Financial Markets Evolutionary Adaptive Dynamics

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October 2018

Abstract

Modeling economic agents is quite often naively rooted in the maximization of a non-trivial and sometimes subjective utility function. Finding statistical evidences of whose economic features explain what in the financial world is most of the times, if not impossible, challenging. Nonetheless, in the recent years, developments in Evolutionary Game Theory (EGT) have endorsed those who believe markets are nothing but a giant and complex evolutionary game bounded by both behavioral and economic rules. In this thesis, we firstly develop an adaptive model with 2 strategies as Market participants such as Momentum and Fundamental Traders. Through their investment decisions, we measure how population collective behavior evolve in time, through the dynamics of peer-influence and changes in the Market. We show that even in a oversimplified population dynamic, Momentum Strategy dominate as spreads out to the entire population, under several different settings. Also, inequality and power-law distributions tend to constantly surge.

Keywords: Evolutionary Game Theory, Financial Markets, Adaptive Agents, Investment Strategies

1. INTRODUCTION

Financial Industry is, by definition, always searching for better accuracy in market predictability. If someone can understand and replicate better what happens in real live-markets, then it has an edge that can be taken profit out of it [4]. In such environment, several thousand of agents compete to outperform its peers, by having not only more money, but also lower levels of risk per unit of profit made. This concept of competition in a population sounds quite familiar to those who study, since long time ago, EGT. The hypothesis underneath the present reasoning comes from the fact that a population with interchangeable behaviors in a Darwinian Evolutionary dynamic is as close to biology as to Financial Markets. Our model replication power, therefore, can be derived from the simple aggregation of agents with few properties or economic features [19].

As William A. Brock [6] asked in his study on the impact of different agents trading together in the market, is there a chance that investors expectations on the future deviate prices long enough allowing profitable opportunities to smart investors? Can we have bearish[1] or bullish[2] views that "cluster together" [6] and lead prices to spike or fall without any "rational evidence"? Different Market expectations are indeed one of the motivation sources to the present study. Whenever we have agents making decision based on different activation functions, we are in a presence of a conflicts of interests [5, 6], and contrarily views about future markets returns (having, therefore, opposite trading and investment decisions that leads ultimately, to opposite performance in the short-term). Having financial Markets framework in presence, agents are competing to have the strategy that can produce higher future expected risk-adjusted sustainable returns. In Farmer and Lo "Frontiers of finance: Evolution and efficient markets" [13] they pose a simple example: assuming 2 populations (from 1926 to 1976) where the first one can only buys the S&P Index and the second one is simply buying the U.S. Treasury bills (one of the safest securities in the world), the accumulated gain of investing 1 dollar is $1,370 and $14, respectively. However, just by letting agents to copy peers' strategy and if perfect foresight was possible, the expected gain would be more than 2 million dollars. Therefore, an ecology of strategies seems to be relevant enough to study and measures its impact on individual and global performance.

Using S&P real historical data (from 1927 until 2018), we measure which of the main strategies is better, and which one is more likely of spreading across investors and Financial Professionals. Additionally, We investigate the disparity in income distribution of the agents in the population. Therefore, the investigation questions that the present problem can arise are the following: *i)* Is there a Dominant strategy in Financial Markets?; *ii)* As time goes by, Does the Domi-

[1] Pessimistic view of the Economy. Most of the times followed by a short-selling position in the Stock Market, meaning, betting that the market is about to fall;

[2] Optimistic belief in the Market and Economy. Most of the times in line with a long position in securities underlying a upper movement in price in the recent future;

nant/dominated Strategies change over time?; *iii)* Can Evolution promote wealth distribution or it is promoting inequality in income distribution?

1.1. Structure

The structure will be the following: **Chapter 2** describes the state of the art in Evolutionary Game Theory, Finance as well as related work in economically driven Multi-Agents systems such as the experiment we are conducting here. Then, **Chapter 3** addresses our proposal and methods employed, namely, the necessary analytic components in order to build a population that fits into the financial reality. Finally, in **Chapter 4**, we show the results and further questions that those results brought, and **Chapter 5** is where we debate our findings and the necessary next steps to be made in the presented area of study.

2. RELATED WORK

Designing economic simulations in a multi-agent system is rooted in several different fields and areas of study. This multidisciplinary comes up naturally as we are dealing with a system made by different types of players when it comes to invest (field of **Traditional Finance**), where those players may not be the most rational ones since they are driven by behavioral human deceptions, (**Behavioral Economics** area). Finally, all investors are thirsty not only to outperform their peers but mostly the overall market. So, it is undeniable the need to reflect the state of Art in **Evolutionary Game Theory** to fully capture the essence of competition relations [17].

2.1. Questioning the Efficient Market Hypothesis

In the center of Economic Theory regarding Asset Prices is the Efficient Market Hypothesis. Firstly proposed by Eugene Fama et al in 1969, EMH is the concept of having markets that quickly react and adjust to new information [12]. By this it is meant that efficient Markets are consistently "reflecting all the available information" [11], where theirs players are constantly processing the information rationally, ignoring the irrelevant one and avoiding systematic errors and deviations from the fundamental value of the Economy. This concept of "fundamental valuation" is the currently fair value of all assets in the economy and the present value of future expected income from current asset allocation. In other words, it measures how much the economy is worth.

