Remaining Useful Life Prediction of Laminated Composite Materials using Thermoelastic Stress Analysis

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Dedicated to my parents, brother and grandparents.
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Resumo

Com a redução dos tempos de processamento de dados e aumento das resoluções de temperatura alcançados com os actuais detectores infravermelhos, técnicas experimentais de longa distância para a medição de extensão, como a Análise Termoelástica de Tensão (TSA), têm sido aplicadas com sucesso em ambiente laboratorial na monitorização do início e propagação de diferentes formas de dano comumente atribuídas a materiais compósitos. A sua extrapolação para rotinas de manutenção de aeronaves no solo e monitorização da saúde estrutural em cenários de voo é antecipado como uma forma mais viável e menos dispendiosa, em tempo e recursos, em relação aos métodos tradicionais de Inspeção Não-Destrutiva. As expectativas aumentam com os recentes desenvolvimentos no rendimento e dimensões dos microbolômetros, provando serem uma alternativa válida aos detectores de fotões.

A previsão da vida útil remanescente de estruturas em compósito pode melhorar a eficiência do agendamento de inspeções e permitir intervenção imediata em caso de falha inesperada e iminente. Em resultados anteriormente obtidos com recurso a Fibras de Grade Bragg instalados em compósitos reforçados com fibra de vidro E-glass e resina epoxy, a exactidão do cálculo de degradação do material baseada na energia elástica libertada em cada ciclo de fadiga foi afetada pela redução da magnitude da extensão medida devido ao efeito de divisão do pico. Para além disso, uma avaliação quantitativa do número de ciclos até ruptura não foi, até ao momento, endereçada através do uso da TSA.

Esta tese propõe readaptar a mesma metodologia no cálculo da energia elástica remanescente em compósitos reforçados com fibra de vidro carregados em estado elevado de ciclo de fadiga sob condições de tração-tração usando as capacidades termográficas da TSA. Os resultados pretendem evidenciar as vantagens de substituir aparelhos de medição local de extensão por técnicas à distância que varrem uma área maior e fornecem múltiplos pontos para avaliação da actual vida útil remanescente do material.

Palavras-chave: Análise Termoelástica de Tensão, Materiais Compósitos Laminados, Monitorização de Saúde Estrutural, Previsão da Vida Útil Remanescente
Abstract

With decreasing infrared detector technology processing times and increasing temperature sensitivity, non-contacting strain experimental measurement techniques such as Thermoelastic Stress Analysis (TSA) have been successfully used under laboratory conditions in monitoring the onset and propagation of different types of fatigue damage commonly attributed to composite materials. Its extrapolation to aircraft ground maintenance routines and in-flight structural health monitoring is anticipated as a more reliable, less time consuming and cost saving technique in comparison with traditional Non-Destructive Inspection (NDI) methods. In addition, expectations are increasing regarding the latest developments in microbolometer's size and performance, hence standing out as a valid alternative to photon detectors.

Prediction of the remaining useful life of composite structures can improve inspections scheduling efficiency and allow timely intervention in case of unexpected and imminent failure. In previous work using Fiber Bragg Grating (FBG) sensors embedded within E-glass fiber/epoxy resin composites, the material degradation calculation accuracy based on the cycle-by-cycle elastic strain energy released was affected by the reduction in magnitude of the measured strain due to the peak splitting effect. Moreover, a quantitative assessment of the number of cycles until failure has not been addressed using TSA.

This thesis proposes to readapt the same methodology into a remaining elastic strain energy density calculation for glass fiber-reinforced composites cyclically loaded under tensile-tensile high-cycle fatigue conditions using the TSA capabilities. The results are intended to state the advantages of replacing preallocation of strain measuring devices by full-field and non-contacting techniques that scan a wider area and provide multiple data points to assess the actual structural remaining useful life.

**Keywords:** Thermoelastic Stress Analysis, Laminated Composite Materials, Structural Health Monitoring, Remaining Useful Life Prediction
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Nomenclature

Abbreviations

AFP  Automatic Fiber Placement
ASTM American Society for Testing and Materials
ATL  Automatic Tape Laying
Avg  Average
CFM  Composite filling material
CFRP Carbon Fiber-Reinforced Polymer
CLPT Classical Laminate Plate Theory
CP   Cross-ply
CTE  Coefficient of Thermal Expansion
DCB  Double-Cantilever Beam
DIC  Digital Image Correlation
DL   Digital Level
DSC  Differential Scanning Calorimeter
ESPI Electronic Speckle Pattern Interferometry
FE   Finite Element
FFT  Fast Fourier Transform
FPN  Fixed Pattern Noise
FRP  Fiber-Reinforced Polymer
FSFT Full-Scale Fatigue Test
GAG  Ground-Air-Ground
GFRP Glass Fiber-Reinforced Polymer
GRP  Glass-Reinforced Polymer
IR   Infrared
MCM  Markov Chain Method
NDI  Non-destructive Inspection
NETD Noise Equivalent Temperature Difference
NI   National Instruments
QI   Quasi-isotropic
RUL  Remaining Useful Life
SD   Standard Deviation
SHM  Structural Health Monitoring
SPATE Stress Pattern Analysis
TMA  Thermomechanical Analysis
TSA  Thermoelastic Stress Analysis
UD   Unidirectional
UTS  Ultimate Tensile Strain

List of symbols

$[T]_{\sigma}$  3x3 transformation matrix for in-plane stress components
$[T]_e$       3x3 transformation matrix for in-plane strain components
$\alpha$      Coefficient of thermal expansion
$\alpha_i$    Coefficient of thermal expansion tensor
$\alpha_r$    Resin's coefficient of thermal expansion
$\alpha_{ref}$ Coefficient of thermal expansion of reference
$\bar{Q}_{ij}$ Off-axis stiffness tensor
$\beta_i$     Volumetric thermal expansion tensor
$\Delta, \delta$ Change
$\lambda$     Wavelength
$|u|, |v|, |w|$ Displacements along $x$, $y$ and $z$ directions, respectively
$\nu_{12}, \nu_{21}, \nu_{xy}$ In-plane Poisson's ratio
\( \nu_r \)  Resin's Poisson's ratio

\( \partial \)  Partial derivative

\( \rho \)  Density

\( \rho_r \)  Resin's density

\( \rho_s \)  Surface reflectivity

\( \rho_w \)  Density of water

\( \sigma \)  Stress

\( \sigma_b \)  Stefan-Boltzmann's constant

\( \sigma_i \)  Stress tensor

\( \sigma_{\text{max}} \)  Maximum axial stress

\( \theta \)  Fiber angle orientation

\( \varepsilon_i \)  Strain tensor

\( \varepsilon^0 \)  Strain tensor at the laminate's mid plane

\( \varepsilon_{\text{max}} \)  Maximum axial strain

\( \varepsilon_{\text{min}} \)  Minimum axial strain

\( \varepsilon_{\text{ult}} \)  Ultimate axial strain

\( \varepsilon_{\text{yield}} \)  Axial yield strain

\( \xi_r \)  Relative error

\( a, b \)  Fatigue coefficients

\( A^* \)  Calibration constant

\( A_{ij} \)  Extensional stiffness tensor

\( B, n \)  Coefficients for temperature correction

\( B_{ij} \)  Coupling stiffness tensor

\( c \)  \( \cos(\theta) \)

\( c_0 \)  Speed of light in the vacuum

\( C_p \)  Specific heat at constant pressure

\( C_e \)  Specific heat at constant strain

\( C_{ij} \)  Elastic isothermal stiffness tensor
$C_{pr}$  Resin’s specific heat at constant pressure
$C_v$  Specific heat at constant volume
$D$  Parameter for RUL assessment
$d$  Exact derivative
$D_t$  Damage parameter
$e$  Emissivity
$E_{1,2}$  In-plane Young’s modulus
$E_b$  Total power emitted by a perfect black body
$E_{\lambda,b}$  Total emitted energy per wavelength and per surface temperature by a blackbody
$E_r$  Resin’s Young’s modulus
$F$  Fatigue modulus
$f$  Frequency
$F_0$  Fourier’s number
$g$  Gravitacional acceleration
$G_{12}$  In-plane shear modulus
$G_r$  Resin’s shear modulus
$GL$  Gauge length
$h$  Thickness
$h_{la}$  Lamina’s thickness
$h_P$  Planck constant
$I_{\lambda,b}$  Spectral intensity
$k_B$  Boltzmann constant
$K_i$  Thermoelastic constant tensor
$k_m$  Curvature tensor
$m$  Mass
$m_{s,a}$  Mass of the structure + sample suspended in the air
$m_{s,l}$  Mass of the structure + sample immersed in the liquid
$m_s$  Mass of the sample
$N$ Number of cycles

$N_m$ Force per unit width tensor

$N_n$ Number of cycles until failure in step $n$

$N_b$ Number of photons per unit area and time

$N_f$ Number of cycles until 90% of the fatigue life

$N_t$ Total number of cycles until failure

$P$ Force

$P_b$ Buoyant force

$P_{ult}$ Ultimate tensile force

$P_{yield}$ Tensile force at the yield point

$q_m$ Heat exchanged per unit mass

$q_v$ Heat transferred per unit volume

$Q_{ij}$ Reduced stiffness tensor

$R$ Temperature correction factor

$r$ Ratio between energies at mean strain level and end of stage II

$R_a$ Grid alloy’s resistivity

$S$ Uncalibrated thermoelastic signal

$s$ $\sin(\theta)$

$S_c$ Uncalibrated thermoelastic signal after temperature correction

$S_m$ Uncalibrated thermoelastic signal before temperature correction

$s_m$ Entropy per unit mass

$SM$ Stress Metric

$T$ Absolute temperature

$t$ Time

$T_0$ Mean absolute surface temperature

$T_1$ Tension in the wire when sample is suspended in the air

$T_2$ Tension in the wire when sample is immersed in water

$T_f$ Final absolute temperature
$T_i$  Initial absolute temperature
$T_m$  Mean absolute surface temperature after undamaged state
$T_{amb}$  Ambient absolute temperature
$U$  Strain energy
$U_0$  Strain energy at the mean strain level
$u_0$  Elastic strain energy density at the mean strain level
$u_f$  Strain energy density at 90% of the fatigue life
$u_{em}$  Strain energy per unit mass
$u_v$  Strain energy density
$v_f$  Fiber volume fraction
$v_m$  Matrix volume fraction
$v_v$  Void volume fraction
$V_s$  Volume of the sample
$w$  Width
$w_v$  Work per unit volume
Chapter 1

Introduction

1.1 Composites in aerospace industry

Over many years, aluminium alloys provided the high strength and stiffness required for structural aircraft components at a relative low cost. However, vulnerability to corrosion and reduced fatigue lives led to increased maintenance costs [1]. Polymer matrix composite materials, in particular glass and carbon fiber-reinforced, started to emerge as an alternative capable of mitigating these problems, thus reducing fuel consumptions and the operational costs due to the higher stiffness achieved by the light-weight manufactured parts [1]. The Boeing 787 Dreamliner and the Airbus A350 XWB are currently in the front lead of higher weight percentage of composite material applied to structural components (fuselage, wings and tail) of commercial aircrafts [1, 2].

1.2 Fatigue of aircraft structures

By definition, fatigue is the progressive failure of a part under repeated cyclic or fluctuating loads whose generated stresses are lower than those responsible for static failure [3]. Within a flight course, aircrafts are submitted to different variable load environments [3, 4]:

- Flight loads induced by pressurization, gusts and maneuvers;
- Ground-Air-Ground (GAG) cycle, defined as the cycle from the minimum (largest negative or smallest positive) to maximum stress value experienced once per flight;
- Taxiing loads;
- Landing impact loads;

1.3 Structural Health Monitoring

As an aircraft accumulates flight hours, the combination of the aforementioned loads with extraneous environmental effects reduces its service lifetime. Current damage identification is still carried out using
preventative replacement and periodical Non-Destructive Inspection (NDI) techniques [5] that rely on a priori knowledge of prone-to-damage or already damaged sites (e.g. by first performing a Full-Scale Fatigue Test (FSFT)) [6].

A different approach is to extract damage sensitive measurements (e.g. stress and strain) while the structure is under service. This procedure is denoted as Structural Health Monitoring (SHM) [6]. Life-saving opportunities emerge from the possibility to assess the structural integrity immediately after the occurrence of an unexpected severe load case scenario [6]. Besides, switching the maintenance scheduling from a fixed time-based approach to a more flexible and only dependent on the damage severity level delivers important cost efficiencies [6].

1.4 Thermoelastic Stress Analysis (TSA)

1.4.1 Thermoelastic effect

Consider a perfect gas initially at temperature $T_i$ inside a closed container. The system is isolated i.e. neither heat nor mass is allowed to be transferred from or to the surroundings. In addition, the movement of the piston is lubricated such that hypothetically no friction effects occur. If the gas is compressed as a consequence of a force $P$, its averaged kinetic energy is increased, thus the temperature ($T_f > T_i$). An opposite temperature evolution occurs if the gas is expanded. Moreover, no additional entropy is generated and the process is considered to be reversible and adiabatic, i.e. isentropic. Therefore, by removing the force $P$, the system is able to return to the exact same initial conditions.

The analogous mechanical-thermal coupling in solids is denoted as thermoelastic effect. John Gough’s experiments reported in 1803 using Indian rubber (or caoutchouc) were pioneer in the qualitative observation of the phenomenon [7]. A solid at the initial temperature $T_i$ under a compressive force $P$ increases its temperature to $T_f$. The initial state can be further recovered if a tensile force with the same magnitude is applied under an isentropic process assumption. Reversible conditions are achieved if the linear elastic domain of the material is not overcome. Moreover, adiabaticity is reliant on the loading frequency parameter. Therefore, the material must be dynamically deformed. The idea is to generate and detect temperature variations solely resultant from the imparted mechanical load and attenuate heat diffusion effects as much as possible.

The mathematical interpretation of the surface principal material stresses ($\Delta \sigma$) based on the temperature difference ($\Delta T$) measured between two loading states in isotropic materials was originally derived by William Thomson around 1850's [7]:

$$\Delta T = -\frac{\alpha}{\rho C_p} T_0 \Delta \sigma$$

(1.1)

where $\rho$ is the density, $\alpha$ is the coefficient of thermal expansion, $C_p$ is the specific heat at constant pressure and $T_0$ is the mean absolute surface temperature. The first three variables are assumed to be temperature independent. A better understanding of the physical meaning of the temperature terms built-in equation 1.1 is presented in figure 1.2.
Figure 1.1: Mechanical-thermal isentropic conversions in perfect gases and linear elastic solids.

Figure 1.2: Schematic representation of the temperature terms built-in equation 1.1.

Under the conditions previously described, very small cyclic surface temperature changes are generated. Detecting the thermoelastic effect only became possible with the enhancement and commercial availability of highly sensitive infrared detectors. The idea of applying these technologies for stress measurements is the baseline of the Thermoelastic Stress Analysis (TSA) technique.

### 1.4.2 Photon Detectors

Any body with an absolute temperature value above 0 K emits radiation in the infrared domain. This radiation contains photons carrying a certain amount of energy. Photon detectors employ a semiconducting material characterized by the ionization energy necessary to remove a valence electron from the atom [8]. Whenever the absorbed energy is equal to or greater than the aforementioned value, an output voltage is generated and a temperature value of the surface being measured is obtained [9].

