Forecasting the demand of spare components for a better stock management: a case study at Portugália Airlines
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November 2017

Abstract: In the last years, the forecasting methods have been established as an important tool within companies allowing better inventory management, allowing reducing the stock levels as well as preventing the occurrence of stock out, thus avoiding greater losses. This work begins by identifying the main difficulties of the company in the case study, which are related to the forecasting methods available for inventory management and which are best suited to the different components and needs of the company. The experimental part of this work begins with the categorization of the demand pattern of the components, and the identification of the forecasting models that best fit each categorization. The accuracy of the different forecasting methods is evaluated, resorting to conventional error measures so that it is possible to compare the methods studied. It will be considered that the most accurate method is the one that have the lowest forecast error. For this particular case studied the forecasting method that presented the lowest forecast error was the ARMA model. Finally, an inventory management model is proposed based on the forecasting method that presents the best accuracy for the sample studied. This model revealed to be an added value for the company, since it can predict a possible occurrence of stock out, which often leads to significant financial losses.

Keywords: Forecasting Methods, Inventory Management, Demand Pattern, Stock Optimization

1) Introduction
In the last years, there has been a great growth in the aviation industry, a consequence of the increasing number of passengers using this means of transport. This growth has led many airlines to increase their fleet in order to meet growing demand.

The investment of airlines in the acquisition of more aircraft is not an isolated investment. Investment in maintenance facilities, skilled labor as well as components essential for maintenance tasks should also be considered.

The inventory management of spare parts is important and necessary for the maintenance of the aircraft since this requires a great investment by the company, which could imply the inoperability of the aircraft, also known as Aircraft on Ground (AOG). However, the management of these components proves to be complex due to the high number of different components present in stock and the low demand of some.

The demand for spare parts presents an intermittent pattern due to its variation, making it difficult to predict the demand for these as well as the management of these components. Therefore, it is considered important to make a detailed study of the forecasting methods, based on the historical data of demand, in order to improve the aircraft availability.

The subject under study in this thesis focuses on the improvement of the inventory management system of Portugália Airlines. Although this is a cross-cutting theme for several industrial sectors, this project focuses on the aviation industry. Therefore, this thesis aims to answer related questions such as the demand forecasting methods to be applied, inventory management strategies and parameters to be used. The work done also addresses the development of a program that models the consumption of the inventory and defines the management parameters allowing better control and management of maintenance consumable components.

The industrial objective of this project is to improve the current practice for maintenance consumables inventory management of Portugália Airlines.

1.1) Methodology
This thesis results of an internship at Portugália Airlines that lasted 6 months. This internship allowed the development of this study, where the first phase of this project passed through the contact with the IT platforms, and at the same time, the author had the opportunity to understand how an airline is organized. The internship took place at the Engineering Department integrated with the engineering team responsible for the management of inventories of maintenance spare components. The industrial challenge for this work was to provide solutions for a better management of the components in order to reduce stock-out situations and eventual aircraft on ground conditions.

The internship was initiated with an overview on the current procedures involved and an
understanding of the information technological platform the company uses to support the procedures. Based on the knowledge acquired two major questions were elected as deserving detailed work: what are the most appropriated forecasting methods to predict future demand of maintenance spares, and how to use demand predictions to control acquisitions and replenishment orders. The company validated and considered that these objectives were very relevant for the company in a formal meeting involving the author, supervisors and company staff.

The analysis of the forecast methods was performed based on the data set of components demand involving the period from sept/2012 to dec/2016. A stratified and significant sample was used to allow the evaluation of a set of relevant forecasting methods. All the algorithms involved were programmed using R, which is a language and environment for statistical computing and graphics. This allowed the full control of the methods, the fit of their parameters and the identification of the forecasting method according to relevant error measures. At the same time, the literature review started with the study of the demand patterns and forecasting methods. A special attention was dedicated to the methods implemented in the information platform of the company.