Since its origin, EMH began to be tested in the majority of markets and different environments and it has as many critics as supporters. A good survey related with this topic was written by Meredith Beechey, David Gruen and James Vickery [3]. On the top of critics of EMH is the definition of information availability and fundamental value of the Economy. On one hand, information is not costless to obtain [3]. Thus, Market players need to perform research and develop methods and tools to better capture what is happening in the market. Grossman and Stiglitz said that even if, by absurd, all the information was constantly reflected in asset prices, there would not be any incentive to obtain it, being price fluctuations close to a random walk which is far from being accurate [16].

It was not until the beginning of the XXI century that CAPM (Capital Asset Pricing Model) benchmarks previously stated started to be profoundly questioned by both psychological and economic researchers. Even though the criticisms were rooted in human behavior and known deceptions, it had an enormous impact in Finance and Economics. It is good to remember that in this same period, Daniel, Hirshleifer and Subrahmanyam presented statistical proofs that something greater than the Adam Smith "invisible hand" was moving the markets to odd "corners" [10]. Abnormal fluctuations in prices, spikes in volatility, long-term reversals[3], short-term momentum[4] and event-based returns were responsible to outperform what the EMH would predict [4]. Moreover, in 2000 the awarded Nobel-Prize Robert Shiller published his "Irrational Exuberance" book, showing persistent deviations from a "normal path" in Stocks [25].

Despite lacking some more statistical evidences [2], a behavioral approach is getting widespread throughout all academic studies and research. This Behavioristic view of Financial Markets is rooted at rejecting a perfectly rational population of investors and limiting the power of arbitrage. The greatest contribution on the stated issues, came from Kahneman and Tversky [18] whose impact in the economic theory is still today far from being accomplished. These psychologists' earlier research helped to conclude that even in a mix population of fully rational investors with noisy traders[5], the latter ones can be responsible for consistently deviating asset prices (causing for instance bubble cycles) being the first ones powerless to arbitrage[6] those incomprehensible price fluctuations. Furthermore (and perhaps more important) their research made on choices prove that humans are at the most partially rational due to their beliefs and preferences [18]. These systematic biased and "bounded rationality" (concept firstly proposed by Nobel-prize-winning economist Herbert Simon) drive prices to a permanently quasi-random dynamic process due to the unstable and not universal investors' expectations.

The main responsible of connecting these two contrarily Worlds were, with no doubt, Andrew W. Lo.

[3]Positive short-term auto-correlation between stock returns;

[4]Well-known as "Overreaction", it is a negative autocorrelation between short-term returns and longer lagged ones;

[5]Investors that somehow do not follow any perceived economic rule, being responsible to add "noise" or irrationality into daily prices.

[6]Power to Arbitrage is comprehended as the power of smart investors to price correctly asset prices, driving irrational traders out of market;

Within his paper on "The Adaptive Markets Hypothesis" which can be described as the study of "Market Efficiency from an Evolutionary Perspective" [20], Lo launched a disruptive view of Markets based on "Socio-Biology" and "Evolutionary Dynamics". His approach to Finance is heavily influenced by the principles of competition and natural selection between investors or investment strategies. By connecting the dots of multiple diverge areas of study, he developed a framework that allows economic agents to be modeled as a social entity (rather than a binary "Homo Economicus" [17]) with behavioral characteristics while operating in a constantly changing system far less homogeneous that what its predecessors believed [20]. For the present work, Lo's research is by far a central piece. It is used throughout this study his concept that "individuals make choices based on past experience and their "best guess" as to what might be optimal, and they learn by receiving positive or negative reinforcement from the outcomes" [20].

2.2. Economics within Evolutionary Game Theory

2.2.1 Game Theory

After reaching this stage, it seems reasonable to do a step-back to specify how economic reality can be described under the laws of Game Theory. Game Theory is the study on how outcomes change when agents are interacting with each other and making strategic decisions based on that. The first breakthroughs on GT were proposed by Von Neumann and Morgenstern [29]. Actually few years later, John Nash's discoveries (one of the fathers of Game Theory) were deeply used in all sorts of economic studies and well-known dilemmas. His proposal was the idea of an equilibrium (which is a core concept in Economics) where none of the players have any incentive to change their strategy, regarding the co-player decisions [22]. The introduction of an equilibrium is crucial due to the fact that if investors have no gain in changing their strategy towards an asset price, it means that the market fully reflect, at that specific time, all the available information and stock prices' converged to its fair valuation [20]. Knowing that reality it is slightly different from a 2-by-2 scenario, Game Theory needed to be reconfigured to move from pair interaction to group interaction. Games with this extension are also called N-Person Games. Those are most of the times a generalization of Prisoner's Dilemma with more than two persons. Nonetheless, in the prior framework we are neither considering what are the changes if both players would have played a series of games (instead of a one-off decision) nor the impact of a particular outcome on his/her future decision. This likelihood of different dynamics constraint to fitness and population characteristics are crucial to the "survival" of an individual throughout time.