The camera’s output signal results from the combination of different thermal sources [9]:

- **Contribution from the object:** the total power emitted by a perfect black body \( E_b \) at a tem-
perature $T$ is defined by the Stefan-Boltzmann’s law ($E_b = \sigma_b T^4$, where $\sigma_b$ represents the Stefan-Boltzmann’s constant). However, real surfaces emit a fraction of the maximum energy depending on their emissivity, $\varepsilon$.

- **Contribution from reflected sources**: for an opaque surface, $\rho_s = 1 - \varepsilon$, where $\rho_s$ is the surface reflectivity. Reflected radiation from the surroundings is reduced if the material presents a high emissivity, thus more accurate temperature measurements are achieved.

- **Contribution from the atmosphere**: the medium behaves as a filter by transmitting a fraction of the total emitted radiation from the surface and surroundings. In addition, it emits radiation at the temperature $T_{amb}$.

![Figure 1.3: Different sources of radiation during a TSA test (adapted from [9]).](image)

The first thermographic prototype became commercially available in 1982 under the denomination of SPATE 8000 [7]. In the seek of better temperature resolutions (SPATE’s sensitivity was around 1 mK [10]) at reduced scanning times, the forthcoming Deltatherm and AltaIR systems turned TSA into an almost real-time structural health monitoring technique. A cooling system (with liquid nitrogen or a Stirling-cycle engine) is built-in to reduce background noise effects and improve the signal-to-noise ratio [10]. This came at the cost of increasing the system’s weight which made them impractical for installation in aerospace structures [10].

### 1.4.3 Data acquisition and post-processing

A sinusoidal variation in load or displacement is imparted to the test specimen using a servo-hydraulic machine. Ideally, the thermal response is expected to be sinusoidal with the same loading frequency. However, the measured signal can contain external noise and additional non-stress related thermal
emissions originated at the specimen’s surface with different frequencies. To extract the relevant thermoelastic data, a reference signal provided by the load transducer or strain measuring device is combined with the thermal response either using a lock-in procedure or solely relying in digital processing using a Fast Fourier Transform (FFT) [11].

Integration of the several captured images is performed within a user-defined period of time denoted as cross-correlation process. For longer integration times, a reduction in the noise-to-signal ratio is obtained until an asymptotic limit defined by the Fixed Pattern Noise (FPN) is reached, as shown in figure 1.4 [12].

![Figure 1.4: Schematic of the noise-to-signal ratio ($\sigma$) against number of cycles for the cross-correlation process ($n_l$) in log-log scale [12].](image)

A final image will contain the in-phase thermoelastic (i.e. the temperature amplitude) and any non-adiabatic response shifted from 0° or 180° depending upon software configurations. A schematic overview of the data processing is presented in figure 1.5.

![Figure 1.5: TSA’s Experimental apparatus.](image)
1.4.4 Prospectives of TSA as SHM technique

When compared to other existing full-field stress/strain measurement techniques applicable using static loading conditions (for instance, Electronic Speckle Pattern Interferometry (ESPI) and Digital Image Correlation (DIC)), TSA's output is less vulnerable to noise generated during conversion from displacement to stress/strain data [13]. Moreover, TSA provides a good quality signal regardless of the structural geometric complexity if appropriate positioning of the camera is assured [14]. No laborious camera calibration procedures are required therefore fast positioning readjustments for every new measurement are permissible.

However, TSA is mainly enclosed to laboratorial environment applications. One of the reasons is due to the excessive weight of photon detectors. A valid alternative came with the application of thermal detectors. Microbolometers typically employ a vanadium oxide (VO$_x$) detector type and rely on the changes in its electrical resistance to generate a signal output. With the absence of a cooling system, the forthcoming devices became smaller, lighter, higher shock tolerant and less power consumable [10].

![Comparison between thermal and photon detectors' dimensions](image)

The sensitivity of a microbolometer in relation to an Indium Antimonide (InSb) array detector can be mislead by the simplistic sole interpretation of the Noise Equivalent Temperature Difference (NETD) parameter. A study of the noise-to-signal ratio derived from the cross-correlation process throughout the fatigue cycle of an open-hole aluminium alloy plate was carried out using different photon and thermal detectors (respectively X-type and A-type) [12]. Cooled focal-plane array systems presented an approximate 30% initial lower signal-to-noise ratio value. However, this advantage was gradually suppressed with increasing integration times until the point its performance was surpassed by microbolometers (fig. 1.7).

Nevertheless, the first step towards reproducibility of TSA during a flight case scenario has been envised [5]. An utopic concept combining several microbolometers into a wireless network capable of providing multiple stress and strain data points in real-time (see figure 1.8), even in locations of difficult
Figure 1.7: Correlation between noise-to-signal ratio \( (c_v) \) and the number of fatigue cycles \( (n_l) \) for different photon and thermal detectors [12].

access, might boost the interest for practical applicability of TSA

Figure 1.8: Concept of multi-connected microbolometers via wireless for real-time flight SHM [5].

1.5 Applications to composite materials

1.5.1 Damage Assessment

Quantitative surface stress and strain redistributions due to damage evolution within polymer matrix composite materials have been captured from the measured temperature changes. Krstulovic-Opara et al [15] identified null load carrying capability locations representative of fractured fibers. A further
insight into sub-surface damage mechanisms (namely delamination, matrix transverse cracking or fiber breakage) in glass fiber-reinforced composites with different stacking sequences was made possible after converting the classical stress output to global strain values \[16, 17\]. Alternatively to the lock-in and FFT data processing methodologies, Plum et al \[18\] determined the temperature amplitude resultant from friction effects by fitting, pixel-by-pixel, the reference to the measured signals and subtracting with the original reference signal. The delamination propagation front in a Carbon Fiber-Reinforced Polymer (CFRP) material previously submitted to a double cantilever beam test (DCB) was evidenced. Mixing the measured signal with a sinusoidal reference at twice the loading frequency enabled Paynter and Dutton \[19\] to successfully identify and quantify the severity of initial imparted damages and subsequent evolved cracks in a wind turbine blade containing skins made of glass-polyester material. Jones et al \[20\] quantified the damage severity after impact as well as for fatigue after impact using a parameter correlating the surface temperature change from undamaged to damaged states, \(D_t\). Higher values obtained at the non-impacted surface resulted from the proximity of damage to these areas. Moreover, with increasing number of cycles performed, \(D_t\) monotonically increased with a profile identical to the material’s compliance.

Between two crack faces, the friction resultant from their relative motion generates a local non-stress related temperature increase. This non-adiabatic behaviour can be detected by analysing the phase data. If adiabatic conditions were attained, the thermal response should be in-phase with the applied load (interpreted either as \(0^\circ\) or \(180^\circ\) depending upon software specifications). Therefore, any phase gap will determine the appearance of a crack. Fruehmann et al \[21\] determined the crack pattern along a weft yarn in a 2x2 twill woven roving e-glass fiber reinforced composite material by using the phase difference map. This data set provided similar results to the non-dimensional \(\Delta T/T_0\) values subtracted from the virgin material state. Kakei et al \[22\] proposed a delamination crack length estimation from the distance between positive peak-to-peak quadrature signal values and validated the results against optical microscope images.

The connection of large composite structures using adhesive bonding offers a simpler alternative than mechanical fastening or welding \[23\]. Initial flaws derived from a poor implementation of joints in GRP structures have been detected using TSA \[24, 25, 26\]. Moreover, the technique provided a better understanding of the crack propagation towards the inner region of the adherend material in single bonded carbon fiber/epoxy composites submitted to different static load levels \[27, 28\]. Thermoelastic work can be also used as a validation method for simplified and inexpensive 2-D plane elasticity FE models. The first stress invariant obtained in different types of joints applied to GRP structures presented an overall good correlation with identical experimental thermoelastic conditions \[23, 24, 25, 26, 29\].

### 1.5.2 Hybrid Approach

The requirement for cyclic loading conditions presents another barrier to the extrapolation of TSA out from laboratory environment. An alternative to the servo-hydraulic machines came with the application of a single impact transient load using a pendulum system (see figure 1.9). Quinn et al \[30\] successfully
validated the stress derived from this approach against cantilever beam theory values in E-glass fiber-reinforced composites. Furthermore, the same work has evidenced a lower resolution capability in comparison with the traditional TSA procedure to detect small scale defects in weave structures. Using the same material and geometry, Quinn et al [31] obtained stress curves along three deflected beams with slope offsets in accordance with the imparted damage severity visually perceived.

![Figure 1.9: Schematic of the impact rig [30].](image)

### 1.5.3 Prediction of Remaining Useful Life (RUL)

The potential to use TSA’s output to directly predict fatigue lives was anticipated by Wong [32] as an emergent research topic. Emery and Dulieu-Barton [33] correlated both Young’s Modulus and first strain invariant trend evolutions with an imminent failure scenario of glass fiber composite laminates under fatigue loading. In addition, Johnson et al [34] applied the MCM probabilistic cumulative damage method to determine limiting stochastic S-N curves for a Fiber-Reinforced Polymer (FRP) single lap joint.

However, there’s still work that can be addressed to a greater extent in the exploitation of the full-field nature of TSA in RUL calculations. This is the scope under development in this thesis.

### 1.6 Objectives

An overview of the main objectives to be fulfilled within the scope of this work are enumerated:

- Provide a thorough literature review on TSA to evidence the lack of research related to remaining useful life calculations and the promising upcoming applications as a SHM technology;

- Validate the strain witness assumption associated to GFRPs containing thick outer resin layers;

- Quantitatively determine the number of cycles until failure of a glass fiber-reinforced/polymer matrix laminated composite material subjected to strain controlled tension-tension conditions using TSA to calculate the strain energy density at the mean strain level;

- Update the results within a user-defined number of steps in the attempt to simulate a real maintenance scenario;
• Generate full-field datasets to evaluate the higher accuracy in predicting the location of critical
damaged sites by analyzing the data for each pixel separately rather than using a single averaged
value approach.

1.7 Thesis Outline

The contents of each chapter following the introduction are now briefly described:

Chapter 2

An introduction to the principles of laminated composite materials is presented. Microstructural com-
ponents and macrostructural definitions, manufacturing techniques employed in prepreg curing and rel-
evant stress-strain relations to be considered for further thermoelastic equations are derived in this
chapter.

Chapter 3

The stress and strain based thermoelastic equations are derived. This chapter also complements the
literature review of chapter 1 by including a section related to non-adiabatic effects and thermal heat
source identification.

Chapter 4

The remaining useful life methodology based on strain energy calculation at the mean fatigue level is
introduced. At the end of the chapter, an algorithm to be implemented for remaining useful life calcula-
tions is included.

Chapter 5

Specimen preparation steps are presented together with a justification for stacking sequence selec-
tion. Tests performed for mechanical, thermal and constituent material characterization are thoroughly
explained and final averaged properties to be input into the thermoelastic models presented.

Chapter 6

This chapter is solely dedicated to TSA. Initially, the different camera and software parameters to
account for prior to any test are explained. Furthermore, final test results involving TSA are presented
and discussed.

Chapter 7

A summary of the conclusions derived from the work developed and future work topics are included
in the final chapter of the thesis.
Chapter 2

Laminated Composite Materials

2.1 Definition

A composite material consists of two or more phases in a macroscopic scale whose mechanical performance and properties are designed to be superior to those of the constituent materials acting independently [35]. Two-phase composites might contain individual layers of reinforcement material embedded in matrix denoted as plies (or laminas). If stacked, a laminate is obtained.

2.2 Laminate Microstructure

2.2.1 Fibers

The reinforcement material is intended to present high strength and stiffness in order to sustain the applied loads to the laminate. Continuous filaments mechanically provide the most efficient fiber shape [35]. Within a single lamina, the fibers can be either steered in a single direction (unidirectional) or in two mutually perpendicular directions (fabric woven).

Aerospace components typically employ two material types:

- **Carbon**: more expensive and used for structural components where Young's Modulus is critical;

- **Glass**: E-glass fiber material is the cheapest among all glass types. The short fatigue life and low stiffness dictates its usage in non-structural aircraft components;

2.2.2 Matrix

The matrix material protects the reinforcement from extraneous environments (e.g. corrosion, humidity) and evenly distributes the stress carrying capability among all fibers. The fiber orientations are also preserved within the stabilized resin medium. Polymeric matrices, namely thermosets (e.g. epoxy, polyimide, polyester) are the most widely used in aerospace applications.
2.3 Laminate macrostructure

When unidirectional plies are stacked, different notations can be attributed to the global laminate structure. Highlight is provided to the stacking sequences further mentioned within the scope of this work:

- **Symmetric**: contains pairs of layers with the same fiber orientation, material properties and thickness equally distant from the mid-plane of the composite;

- **Balanced**: contains pairs of layers with opposite fiber orientations but equal material properties and thickness located anywhere along the laminate’s depth [36].

- **Unidirectional (UD)**: all plies have the same fiber orientation. Short notations are attributed to laminates with all fibers aligned along or transversely to the loading direction (UD(0) and UD(90), respectively);

- **Cross-Ply (CP)**: contains plies oriented perpendicularly to each other (i.e. 0 or 90 deg);

- **Quasi-isotropic (QI)**: multidirectional laminate with equal mechanical and thermal properties regardless of the direction considered;

2.4 Manufacturing processes

2.4.1 Fabrication

In this stage, the fibers and matrix are stacked on top of a tool (typically made of steel, aluminium or ceramics). They can be provided either in preimpregnated shape or separately to be further combined during the stacking process. Prepregs offer fabrication cost efficiencies and light-weight final products with good mechanical performance. Focus hereinafter will be given to the different stages of prepreg-based manufacturing.

Tool surface preparation is required prior to stacking. A sealer liquid is firstly applied to cover any gaps in the mold followed by a release agent to ease the removal of the laminate after curing. Hand lay-up is the old-fashioned but rather preferred technique to stack the different plies in the desired orientations. Automated processes include Automatic Tape Laying (ATL) and Automatic Fiber Placement (AFP).

Other secondary materials are useful in composite manufacturing [36]:

- **Breather fabric**: a porous polyester material that absorbs the excess of resin and improves the removal capability of air and volatiles away from the assembly;

- **Peel-ply**: also helps volatiles and excess of resin to be easily removed from the laminate. It provides a non-sticking medium between the laminate surface and breather. Typically made of nylon, fiberglass or polyester (see figure B.1 in appendix B);

- **Release Films**: an alternative to the peel-ply hence providing a shiny finishing if applied to the surface that is not in contact with the mold. Additionally can be used as an intermediate layer between different prepreg layers to obtain double cantilever beam test specimens;
• **Bagging film**: converts the whole setup into a closed system. A sealant tape is used to attach the film to the mold and avoid any air leakage. It is commercially available with nylon or polyolefin material and should be flexible enough to not rupture after vacuum is applied to the assembly;

A vacuum port is finally installed in the bag to allow air suction from the inside of the assembly using a pump.

![Figure 2.1: Vacuum bagging setup](image)

2.4.2 Processing

The fiber reinforced polymer matrix material is obtained after performing a curing cycle. It is characterized by different temperatures and pressures remained constant during dwell periods. A typical curing cycle consists in two steps. In the first dwell period, the resin's viscosity should be at its lowest value to allow entrapped gases inside the stacked plies to be removed. During the second dwell period, the strength and stiffness properties are further enhanced.

