hat{M}d_{n-i}) + (\hat{M}z_n)
\]

(49)

where

\[
(G_n) = \begin{bmatrix}
(1,1)G_n & (2,1)G_n & \cdots & (N_{t,1})G_n \\
(1,2)G_n & (2,2)G_n & \cdots & (N_{t,2})G_n \\
\vdots & \vdots & \ddots & \vdots \\
(1,N_{t})G_n & (2,N_{t})G_n & \cdots & (N_{t,N_{t}})G_n
\end{bmatrix}
\]

(50)

is the MIMO phase matrix for each vector and contains the phase matrix for each channel, \(\circ\) denotes a block Hadamard product and a similar approach to the one used for a single transmitter may thus be adopted at the receivers.

As in the single-transmitter case an iterative procedure may be used to refine the channel and symbol estimates. Now, however, MIMO cancellation is used to deal with the superimposed transmissions. The signal observed at the \(n\)th receiver is given by

\[
(x_n) = \sum_{c=1}^{N_c} (\hat{^c}^p H_n)(\hat{^c}^p d_n) + (\hat{^c}^p z_n).
\]

(51)

From this description, MIMO cancellation is simply, for the \(c\)th transmitter,

\[
(x_n) = \sum_{c \neq ct} (\hat{^c}^p H_n)(\hat{^c}^p d_n)
\]

(52)

where \((\hat{^c}^p d_n)\) are the symbol vectors estimates. The refinement of the CIR estimates using the symbol vector estimates is done by solving an optimization problem

\[
\minimize \sum_{n=0}^{N_t} \left\| (\hat{^c}^p x_n) - \sum_{c=1}^{N_c} (\hat{^c}^p H_n)(\hat{^c}^p d_n) \right\|^2.
\]

(53)
By using MIMO cancellation the optimization problem can be rewritten as
\[
\text{minimize} \quad \sum_{n} \sum_{\rho} \sum_{\varsigma} \left\| \left( \begin{array}{c} \varsigma \rho x_n \\ \varsigma \rho \hat{H}_n \rho d_n \end{array} \right) \right\|^2.
\]
(54)

By explicitly using MIMO cancellation, this optimization can be done by solving for each MIMO cancelled signal as if it was a single transmitter, specifically using (30). As each CIR is computed the cost function (54) is evaluated using the new estimates until the fit does not cause a significant drop in the cost function or a maximum number of iterations is reached.

With both the phase and CIR estimates available a MIMO inter-vector interference cancellation can be performed as follows
\[
\left( \begin{array}{c} \varsigma \rho x_n \\ \varsigma \rho \hat{H}_n \rho d_n \end{array} \right)\]
where (55) is evaluated using the new estimates until the fit
\[
\left( \begin{array}{c} \varsigma \rho x_n \\ \varsigma \rho d_n \end{array} \right)
\]
By stacking the nth transmitted symbol vectors from all \( \varsigma = 1, \ldots, N_t \) as \( \varsigma \rho d_n \) and by stacking the received signals as \( \varsigma \rho x_n \) then the previous equation becomes
\[
\varsigma \rho x_n = \varsigma \rho H_n \varsigma \rho d_n
\]
(57)

4.1. Simulated Channel

The simulated UWA channel has a symbol spaced memory length of \( L = 20 \) corresponding to the multi-path delay. It was simulated using a Rayleigh channel function in MATLAB and is time invariant. The residual Doppler is added to the signal after the Rayleigh computations, and is generated for each \( k \) symbol of a block using
\[
\theta_{k+1} = \theta_k + 2\pi c T_s + \xi_k
\]
(59)

for \( k = 1, \ldots, K \), where \( c = a_s f_c \) is the residual Doppler factor and \( \xi_k \) is an additional phase distortion randomly generated from a Gaussian distribution \( N(0, \xi^2) \). White Gaussian noise is added to the signal resulting in the channel model described by (3), and as such the described structure for equalization can be used.

4.2. OSDM Shape

As described before, the choice of the shape of the OSDM matrix is one of the most important design decisions. By choosing a small \( N \) the number of columns can be increased and with it the pilot vectors are larger and provide a better estimate for the CIR. As seen in Figure 3, the choice of the number of rows and number of transmitters has a big influence on the efficiency of each OSDM block. This is not the only constraint, as different OSDM shapes might have different performances. The channel used was described in the previous section, with no Doppler noise, and the following parameters are set to \( J = 8, I = 3, \beta_{\text{max}} = 3 \) and \( \gamma_{\text{max}} = 3 \). The number of columns must be greater than the length of the channel taps and as such three configurations were tested, namely, \( N = 8, 16, 32 \). These configurations are the most efficient for an OSDM block.

From the results seen in Figures 4 and 5, as the number of rows increases the BER increases also, which might be due the fact that the initial channel estimation has fewer pilot symbols, as there are fewer columns, and as such the initial CIR estimation is not sufficiently detailed.
good. Given that the system’s performance is best for $N = 8$ and the increase in available bits is negligible when using $N = 16$, then the most useful OSDM shape for this situation is $N = 8, M = 128$ for both the single transmitter and MIMO cases.

4.3. Iterative process

The receiver has two main iterative processes running: an inner one where there is a computation of the CIR and phase and an outer one for equalization and decoding. For each, a limit on the maximum number of iterations must be set, chosen in such a way that there are sufficient iterations without unnecessary increase in computation complexity.