2.2.2 Evolutionary Game Theory

The combination of payoffs and fitness allow us to empower and enlarge the previous model and its applications. By doing so, agents with greater payoffs trough time, are perceived as more fit, leading to a higher level of reproduction (in the eyes of a biologist) or higher level of being copied by his fellows (in social sciences). Therefore, adding these new features brings us to an Evolutionary Game Theory (EGT) where it is possible to evaluate evolution of agents throughout time [26]. A key aspect is the fact that EGT focuses more on the dynamics of the strategic change rather than on the properties of strategy equilibria and avoids a set of strong assumptions on rationality [19]. Therefore, the goal moved from predicting how agents behave when interacting with others like them, to the study of the evolutionary dynamics regarding how agents' behavior changes through time.

Nonetheless, by considering a model that is played by in large populations of arbitrary compositions, there is the need to describe some necessary steps and notation. Let be $e = (e_1, e_2, ..., e_n)$ the set of all possible strategies to use, while x is a vector representing the portion of the population using a given strategy, $x = (x_1, x_2, ..., x_n)$. Thus, we are interested to acknowledge the evolution of vector x, while individuals are making decisions regarding strategies.

Designing evolutionary dynamics within EGT is slightly different from modeling pure rational agents, that fully know the payoff structure of the game and choose actions following heuristics such as best response. In EGT evolution is driven by the best individuals being copied more often than the rest of the population. Likewise, being copied more often can be directly translated by the increase of the fraction of individuals that share the most fit strategies. Mathematically speaking, this process can be processed through a replicator equation. This replicator tells us which fraction of the population (using a given strategy) will increase/decrease proportionally to the adaptivity provided. Therefore, strategies that yield some additional advantage to the population will spread faster as they are more copied over time. The replicator equation can be as the following:

$$\dot{x}_i = x_i[(Ax)_i - x.Ax] \tag{1}$$

The presented formula can be simplified, since we are dealing with only two different strategies $(n = 2)$. If M and F stand for momentum and fundamental populations, respectively, the system of differential equations in (5) become equivalent to:

$$\dot{x}_i = x(1 - x)(f_m - f_f) \tag{2}$$

where x is the portion of momentum traders in the system and $1 - x$ the fraction of fundamental investors. Additionally, f_m and f_f are the fitness of momentum

and fundamental strategies. Nonetheless, the equation above has some restrictions. Firstly, it only has a single solution given an initial fraction of x strategy and the distribution of strategies in the population. Secondly, it assumes a system with an infinitely large number of agents where all can interact with each other (in other words the replicator equation is configured for a well-mixed and fully connected network with infinitum amount of individuals). Due to the need of considering finite populations within scale-free networks, the next section will address the introduction of stochastic effects.

2.2.3 Stochastic Effects in Evolution

Moving from an infinite population to a computer EGT simulation based on a finite population forces the introduction of a stochastic process. With infinity populations we know, deterministically, the dynamic of the population (using the replicator equations). Nonetheless, for finite populations it is needed to consider the probability of an agent moving from one strategy to the other. Likewise in many other studies [24, 28], in this work it is proposed to use Fermin function as a pairwise comparison rule:

$$P(x\, imitates\, y) = \frac{1}{1 + e^{\beta(f_x - f_y)}} \tag{3}$$

As seen in the equation above, when using this update rule, imitation will occur with a probability proportional to the gap between agent's fitness and the one he is comparing to $(f_x - f_y)$. Note that if both have the same fitness, the imitation probability is 0.5, implying that fitness does not influence the decision making process of changing agent's strategy. Moreover, β parameter represents the strength of population selection. For lower values, imitation will be close to happen randomly, whereas for larger β, imitation will strongly depend on the differences between fitness [24].

2.2.4 Financial Evolutionary Game Theory

There is a profound research to be mention regarding Economic EGT applications, with a special attention to Daniel Friedman works [14, 15]. Dedicated to prove that Evolutionary Stable Strategies could be applied to economics, he presented several studies to demonstrate his point of view. A population in a ESS is the situation where not only there is no incentive to change strategy, but mostly that even if a small population of "mutants"[7] invade with an abnormal behavior, they will eventually disappear due to lack of fitness and natural selection.

[7] Concept brought by biologists to ADN and Cell Studies. As for this work it is understood as a simple change of opinion;

Despite the fact of being rooted in Biology, as aforementioned when we are dealing with economic realities, it is undeniable the need to move from this "mathematical" genetic mechanism to a more complex learning and copy mechanism where social interactions and a broader imitation processes variety [14] can exists. In his study, it was developed a population whose actions were a product of a strategy profile set. To define those strategies he mentioned several examples with a growing level of complexity. If, for instance, in a population of Buyers and Sellers where the latter ones can cheat or be honest about what they are selling and the first ones be naive or suspicious, each one of the elements of the populations desire to know who they are dealing with to take advantage out of it (cheating if far less attractive to sellers when they are dealing with more suspicious buyers).

Knowing the diversity of trading investment profiles, additional studies were done to better picture whose profiles are more determinant and how they evolve throughout time. William A. Brock et al [6], under the notion of a theoretical ABS (Adaptive Belief System) [5], developed an artificial market with several competitive (and different) buyers and sellers that can randomly interchange their beliefs and actions based on two different asset classes (one relatively risk-free and one close to a stock market index, thus riskier). These works shown that as we add more "intensity of adaptation" or diversity of beliefs in those market types, price deviations or fluctuations appear to be more persistent relative to fundamental benchmarks and responsible for generating excess volatility [6].