Once the forecasting method that best suits to the company's reality have been chosen and the results presented to the company, the following phase evolve to the study of how to consider the demand as an important input to control the acquisition of spares. Again the author made a detailed analysis on the information platform and interview company staff involved in deciding the acquisitions. In parallel, the most important stock control models present in the literature were studied. This allowed the proposal of a specific stock control model that integrates a continuous revision strategy of the stock level with the demand forecast for the following period.

The realization of this internship allowed a better comprehension of the importance of inventory management for the company, having a considerable impact on maintenance tasks and interfering with flight schedule.

2) Literature Review and State of the Art

2.1) Component Demand Pattern Categorization

Initially, the idea of categorizing demand patterns was suggested by Williams [1] in 1984. Later this model was modified by Syntetos [2], which categorized the pattern into four categories [3].

The categorization of the demand pattern can be made based on the the calculation of the parameters [2] [3].

- **Average Inter-Demand Interval (ADI)**, which is the average number of time periods between two successive demands.
- **Square Coefficient of Variation ($CV^2$)** defined by the ratio between the standard deviation of non-null period demands ($\sigma_a$) and the average period demands ($\bar{a}$).

$$CV^2 = \left(\frac{\sigma_a}{\bar{a}}\right)^2$$

The four categories of the demand pattern can be represented in a diagram divided into 4 quadrants, which are defined by two cut-off values obtained by the theoretical results of [2].

$$ADI = 1.32$$

$$CV^2 = 0.49$$

The figure 1 presents the diagram that relates the demand patterns and the forecasting methods that should be applied to each component according to its categorization, proposed by Boylan and Syntetos [4] based on two forecast models Single Exponential Smoothing (SES) and Croston [3].

![Figure 1: Categories of demand pattern](image)

2.2) Description of Forecasting Methods

The time series are the forecasting methods that will be studied, since their application is simple and most often used in supply chain planning [6].

The forecasting methods that will be described are: Single Exponential Smoothing (SES); Holt's Exponential Smoothing (HES); Holt-Winters Exponential Smoothing (HWES); Auto Regression - Moving Average (ARMA) Croston Method (CM).

2.2.1) Single Exponential Smoothing

The SES forecasting method is given by equation (4) [7] [8].

$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t$$

2.2.2) Holt's Exponential Smoothing

The Holt’s Exponential Smoothing also known as Double Exponential Smoothing, is an extension of the Single Exponential Smoothing with Trend. The Trend is a smoothed estimate of
the average growth at the end of each period, being defined by the parameter $\beta$.

The level is defined as the smoothed estimation of the value of the data at the end of each period [9], equation (5). The trend is the smoothed estimation of the average growth of the end of the period $t$ [24], equation (6) [7] [8].

The forecast of this model, equation (7) [7] [8], is the sum of the values of the level (5) and trend (6) equations.

$$L_t = \alpha \cdot D_t + (1 - \alpha) \cdot (L_{t-1} + T_{t-1}) \quad (5)$$

$$T_t = \beta \cdot (L_t - L_{t-1}) + (1 - \beta) \cdot T_{t-1} \quad (6)$$

$$F_{t+1} = L_t + T_t \quad (7)$$

### 2.2.3) Holt-Winter’s Exponential Smoothing

HWES is used when the data show trend and seasonality. Thus, this forecasting method will be characterized by three parameters that are $\alpha$, $\beta$ and $\gamma$ that correspond respectively to level, trend and seasonality.

#### 2.2.3.1) Multiplicative Holt-Winters (MHW)

The MHW is used when the data present multiplicative seasonality. Typically, this method is applied to time series with proportional seasonal effects in size to the local average of deseasonalized.

Multiplicative Holt-Winters model is based in three smoothing equations, one for level (8), other for trend (9) and another for the seasonality (10), and a final equation to estimate the future demand (11) [7] [8].

$$L_t = \alpha \cdot \left( \frac{D_t}{S_{t-s}} \right) + (1 - \alpha) \cdot (L_{t-1} + T_{t-1}) \quad (8)$$

$$T_t = \beta \cdot (L_t - L_{t-1}) + (1 - \beta) \cdot T_{t-1} \quad (9)$$

$$S_t = \gamma \cdot \left( \frac{D_t}{L_t} \right) + (1 - \gamma) \cdot S_{t-s} \quad (10)$$

$$F_{t+1} = (L_t + T_t) \cdot S_{t-s} \quad (11)$$

#### 2.2.3.2) Additive Holt-Winters (AHW)

The AHW model is applied when the data exhibits additive seasonality. This model is used when time series with seasonal effects of constant size are verified.