Two distinct processes can be carried out depending on the chosen device to perform the cycle [36]:

- **Autoclave Curing**: an autoclave is an enclosed and pressurized cylindrical metallic structure. Due to the externally applied pressure, final products achieve higher fiber volume fractions and reduced void contents. However, the required equipment and operating investment make this process very costly;

- **Vacuum Bag Curing**: the assembly is only submitted to a temperature profile and constant ambient pressure. It can be performed either inside an oven or by evenly heating the mold. It is cheaper yet lower quality products are expected in relation to autoclave curing.
2.5 Stress-strain relations

2.5.1 Reference axes

For laminates made of unidirectional plies, two different systems of axes will be used hereinafter to represent the stress/strain field under in-plane stress conditions:

- Local reference aligned with the fiber direction in each ply i.e. the principal material directions (1,2);

- Global reference aligned with the laminate’s loading direction and principal geometric dimensions i.e principal stress directions (x,y);

The orientation of each ply, $\theta$, is defined by the rotation angle between the two aforementioned systems of axes.
2.5.2 Lamina level

An orthotropic material has three mutually perpendicular planes of symmetry. A total of 9 elastic constants describe the stress-strain relation:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_4 \\
\tau_5 \\
\tau_6 \\
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_4 \\
\gamma_5 \\
\gamma_6 \\
\end{bmatrix}
\]  
(2.1)

where \(C_{ij}\) is the elastic isothermal stiffness tensor. Under plane stress state conditions, from the second condition mentioned in section 2.5.4:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
0 \\
0 \\
\tau_6 \\
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_4 \\
\gamma_5 \\
\gamma_6 \\
\end{bmatrix}
\]  
(2.2)
A system of 4 equations is obtained:

\[
\begin{align*}
\sigma_1 &= C_{11}\varepsilon_1 + C_{12}\varepsilon_2 + C_{13}\varepsilon_3 \\
\sigma_2 &= C_{12}\varepsilon_1 + C_{22}\varepsilon_2 + C_{23}\varepsilon_3 \\
\tau_6 &= C_{66}\gamma_6 \\
0 &= C_{13}\varepsilon_1 + C_{23}\varepsilon_2 + C_{33}\varepsilon_3
\end{align*}
\]  

(2.3)

Considering only in-plane motion (1-2), the strain component \(\varepsilon_3\) in equations 2.3 is suppressed after rewriting as a function of the remaining strain components:

\[
\begin{align*}
\sigma_1 &= (C_{11} - \frac{C_{13}^2}{C_{33}})\varepsilon_1 + (C_{12} - \frac{C_{13}C_{23}}{C_{33}})\varepsilon_2 \\
\sigma_2 &= (C_{12} - \frac{C_{13}C_{23}}{C_{33}})\varepsilon_1 + (C_{22} - \frac{C_{23}^2}{C_{33}})\varepsilon_2 \\
\tau_6 &= C_{66}\gamma_6
\end{align*}
\]  

(2.4)

The stiffness terms can be combined in reduced stiffness components \(Q_{ij}\):

\[
\begin{align*}
Q_{11} &= (C_{11} - \frac{C_{13}^2}{C_{33}}) \\
Q_{12} &= (C_{12} - \frac{C_{13}C_{23}}{C_{33}}) \\
Q_{21} &= (C_{22} - \frac{C_{23}^2}{C_{33}}) \\
Q_{22} &= (C_{22} - \frac{C_{12}C_{23}}{C_{33}}) \\
Q_{66} &= C_{66}
\end{align*}
\]  

(2.5)

In matrix form yields:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6
\end{bmatrix}
\]  

(2.6)

The aforementioned stiffness terms can be expressed as function of engineering constants:

\[
\begin{align*}
Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{12} &= \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \\
Q_{21} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \\
Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\
Q_{66} &= C_{66}
\end{align*}
\]  

(2.7)

where the Poisson’s coefficients \(\nu_{12}\) and \(\nu_{21}\) are correlated by the condition of symmetry of the stiffness matrix i.e. \(Q_{12} = Q_{21}\):

\[
\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}
\]  

(2.8)
Transformation matrices are used to convert the stress and strain vector components between the two reference axes. For the stress components' case:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6 
\end{bmatrix}
= \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_s 
\end{bmatrix} = \left[T\right] \sigma 
\] (2.9a)

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_s 
\end{bmatrix}
= \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_6 
\end{bmatrix} = \left[T\right]^{-1} \sigma 
\] (2.9b)

where:

\[
\left[T\right] = \begin{bmatrix}
c^2 & s^2 & 2cs \\
s^2 & c^2 & -2cs \\
-cs & cs & c^2 - s^2
\end{bmatrix} 
\] (2.10a)

\[
\left[T\right]^{-1} = \begin{bmatrix}
c^2 & s^2 & -2cs \\
s^2 & c^2 & 2cs \\
cs & -cs & c^2 - s^2
\end{bmatrix} 
\] (2.10b)

Whereas the strain components' case:

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6 
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_s 
\end{bmatrix} = \left[T\right] \varepsilon 
\] (2.11a)

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_s 
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_6 
\end{bmatrix} = \left[T\right]^{-1} \varepsilon 
\] (2.11b)

The matrices for strain conversion yield:

\[
\left[T\right] = \begin{bmatrix}
c^2 & s^2 & cs \\
s^2 & c^2 & -cs \\
-2cs & 2cs & c^2 - s^2
\end{bmatrix} 
\] (2.12a)

\[
\left[T\right]^{-1} = \begin{bmatrix}
c^2 & s^2 & -cs \\
s^2 & c^2 & cs \\
2cs & -2cs & c^2 - s^2
\end{bmatrix} 
\] (2.12b)
2.5.3 Resin level

For isotropic materials, the properties are independent of the direction. Therefore, the number of independent variables is reduced to two:

\[
\begin{bmatrix}
\sigma_1 \\ \sigma_2 \\ 0 \\ 0 \\ \tau_6
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{11}-C_{12}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6
\end{bmatrix}
\]

(2.13)

Four valid equations can be represented under in-plane stress state conditions:

\[
\begin{align*}
\sigma_1 &= C_{11}\varepsilon_1 + C_{12}\varepsilon_2 + C_{12}\varepsilon_3 \\
\sigma_2 &= C_{12}\varepsilon_1 + C_{11}\varepsilon_2 + C_{12}\varepsilon_3 \\
\tau_6 &= \frac{C_{11}-C_{12}}{2}\gamma_6 \\
0 &= C_{12}\varepsilon_1 + C_{12}\varepsilon_2 + C_{11}\varepsilon_3
\end{align*}
\]

(2.14)

Rewriting the equations only as a function of in-plane strains:

\[
\begin{align*}
\sigma_1 &= (C_{11} - \frac{C_{12}^2}{C_{11}})\varepsilon_1 + (C_{12} - \frac{C_{12}^2}{C_{11}})\varepsilon_2 \\
\sigma_2 &= (C_{12} - \frac{C_{12}^2}{C_{11}})\varepsilon_1 + (C_{11} - \frac{C_{12}^2}{C_{11}})\varepsilon_2 \\
\tau_6 &= \frac{C_{11}-C_{12}}{2}\gamma_6
\end{align*}
\]

(2.15)

The number of distinct stiffness terms, \(Q_{ij}\) for the isotropic case is reduced to three:

\[
\begin{align*}
Q_{11} &= Q_{22} = \frac{E_r}{1-\nu_r^2} \\
Q_{12} &= Q_{21} = \frac{E_r\nu_r}{1-\nu_r^2} \\
Q_{66} &= G_r = \frac{E_r}{2(1+\nu_r)}
\end{align*}
\]

(2.16)

where the subscript \(r\) stands for resin, the only composite component whose mechanical behaviour is considered isotropic.

2.5.4 Classical Laminate Plate Theory (CLPT)

The global mechanical behaviour prediction of laminates based on the classical laminate plate theory is established upon the following assumptions:

- Each lamina is mechanically orthotropic and homogenized in its properties. As it is virtually
impossible to identify the exact distribution of the fibers within the resin medium, the latter condition holds true;

- Thickness is much smaller than the width and length of the material, so that plane stress conditions can be applied i.e. $\sigma_3 = 0$, $\tau_4 = \tau_5 = 0$;

- All displacements are small compared to the thickness of the laminate i.e. $|u|, |v|, |w| < h$;

- A straight line normal to the mid-plane preserves its condition after deformation;

- The material is deformed under its elastic domain;

- Displacements are continuous throughout the laminate;

### 2.5.5 Laminate level

Considering an external in-plane applied force per unit width, $N_m$, with $m = x, y, s$, the mid-plane strain field and curvature vectors ($\varepsilon^0$, and $k_m$, respectively) can be derived using the extensional stiffness components, $A_{ij}$, and coupling stiffness matrix terms, $B_{ij}$, with $i, j = 1, 2, 6$:

$$
\begin{bmatrix}
N_x \\
N_y \\
N_s
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{21} & A_{22} & A_{26} \\
A_{61} & A_{62} & A_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon^0_x \\
\varepsilon^0_y \\
\gamma^0_s
\end{bmatrix} +
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{21} & B_{22} & B_{26} \\
B_{61} & B_{62} & B_{66}
\end{bmatrix}
\begin{bmatrix}
k_x \\
k_y \\
k_s
\end{bmatrix}
$$

(2.17)

For symmetric and balanced laminate configurations, $B_{ij} = 0$ and $A_{16} = A_{26} = 0$, respectively. Therefore, the simplified matricial equation 2.17 yields:

$$
\begin{bmatrix}
N_x \\
N_y \\
N_s
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & 0 \\
A_{21} & A_{22} & 0 \\
0 & 0 & A_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_s
\end{bmatrix}
$$

(2.18)

where $A_{ij}$ is function of the off-axis stiffness matrix components $Q_{ij}$ and thickness of each lamina, $h_n$:

$$
[Q] = [T]^{-1}[Q][T]$$

(2.19a)

$$
A_{ij} = \sum_{n=1}^{N} (Q_{ij})_n h_n
$$

(2.19b)
Chapter 3

Thermoelastic Stress Analysis (TSA)

3.1 Thermoelastic equations

3.1.1 Derivation

An Hookean elastic solid deforms under a reversible process between two fixed states. Each state can be described using a thermodynamic function denoted as state function. The strain energy, $u_m$, and entropy, $s_m$, (both per unit mass) can be considered state functions. To obtain a temperature-mechanical relation, the state functions must be characterized by the temperature and strain tensor, $s_m = s_m(T, \varepsilon_i)$ and $u_m = u_m(T, \varepsilon_i)$.

From the first and second laws of thermodynamics, as well as the Helmholtz potential function, one can obtain the following differential equation \[37\]:

$$\frac{\delta q_m}{T} = -\frac{1}{\rho} \frac{\partial \sigma_i}{\partial T} d \varepsilon_i + C_\varepsilon \frac{dT}{T} \tag{3.1}$$

where $\delta q_m$ is the heat exchanged per unit mass, $T$ is the temperature in absolute units, $\rho$ is the density, $\sigma_i$ is the stress tensor and $C_\varepsilon$ is the specific heat at constant strain (equivalent to the specific heat at constant volume, $C_v$).

The Duhamel-Neumann constitutive law for thermoelasticity should now be introduced \[38\]:

$$\sigma_i = C_{ij} \varepsilon_j - \beta_i (T - T_0) \tag{3.2}$$

where $\beta_i$ is the coefficient of volumetric thermal expansion vector defined as:

$$\beta_i = C_{ij} \alpha_j \tag{3.3}$$

Substituting equation 3.2 differentiated in relation to $T$ into equation 3.1 and considering a process
under adiabatic conditions (i.e. $\delta q_m = 0$), one obtains:

$$C_\varepsilon \frac{dT}{T} = -\frac{1}{\rho} C_{ij} \alpha_j d\varepsilon_i$$

where $\alpha_j$ is the coefficient of thermal expansion tensor. Up to this point, all the temperature dependent terms have been neglected. The errors deriving from this approximation were proved to be only significant when considering compression-compression or tension-compression loading conditions [39].

Integrating equation 3.4 between the peaks of the cyclic strain change considering that the mechanical and thermal properties are temperature independent:

$$-\frac{1}{\rho} C_{ij} \alpha_j \Delta \varepsilon_i = C_\varepsilon \ln \left( \frac{T}{T_0} \right)
= C_\varepsilon \ln \left( 1 + \frac{\Delta T}{T_0} \right)$$

Aware that $\Delta T \ll T_0$, the first term of a Taylor series expansion of the logarithmic function can be retained without loss of accuracy. In addition, it is experimentally more practical to determine the specific heat at constant pressure, $C_p$, and the difference to $C_\varepsilon$ can be neglected [37]. Therefore, the general form of the recorded temperature change during the load cycle yields:

$$\Delta T = -\frac{T_0}{\rho C_p} C_{ji} \Delta \varepsilon_i \alpha_j = -\frac{T_0}{\rho C_p} \Delta \sigma_j \alpha_j$$

### 3.1.2 Stress-based equations for orthotropic materials

The thermoelastic effect detected on the surface of multidirectional laminates under in-plane stress conditions can be expressed in relation to the principal stress and principal material directions by expanding equation 3.6:

$$\Delta T = -\frac{T_0}{\rho C_p} (\alpha_1 \Delta \sigma_1 + \alpha_2 \Delta \sigma_2 + \alpha_6 \Delta \tau_6)
= -\frac{T_0}{\rho C_p} (\alpha_x \Delta \sigma_x + \alpha_y \Delta \sigma_y + \alpha_s \Delta \tau_s)$$

The coefficient of thermal expansion and mechanical stress are the only tensor quantities. When loading along the principal stress directions, $\tau_s$ component is null, whereas along the principal material directions, $\alpha_6$ is zero [29]:

$$[\sigma]_{x,y} = \begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \end{bmatrix};
[\sigma]_{1,2} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_6 \end{bmatrix}$$

$$[\alpha]_{x,y} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_s \end{bmatrix};
[\alpha]_{1,2} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix}$$

Therefore, equation 3.7 is simplified into the well-known TSA surface stress-based formulation:
\[ \Delta T = -\frac{T_0}{\rho C_p} (\alpha_1 \Delta \sigma_1 + \alpha_2 \Delta \sigma_2) = -\frac{T_0}{\rho C_p} (\alpha_x \Delta \sigma_x + \alpha_y \Delta \sigma_y) \]  \hspace{1cm} (3.9)

Equation 3.9 can be reformulated by grouping the thermal and mechanical properties in a single thermoelastic constant, \( K_i \):

\[ \Delta T = -T_0 (K_1 \Delta \sigma_1 + K_2 \Delta \sigma_2) = -T_0 (K_x \Delta \sigma_x + K_y \Delta \sigma_y) \]  \hspace{1cm} (3.10)

From the thermoelastic point of view, each lamina can be homogenised in the properties within \( K \) parameter. The almost instantaneous thermal equilibrium that occurs in the fiber-matrix interface confirms the feasibility of this approach [40].