Nevertheless, as aforementioned, updating information anytime something change is costly. Few Market players have the capability of reacting to every event in the Stock Market. The reasons for the stated come from quite different sources: from Economic to population and behavioral constraints. On one hand, some players (as Mutual Funds and Trackers) are by construction constraint to follow strict rules regarding investment. As Chan pointed out on his survey about the main characteristics of major Market players, those who are constraint tend to broadly cluster their investment strategy to the overall market performance, without taking quite extreme positions and preferentially favor higher past winners [27, 9]. Therefore, even if the presented strategy is not the optimal under a given time horizon, several investors are "stubborn" fundamentalists or trend followers [19].

3. MODEL METHODOLOGY

3.1. Investors Decision Making Process

In order to understand how Markets behave, it is crucial to define how agents react to changes in the environment. Based on the Chan's book [8], a trading strategy is a pre-set of methods to make money out of the fluctuations of the market. Defining investment strategies is challenging since the disparity in types of

trading profiles. Yet, a strategy can only be profitable if is either mean-reverting or following a trend. Therefore, Market participants can be grouped in 2 different profiles: Momentum and Fundamental Investors. Moreover, the usage of daily prices might seem limited considering Financial Market complexity. Nonetheless, it is the belief of this proposal that investors tend to rely on prices to make investment decisions. In Karl Marx's Capital [21], it is mentioned as a major criticism of Capitalism the fact that 2 products with the same price are perceived as similar despite the differences in sources of value (like the intensity or type of labor used). Therefore, even if some Market players use complicated methods and models to produce their investment "activation function", those tools tend to be dependent and mostly a function of past prices.

3.1.1 Strategy Based On Past Performance

When it comes to designing any investment strategy, it is fundamental to define a reference point. Without having a clear fundamental value of the economy, today's fair valuation can only be priced relative to past performance and to future expectations. Therefore, investors compare today's prices to a given look back period in time. This oversimplification of a moving average is the same to say that the cheapness or expensiveness of a given asset is constraint to what it has been doing in the past. So, despite the strategy used, investors have a specific look back period, κ, that they used at performing investment decisions. For instance, if a given investor has a $\kappa = 25$, then his decision of buying or selling in the Stock Market will be derived from the relation between today's price and the price of in κ, meaning, $P_t - P_{t-25}$. Therefore, Investment trading decisions can be summarized as two contrariwise boolean activation's functions, such as, $b(P_t, P_{t-\kappa})$ and $s(P_t, P_{t-\kappa})$, for buying and selling, respectively.

Momentum Investment Strategy Market Agents that use momentum strategies are those in favor of "surfing the trend waves". Using their own indicator (κ), they are willing to buy if prices are trending up relative to historical prices, and sell if they are already falling. Their reasoning comes from believing that the aggregate market knows better where the value is. Thus, a single investor can only follow Market tendency and buy/sell what is fashionable. Consequently, Momentum investors are responsible for increase bubble cycles if prices are rising and augment panic cycles in the Stock Market when prices are going down.

$$b(P_t, P_{t-\kappa}) = \begin{cases} 1 & \text{if } P_t > P_{t-\kappa} \\ 0 & \text{if otherwise} \end{cases} \tag{4}$$

$$s(P_t, P_{t-\kappa}) = \begin{cases} 0 & \text{if } P_t > P_{t-\kappa} \\ 1 & \text{if otherwise} \end{cases} \tag{5}$$

Fundamental Investment Strategy Contrariwise to Momentum, Fundamental investors have an opinion on Market Valuation. They think the economy has an intrinsic value and it is more robust than the "irrational" short-term ups-and-downs. Therefore, they follow a trend reversal strategy. Fundamental traders invest on the logic of capturing the local minimum/maximum, believing that if a given stock is trending up will eventually converge to a broader moving average which is its fair value. So, using the same reasoning as above, with an indicator (κ), they are willing to sell if prices are trending up relative to historical prices, and buy if they are already falling. Agents following such strategy are those that are betting against big swings in prices and trust in price stability over time.

3.2. Our Proposal

Thus, in the present work, it is proposed a sequential dynamic model composed in three stages. Focusing on an arbitrary agent or investor i, these stages can be described as follows: *1)* Investor i based on his strategy, takes an action of buying or selling a given product (in the present study it is used as asset to be exchangeable, the S&P 500 Index[8]; *2)* Based on his trading operations, a daily positive or negative income flow is added on his fitness, based on the following equation:

$$f_t = f_{t-1} \times Return(t); \tag{6}$$

$$Return(t) = 1 + log(\frac{P_t}{P_{t-1}}) \times Strategy(t-1); \tag{7}$$

3) After the market is closed for the day, Investor i has the possibility of copying a random neighbor if the gap in fitness is large enough (using a β of 1):

$$P(i\, imitates\, j) = \frac{1}{1 + e^{(f_i - f_j)}}; \tag{8}$$

4. Results & discussion

4.1. Homogeneous Populations

Consider a population of 10^4 similar individuals, whose initial fitness [9] is set to 100 and all sharing the same strategy. Within such simulation framework, coping each other strategy does not have any impact, since all agents have the same profile. Nonetheless, it is important to acknowledge which strategy generates the highest payoffs in the period under study. This analysis offers a baseline reference to the study of heterogeneous cases. As summarized in **Figure 1**, we can argue some of the strategies are considerably better relative to its peers, individually speaking.