Using the channel described in Section 4.1 with no Doppler noise, and setting $N = 8, J = 8, I = 3$ and $\gamma_{\text{max}} = 3$ there can be an evaluation of the $\beta_{\text{max}}$ maximum number of detection iterations without compromising the performance. By evaluating BER results for the single transmitter and 2x2 MIMO for $\beta_{\text{max}} = 0, 1, 2, 3$ there is a clear improvement for $\beta_{\text{max}} > 1$ and a small improvement from $\beta_{\text{max}} = 2$ to $\beta_{\text{max}} = 3$, as seen in Figures 6 and 7. These results suggest that for practical solutions $\beta_{\text{max}} = 2$ is sufficient to obtain a mostly correct detection.

For the joint channel phase estimation a different approach was taken to evaluate the impact of the maximum number of iterations $\gamma_{\text{max}}$. Considering the described channel, the same fixed parameters as before, with $\beta_{\text{max}} = 3$ and $SNR = 25dB$ the output SNR is evaluated. By introducing a variation on the Doppler noise, the results for the output SNR are shown in Figures 8 and 9. The results suggest that for this situation there is an advantage on using the $\gamma_{\text{max}} = 3$ for the joint channel and phase estimation, as the increase in complexity is justified by the gain of the output in both the single transmitter and MIMO.

For the multiple transmitter case, there is a need for cancellation when performing the joint channel and phase estimation. This cancellation allows for each transmitter/receiver pair to be treated individually. However, in each cancellation the resulting estimate could be an improvement on the previous one and as such these new estimations should be used to perform an improved cancellation and result in better CIR estimates.

This process is done for both the CIR and phase estimates, and a BER evaluation of different values for the max iterations for each cancellation process can be seen in Figure 10. The results suggest that for two iterations there is an improvement on the estimates and for more iterations there is a fit on the noise resulting in a worse estimate.

4.4. Phase Estimation

With regards to the phase estimation method there are two relevant parameters to evaluate: the Doppler span $I$ and the length of the quasi-static subvector $J$. In the single transmitter case, by fixing $SNR = 25dB$ and varying the Doppler noise the resulting output SNR is seen in Figure 11. It would be expected that for larger Doppler spans and shorter length of quasi-static sub-
vectors there would be an increase on the output SNR with an increase on the computation time. However the results do not reflect any significant improvement on the output SNR with the increased SNR. By evaluating the phase of the CIR it seems to be compensating for much of the phase distortion introduced by Doppler, resulting in a small contribution of the phase estimation part of the algorithm.

5. Ray Tracing Simulation
A numerical simulation of channel was done using the Bellhop ray tracing program from the ocean acoustics toolbox [1]. This simulation replicates the conditions of a previous underwater acoustic trial, CALCOM10, performed in Vilamoura, Portugal, in June 2010. The sound speed profile used in Bellhop was measured during the trials and the bathymetry of the location was obtain from the General Bathymetric Chart of the Oceans (GEBCO). The simulated channel has no Doppler noise and signal to noise ratio $SNR = 25dB$ was set.

In these simulations the block length and duration is the same as in the previous case and the following parameters were set to $J = 8$, $I = 3$, $\beta_{max} = 2$, $\gamma_{max} = 3$. The CIR in this simulation has a tap length of about $L = 120$, which limits the available OSDM shapes that ensure $M > L$. Considering the previous analysis of the best shapes $N = 8$ was chosen.

Using the OSDM modulation in both the single transmitter and multiple transmitter configuration the system was able to accurately estimate the CIR, as seen in Figures 12 and 13 for a single transmitter and in Figures 14 and 15 for MIMO. These estimates then lead to accurate equalization and decoding and as such suggest that the considered system is robust to different underwater scenarios. As long as the channel tap length is known, then an appropriate OSDM shape can be chosen to maximize the number of bits per OSDM block with the limitation that $M > L$.

6. Conclusions
The proposed OSDM communications system was described, including the modulation and demodulation process, the channel description for the transmitted signal and how the modulation can be used to provide a per-vector equalization strategy. A turbo like iterative process on the symbol detection was presented, where a the channel and residual phase are jointly estimated, followed by an equalization process and decoding. This process allows an improvement on the confidence of the decoded bit stream. A MIMO extension of the OSDM modulation was proposed where the spectral efficiency is improved by using simultaneous transmitters. It is possible to design a MIMO system which takes advantage of the OSDM per vector structure, and as in the single transmitter case reduce the complexity of equalization.

The results of this work were presented, the OSDM parameters were assessed to find a suitable set for efficient transmission given some channel constraints. It was shown how the iterative processes can improve
the system. The simulation results suggest that the joint channel and phase estimates hinder each other, as the channel equalization also compensates for some of the Doppler noise. This compensation affects the Doppler structure, causing the consequent phase estimates to be largely irrelevant for actual Doppler compensation. These results were somewhat unexpected, as they do not agree with the ones shown in [5].

In conclusion, the OSDM modulation has interesting characteristics for underwater acoustics communications, namely the flexibility regarding the shape, which allows deployment in various underwater environments, the per vector structure that allows an intuitive turbo like detection and the adaptive frequency domain size which allows better management of the peak to average power ratio.

Future work on this theme involves: Testing MIMO cancellation schemes which use the available information regarding the correctly decoded vectors to improve on the CIR estimates; Developing an algorithm which allows better phase estimation and Doppler compensation, as well as alternative criteria for performing joint channel and phase estimation in a non-iterative way; Extra testing, including a field or tank test to assess under which conditions significant performance improvements relative to single carrier or OFDM might be obtained; Developing adaptive modulation strategies for selecting suitable parameters (namely, number of subcarriers and length of vectors) based on feedback information in bidirectional underwater acoustic links.

References