Nevertheless, there’s a level of anisotropy inherent to composite materials that leads to a dependency of the thermoelastic constant with the constituent volume fraction. Several works accounted for this factor by using a rule of mixtures of the microconstituent deformations and material properties [41, 42].

Another useful form of equation 3.9 is presented in terms of a stress metric [29]:

\[ SM_1 = \frac{\Delta T}{K_1 T_0} = \Delta \sigma_1 + \frac{K_2}{K_1} \Delta \sigma_2 \]  \hspace{1cm} (3.11a)

\[ SM_2 = \frac{\Delta T}{K_x T_0} = \Delta \sigma_x + \frac{K_y}{K_x} \Delta \sigma_y \]  \hspace{1cm} (3.11b)

Radiometric uncalibrated devices are uncapable of directly converting the collected signal \( S \) - in Digital Level units (DL) - to temperature (e.g. SPATE, Deltatherm). Hence, equation 3.9 must be rearranged such that:

\[ A^* S = \alpha_1 \Delta \sigma_1 + \alpha_2 \Delta \sigma_2 = \alpha_x \Delta \sigma_x + \alpha_y \Delta \sigma_y \]  \hspace{1cm} (3.12)

where \( A^* \) is the calibration constant. Prior to any quantitative stress measurement, \( A^* \) should be determined at the material’s undamaged state.

### 3.1.3 Stress based equations for isotropic materials

The thermoelastic equation for isotropic materials can be obtained from equation 3.9 by considering the coefficient of thermal expansion as constant for any considered direction:

\[ \Delta T = -T_0 \frac{\alpha}{\rho C_p} (\Delta \sigma_x + \Delta \sigma_y) \]  \hspace{1cm} (3.13)

Thus following the same simplified approach of equation 3.10, one can obtain:

\[ \Delta T = -T_0 K (\Delta \sigma_x + \Delta \sigma_y) \]  \hspace{1cm} (3.14)
3.1.4 Strain based thermoelastic equation for orthotropic materials

An externally loaded multidirectional laminate with perfectly bonded layers yields a constant displacement field along the through-thickness direction, hence a constant strain field in the principal stress directions. Therefore, reformulating the thermoelastic equations in terms of strain components instead of stresses is a more useful approach. Taking the principal material directions of the orthotropic surface layer (1,2) as the initial reference axis, from equation 3.1 one can obtain:

$$\Delta T = -\frac{T_0}{\rho C_p} [\alpha_1 Q_{11} + \alpha_2 Q_{12}][\Delta \varepsilon]_{x,y}$$

(3.15)

Or in matrix form:

$$\Delta T = -\frac{T_0}{\rho C_p} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \Delta \varepsilon_x \\ \Delta \varepsilon_y \\ \Delta \gamma_s \end{bmatrix}$$

(3.16)

where \( c = \cos(\theta) \) and \( s = \sin(\theta) \).

After simplification and factorization of equation 3.16 in terms of material constants and strain terms:

$$\Delta T = -\frac{T_0}{\rho C_p} [(\alpha_1 Q_{11} + \alpha_2 Q_{12})(\Delta \varepsilon_x \cos^2 \theta + \Delta \varepsilon_y \sin^2 \theta + \Delta \gamma_s \cos \theta \sin \theta)$$

$$+ (\alpha_1 Q_{12} + \alpha_2 Q_{22})(\Delta \varepsilon_x \sin^2 \theta + \Delta \varepsilon_y \cos^2 \theta - \Delta \gamma_s \cos \theta \sin \theta)]$$

(3.17)

Using the same unified constant notation as presented in equation 3.10, the final form yields:

$$\Delta T = -T_0[(K_1 Q_{11} + K_2 Q_{12})(\Delta \varepsilon_x \cos^2 \theta + \Delta \varepsilon_y \sin^2 \theta + \Delta \gamma_s \cos \theta \sin \theta)$$

$$+ (K_1 Q_{12} + K_2 Q_{22})(\Delta \varepsilon_x \sin^2 \theta + \Delta \varepsilon_y \cos^2 \theta - \Delta \gamma_s \cos \theta \sin \theta)]$$

(3.18)

3.2 Non-adiabatic effects

3.2.1 Heat Diffusion

The aforementioned equations are valid only if the measured temperature is generated at the laminate’s first ply. However, in a multidirectional laminate, depending upon the chosen stacking sequence, each lamina with a different fiber orientation will generate a different stress field, thus a locally distinct stress induced temperature change. Between lamina, heat conduction is, therefore, a distinct possibility. The short-time and small amplitude temperature fluctuations associated to TSA impart negligible radiation and convection heat losses to the environment [12].

3.2.2 Loading Frequency

Minimizing interlaminar heat diffusion requires cyclic changes in the applied load with a time scale lower than that associated to the heat conduction [43], i.e. loading frequencies as high as possible,
limited by the capabilities of the available equipment. Diffusion is never eliminated but its rate is rather slowed down so that quasi-adiabatic conditions are attained. Wong [40] presented a first analytical assessment of the Fourier Number’s ($F_o$) effect on the temperature profiles for an infinite plate submitted to a sudden change in the initial conditions at both of its surfaces. For a typical CFRP lamina, pure adiabatic conditions are impractical to reach since a $t < 2 \times 10^{-5}$ s would be required. In the same work, another non-adiabatic analytical model considering a finite plate with time-dependent temperature boundary conditions was evaluated. Wong observed that the phase difference approached the SPATE’s out-of-phase condition for decreasing loading frequencies together with a more homogeneised temperature profile along the through-thickness direction. The influence of subsurface plies in the surface thermal response therefore decreases with increasing frequency [40]. Increasing number of plies leads to a more homogeneous thermal response [44]. The latter conclusion is reported in figures 3.1(a) and (c) which represent CP laminates with a zero degree surface fiber orientation, [0/90], and [0/3/90/3], respectively. For the 12-plied case, one can denote the less pronounced influence of the surface fiber orientation in the image pattern.

![Figure 3.1: Thermoelastic images obtained from GFRP specimens with different stacking sequences at 10 Hz loading frequency](image)

(a) CP with surface ply fibers oriented at 0° (b) CP with surface ply fibers oriented at 90° (c) CP with surface ply fibers oriented at 0° and 3 layers equally oriented consecutively stacked (d) CP with surface ply fibers oriented at 90° and 3 layers equally oriented consecutively stacked (e) Angle-Ply (AP) (f) AP with 3 layers equally oriented consecutively stacked (g) QI with surface ply fibers oriented at 45° (h) QI with surface ply fibers oriented at 90° [44].

The higher thermal conductivity of the carbon reinforcement in relation to glass can explain the adiabatic condition incompatibility for a range of working frequencies between 5 and 50 Hz found in [41]. In another reference to carbon reinforced composite materials, the output signal proved to be frequency dependent below the 4 Hz threshold [39], whereas beyond a 5 Hz limit, an almost constant trend of the same signal was observed by Pitarresi et al [45] in unidirectional E-glass roving fabric composites. A 2D heat transfer computational model supported by experimental results on different stacked GFRPs allowed Sambasivam et al [44] to prove that, for a 10 Hz loading frequency, the temperature amplitude images present a marked pattern of the surface layer fiber orientations (see figure 3.1). This frequency value derives from previous thermoelastic work in metallic structures [46] and has been extrapolated to
most research work applied to glass fiber reinforced composites.

When working under combined load-frequency conditions inefficient to prevent heat diffusion from occurring, treating the laminate thermal behaviour as globally homogeneous has enabled good computational reproducibility of the magnitude and distribution of the experimental full-field stress map obtained in carbon made sandwich panels (see figure 3.2) [47]. The same treatment applied in [48] for determining the maximum stress concentration factor around holes in unidirectional glass/epoxy specimens revealed close agreement with experimental results.

![Figure 3.2: CP panel calibration using global approach (a) homogeneous FE model (b) TSA [47].](image)

### 3.2.3 Resin-rich layer

Current composite manufacturing processes lead to the accumulation of resin matrix in the outer surface of the laminate, immediately after the first ply (see figure 3.3). For increasing loading frequencies, the wavelength of the heat wave generated in the laminate substrate is reduced. Since the epoxy resin is low conductive, thicker layers present a barrier to heat conduction towards the surface. This reflects in a decreasing temperature amplitude reading and increased phase shift of the measured signal in relation to the reference [49]. This phenomena is denoted as thermal drag-down [50] and the resin-rich layer is said to act as a strain witness of the laminate thus becoming the sole responsible for the detected signal. It is an advantageous approach due to the simpler thermoelastic formulations attributed to isotropic materials and possibility to overcome the variability in reported material properties as a consequence of the heterogeneous structure [51, 52]. El-Hajjar et al [53] idealized the combined external veil/CFM layers as in-plane transversely isotropic and responsible for the thermoelastic signal in GRP composites manufactured using a pultrusion method. The first strain invariant obtained both experimentally and using a FE analysis for two stress states (along and transversely to the roving orientations) presented good correlation.
Minimum thickness thresholds of 25 \( \mu m \) and 30 \( \mu m \) have validated the strain witness assumption in glass and carbon fiber reinforced composites, respectively [54, 55]. Low conductive reinforcement materials (in particular, glass fibers) prove to better fit the strain witness model and more effectively whenever the surface ply is oriented transversely to the applied load [41, 45, 49]. The influence reaches a greater extension whenever the ratio between coefficients of thermal expansion, \( \alpha_y/\alpha_x \), is increased. For CFRPs, it is typically equal to 65 [56]. Pitarresi and Galietti [57] evidenced the deviation of CFRPs thermoelastic data from the strain witness predictions. The authors believe the resin extent might influence the \( \alpha_x \) value such that plausible values provide a better fitting to the bulk model (to be referred in the following sections).

Local spots of resin accumulation lead to an increased temperature generation and phase shift as a result of its low conductivity. This non-adiabatic behaviour is mostly detected in surfaces made of woven or unidirectional stitched fibers which are prone to develop resin pockets [43, 57].

![Figure 3.3: Surface resin-rich layer [55.](image)](image)

### 3.2.4 Temperature correction methodology

Polymer matrix composite materials are good thermal insulators. Any heat generated is hardly dissipated with increasing frequencies leading to a non-adiabatic response. The crimp structure of woven fibers allocates resin accumulated regions that generate a locally increased thermoelastic response [43]. After crack formation, friction between crack surfaces locally increases the temperature. This viscoelastic heating phenomenon also affects the precision of TSA’s measurements.

Separating these non-stress related temperature increases from the actual thermoelastic effect using a calibration procedure has been successfully applied to a damaged FRP by reducing the maximum measured signal in 10\% [58]. The same methodology is replicated during the RUL calculations.

For a blackbody, the spectral intensity, \( I_{\lambda,b} \), is known as the Planck distribution [59]:

\[
I_{\lambda,b}(\lambda, T) = \frac{2hc}{\lambda^5}\frac{c^2}{e^{hc/\lambda k_B T} - 1}
\]

(3.19)
where $h_P = 6.626 \times 10^{-34}$ J.s is the Planck constant, $k_B = 1.381 \times 10^{-23}$ J/K is the Boltzmann constant, $c_0 = 2.998 \times 10^8$ m/s is the speed of light in the vacuum and $T$ is the absolute temperature of the blackbody. One can consider the specimen as a diffuse emitter i.e. the radiation intensity is independent of the considered direction. Considering the region of influence as a semi-hemisphere, the total emitted energy per wavelength and per surface temperature of a blackbody, $E_{\lambda,b}(\lambda, T)$ is [59]:

$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}(\lambda, T)$$ (3.20)

The FLIR camera contains a photon detector therefore providing an output signal from counting the number of photons received. For a single wavelength, $N_{\lambda,b}$ is defined as:

$$N_{\lambda,b} = \frac{E_{\lambda,b}(\lambda, T)}{hc_0/k_B}$$ (3.21)

However, the wavelength range captured by this camera model lies between $\lambda_1 = 1.5 \mu m$ and $\lambda_2 = 5.5 \mu m$. Integration of equation 3.21 between these two limits yields the total amount of photons, $N_b$ [58]:

$$N_b = \int_{\lambda_1}^{\lambda_2} \frac{2\pi c_0}{\lambda^4[e^{h_0c_0/\lambda k_B T} - 1]} \partial \lambda$$ (3.22)

A final formulation only dependent on temperature after integration is not possible to be obtained. Yet, an approximation to a power law function will be valid if the maximum operating wavelength of the camera is smaller than the maximum defined by the Wien’s displacement law at the TSA’s working temperature. Equation 3.23 verifies to be valid since $\lambda_{max} = 2898 [\mu m.K] / 297 [K] = 9.75 \mu m$ [60]:

$$N_b = BT^n$$ (3.23)

where $N_b$ is the number of photons per unit area and time. Taking $ln$ of the terms in equation 3.23, the coefficients $B$ and $n$ are derived from the linear regression of equation 3.24 for a set of data points $(T, N_b)$ calculated from integration of equation 3.22:

$$ln (N_b) = ln(B) + nln(T)$$ (3.24)

The linear regression results for temperatures ranging between 293 K and 323 K are presented in figure 3.4 with a R-squared coefficient of 0.9999.

Focus shall be given to the parameter $n$ that is built-in the correction factor, $R$, defined in [58]:

$$R = (T_0/T_m)^{0.74}$$ (3.25)

where $T_0$ and $T_m$ are the mean surface temperature results at the undamaged state and after imparting damage to the specimen, respectively.

Although structurally the same, the methodology here applied to TSA needs to consider the radiometric calibrated nature of the FLIR device:
• At the undamaged state, $T_0$ averaged over the specimen’s surface is stored;

• For every new TSA measurement ran after plastic work has been imparted to the specimen, local $T_m$ and temperature change $\Delta T_m$ values should be updated;

• $T_0$ and $T_m$ are then combined in equation 3.25 to obtain single calibration constants for each point of the image;

• In [58], the point-wise corrected and uncalibrated thermoelastic signals ($S_c$ and $S_m$ respectively) were correlated:

$$S_c = R S_m \tag{3.26}$$

From the thermoelastic equation 3.12 for orthotropic materials using radiometric uncalibrated devices, one can deduce the following relation by similarity:

$$A^* S = -\frac{\Delta T}{T_0} \rho C_p \tag{3.27}$$

Substitution equation 3.27 in 3.26 and considering $A^*$, $\rho$ and $C_p$ as independent of the damage level, the corrected temperature ratio is attained:

$$\left( \frac{\Delta T}{T} \right)_c = R \left( \frac{\Delta T}{T} \right)_m \tag{3.28}$$

A schematic of the steps above described is shown in figure 3.5.
3.3 Strain based thermoelastic models

The anisotropic nature of laminated composite materials at the micro constituent level together with the numerous fiber-matrix material type arrangements and stacking sequences will dictate different thermal interactions between layers, hence the final surface temperature field. Understanding the source of the thermoelastic signal is a topic of major importance in TSA discussions. Four thermoelastic strain models have been presented in the literature [8] and will be considered hereinafter.