[8] Despite the fact of being unrealistic having a population of investors focused on buying/selling one single product, it can be understandable as a study that it is only interested in capturing investors' trading decision on a given index either through buying any Index constituent or an Exchange-Traded Fund that aggregates Market members.

[9] It can be understood as the monetary disposition that an agent has to invest in a given period of time.

Figure 1: Strategy performance relative to a "Buy and Hold" benchmark (values above 100% outperform the strategy). Each bar represent a decade and the right-most one includes the entire sample.

One can say that this may indicate that if we mix these strategies into a well-mixed population, dominant strategies must appear. Momentum Strategy seems to constantly outperform a fundamental one. This confirms the well-documented Carhart's Momentum factor [7] proving that following a momentum strategy yields superior risk-free returns, all else equal. Nonetheless, it is important to refer the fact that the performance of a strategy is not constant throughout time. For instance, an investor i in the period of 1998-2008 that blindly followed a Fundamental strategy would have higher payoffs than any other strategy, for the same period. This results can be easily confirmed by the data, since it was the period of the "dot.com" bubble and the Financial Crisis in 2008. Finally, it is interesting that, relative to a simple "Buy and Hold" investment reasoning, most strategies not only under-performed but also lost money in most of the sample periods, as described in **Figure 1**.

4.2. Semi-Heterogeneous Populations

In the present section, we tested the impact of population dynamics and evolution. If a strategy is far better than any other, it should spread and be increasingly adopted by the agents in the population. Again, we are considering a population of 10^4 similar individuals, but assuming that 50% of them were initially following one strategy and the rest another one. For instance 50% were Fundamental investors while the rest were momentum traders. We are now considering a population with a semi-heterogeneous profile assuming that agents can only adopt one out of two strategies and individuals with the same strategy are equal. Additionally, every agent is allowed to pick in a given time-stamp any random investor to (re)evaluate his investment strategy[10]. Therefore, the number of interactions (and possibilities of changing strategy) is equal to the number of periods of each simulation. In the case of the entire sample, the number of possibilities is above the 2.3×10^3 daily iterations. Within such context, we measured the impact of the under and out-performance of a strategy relative to the other. Moreover, one can evaluate how fitness distribution is across time and test how the population evolves.

4.2.1 Momentum Strategy Dominance

Given the confirmed fact that, individually, Momentum outperform most strategies throughout time, we tested whether that would be valid if agents can copy other investors' investment decisions, leading to a convergence to a fully Momentum population. Thus, we run a set of 100 simulations for a population of Fundamental and Momentum investors. Yet, it is important to mention that we are assuming fully-committed agents that live forever and the number of players is fixed.

Fundamental Vs. Momentum Strategy ($\kappa = 50$) As it can be seen in **Figure 2**, after running no more than 1×10^3 iterations, Fundamental investors are completely dominated by Momentum investors. The results seem to be in line with both **Figure 1**, and

[10] It is assumed a complete graph where all nodes are linked between each other.

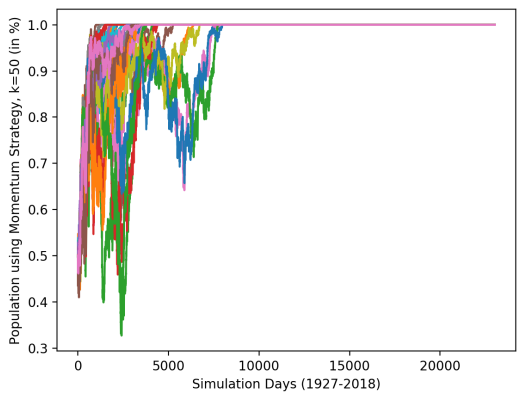

Figure 2: Full Momentum Strategy Dominance (Initial condition of 50/50 Fundamental and Momentum Strategy, $\kappa = 50$)

Carhart's Momentum factor [7], given the discrepancies in performance. Furthermore, after confirming momentum dominance, we were also interested in understanding the behaviour of fitness/wealth or income distributions were, on average, for the simulations made.

As displayed in **Figures 3** and **4**, where we have the cumulative log-log income distribution, we can argue that we are in the presence of a power-law distribution or Pareto Law income inequality where a few wealthy nodes accumulate most of the earnings from the market whereas the large majority has relatively under-performed, for the period in study. The presented Figures allow us to make some comments about the inner characteristics of our financial model simulations. In the first place, however incipient, allowing individuals to copy other people strategy changes the population dynamics. We can argue that whenever the model introduces evolution, income distribution changes. Secondly, comparing both Figures, we can see power-law tend to smooth as agents can copy someone else way of investing. For the same period, adding the presented feature, it empowers equality on investors fitness. Therefore, one can say that evolution tries to mitigate the "rich gets richer" effect, where flowing most of the profits from the markets to the ones that are wealthiest. This "evolution effect" can be somehow explained by the simple fact that, by allowing less fit agents to copy others' strategy, the likelihood of also performing better increases.