Additive Holt-Winters has the same equations as the multiplicative model, however in the equation of the level and seasonality the first division is replaced by a subtraction (8) and (9), and in equation (11) the multiplication is replaced by an addition [7].

### 2.2.4) ARMA model

The ARMA model will be defined by $p$ and $q$, which defined the order of the model, ARMA ($p$, $q$). This can be defined by the following equation (12) [10] [11] [12].

$$X_t = c + \alpha_t + \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \alpha_{t-i} \quad (12)$$

### 2.2.5) Croston Method

This method is based in two procedures. The first one depending on the size of the demand, equation (13), and the second on the interval between demands, equation (14). The forecast for future demand results from the quotient between the averages of the size of the demands and the interval between the demands is presented in equation (15) [5] [13] [14].

$$Z_t = \alpha \cdot D_{t-1} + (1 - \alpha) \cdot Z_{t-1} \quad (13)$$

$$P_t = \alpha \cdot q + (1 - \alpha) \cdot P_{t-1} \quad (14)$$

$$F_t = \frac{Z_t}{P_t} \quad (15)$$

Where, $Z_t$ is the average of the demand size and $P_t$ is the average interval between demands.

### 2.3) Forecasting Methods Evaluation

The comparison of the errors associated with each forecasting method allows the identification of the best method.

According to Hyndman and Koehler [15], the best error measures to compare different forecasting methods for the same data are the accuracy methods that depends on the scale of the data.

Thus, the error measures that will be considered to compare forecasting methods is RMSE and MAE.

**Root Mean Square Error (RMSE):**

$$RMSE = \left[ \frac{\sum_{i=1}^{n} (F_t - Z_t)^2}{n} \right]^{1/2} \quad (16)$$

**Mean Absolute Error (MAE):**

$$MAE = \frac{\sum_{i=1}^{n} |F_t - Z_t|}{n} \quad (17)$$

### 2.4) Inventory Management Models

Inventory management models are associated when to trigger a component replenishment order and in what quantity. The basic models can consider (a) a periodic review of the stock level and a variable order quantity to fill up the stock to its maximum level, (b) a continuous stock monitoring to trigger the replenishment with a fixed quantity as the stock reaches a certain level.

#### 2.4.1) Fixed-Order Quantity Model (FOQM)

The demand of expendable maintenance components is not constant. So it is necessary to define the Service Level (SL) and the Safety Stock (SS). The SS is defined as the quantity of components in stock that allows to reduce the risk of incurring stock out due to fluctuations in demand that may occur during the Lead Time (LT) [6] [16], being defined by equation (18) [6].

$$SS = \sigma \cdot z \quad (18)$$

Where, $\sigma$ is the standard deviation of demand and
where \( z \) is the number of standard deviations for a specified service level.

For the case in which a forecasting method is considered to predict the demand of components, and knowing the period for which the forecast is being made as well as the lead time it is possible to define the safety stock by the equation (19) [16].

\[
SS = \sigma \cdot z \cdot LTF
\]  

Where, \( \sigma \) is the standard deviation of demand, \( z \) is the number of standard deviations for a specified service level and \( LTF \) is the Lead Time Factor.

The Lead Time Factor (LTF) is given by the equation (20) [16].

\[
LTF = \frac{\text{Lead Time}}{\sqrt{\text{Forecast Period}}}
\]  

The Lead Time (LT) is defined by the time from the moment the order is placed to the moment it is received [25] [26].

The forecast period is the time for which is being considered the future forecast [16].