3.3.1 Surface Resin-Rich Layer model (RRL model)

The RRL model is based on the previously mentioned hypothetical 'strain witness' behaviour of the resin-rich layer, i.e. the strain field in the resin is the same as the laminate's. The global Poisson’s ratio, $\nu_{xy}$, shall be also introduced to eliminate the influence of the transverse strain change from the thermoelastic equations hereinafter:

$$
\Delta \varepsilon_{xr} = \Delta \varepsilon_{xc}
$$

$$
\Delta \varepsilon_{yr} = \Delta \varepsilon_{yc} = -\nu_{xy} \Delta \varepsilon_{xc}
$$

(3.29)

From the stress-strain formulation presented in 2.6 and using the reduced stiffness terms for an isotropic medium from section 2.5.3, the first stress invariant can be obtained:

$$
\Delta \sigma_x + \Delta \sigma_y = \frac{E_r}{1 - \nu_r} (1 - \nu_{xy}) \Delta \varepsilon_{xc}
$$

(3.30)
where the subscript $r$ stands for resin. Substituting equation 3.17 into equation 3.8b:

$$\Delta T = -T_0 \frac{\alpha_r}{\rho_r C_{pr}} \left[ \frac{E_r}{1 - \nu_r} (1 - \nu_{xy}) \Delta \varepsilon_{xc} \right]$$  \hspace{1cm} (3.31)

### 3.3.2 Orthotropic surface layer (Bulk model)

When uniaxial tensile stress is applied to a balanced and symmetric orthotropic laminate, the $\Delta \gamma_s$ component is null. This simplifies equation 3.17:

$$\Delta T = -T_0 \frac{\rho C_p}{\rho C_p} \left[ (\alpha_1 Q_{11} + \alpha_2 Q_{12}) (\cos^2 \theta - \nu_{xy} \sin^2 \theta) + (\alpha_1 Q_{11} + \alpha_2 Q_{22}) (\sin^2 \theta - \nu_{xy} \cos^2 \theta) \right] \Delta \varepsilon_{x}$$  \hspace{1cm} (3.32)

### 3.3.3 Coefficient of Thermal Expansion coupled in the stack (CTE model)

This model has been suggested by Sambasivam [8] and is applicable at the orthotropic surface layer level. The novelty in relation to the previous described model is assuming the thermal strains have a constant through-thickness behaviour as the mechanical strains:

$$\Delta T = -\frac{T_0}{\rho C_p} \left[ \alpha_x (Q_{11} + Q_{12}) - \alpha_y (Q_{12} + Q_{22}) \nu_{xy} \right] \Delta \varepsilon_{x}$$  \hspace{1cm} (3.33)

### 3.3.4 Homogeneous orthotropic material (Homogeneous model)

This model assigns the mechanical and thermoelastic properties as global values. Considering a balanced and symmetric laminate, after replacing the stress-strain relation presented equation 2.18 in matrix form into equation 3.9, one obtains:

$$\Delta T = -\frac{T_0}{\rho C_p h} \left[ \alpha_x (A_{11} + A_{12}) - \alpha_y (A_{12} + A_{22}) \nu_{xy} \right] \Delta \varepsilon_{x}$$  \hspace{1cm} (3.34)
Chapter 4

Remaining Useful Life Methodology

4.1 Fatigue Modulus

Under constant cyclic strain amplitude conditions, as a consequence of the damage evolution within a laminate throughout a fatigue process, the maximum applied stress gradually decreases as schematically represented in figure 4.1. For cycle $N$, the fatigue modulus is defined as the ratio between the current maximum peak stress state, $\sigma_{\text{max}}(N)$ and the maximum strain value, $\varepsilon_{\text{max}}$ [61]:

$$F(N) = \frac{\sigma_{\text{max}}(N)}{\varepsilon_{\text{max}}}$$  \hspace{1cm} (4.1)

It is only a function of the cycle number $N$, therefore independent of the applied stress or strain ratio [61].

![Figure 4.1: Fatigue Modulus.](image)
The area of the region below the slope represents the strain energy density, $u_v(N)$:

$$u_v(N) = \frac{1}{2} \sigma_{max}(N) \varepsilon_{max}$$  \hspace{1cm} (4.2)

### 4.2 Analytical formulation

Previous work [62] observed a growing trend of the expended strain energy rate, $dU/dN$, for FRP's submitted to constant strain amplitude conditions under fatigue. It can be split into three segments:

- **Stage I** - initial steep evolution of the energy curve until approximately 10% of the composite's life;
- **Stage II** - linear evolution until 90% of the fatigue life, considered the upper limit of its useful life;
- **Stage III** - final region where abrupt failure is imminent.

![Figure 4.2: Expended strain energy as a function of the number of cyclic loads [62].](image)

Natarajan et al [62] developed a promising method based on the linear evolution of the strain energy release rate during the intermediate stage. The slope was found to be correlated with different applied strain ratios ($\varepsilon_{max}/\varepsilon_{ult}$) by a power law function:

$$\frac{dU}{dN} = a \left(\frac{\varepsilon_{max}}{\varepsilon_{ult}}\right)^b$$  \hspace{1cm} (4.3)

where different $a$ and $b$ coefficients must be attained according to the stacking sequence, fiber volume fraction, specimen thickness and loading type (e.g. for tension-tension, tension-compression, bending).

Integrating the previous equation between the moment before fatigue initiation and the end of stage II, the remaining useful life equation is obtained:

$$N_f = \frac{U_0(r - 1)}{a \left(\frac{\varepsilon_{max}}{\varepsilon_{ult}}\right)^b}$$  \hspace{1cm} (4.4)

where $r = U_f/U_0$ and $U_0$ is the strain energy at the initial mean load level.
Keulen et al [63] were pioneers in replicating this methodology to SHM techniques. The strain energy released at each fatigue cycle was determined from locally embedded fiber Bragg gratings (FBGs) capable of measuring the peak cyclic strain. When summed over the entire life of the specimen under test, although over predicting the remaining useful life in the early fatigue stage, its mid-life was well captured for an applied strain ratio equal to 0.4. The key remarkable factor evidenced by the authors was the locally differing strain readings from the global values provided by the extensometer. In fact, this confirmed the onset and growth of matrix cracks as well as other micron- or submicron-level cracks imperceptible when solely interpreting the extensometer’s strain data. Disadvantages come from the fragile nature of the fibers - requiring special care during manufacturing - and the need to implement the fibers in a highly plausible location for damage growth thus arising more difficulties when confronted with uniform geometries (e.g. without stress concentration raisers, as holes).

Overcoming the latter limitations might be possible using TSA and by readjusting the methodology first derived by Natarajan:

- Calculation of the total number of cycles until failure $N_t$ is more intuitive than defining calculations up to the end of stage II;
- To avoid dependency of the RUL calculations upon the specimen’s area selected for strain averaging, strain energy density, $u_v$, will be considered hereinafter.

Aware that stage III is reached at 90\% of the fatigue life, from equation 4.4 the remaining useful life (RUL) calculation methodology using TSA yields:

$$N_t = \frac{u_0 (r - 1)}{0.90 \left[ \frac{a (\varepsilon_{max}/\varepsilon_{ult})^b}{(\varepsilon_{ult}/\varepsilon_{max})^b} \right]}$$

(4.5)

If linear elastic behaviour is not overcome, $u_0$ can be determined using the strain-based thermoelastic equations.

### 4.3 Elastic strain energy density calculation

From the first law of thermodynamics, the work per unit volume, $\delta w_v$, applied to a multidirectional laminate composite material is equivalent to:

$$\delta w_v = du_v - \delta q_v$$

(4.6)

where $\delta q_v$ is the heat generated and $du_v$ is the internal energy, both per unit volume and in absolute values. Note that the internal energy is taken as an exact derivate since its integration is independent of the path unlike the heat exchange and applied work.

If adiabatic conditions are attained, all the applied work along the principal geometric dimension, $x$, is fully converted to strain energy density:

$$du_v = \delta w_v = \sigma_x d\varepsilon_x$$

(4.7)
Moreover, under the linear elastic stress-strain behaviour, an energy formulation including the Young’s Modulus can be rewritten from equation 4.2. The stiffness degrades as a function of the number of fatigue cycles, $E_x(N)$:

$$u_0(N) = \frac{1}{2} E_x(N) \Delta \varepsilon_x^2$$

(4.8)

where $\Delta \varepsilon_x$ is derived from the four thermoelastic models described in section 3.3 of chapter 3.

### 4.4 Algorithm for remaining useful life calculations

A summary of the adopted strategy for remaining useful life implementation is presented:

- Determine all the necessary in-plane mechanical and thermal properties for the UD lamina, global laminate and resin matrix to input as data in the model equations;
- Perform exploratory tests on the required frequency to reach quasi-adiabatic conditions;
- Perform a calibration of the thermoelastic signal at the selected frequency i.e. determine which of the models presented in chapter 4 better describes the signal measured by the infrared camera;
- Determine the global Young’s modulus ($E_x$) and Poisson’s ratio ($\nu_{xy}$) using the apparatus described in Test Method D 3039 [64];
- ‘TSA’ step: Perform a cyclic strain-controlled test under tensile-tensile conditions with a maximum applicable strain level limited by the yield point threshold;
- ‘Fatigue’ step: in this stage, strain is again the controlling parameter and its maximum should be equal to the strain level at the yield point plus the amplitude at which the previous step is performed. Damage is now allowed to initiate and evolve in the laminate;
- The procedure restarts from the 4th step if complete failure has not occurred or damage has not propagated extensively. $N_t$ is, therefore, updated every step.

Two schematics regarding the described methodology and sequence of calculations until $N_t$ is obtained is presented in figures 4.3 and 4.4
Figure 4.3: Schematic of the experimental apparatus for determining the remaining useful life using the strain-based methodology.

Figure 4.4: Schematic of the calculations for determining $N_t$. 

Chapter 5

Implementation

5.1 Test Samples Preparation

5.1.1 Manufacturing Process

Layers of stitched unidirectional E-glass fibers with a total areal weight of 330 g/m$^2$ preimpregnated with epoxy resin were used during the fabrication stage of the test specimens. An aluminium mold surface of 400 x 450 mm was prepared for vacuum bag curing inside a temperature controlled oven.

A two-step temperature profile consisted of a first dwell period of 1h30 at 70$^\circ$C followed by a second dwell period of 2h00 at 120$^\circ$C. Finally, the assembly cooled down to room temperature at a slow rate to avoid generating residual stresses.

5.1.2 Tabbing Process

Shear forces are applied by the machine grips to the specimen's surface during static and dynamic tension tests. To prevent slippage, the grips' surfaces present some roughness. Protecting the specimen from premature damage in the gripping area is the driving motivation for the use of tabs.

When stiff unidirectional stacking sequenced structures are expected to fail (i.e. under fatigue or when determining ultimate static properties), the selection of the tabbing geometry and tabbing material become crucial to relieve the stress concentrations away from the tab terminations towards the gage section area. Conclusions derived from previous finite element analysis work [65] provided the required background knowledge to define the following set of parameters:

- Tabs should be made of a low strength material. GFRP provides the optimum mechanical solution, yet at a relative high cost when applied to a large number of specimens, together with a more laborious machining process. An alternative is to consider aluminium-made tabs whose strength is of the same order of magnitude as GFRP. Stress concentrations along the two normal directions obtained with aluminium tabs proved to be within less than 5% difference in relation to the ideal case scenario. Therefore, aluminium was the selected material;
• The ratio between the tab and specimen thicknesses should be comprehended between 1 and 4. This criteria is only to be considered for UD(0) stacking sequences static or dynamically loaded above the yield point;

• Recommended tab length should lie between 40 and 90 mm. A value of 50 mm was set by default;

• The inclusion of a taper angle regarding the involved thicknesses becomes impractical. Therefore, for simplicity, all tabs remained with a constant cross sectional area;

A slight curvature was initially detected in most alluminium tabs and corrected after compression against two small metallic plates as shown in figure 5.1 (a). Moreover, sharp edges were found to potentially cause premature damage on the specimen when submitted to the grip force. The tab smoothing process is shown in figure 5.1 (b).

![Figure 5.1: (a) Tab alignment (b) Sharp edge smoothing process.](image)

An homogeneised rough surface was prepared using a sanding machine (figure 5.2) for increasing the shear force transmissibility between the tabs and specimen. As a consequence of the sanding process, there remained alluminium scraps in the tab surface easily removed using an ultrasonic cleaning device.

![Figure 5.2: (a) Tab sanding process (b) Final surface pattern.](image)
For an increased control on the amount of material removed from the specimen’s bonding area, the sanding machine was replaced by a wet hand sanding procedure as shown in figure 5.3. After sanding, the bonding area was cleaned using acetone or isopropyl.

Figure 5.3: Specimen’s surface sanding process.

A thin layer of the 2-component Araldite 2000 epoxy-resin was spread in both tab and specimen’s surfaces. Any excess of adhesive should be removed using a dry tissue. While setting the tabs position, particular focus should be given to the alignment of the tab terminations from opposite faces. It is important to guarantee geometric symmetry, thus stress equilibrium.

A pressure between 1 to 3 kPa is recommended to be applied while the resin is curing at room temperature [65]. Two small metallic plates of 380 g each were positioned on top of each sides of the assembly (figure 5.4). An interval time of 24h was set to ensure the curing process completeness.

Figure 5.4: Adhesive curing process at room temperature.

The aforementioned methodology was set to proof in two tensile tests until final rupture of the specimen. A first comparison of the ultimate static properties and failure location for tabbed and untabbed quasi-isotropic ([0/±45/90]), E-glass fiber reinforced specimens is reported on table 5.1. For low stiff
materials along the loading direction, the performance of both configurations is similar, meaning that tabbing is not a mandatory requirement.

<table>
<thead>
<tr>
<th></th>
<th>$h$ (mm)</th>
<th>$w$ (mm)</th>
<th>$GL$ (mm)</th>
<th>$P_{ult}$ (kN)</th>
<th>$\varepsilon_{ult}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Tabs</td>
<td>2.15</td>
<td>25.00</td>
<td>150.00</td>
<td>15.76</td>
<td>3.03</td>
</tr>
<tr>
<td>Without Tabs</td>
<td>2.19</td>
<td>25.20</td>
<td>150.00</td>
<td>15.46</td>
<td>3.07</td>
</tr>
</tbody>
</table>

Table 5.1: Ultimate static properties of tabbed and untabbed QI E-glass reinforced specimens.

A second test was ran on unidirectional (UD(0)) carbon and glass fiber specimens whose tab over specimen thicknesses ratio fell outside the limits recommended in [65]. Figures 5.5 (a) and (b) reveal the failure located at the end terminations of the grip faces inside the tabbing region. During the tensile test course, the applied pressure at the grip region is increased up to the maximum transverse compressive force is attained where final failure occurs.

Figure 5.5: (a) Front view of the specimen's tab region; (b) Side view of the specimen's grip area.

### 5.2 Micromechanical Properties

#### 5.2.1 Reinforcement

The relevant fiber properties provided by the manufacturer are presented in table 5.2

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$E_1$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-glass</td>
<td>2480 - 2600</td>
<td>72 - 82</td>
</tr>
</tbody>
</table>

Table 5.2: Fiber properties.