4.3. Heterogeneous Populations

Adding the possibility of having many different investment profiles is moving a step closer to the real-live markets. Firstly because, bearing in mind the amount of different types of players that Financial Markets can have, it is challenging to accept that they all share the same reasoning when it comes to investing. Secondly, being difficult to access the investment time horizon an investor has, sampling a larger set and leaving the work in hand of evolution, is, if not accurately what happens in reality, the closest guess we can find. As in the previous contexts, we considered a population of 10^4 individuals, but giving to each one of them a "unique investment profile" (κ), ranging from 1 to 500 trading days. The chosen range was thought to include a large spectrum of investors, since High-frequency traders to the most conservative funds. Therefore, instead of having a population with only two ways of looking into the environment and act, now we have 1000 different strategies, where half of them follow a Momentum mindset of trading, while the rest is Fundamental investors.

4.3.1 Momentum Strategy Dominance

Having a population with a larger set of strategies, it is not only interesting to understand which (if any) of the strategies dominates but also which is the prevailing and most profitable investment time horizon (κ).

The impact of Look-Back Investment Period (κ)
Most of what was discussed so far relied on the fact that Momentum Strategy is dominant in most environments. Despite the introduction of evolution, other strategies cannot perform and spread as effective as Momentum. However, as we discussed earlier, financial population is quite diverse in strategies and profiles. Having said that, measuring the impact on the overall wealth of allowing investors to copy, has key importance. In **Table 1** it is the wealth of a mixed population versus one with only Momentum investors and other with Fundamental individuals. This is relevant to check how populations benefit from copying someone else strategy. In the first half of the table we can see, once more, the disparity between Momentum/Fundamental Strategies but mostly we see the impact of a mixed population. Despite being reasonably better off for Fundamental investors to be in a mixed environment, Momentum investors that temporarily changed to this losing strategy, will be worse off in the end of the simulation. Between 1947-1977 and from 2007 until nowadays, the mixed population lost value compared with a scenario where evolution and copy was forbidden and the population of Momentum and Fundamental were separated in a 50/50 composition. This seems to be in line with what was discussed earlier about the "survival of the fittest" and dominant strategies spreading out in all cases. In the process of evolution, we find that there is a loss on the overall population wealth. Lastly, it is important to mention the fact that within the mixed population context, only two of the periods considered were profitable for the average investor. This can be seen on the second half of **Table 1** where all the values below 100 represent a lost in wealth (since all agents start with an initial fitness or income of 100). The reason is

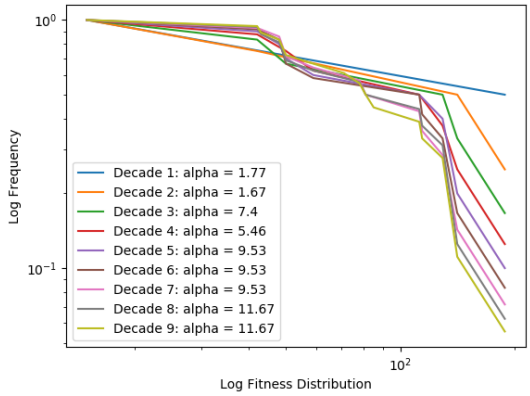

Figure 3: Log-Log Income Distribution over a Population with a 50/50 Momentum and Fundamental Strategy without evolution (allowance to copy others). Each line represents a decade starting in 1927 and finishing in 2018

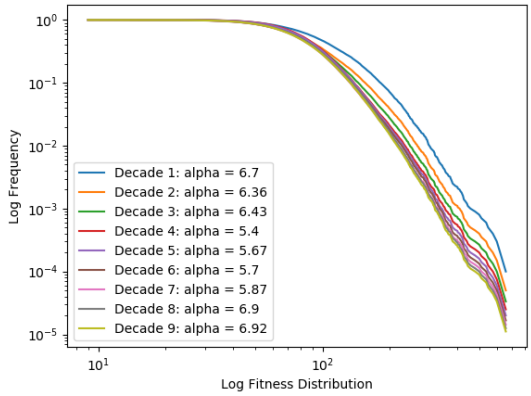

Figure 4: Log-Log Income Distribution over a Population with a 50/50 Momentum and Fundamental Strategy with evolution (every agent is allowed to copy). Each line represents a decade starting in 1927 and finishing in 2018

quite easy to understand: whereas an investor copies the losing strategy it erases his future likelihood of making more income in the future.

Finally, one can say that, within the presented results and information, it could be extrapolated what would be the best combination of look-back investment periods (κ) for the sample in study. We studied the stability of performance of each look-back investment period (κ), when combined with a Momentum Strategy. Thus, we measured not only the average performance of each single strategy, but also their standard deviation and quartiles. Perhaps, considering the reasoning developed, one can argue that a population with a strategy composed by lower κ will have higher fitness ranges, whereas a higher investment look-back period Strategy will be more constant. Therefore, we can extrapolate that an investor i that copies more often wealthy investors (without taking into account the risk-reward of the strategy he intends to copy), will have a greater likelihood of larger spikes in fitness. Thus, this feature brings an all new area of study, which is the impact of investors network in their performance. The individual's set of ties or connections, impacts on his future performance. The population a given agent can be connected not only impacts on his performance, but also on the variation of his wealth throughout time. In the case of our model, agents are only looking to copy someone (randomly) better in terms of fitness. Future studies need to study the impact of network structure in investors relations and its impact on fitness and risk-reward.