The order is made when the Reorder Point (ROP) is reached and this is given by the equation (20) [16].

\[
ROP = LTD + SS
\]  

Lead Time Demand (LTD) is defined by the demand during the delivery period and given by the equation (21) [16].

\[
LTD = DD \cdot LT
\]  

Where, \( DD \) is the Daily Demand and \( LT \) is the Lead Time.

In figure 2, it is presented the q-model when the demand is not constant.

![q-model for no constant demand](image)

The Service Level (SL) is a parameter that measures the frequency of requests that are satisfied immediately from the stock. Due to the stock control strategy one can perceive that only during the replenishment lead time a stock out can occur [6].

The Cycle Service Level (CSL) is defined as the level of service that would reduce the costs of incurring stock-out and the cost of holding stock.

The service level is seen as a probability, then the CSL is given by the equation (22) [6].

\[
CSL \leq P \leq \frac{C_u}{C_u + C_o}
\]  

3) Case Study

For maintenance logistics, the company uses an application known as AMOS. However, this application does not define the best forecasting method to be applied to the multiple components involved. AMOS has to manage thousands of components with different demand patterns. The main problem that the company faces is related to inventory management system, since the forecasting method that provides the most accurate results is unknown, and important parameters such as Safety Stock, Reorder Point, and Economic Order Quantity seem not to be defined appropriately. Due to this fact, the need to study the consumption pattern of the different components emerged in order to find the best forecasting method and to define inventory management strategies and inventory control parameters to accommodate the required service levels with low stock associated cost.

3.1) AMOS description

The AMOS software is an application that provides support to the company, helping in the fleet management, maintenance tasks and management of the inventory [27]. In relation to the inventory management system provided by this application, one of the implemented functionalities is the prediction of the consumption of a component for the next month based on past demand data.

AMOS has several forecasting methods that can be applied to the components in stock such as the Single Exponential Smoothing, Holt’s Exponential Smoothing, Linear Regression, Holt-Winter’s Exponential Smoothing, Simple Moving Average, ARMA model, Non-Linear Regression and Time Series Decomposition. Each forecasting method can be applied to the components in stock, but one cannot assure which at those forecasting methods is the most accurate. Considering Boylan and Syntetos [9] different forecasting methods are chosen based on which is the demand pattern. Since AMOS does not recommend a forecasting method that should be applied to the components, it was decided to study each forecasting method applying this to a sample in order to find the forecasting method that best suits to the Portugália Airlines reality.

3.2) Description of the data

Since the new fleet is operating at less than 12 months, its consumption data of expendable components does not contain yet a number of registers significant for this study. Therefore, the expendable components of the previous fleet will be considered. This allows having 52 months of data consumption in AMOS database, since the software was implemented in September 2012 and the fleet data goes to December 2016. In this period, the AMOS contains 17766 expendable components.

The study started with the extraction of all consumption historical data for all the expendable...
components in the past 52 months. In order to analyze and study all the data collected, an application using programming language R was developed to help with the visualization of components consumption over the time. The consumption historical data of each component during the period of 52 months were carefully analyzed and it was possible to verify that a significant number of components only has a demand during this period, those components were excluded from this study since they are not relevant to it, being then discarded. After this data cleaning, it was observed that each component has its own consumption pattern, which characterizes it.

The categorization of the demand pattern is based on the categorization scheme proposed by Syntetos, Boylan, and Croston (SBC). The study of the categorization of the demand pattern of components requires that each component has more than one demand during the period under analysis, which means that only the components with more than one demand during the period under analysis were considered. Thus, from the universe of 17766 expandable components, only 5350 can be studied.

After the data cleaning, it was possible to verify that remaining components represent a significant number of components that can be studied. In order to simplify this study, it was decided to define a sample that allows making conclusions related to the population of components. Since the demand patterns are very different it was important to consider a sample for this study that contains components of each demand pattern, this means that it will be considered a stratified sample. According to Boylan and Syntetos [4], it was proceeded to the categorization of the demand pattern of all components.

The figure 3, shows the distribution of the components according to the demand pattern.