#### 5.2.2 Density

The measurement is based on the hydrostatic weighing method and is schematically explained in figure 5.6. The combined mass of the sample suspended in the air by an improvised structure, $m_{s,a}$, is first measured. At this initial moment, the weight of the sample and tension on the wire, $T_1$, are in equilibrium:
When fully immersed in water, the buoyant force, $F_b$, will act on the sample:

$$m_s g = T_2 + \rho_w V_s g$$  (5.2)

where $T_2$ is the tension force when the object is immersed, $\rho_w$ is the density of the water and $V_s$ is the volume of the sample. Due to the upward action of the buoyant force, the sample will be apparently lighter i.e. $m_{s,l} < m_{s,a}$. This weight difference, $\Delta m g$, can be explained by the tension reduction:

$$\Delta m g = |m_{s,l} - m_{s,a}| g = |T_2 - T_1| = \rho_w V_s g \Rightarrow \Delta m = \rho_w V_s$$  (5.3)

It is reasonable to consider $\rho_w = 1.0 \text{ g/cm}^3$, hence the volume of the sample is simply determined from the final balance reading. After determining the sample’s mass, $m_s$, the composite’s density can be calculated.

Single measurements on five samples with $[0]_{12}$ accounted for the microstructure variability inherent to composite materials. In contrast, for the higher level of homogeneity in epoxy resins, it was decided to take five measurements using one sample. The final averaged results (denoted as ‘Avg’) with standard deviation (‘SD’) are presented in table 5.3.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (Kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>1834</td>
</tr>
<tr>
<td>Epoxy</td>
<td>1142</td>
</tr>
</tbody>
</table>

Table 5.3: Density results for epoxy resin and glass fiber-reinforced polymer matrix composite.
5.2.3 Volume Fraction and Void Content

Each constituent of a bi-phase composite material is a fraction of the total volume. Ideally, the sum of the reinforcement and matrix volume fractions should equal the unity \((v_f + v_m = 1)\). However, due to defects inherent to the prepreg or generated during the fabrication and processing stages of manufacturing, void locations which contain neither reinforcement nor matrix material will appear, \(v_v\).

A matrix carbonization inside a nitrogen-purging furnace allows the determination of the void content [66]. After having determined the density, five composite samples and one epoxy resin sample were dried inside an oven for 24h at 70°C. Each sample was weighed and placed inside a crucible with known mass (figure 5.8).

The carbonization process was held for 1h at 560°C after which the residue's mass was remeasured inside the crucible. For the range of e-glass density values provided in table 5.2, bonding limits for the fiber volume fraction and void contents are shown in table 5.4 for each tested sample.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>(v_f) (%)</th>
<th>(v_m) (%)</th>
<th>(v_v) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[47.20, 49.48]</td>
<td>43.83</td>
<td>[6.69, 8.98]</td>
</tr>
<tr>
<td>2</td>
<td>[46.50, 48.75]</td>
<td>45.35</td>
<td>[5.90, 8.15]</td>
</tr>
<tr>
<td>3</td>
<td>[46.44, 48.68]</td>
<td>47.75</td>
<td>[3.56, 5.81]</td>
</tr>
<tr>
<td>4</td>
<td>[46.28, 48.52]</td>
<td>48.12</td>
<td>[3.36, 5.60]</td>
</tr>
<tr>
<td>5</td>
<td>[47.43, 49.73]</td>
<td>46.25</td>
<td>[4.02, 6.31]</td>
</tr>
</tbody>
</table>

Table 5.4: Volume fractions and void content results.

A maximum void content percentage of nearly 9% is prone to negatively affect the mechanical per-
formance of the manufactured specimens. From the manufacturing point of view, alternative processing techniques able to apply higher pressure levels would allow entrapped air to be easily removed from the laminate hence decreasing the void content percentage and achieve more compact specimens.

5.2.4 Optical Microscopy

A closer insight to the composite’s microstructure and characterization of the outer resin-rich layer’s thickness is made possible through the magnification of a lens with visible light. Obtaining a stabilised and parallel view to the lens requires embedding the sample within a fast-curing epoxy resin medium mixed with hardener as shown in figure 5.9. Two final grinding and polishing steps applied to the side containing the cross sectional area of the sample guaranteed a better surface quality finishing.
Two relevant through-thickness images taken at the surface of the sample are presented in figure 5.10. The surface ply fibers are clearly identified from their circular cross sectional areas. Within a small area, multiple void areas are visible in figure 5.10 (a), hence confirming the high percentages derived from the matrix burnoff tests. Either fabrication defects, low pressure applied during the processing stage or a combination of both can justify the appearance of these undesirable regions.

Another important factor to account for when determining the surface heat source in thermoelastic measurements is the thickness variability of the epoxy layer formed next to the surface ply. Excessive accumulation of resin is locally noticeable in figure 5.10 (b) where the epoxy is expected to behave closer to the strain witness assumption.

Figure 5.10: Optical microscopy images.
5.3 Thermal Properties

5.3.1 Coefficient of Thermal Expansion (CTE)

If a body remains with a zero-stress field while undergoing a temperature change, $\Delta T$, the strain field along direction $i$ varies linearly depending upon the coefficient of thermal expansion, $\alpha_i$:

$$\varepsilon_i = \alpha_i \Delta T$$

(5.4)

The elongation variation due to an increase in temperature was obtained using a strain gage method [67]. Uniaxial strain gages contain a grid alloy of resistivity $R_a$. The change in resistivity derives from two sources: the mechanical elongation of the sample being measured and the thermal expansion coefficient of the grid material. To eliminate the latter effect from calculations, a material with known thermal properties is used as a reference. The final output is derived from the following calculation:

$$\alpha_i = \alpha_{ref} + \frac{\varepsilon_i - \varepsilon_{ref}}{\Delta T}$$

(5.5)

The experimental apparatus inside a temperature-controlled oven and monitored in real-time using a National Instruments NI system is shown in figure 5.11 (a). Inside the oven, [0]_{12} and [0/±45/90], 20 x 25 mm samples were positioned on top of a low-friction medium to unconstrain dilatation and contraction effects (figure 5.11 (b)). Aluminium provided the initial CTE reference of $2.40 \times 10^{-5}$ obtained from a Thermomechanical Analysis (TMA). One K-type thermocouple was attached to the sample surface immediately adjacent to the gage. Data collection for temperatures ranging from 25 to 45°C was representative of the TSA’s working conditions.

![Figure 5.11](image)

(a) Overall experimental apparatus for CTE determination; (b) Samples positioning inside the oven.

Five equal measurements assured the test repeatability for a single stacking sequence and reinforcement material along a specific direction. Averaged values are presented in table 5.5. As previously reported in the literature [57], the proximity of the coefficient of thermal expansion in the fiber direction
around zero is again confirmed.

<table>
<thead>
<tr>
<th></th>
<th>UD(0)</th>
<th>QI</th>
<th>Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$ ($^\circ$C)</td>
<td>$7.00 \times 10^{-6}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\alpha_2$ ($^\circ$C)</td>
<td>$2.60 \times 10^{-5}$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\alpha_x$ ($^\circ$C)</td>
<td>—</td>
<td>$1.20 \times 10^{-5}$</td>
<td>—</td>
</tr>
<tr>
<td>$\alpha_y$ ($^\circ$C)</td>
<td>—</td>
<td>$1.20 \times 10^{-5}$</td>
<td>—</td>
</tr>
<tr>
<td>$\alpha_r$ ($^\circ$C)</td>
<td>—</td>
<td>—</td>
<td>$6.60 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 5.5: Averaged lamina and global coefficient of thermal expansion for GFRP and epoxy samples.

5.3.2 Specific Heat at Constant Pressure

The Differential Scanning Calorimetry method is thoroughly described in standard ASTM E1269 [68]. This method allows the determination of the specific heat capacity using a Differential Scanning Calorimeter (DSC) shown in figure 5.12.

An initial heat flow calibration must be performed using a synthetic sapphire disk whose tabulated specific heat capacity values over a range of temperatures are known. Inside the test device, the specimen of interest is then heated up at a controlled rate. For each temperature value, the difference between the heat flow to the sample and reference sapphire standard is continuously recorded.

$C_p$ results were provided for temperatures ranging between 10 and $50^\circ$C. During the TSA tests, the surface temperature is expected to lie within the aforementioned limits. From the best linear fit to the experimental data (figure 5.13), the final values of 933 J/Kg.$^\circ$C and 1291 J/Kg.$^\circ$C for the GFRP and
epoxy, respectively, were extrapolated.

Figure 5.13: Specific Heat at Constant Pressure curves for epoxy resin and GFRP between 10 and 50°C.

5.4 Static Mechanical Properties

5.4.1 Young’s Modulus and Poisson’s Ratio

Equal global and lamina mechanical properties are attained for UD stacking sequences. When a UD(0) specimen is stretched along the fiber direction, $\sigma_1$ is the only non-zero stress component. Therefore, $E_1$ and $\nu_{12}$ can be determined. On the other hand, $\sigma_2$ is the only stress component of a UD(90) loaded transversely to the fiber direction. This allows the remaining $E_2$ and $\nu_{21}$ values to be calculated.

Tests were conducted in accordance with the ASTM D3039 standard test method [64]. 12 layered UD(0) and UD(90) plates were prepared. 5 specimens from each stacking sequence were cut and bonded with 25 x 50 x 1.0 mm aluminium tabs with constant cross-sectional area.

An Instron 5983 servo-hydraulic machine equipped with a 100 kN load cell served the purposes of this test. Strain measurements were provided by an Instron 2650-563 biaxial extensometer with 50 mm axial and 25 mm transverse gauge lengths. Data collection was triggered for longitudinal strain values set between 1000 $\mu$ε and 3000 $\mu$ε. Output repeatability was assured by performing 5 measurements for each specimen. The averaged with standard deviations results for the Chord Modulus and Poisson’s ratio are presented on table 5.6. The $\nu_{21}$ value was compared against calculated values from equation 2.8 and exact correlation was found.

For the epoxy resin characterization, one dog-bone specimen was prepared according to the recommendations from ASTM D638 standard test method [69]. The general test control conditions and measuring device were the same as proceeded in the aforementioned tests. The final results can be
also found on table 5.6.

<table>
<thead>
<tr>
<th></th>
<th>GFRP</th>
<th>Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg</td>
<td>SD</td>
</tr>
<tr>
<td>$E_1$ (GPa)</td>
<td>34.15</td>
<td>0.48</td>
</tr>
<tr>
<td>$E_2$ (GPa)</td>
<td>12.33</td>
<td>0.64</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>3.75</td>
<td>0.03</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.23</td>
<td>0.01</td>
</tr>
<tr>
<td>$\nu_{21}$</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>$E_r$ (GPa)</td>
<td>——</td>
<td>——</td>
</tr>
<tr>
<td>$\nu_r$</td>
<td>——</td>
<td>——</td>
</tr>
</tbody>
</table>

Table 5.6: Young’s modulus, Poisson’s ratios and shear modulus for GFRP and epoxy resin.

5.4.2 In-plane Shear Modulus

In the 1-2 plane, a shear stress $\tau_6$ applied within the elastic domain proportionally deforms the orthotropic material by an engineering shear strain $\gamma_6$:

$$\tau_6 = G_{12}\gamma_6$$

where $G_{12}$ is the in-plane shear modulus.

The special fixture of figure 5.14 applies an asymmetrical four-point bend loading to flat strips with symmetrical centrally located v-notches. The notches guarantee a uniform distribution of the shear strain in the middle region and are aligned with the compressive force’s line of action. In addition, the bending moment is null. The resultant shear strain is measured by attaching a strain-gauge rosette to the middle section of the specimen.

The v-notched test parameters and specimens dimensions were defined in accordance with the ASTM D5379 standard test method [70]. 12-layered CP plates ([(90/0),3]) were cut. This stacking sequence proved to ensure the best overall results. Five tests for each four different specimens were conducted. Table 5.6 presents the test results.

5.4.3 Stacking sequence selection

Two criterias were taken into consideration when selecting the stacking sequences for remaining useful life predictions:

- The chosen configurations must be adequate to common practices in aerospace industry;
- Obtain temperature difference readings closer to the strain witness model. It is more advantageous to deal with isotropic material properties due to their lower degree of variability in relation to the orthotropic case.
QI laminates are common practice within aerospace structures’ context. They can withstand loading conditions evenly along any direction. Care must be taken when 0 and 90 degree laminas are consecutively stacked since delamination process is enhanced thus reducing the material’s operational life.

E-glass fiber reinforced laminates are low thermal conductors. Therefore, the thermoelastic response is expected to be originated in the laminate’s surface, either at the resin-rich layer or the outer-surface ply. A parametric study similar to the one carried out in [57] intended to evidence the better fitting of QI stacking sequences to both models’ results. The dependency of the remaining models on the global coefficient of thermal expansion impossibilitates their computational reproducibility, as one would need to experimentally obtain individual values for every change in the stacking sequence.

A script was written using MATLAB R2016a software to determine the constant through-thickness strain field under external constant amplitude load of 2.4 kN and displacement conditions of $1.68 \times 10^{-4}$ mm by considering the CLPT. The material properties determined up to this point are now used considering a tested specimen with gauge length equal to 50 mm, 2.15 mm thick and 25 mm wide. The temperature differences from equations 3.31 and 3.32 were then normalised by the UD(0) case. Taking $\theta$ as the variable fiber angle orientation to be optimized, two different 8-ply combinations of symmetric and balanced laminates were considered.

When the surface ply fibers are oriented at $90^0$ (figure 5.15(a)), the results from both models follow the same trend yet distant by an offset value. There’s also an overestimation of the bulk model’s results. Predictions are almost coincident throughout the whole range of $\theta$ angles when the surface fibers are aligned with the load direction.

A QI laminate is achieved if $\theta = 45^0$ in both figures. One can conclude that, if a minor influence from the inner plies in the thermoelastic response is verified, the first strain invariant in $[0/45/-45/90]$, laminates (hereinafter simply referred as QI(0)) can be determined using only surface properties and, more specifically, the resin’s isotropic properties.
Figure 5.15: Comparison between 'bulk' and resin-rich layer models’ temperature difference for different GFRP lay-ups

5.4.4 Off-axis stiffness matrices

For the orientations considered in the quasi-isotropic stacking sequences of this work, the off-axis stiffness matrices for each of the four different $\theta$ fiber orientations ($[Q]_\theta$) can now be calculated from the
static test results presented in sections 5.4.2 and 5.4.1 and using equation 2.19a:

\[
\begin{align*}
[\bar{Q}]_{\theta}^0 &= \begin{bmatrix}
34.78 & 2.80 & 0 \\
2.80 & 12.56 & 0 \\
0 & 0 & 3.75
\end{bmatrix} \text{ (GPa)} \\
[\bar{Q}]_{45^\circ} &= \begin{bmatrix}
23.79 & 9.48 & 5.55 \\
9.48 & 16.98 & 5.55 \\
5.55 & 5.55 & 10.44
\end{bmatrix} \text{ (GPa)} \\
[\bar{Q}]_{-45^\circ} &= \begin{bmatrix}
23.79 & 9.48 & -5.55 \\
9.48 & 16.98 & -5.55 \\
-5.55 & -5.55 & 10.44
\end{bmatrix} \text{ (GPa)} \\
[\bar{Q}]_{90^\circ} &= \begin{bmatrix}
0 & 2.80 & 0 \\
2.80 & 34.78 & 0 \\
0 & 0 & 3.75
\end{bmatrix} \text{ (GPa)}
\end{align*}
\] (5.7a)

Since \( A_{ij} \) is a function of the lamina’s thickness, in section A.1 of appendix A one can find the matrices for the corresponding specimens used during the upcoming TSA tests.