5. CONCLUSIONS

In the present work, we took the challenge of contributing to the many efforts that try to provide an explanation for the complexity of Financial Markets. Through this project we conclude that there are strategies that outperform others and income inequality based on investment decision tend to constantly appear when using real US data. Despite being just a starting point, the present preliminary results provide a promising starting point regarding the power of evolutionary game theory to understand and characterize financial population dynamics.

5.1. Summary of contributions

In view of the results obtained, we delineate here what we believe are lines in what concerns future studies:

- **Wisdom of the Crowds or Momentum:** Despite being counter-intuitive, the present work gives strength to the argument that following the Market trends is not only profitable but also the best simple guess of how to outperform the Market. Therefore, following the "Wisdom of the Crowds" is the best strategy. To some extend, these conclusions put in question those believing in the EMH and highlights the benefits of using of Carhart's Momentum factor [7] into Asset allocation Models and Stock picking analysis.

- **Investment look-back Period matters:** Besides using a Momentum Strategy, how far we look back to make an investment decision is crucial to overpass the results of a fully-committed and blind Momentum. As it appears in our results, for the sample period in study, one can say SPX has a short-term period of 2 trading day and a longer one around 420 trading days. Nonetheless, It is the short-term figure where we find the best results showing that having as activation or action function a simple "tit-for-tat" momentum strategy ("Buy if it went up, Sell it otherwise"), is the best response. In our view, this is a quite remarkable conclusion, since these experiments are rooted in EGT and the results seem to be in line with Anatol Rapoport findings in the Prisoner's dilemma and its good performance in Axelrod's Tournament [1, 23].

	Total Wealth			Average Wealth		
	Momentum Strategy	**Fundamental Strategy**	**Mixed Population (Allowance to Copy)**	**Momentum Strategy**	**Fundamental Strategy**	**Mixed Population (Allowance to Copy)**
1927-1937	82 623	21 077	55 848	165	42	111
1937-1947	60 273	33 490	48 411	120	67	97
1947-1957	71 984	29 964	50 560	144	60	101
1957-1967	51 679	42 679	43 925	103	85	88
1967-1977	58 216	35 959	45 441	116	72	91
1977-1987	56 844	35 986	46 765	113	72	93
1987-1997	51 757	40 739	47 172	103	81	94
1997-2007	44 507	42 574	46 285	89	85	92
2007-2018	47 102	42 581	43 981	94	85	88

Table 1: Summary Table of Wealth Distribution for mixed and non-mixed Population of Fundamental and Momentum Strategies (All Values are, an average from 100 runs, in Dollars). The investment look-back period considered ranged from 1-500.

- **"My network empowers me!":** The prevalence of inequality that we observe, even in a population constituted by similar strategic profiles, has key importance. These differences can only be understood and rooted in timing of change and network Ties. With timing of change we mean the time-stamp in which an investor *i* changes his trading strategy (either through copy a given neighbor or just by following his own strategy). Having the luck to change an investment decision in the right time is key to outperform the rest of the community. However, an investor *i* surrounded by a group of "bad traders" would eventually end up worse off than an investor *j* whose neighbors are, on average, making better decision calls when it comes to invest. Despite the concept of "trading network" was not used in the present work, we found that even in a scenario where every investor can be connected to the entire population, those to whom I connect during the simulation are responsible to define my performance. Additionally, not only copying is important to define fitness performance but also its prospect of future growth and variance, over time. Thus, those investors who were lucky enough to pick an agent with a Momentum Strategy with low investment look-back period, will outperform considerably all others. On the other hand, by changing to some peers' strategy based on longer-term investment look-back period would decrease its potential gains, but empower wealth stability.

5.2. Future Work

The questions raised by the present work are far greater than the work itself. It is our belief there are some topics that deserve future and careful study:

- **My actions impact on Market Prices:** One can say that we are only in a presence of a EGT environment whenever agents can change, with their actions, the game they are part of. Therefore, for future reference, we should study the impact that our modeled agents bring to the real prices, or even, how different it is the time series produced by such population of oversimplified investment reasoning relative to real-live prices.

- **The impact of complex networks:** As we tried to show in this present contribution, networks have a central role in the studied reality. Firstly, even with a homogeneous population, the simple fact of copying investor i and not investor j, can be decisive at explaining higher fitness relative to its peers. Studying the impact of different networks is, with no doubt, the way to go in order to fully measure the impact evolution can have and which one of the networks can produce higher or lower degrees of inequality. Moreover, it would be interesting to understand what would be the impact of a single wealthy agent in the history of relations of a given investor i and what needs to happen in order to this investor i become wealthy as well. Finally, following the reasoning of a cooperator-defector game, it would be relevant to quantify how many agents are needed to start an upward or downward movement in financial Markets that can spread out and contaminate the entire network.