![Figure 3: Components distribution by consumption pattern](image)

The results obtained revealed that the majority of components have an intermittent demand pattern, and 3% of the components have a Smooth and Erratic demand pattern. Since there are many components that can be studied, it was decided that a stratified sample would be studied. According to Lohr [17], the size of the sample will be calculated using the formula used to determine the size of a sample for a binomial case, because in this study it is important to know whether or not the forecasting method is the most accurate. Considering a margin of error of 5% and a confidence level of 95% the size of the sample is equal to 360 components. In table 2 is presented the stratification of the sample considered in this study.

In the table 1 is presented the stratification of the sample considered in this study.

<table>
<thead>
<tr>
<th>Category</th>
<th>Quantity</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>7</td>
<td>1.8</td>
</tr>
<tr>
<td>Lumpy</td>
<td>83</td>
<td>23</td>
</tr>
<tr>
<td>Erratic</td>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>Intermittent</td>
<td>265</td>
<td>73.9</td>
</tr>
<tr>
<td>Total</td>
<td>360</td>
<td>100</td>
</tr>
</tbody>
</table>

**3.3) Forecasting Methods Results**

After the categorization of the component consumption pattern, it is necessary to identify the best forecasting method for each category and find the parameter that allows obtaining the best accurate model. The study of the forecasting methods can be carried out without a previous study of the demand pattern of each component. However, the application of a forecasting method to a demand pattern for which the method is not the best one that can lead to inaccurate results. According to Boylan and Syntetos [4], the forecasting method to be applied depends on the characteristics of the demand pattern.

3.3.1) SES

The SES is a forecasting method that allows to obtain better predictions for components categorized as Smooth, it was considered 7 components because these are the components that should be considered for a stratified sample of 360 components.

The table 2 presents for each component the $\alpha$ parameter for the SES method that minimizes the forecast error measures.

<table>
<thead>
<tr>
<th>PARTNO</th>
<th>RMSE</th>
<th>MAE</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.81</td>
<td>19.23</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>36.17</td>
<td>24.32</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>62.82</td>
<td>48.22</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>9.48</td>
<td>7.36</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>2.33</td>
<td>1.84</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>4.53</td>
<td>3.8</td>
<td>0.4</td>
</tr>
<tr>
<td>7</td>
<td>2.85</td>
<td>2.23</td>
<td>0.6</td>
</tr>
</tbody>
</table>

In figure 3, the RMSE and MAE are presented for each smooth demand component in the
The AMOS software uses a smoothing parameter, α, equal to 0.1. Based on the results achieved it is suggested to change the α parameter to 0.3 as far as for the smooth demand components the forecast errors are lower. In fact, setting the SES smoothing parameter to 0.3 result in a reduction of 1.35% and 4.1% on the demand forecast MAE and RMSE, respectively, when compared to the company currently set parameter.

3.3.2) HES
This forecasting method is often applied to components that have a Smooth demand pattern, in this way this method will be applied to the 7 components classified as Smooth of the stratified sample.

In the table 3 are presented for each component the α and β parameter of the HES method that minimizes the forecast error.

Table 3: α and β that minimizes forecast error measures for each component

<table>
<thead>
<tr>
<th>PARTNO</th>
<th>RMSE</th>
<th>MAE</th>
<th>α</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.48</td>
<td>20.78</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>39.11</td>
<td>27.49</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>74.9</td>
<td>59.22</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>9.84</td>
<td>7.49</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>2.37</td>
<td>1.9</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>5.5</td>
<td>4.28</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>7</td>
<td>3.16</td>
<td>2.38</td>
<td>0.7</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Since, one of the limitations of the AMOS software is that it is not possible to define the parameters α and β for each component. The software only allows defining the parameters α and β that will be used by the forecasting method. To solve this problem, were studied all the combinations of α and β, varying these parameters between 0 and 1.

3.3.3) Holt-Winters Exponential Smoothing
The HWES is a forecasting method defined by three parameters, smoothing α, trend β, and seasonality γ.

The next table 4, presents for each component α, β and γ the parameters that minimizes the forecast error measures for the HWES method.