### 5.4.5 Ultimate tensile static tests

Two relevant properties to be determined from the stress-strain \((\sigma \text{ vs } \varepsilon)\) curve are the yield and ultimate tensile stresses and strains. The yield point will limit the maximum allowable loading values impartable while performing TSA whereas properties at break will be useful to define a ratio between applied and ultimate strains \((\varepsilon_{\text{max}}/\varepsilon_{\text{ult}})\).

5 QI(0) GFRP samples bonded with 25 x 50 x 1 mm aluminium tabs were continuously stretched at a constant grip head’s displacement rate of 2 mm/min while the material strains were monitored using a 50 mm gauge length dynamic extensometer. One of the stress-strain curves obtained after data processing is shown in figure 5.16. Elastic-plastic transition is very slight as generally attributed to FRP materials. A line with a slope coincident with the initial elastic stress-strain evolution was drawn for every specimen. When the experimental curve crossed the drawn line, the yield point was defined. Averaged final results with standard deviation are summarized in table 5.7.

The ratios \(\varepsilon_{\text{yield}}/\varepsilon_{\text{ult}} = 0.15\) and \(P_{\text{yield}}/P_{\text{ult}} = 0.21\) establish the upper boundaries for further TSA tests. The latter is in accordance with the value mentioned in [49].

### 5.5 Fatigue Mechanical Properties

A cyclically loaded FRP is characterized by a linear strain energy release stage between 10\% and 90\% of its useful life. The slope \(du_{\varepsilon}/dN\) obtained for different applied strain ratios, \(\varepsilon_{\text{max}}/\varepsilon_{\text{ult}}\) follows a
Three tension-tension strain-controlled cyclic tests were ran on QI(0) specimens at each considered strain ratio (0.4, 0.5, 0.6, 0.7 and 0.8) until failure. Heat generated at frequencies higher than 5 Hz influences the fatigue life of GFRP’s [62, 63]. Therefore, the maximum threshold was kept constant for all tests. As damage progresses while keeping the strain range constant, the maximum and minimum peak load values decrease as the specimen’s axial stiffness degrades. A minimum strain level of 8% of the ultimate tensile strain assured positive load values until failure. Final test results together with the specimen’s dimensions are presented in table 5.8.

Final plots of $\frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{ult}}}$ = $f(N_t)$ and $d\varepsilon_{\text{ult}}/dN = f(\varepsilon_{\text{max}}/\varepsilon_{\text{ult}})$ and the respective power law fitting curves are shown in figure 5.18. The coefficients $a$ and $b$ determined from figure 5.18(b) are 4654.6 and 7.3964, respectively.
Table 5.8: Fatigue test results at different strain ratios.

Table 5.8 also presents the ratio $r$ between the strain energies at the end of stage II and mean strain level ($u_f/u_0$) defined in equation 4.4. The equation resultant from the power law fitting to $\varepsilon_{\text{max}}/\varepsilon_{\text{ult}} = f(r)$ plotted in figure 5.17 will provide the value to be further considered in the RUL calculations.

Figure 5.17: Experimental results and fitting plot for $\varepsilon_{\text{max}}/\varepsilon_{\text{ult}} = f(r)$.
Figure 5.18: (a) Experimental results and fitting plots for (a) $\frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{ult}}} = f(N_f)$ (b) $\frac{du_s}{dN} = f(\frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{ult}}})$. 
Chapter 6

TSA Results

6.1 Overall experimental apparatus

Commonly to all TSA experiments, the necessary equipment is shown in figure 6.1. A description of each component is given in the next paragraphs.

![TSA experimental apparatus](image)

Figure 6.1: TSA experimental apparatus.

6.1.1 FLIR Camera

The FLIR X6580sc model contains an Indium antimonide (InSb) detector built-in. It is capable of streaming out 13-bit thermal images and detects differences typically smaller than 20 mK with an accuracy of ±1 K. 25 mm and 50 mm focal length lenses were provided with the camera. Longer lengths capture narrower angles of view with higher magnification. However, motion effects associated to thermoelastic measurements are also amplified thus translating into erroneous results. Available motion
compensation softwares track the displacement of a vector initially defined between two fixed features throughout the sequence of recorded images and replicate to the remaining pixels [43]. In the absence of motion compensation softwares in the current work, setting the camera with a lower magnification lens and correct stand-off distance becomes more practical and reliable. Therefore, all TSA measurements were performed with the 25 mm lens.

6.1.2 DisplayImg 6

This software allows user-control of the data acquisition process, data processing and visualization of the final results. An initial signal analysis determines the reference signal's frequency prior to the digital processing of the thermal data using a FFT.

Before starting any TSA measurement, the following parameters should be defined according to the test specifications:

- **Frame Rate**: the number of images collected per second and adjusted according to the window of imaging. Smaller windows allow increased frame rates. A constant value of 350 Hz was defined for all forthcoming tests.

- **Integration time**: the inverse of the frame rate i.e. defines the time during which a single image is taken. It is selected according to the range of temperatures expected to be measured during the experiment. Calibration curves provided by the manufacturer for the 25 mm and 50 mm lenses are shown in figure 6.2;

- **Emissivity**: ratio between the radiated power emitted by a real and black bodies at the same temperature and same wavelength [59].

- **Reflected Temperature**: since the surface under test is not a perfect blackbody, background reflections are captured by the camera. The surrounding temperature is determined by reading the averaged apparent temperature from a piece of alluminium - considered to be a perfect reflector - positioned in the camera's field-of-view;

- **Measurement duration**: time during which the cross-correlation process integrates all collected thermal images. Longer integration times lead to a reduction in the noise-to-signal ratio as previously shown in figure 1.7.

6.1.3 Servo-hydraulic machine

The Instron 8803 model is capable of imparting dynamic loads with a $\pm 250$ kN load full-scale. Either under strain or load control, the frequency and amplitude of the sinusoidal wave form are transmitted to the camera through a 0.150-10 V analog output signal.
Figure 6.2: Range of measurable temperatures according to the integration time permitted by the specific FLIR X6580sc model used using a (a) 25 mm lens (b) 50 mm lens.

6.2 Emissivity Tests

TSA is a surface temperature measurement technique. Therefore, any changes in the camera’s output signal are derived from the external epoxy layer. An accurate correlation between the absorbed energy carried by the photons and the surface temperature using the Stefan-Boltzmann law is dependent upon the correct input of the emissivity value in the DisplayImg 6 software. Different values of emissivity can be found in the literature [55, 71]. To find an unique and consensual value for further tests, an experiment to qualitatively characterize the emissivity was carried out.

The method considers the material under observation and an additional one of known emissivity. Both should be evenly heated up to values at least 20 K higher than room temperature. While keeping the software’s emissivity parameter equal to the tape’s, the averaged apparent temperature measured
by the camera on the tape is determined. Afterwards, the same averaged temperature reading should be attained in the specimen's surface by modifying the emissivity parameter.

A QI E-glass fiber-reinforced laminate dimensionally equal to the specimens used for ultimate tensile properties determination was vertically positioned inside a temperature controlled oven. The general experimental apparatus is shown in figure 6.3(a). One M3 black tape and one small rectangular aluminium strip were attached to the specimen's surface.

The FLIR camera was positioned at a tilted angle to avoid visualizing its own reflection i.e. the Narcissus effect. The 50 mm lens was the most suitable lens configuration for this setup since no motion was expected to occur. In addition, while obtaining visual access to the oven's interior, a sudden variation in temperature occurs near the oven's outlet when opened, leading to a sudden temperature gradient nearby. Therefore, to assure a time-constant homogenised surface temperature, the specimen was positioned as further from this region as possible while maintaining a minimal field-of-view for data collection as shown in figure 6.3(b).

Initially, the oven's temperature was increased up to 55 \(^{\circ}\)C and allowed to stabilize. For increased temperatures, higher undesirable temperature gradients during data collection would be generated, hence generating higher data variability. From figure 6.2, setting the camera's integration time to 1000 \(\mu\)s would cover the expected upper temperature limit with a safe margin. The reflected temperature determination using the attached aluminium strip was repeated for every new measurement.

![Figure 6.3: (a) Emissivity experimental apparatus overview (b) FLIR camera's point of view.](image)

However, the method couldn’t be applied at its full capability since the emissivity of the black tape was unknown. Without loss of accuracy, a range of values between 0.95 and 0.97 was considered.

In figure 6.4, three rectangular areas are presented which cover the different regions of interest for temperature averaging purposes. Area 1 covers the aluminium strip, area 2 the epoxy resin and area 3 the black tape. Qualitatively, the tape's edges are almost undistinguishable from the specimen's surface which reveal the proximity between the epoxy's and black tape's emissivity. This conclusion can be
quantitatively assessed from table 6.1. For the range of emissivity values considered, the tape’s and epoxy’s averaged temperature differs by $0.10 \, ^\circ\text{C}$. Therefore, one can assume that the epoxy resin’s emissivity value of 0.95 provided in [71] is a valid assumption for further application in TSA tests.

The epoxy’s high emissivity value possibilitates the absence of extra black paint coating. Two advantages come from this action: amplitude reduction effects are eliminated [50] and less time is consumed for surface preparation thus accelerating the TSA’s data collection process.

![Figure 6.4: Thermal output and regions of temperature data extraction during an emissivity determination test.](image)

<table>
<thead>
<tr>
<th>$e$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>50.40</td>
<td>48.82</td>
<td>48.91</td>
</tr>
<tr>
<td>0.96</td>
<td>50.30</td>
<td>48.81</td>
<td>48.91</td>
</tr>
<tr>
<td>0.97</td>
<td>50.40</td>
<td>48.30</td>
<td>48.40</td>
</tr>
</tbody>
</table>

Table 6.1: Surface temperature data averaged over different areas for different black tape emissivity values.

### 6.3 Frequency Tests

The two main purposes of this section are:

- Determine the cyclic loading frequency at which quasi-adiabatic conditions can be attained;
- Assess the source of the thermoelastic signal.
6.3.1 Test description

Load control tests were ran at 5, 8, 10, 12, 15 and 18 Hz on two QI stacking sequences with equal extensional stiffness matrix terms, $A_{ij}$, but exchanged surface and mid-plane ply orientations ([0/45/-45/90]$_s$ and [90/45/-45/0]$_s$). A short notations according to the outer surface ply fiber orientation will be also used hereinafter for the latter configuration (QI(90)). The main goal is to understand the influence of the outer fiber orientation on the final thermoelastic data in both cases. From the ultimate tensile tests results (see section 5.4.5 in chapter 5), a maximum peak cyclic load of 3.7 kN assured linear elastic stress-strain conditions. To maintain a tensile-tensile condition throughout the test, a minimum load value of 0.3 kN was chosen. The strain range was measured along and transversely to the loading direction using two uniaxial strain gages bonded to the specimen’s surface.

The camera with a 25 mm lens configuration was fixed at a stand-off distance of 520 mm enough to visualize the specimen with a good quality resolution. Thermal images were collected with an integration time of 1900 µs. Again, from figure 6.2, this definition fixed the maximum measurable temperature to approximately 50°C. The duration of the image integration process was fixed by the number of loading cycles i.e. 1200 cycles. Manual data triggering started 60 s after loading the specimen up to the mean level. The strain gage data sets were averaged from that moment onwards.

6.3.2 Results discussion

Temperature amplitude ($\Delta T/2$), phase angle ($\varphi$) and mean absolute surface temperature ($T_0$) were collected from a box of pixels generated away from the specimen's edges to avoid capturing the effects of the cyclic motion. A first comparison between the experimental (with average and standard deviation) and calculated temperature ratios’ $\Delta T/T_0$ evolution with increasing frequency $f$ is shown in figure 6.5. A ratio decay is continuously observed in both configurations and more abrupt between the 10 and 12 Hz steps (6.9 % and 7.5 % for the QI(0) and QI(90), respectively). Onwards, a more stabilized response evidences the proximity to quasi-adiabatic conditions. As the frequency is increased, the heat transferred between laminas is cyclic in shape but with shorter wavelengths. In combination with the low thermal conductivity of glass fibers, any attempt to reach a temperature equilibrium between neighouring laminas with different fiber orientations is delayed.

The RRL model better approaches the experimental data for the QI(0) which evidences the tendency to define GFRPs as thermoelastically isotropic. However, QI(90) results underestimate all theoretical models. From the reviewed literature on thermoelastic signal source identification, it is also surprising that a configuration with surface ply fibers at 90° straddles the strain witness behaviour. Two arguments can justify the witnessed results, described from highly to less probable:

- The outer resin’s thickness was unable to act as a strain witness of the laminate;
- The loading frequency failed to diminish the heat diffusion rate between laminas.

To aid understanding the influence of the resin in the surface thermal field, the pixel-by-pixel experimental temperature change ($\Delta T$) dependency with frequency is shown in figures 6.6 (a) and (b). In both
cases, the surface ply fiber orientation is clearly visible, possibly indicating an influence of the surface ply in the final results wherever there’s a reduction in the epoxy thickness. The isotropic nature of the epoxy should dictate a more homogeneous overall thermoelastic response capable of diminishing local variations resultant from the orthotropic structure of the laminate. Indeed, an increasing homogenization of the temperature difference field leading to a reduction of the high amplitude regions occurs for both stacking sequences, although an higher degree of scatter is still evident in the QI(90) case at 18 Hz.

The uniaxial strain gage measured ranges, $\Delta \varepsilon_x$ and $\Delta \varepsilon_y$, the experimental averaged temperature ratio with standard deviation, $\Delta T/T_0$, as well as the phase average with standard deviation are presented in tables 6.2 and 6.3. Phase standard deviation results prove the higher variability of the QI(90) data sets in relation to the QI(0) case. This indicates the lower influence of the isotropic epoxy layer in the final results of the QI(90). Moreover, the thermal drag-down effect in the QI(0) can be inferred from the sudden average phase jump between the 12 and 15 Hz tests. Nevertheless, quasi-adiabatic conditions are gradually attained for increasing frequencies as the temperature ratio’s scatter is reduced.

$$
\begin{array}{cccc}
  f \text{ (Hz)} & \Delta \varepsilon \times 10^{-3} & \Delta T/T_0 \times 10^{-4} & \text{Phase (°)} \\
  \text{Average} & \text{SD} & \text{Average} & \text{SD} \\
  5 & -3.5 & 1.0 & 5.39 & 0.61 & -1.51 & 3.16 \\
  8 & -3.4 & 1.0 & 5.27 & 0.60 & -5.26 & 1.64 \\
  10 & -3.4 & 1.0 & 5.23 & 0.58 & -2.38 & 1.45 \\
  12 & -3.2 & 0.91 & 4.87 & 0.54 & -4.04 & 1.89 \\
  15 & -3.3 & 0.93 & 4.87 & 0.57 & -15.27 & 1.46 \\
  18 & -3.2 & 0.90 & 4.65 & 0.56 & -12.28 & 1.75 \\
\end{array}
$$

Table 6.2: Strain range, temperature ratio and phase experimental data obtained for QI(0).