- **The impact of Behavioral Economics:** We argued earlier that agents tend to behave in a similar manner regarding asset prices. Nonetheless, it is undeniable there is still a path to go in order to achieve a live framework. In the future, it may be relevant to test more sophisticated learning algorithms and eventually include in agents' decision making forecast capabilities and the possibility to differentiate their investment strategy depending on particular assets' characteristics. For that, it will be fundamental to consider the different companies in isolation, rather than using an average index (as we have been doing so far). The introduction of behavioral biased, widely studied by Kanhneman and Tversky [18], it would make investor, for instance, over-invest whenever they are having a positive performance (despite the current market conditions) or under-invest or panic even if it is the best time/opportunity to buy cheap. While implementing such sophisticated agents is out of the scope of this thesis, we gave a first step into the study of how SPX

members are connected, and how this network of companies is responsible for changes in the aggregated Index [11].

References

[1] R. Axelrod and W. D. Hamilton. The evolution of cooperation. *science*, 211(4489):1390–1396, 1981.

[2] N. Barberis and R. Thaler. A survey of behavioral finance. *Handbook of the Economics of Finance*, 1:1053–1128, 2003.

[3] M. Beechey, D. W. Gruen, J. Vickery, et al. *The efficient market hypothesis: a survey*. Reserve Bank of Australia, Economic Research Department Sydney, 2000.

[4] Z. Bodie, A. Kane, and A. J. Marcus. *Investment and portfolio management*. McGraw-Hill Irwin, 2011.

[5] W. A. Brock and C. H. Hommes. Heterogeneous beliefs and routes to chaos in a simple asset pricing model. *Journal of Economic dynamics and Control*, 22(8):1235–1274, 1998.

[6] W. A. Brock, C. H. Hommes, and F. O. Wagener. Evolutionary dynamics in markets with many trader types. *Journal of Mathematical Economics*, 41(1):7–42, 2005.

[7] M. M. Carhart. On persistence in mutual fund performance. *The Journal of finance*, 52(1):57–82, 1997.

[8] E. Chan. *Quantitative trading: how to build your own algorithmic trading business*, volume 430. John Wiley & Sons, 2009.

[9] L. K. Chan, H.-L. Chen, and J. Lakonishok. On mutual fund investment styles. *The Review of Financial Studies*, 15(5):1407–1437, 2002.

[10] K. Daniel, D. Hirshleifer, and A. Subrahmanyam. Investor psychology and security market under- and overreactions. *the Journal of Finance*, 53(6):1839–1885, 1998.

[11] E. F. Fama. Efficient capital markets: Ii. *The journal of finance*, 46(5):1575–1617, 1991.

[12] E. F. Fama, L. Fisher, M. C. Jensen, and R. Roll. The adjustment of stock prices to new information. *International economic review*, 10(1):1–21, 1969.

[11] Work already accepted as full paper at the The 7th International Conference on Complex Networks and Their Applications (December 11-13, 2018, Cambridge, UK) and inclusion in the proceedings to be published by Springer Verlag Lectures.

[13] J. D. Farmer and A. W. Lo. Frontiers of finance: Evolution and efficient markets. *Proceedings of the National Academy of Sciences*, 96(18):9991–9992, 1999.

[14] D. Friedman. Evolutionary games in economics. *Econometrica: Journal of the Econometric Society*, pages 637–666, 1991.

[15] D. Friedman. Towards evolutionary game models of financial markets. 2001.

[16] S. J. Grossman and J. E. Stiglitz. On the impossibility of informationally efficient markets. *The American economic review*, 70(3):393–408, 1980.

[17] M. Hellmich. *Bubble formation and crises in complex networks*. 2016.

[18] D. Kahneman and A. Tversky. *Choices, values, and frames*. Cambridge University Press, 2000.

[19] H. Li, C. Wu, and M. Yuan. An evolutionary game model of financial markets with heterogeneous players. *Procedia Computer Science*, 17:958–964, 2013.

[20] A. W. Lo. The adaptive markets hypothesis. *The Journal of Portfolio Management*, 30(5):15–29, 2004.

[21] K. Marx. Das kapital: kritik der politischen ökonomie. *Germany: Verlag von Otto Meisner*, 1885:1894, 1867.

[22] J. Nash. Non-cooperative games. *Annals of mathematics*, pages 286–295, 1951.

[23] M. A. Nowak and K. Sigmund. Tit for tat in heterogeneous populations. *Nature*, 355(6357):250, 1992.

[24] F. P. Santos. Evolution of fairness under n-person ultimatum games.

[25] J. Shiller Robert. Irrational exuberance. *Princeton, New Jersey, Princeon University*, 2000.

[26] J. M. Smith. *Evolution and the Theory of Games*. Cambridge university press, 1982.

[27] P. Souto. Marketopolis: A market simulation based on investors decisions. Master's thesis, 2016.

[28] A. Traulsen, M. A. Nowak, and J. M. Pacheco. Stochastic dynamics of invasion and fixation. *Physical Review E*, 74(1):011909, 2006.

[29] J. Von Neumann and O. Morgenstern. *Theory of games and economic behavior*. Princeton university press, 2007.