Point-wise approximations to the different models might exist within the full-field nature of TSA. The percentage relative error ($\xi_r$) between the experimental and calculated $\Delta T/T_0$ was derived for the QI(0) and QI(90) for the 12 Hz loading frequency:
Figure 6.6: $\Delta T$ dependency with $f$ for (a) QI(0) (b) QI(90).
\[ \xi_r = \frac{|(\Delta T/T_0)_t - (\Delta T/T_0)_e|}{(\Delta T/T_0)_t} \times 100 \]  

(6.1)

where the subscripts \( t \) and \( e \) stand for theoretical and experimental, respectively.

From figure 6.7(a), there's an overall uniformity in the RRL theoretical temperature ratio prediction followed by the homogeneous model whereas the CTE model is the most discrepant of the four under consideration. Moreover, the relative error of the bulk model in the visible surface fibers is lower in comparison with the RRL model. In contrast, the thermoelastic effect in the matrix areas is better captured by the strain witness assumption. Although globally the CTE model performs the worst, locally can be the closest to predict the thermal response in areas where the other models failed. An example is reported in the rectangle drawn in the upper right side of the four images. These full-field representations evidence the incorrect under or over predicting strains derived if one relies on a single model to characterize the overall thermoelastic effect in laminated composite materials. From the optical microscopic images presented in section 5.2.4, the non-uniformity in the microstructure distribution containing voids mixed with resin pockets locally modifies the fiber volume fraction leading to variations in the heat diffusion phenomena. Finally, the thermoelastic effect along the transverse fibers in the Qi(90) surface ply is well captured by the RRL, bulk and homogeneous models.

### 6.4 Remaining Useful Life

#### 6.4.1 Test description

The final stage of this work consists of the calculation of the remaining useful life, \( N_t \), of the Qi(0) specimen based on the elastic strain energy determination. The procedure can be generalized into three main steps that are sequentially repeated every 20,000 fatigue cycles until final failure or gross damage is observed, as schematically presented in figure 4.3.

An initial step consisted of uniaxially loading the specimen up to a maximum strain level of 3100 \( \mu \varepsilon \) for \( \nu_{xy} \) and \( E_x \) calculations. Strains were measured using a bi-axial extensometer with 50 mm axial and 25 mm transverse gauge lengths.

In the following TSA step, the elastic strain energy density at the mean fatigue strain level (\( \bar{\varepsilon}_0 \)) is calculated whereas damage is allowed to initiate and propagate during the subsequent fatigue step. Both are performed under strain-controlled test conditions. A dynamic extensometer with a 50 mm
Figure 6.7: ξ_r(%) at f = 12 Hz for (a) QI(0) (b) QI(90).

gauge length is attached oppositely to the shiny surface of the specimen, the latter expected to yield an homogeneous outer epoxy layer suitable for thermal data recording.
The maximum strain limit for TSA is imposed by the yield point equal to 15% of the ultimate tensile strain value according to the results from section 5.4.5. When selecting the minimum strain level for both cases, one needs to account for the load cell’s sensitivity and material degradation with growing damage while maintaining tensile-tensile conditions. However, by pushing the minimum value above zero, any strain energy calculations - hence RUL predictions - become more conservative. Moreover, the higher the limit is set, the higher mean strain level will imply a lower maximum strain applicable during fatigue, hence leading to more time consuming and laborous experiments. Under high-cycle fatigue, crack initiation and mechanical properties degradation undergo at a slow rate with low energy release at the breakage point. Therefore, a final choice of 2% of the UTS (equivalent to approximately 0.300 kN) satisfied all the mentioned limitations. Having defined the minimum and mean fatigue strain level, the maximum is fixed at 28% of the UTS (7250 με). From figure 5.17, the $r$ value is established to 1.71.

From the TSA’s frequency dependency tests, a 12 Hz value guaranteed an almost stabilized temperature change at the surface. Under fatigue, the 5 Hz frequency used during the fatigue coefficients determination tests was selected. The camera’s integration time was set to 1900 μs whereas the frame rate to 480 Hz. A stand-off distance of approximately 570 mm remained constant for all tests. Table 6.4 summarises the test conditions and specimen dimensions.

<table>
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<tr>
<th>Dimensions</th>
<th>TSA step</th>
<th>Fatigue step</th>
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</thead>
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<tr>
<td>h (mm)</td>
<td>f (Hz)</td>
<td>N $\varepsilon_{min}$ (με) $\varepsilon_{max}$ (με)</td>
</tr>
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<td>2.08</td>
<td>12</td>
<td>1200 520 3880</td>
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</table>

Table 6.4: Specimen’s initial dimensions and strain controlled-test parameters for the TSA and Fatigue steps.

### 6.4.2 Results discussion

A total of 17 steps involving 327,630 cycles were performed. From a first comparison with the fitting curve prediction from figure 5.18, the total number of cycles using this dataset (187,099 cycles) reveals uncapabability to accurately predict the total number of cycles. Step 14 was the last before the material entered stage III. Although complete separation of the specimen at the crack region didn’t occur, the useful life of the specimen was established after overcoming the strain limits defined in table 6.4.

The direct measurements resultant from the static and TSA tests as well as the strain changes averaged over the specimen’s surface calculated for each thermoelastic strain model are presented in table 6.5. In comparison with the imparted strain change from table 6.4, the averaged results derived from the RRL are in better agreement which is in accordance with the conclusions derived from the frequency tests section. In addition, the increase in the standard deviation results regarding the TSA’s experimental data denote the appearance of multiple locations where crack friction effects cause non-adiabatic thermal responses.

Plots of averaged $N_t$ as a function of the performed step for the four different thermoelastic models are shown in figure 6.9. Clearly, any RUL predictions derived from the CTE model become meaning-
<table>
<thead>
<tr>
<th>Step</th>
<th>$E_x$ (GPa)</th>
<th>$\nu_{xy}$</th>
<th>$\Delta T/T_0$ ($\times 10^{-4}$)</th>
<th>Phase ($^{\circ}$)</th>
<th>$\Delta \varepsilon_x$ ($\times 10^{-3}$)</th>
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<td>0.22</td>
<td>5.25</td>
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<td>3.18</td>
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</tbody>
</table>

Table 6.5: Static and TSA tests measurements together with $\Delta \varepsilon_x$ for the different thermoelastic strain models under consideration.

less caused by the unrealistic underpredicted longitudinal strain change. The remaining three follow a decreasing trend similar to the three stages defined in chapter 4. Stage I characterized by an abrupt release of energy is inferred by the steep decrease in $N$ up to step 2, followed by a nearly constant averaged RUL up to $2.60 \times 10^5$ cycles performed, after which a new abrupt change in mechanical properties leads to a linear decay in the specimen’s life before final failure within stage III. The best initial prediction is provided by the bulk model (4.5% error). Despite the closest proximity of the dynamic extensometer readings to the RRL strain change calculations, one can define a range between steps 5 and 9 where the relative errors for this model are minimised (a maximum 16.5% error was attained in step 5). On the other hand, up to step 7, both the bulk and homogeneous models are in good agreement with the experimental results.

However, none of the models is capable of capturing the experimental line trend from step 10 onwards where the predicted averaged results straddle the experimental ones. Firstly, the formation of crack propagation fronts yields higher averaged $\Delta T/T_0$, thus increased $\Delta \varepsilon_x$ values. Since only single $E_x$ and $\nu_{xy}$ values are known, the RUL is overpredicted. This scenario is first witnessed in the transition from step 10 to 11 where the averaged $\Delta T/T_0$ increases by 5.8%. If solely relying in the dynamic extensometer readings, capturing this phenomenon would be impossible. To aid comprehension, figure 6.8 shows the temperature ratio after correction together with the phase data at step 14. The propagating crack fronts in the smaller detailed image can be distinguished by their higher $\Delta T/T_0$, meaning higher deformation levels thus increased life predictions. On the other hand, the crack surfaces already formed no longer sustain the applied displacement, thus yielding local reductions in $\Delta T$ results - and, consequently, in $\Delta T/T_0$ - accompanied by an overall increase in the phase angle.

In addition, the high-cycle fatigue test imparted damage at a very slow rate and the crack responsible for tests completion was confined to a small area. Surrounding fibers, although damaged, could still
withstand the applied deformation levels and contributed to maintain an high averaged result.

Finally, it is important to underline the incapability to update the thermal and mechanical properties built-in the thermoelastic strain equations after severe degradation has been imparted to the specimen. This situation would be diminished if an unidirectional stacking sequence was tested. For instance, static tests in UD(0) would yield $E_x = E_1$ and $\nu_{xy} = \nu_{12}$.

![Figure 6.8: Phase and $\Delta T/T_0$ at step 14 accompanied by detailed image of two crack fronts location.](image)

Assessing the RUL using a single averaged strain value proved to follow the decreasing life trend but failed as an accurate predictor starting from in-between the material's mid-life period. In the seek of seizing the full-field capability of TSA data, pixel-by-pixel maps of RUL results are to be obtained. In the absence and incapability of locally determining $E_x$ and $\nu_{xy}$ using the current methodology, calculations are performed considering an even degradation for any $(x, y)$ positions of the specimen, which turns out to be a misconception. Yet, and for the sake of discussion, whether crack surfaces already formed or under propagation are considered, a local $E_x$ would always turn out to be smaller than the global value, thus yielding a lower RUL prediction in both situations. Therefore, the final results here presented allow a less conservative approach to the problem.

Figures 6.10 and 6.11 separate the locations comprehended inside and outside the last 10% of the specimen’s life (in yellow and blue colours, respectively). A total of 5 intermediate steps together with the specimen’s image after tests completion is presented. The area percentage lying within stage III increases as a consequence of the increased damage level of the specimen. The location and severity of the crack responsible for failure is now well predicted and confirmation only occurs at the last TSA step. In fact, the crack was continuously evolving behind one of the elastics that positioned the dynamic extensometer throughout the course of the strain-controlled experiments and only became visible after propagating downwards. However, whenever an unobstructed line of view is absent, TSA becomes a
vulnerable technique for damage assessment.

For step \( n \), a non-dimensional parameter, \( D \), is defined by the ratio between the local number of cycles until failure, \( N_n(x, y) \), and the average RUL determined at the undamaged state, \( N_0 \):

\[
D = \frac{N_n(x, y)}{N_0}
\]  

(6.2)

The purpose of this parameter is to provide a feasible comparison in a real maintenance scenario where the actual number of cycles until failure is unknown. Since all models depend on the same global mechanical properties and same \( \Delta T/T_0 \), the final outcome is model-independent. This is advantageous for models that straddle the initial experimental measurement. Figure 6.12 presents the calculations for three intermediate steps and the final one using the RRL model. The number of damaged locations is once again seen to increase from the presence of lower \( D \) values represented with a blue colour. From the initial step considered, the surface ply fibers are clearly responsible for withstanding most of the applied load as they gradually lose strength, thus yielding a \( D \) value smaller than the surrounding epoxy resin’s.
Figure 6.10: Locations with remaining number of cycles inside (in yellow) and outside (in blue) the stage III threshold using (a) RRL model (b) Bulk model.
Figure 6.11: Locations with remaining number of cycles inside (in yellow) and outside (in blue) the stage III threshold using the homogeneous model.

Figure 6.12: $D$ at steps 4, 8, 12, 14 and 16 considering the epoxy resin material properties.
Chapter 7

Conclusions

7.1 Achievements

In similarity with previous works focused in the thermoelastic signal source identification, the combined effect of the epoxy layer accumulated near the surface ply with the low thermal conductivity of the e-glass reinforcement led to a diminishing impact of the heat generated at the substrate level in the thermal field measured at the material's surface. This strain witness behaviour offered the possibility to work around the thermoelastic strain models that include mechanical and thermal properties locally dependent upon void content and fiber/matrix volume fractions.

However, when applying the aforementioned findings to averaged longitudinal strain calculations, hence remaining useful life predictions, the RRL model followed the overall decreasing experimental trend yet better agreement was attained during the mid-life steps of the material. Overpredictions during the last stages were attributed to mechanical and thermal properties updating incapability, increased strain levels at the crack front and very localised crack generation due to the low-amplitude imparted deformation levels.

Greater benefit came after applying the TSA's full-field view over the whole surface to locally perceive the magnitude of each defect. Without a laborious experimental preparation and using a reduced number of equipments - although heavy and expensive -, RUL calculations using the strain energy density methodology applied to TSA was successfully reproduced in a geometrical homogeneous surface without a priori knowledge of the final failure location. Within the framework of TSA's literature, these results are neither probabilistic, nor qualitative, therefore adding an innovative quantitative approach to this technique. Once again, SHM technologies prove to be a more reliable, hence more valuable approach for damage assessment and RUL determination.

However, a straightforward applicability to a real maintenance scenario is still not foreseen. The need to redetermine the fatigue coefficients and energy ratio for every new geometrical configuration and laminate micro and macro structures under test still encloses this technique to laboratorial environment.
7.2 Future Work

Ideas to improve the accuracy of RUL using averaged strain values while covering a wider number of real case studies are enumerated:

**Local determination of Young’s modulus and Poisson’s ratio**

Other full-field SHM techniques - namely DIC - can cooperate with TSA to update the mechanical properties in a local manner. Challenges come from the need to exactly match both output images and assess the effect of the speckle pattern in the thermoelastic signal's amplitude reduction.

**Replicate to CFRPs and/or UD(0) stacking sequences**

Reproducibility to other laminate configurations and reinforcement materials will strengthen this methodology's range of validity. Carbon fiber reinforced polymers and unidirectional stacking sequences are highlighted for their importance in aerospace components where high stiffness properties are mandatory.

**Perform load-controlled tests**

The dynamic extensometer is a physical obstacle for a clear full-field surface monitoring. Moreover, during fatigue tests, if slippage of the measuring device occurs, the load range is shifted up or downwards, hence becoming a concern. Load-controlled tests are, not only safer, but more intuitive.

**Replicate to a real structure component under variable amplitude and variable frequency cyclic loading conditions**

The constant frequency strain-controlled tests applied to the simple geometry are an idealized scenario. If extrapolation to a real structure is intended, RUL calculations must be readjusted to account for variable frequencies and amplitudes throughout the test course.
Bibliography


Appendix A

Remaining calculations

A.1 Extensional stiffness matrices

The $A_{ij}$ tensor quantities are now presented for the frequency tests and RUL calculation. The thicknesses considered are 2.15, 2.05 and 2.08 mm, by order:

$$[A] = \begin{bmatrix} 0.044 & 0.013 & 0 \\ 0.013 & 0.044 & 0 \\ 0 & 0 & 0.015 \end{bmatrix} \text{ (} GPa.m \text{)} \quad (A.1a)$$

$$[A] = \begin{bmatrix} 0.042 & 0.013 & 0 \\ 0.013 & 0.042 & 0 \\ 0 & 0 & 0.015 \end{bmatrix} \text{ (} GPa.m \text{)} \quad (A.1b)$$

$$[A] = \begin{bmatrix} 0.043 & 0.013 & 0 \\ 0.013 & 0.042 & 0 \\ 0 & 0 & 0.015 \end{bmatrix} \text{ (} GPa.m \text{)} \quad (A.1c)$$

A.2 Elastic strain energy density

Averaged $u_0$ calculations for the RUL prediction are now presented in table A.1:
### Table A.1: Averaged $u_0$ calculations performed for RUL predictions.

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Appendix B

Airtech Catalogue

Figure B.1: Airtech catalogue