Table 4: α, β and γ parameters that minimizes forecast error measures

<table>
<thead>
<tr>
<th>PARTNO</th>
<th>RMSE</th>
<th>MAE</th>
<th>α</th>
<th>β</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.48</td>
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<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>3.16</td>
<td>2.38</td>
<td>0.7</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Analyzing the results presented in the previous table, it is possible to conclude that seasonality parameter can be neglected. Since the seasonal parameter, γ, can be neglected this means that the HWES method degenerates in the HES method. This can be verified in the equations that characterize the HWES method exposed in the Literature Review. Thus, for this forecasting method the various combinations of α and β that minimizes the forecast error was not studied, because that the HWES method degenerates in the HES method, this method will also have the lowest forecast errors when α = 0.4 and β = 0.4.
3.3.4) Croston Method

The Croston Method is the forecasting method most used for intermittent demand forecasting.

In the figure 6, it is possible to verify the results obtained for the 9 possibilities studied of the smoothing parameter \( \alpha \). It will be considered that the best smoothing parameter is the one which minimizes the error, in this particular case RMSE and MAE.

![Figure 5: Average of RMSE and MAE error](image)

Observing the results obtained, it is possible to conclude that the smoothing parameter equal to 0.1 allows to minimize the forecast error measures for each component with Lumpy and Erratic demand patterns. The Croston method has the lowest forecast error for the Intermittent demand pattern when it is considered the parameter \( \alpha = 0.2 \).

3.3.5) ARMA model

The study of the ARMA model started with finding the parameters \( p \) and \( q \) for which the consider model minimizes the forecast error measures. After obtaining the parameters \( p \) and \( q \) that allow to minimize the forecast error measures for each component, it was verified the maximum of these parameters, in order to establish the hypotheses that will be considered in this study.

The table 5 presents the hypotheses that will be considered in this study.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>3</td>
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</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

The hypotheses mentioned above were all tested for the ARMA model and applied to the 360 components of the representative sample. However the hypotheses \( p=1 \) \( q=2 \), \( p=1 \) \( q=3 \), \( p=2 \) \( q=1 \), and \( p=q=2 \) induced a non stationary solution and according to Brockwell and Davis [12] the solution must be stationary. Since the presentation of the individual results for each component proves to be quite extensive, it was decided to group the components by categories of the demand patterns and present the results for each category. The results are presented in the figure 7.

3.4) Results Comparison and Conclusions

For the components classified as Smooth, the forecasting methods studied were Single Exponential Smoothing, Holt Exponential Smoothing, Holt-Winter’s Exponential Smoothing and ARMA.

The following figure 8, presents the results obtained for the forecasting methods referred previously considering components with Smooth demand pattern.

![Figure 7: Error comparison between the forecasting methods for Smooth demand pattern](image)

Observing the results obtained, it is possible to conclude that the forecasting method that optimizes the prediction, minimizing the error, is the ARMA model.

The components classified as Lumpy, Erratic and Intermittent were studied using Croston’s method and ARMA model. For the forecasting methods referred previously, the results obtained are presented in figure 9.
Analyzing the previous figure, it is possible to conclude that the forecasting method that improves the prediction is the ARMA model, since the RMSE and MAE error measures are lower for this method.

4) Inventory Management Model

4.1) Fixed-Order Quantity Model based on EOQ purchased

This inventory management model is based on continuous inventory monitoring, and when the reorder point is reached (ROP), an order is triggered.

This inventory management model was applied to a component and the evolution of the stock for this model is represented in figure 10.

![Figure 9: Stock Evolution](image)

The previous figure, confirms what it was said before, the average stock of this inventory management model is high due to the high quantity ordered. The time between the next order will be great due to the high level of stock caused by the quantity ordered, raising the holding costs, but the order costs are minimized.

4.2) Inventory Management Model based in Forecasting Methods

The evolution of the stock will be studied over a period of two years, in which case a forecasting method will be used. This forecasting method is the ARMA model, since as studied previously, this is the one that presents more accurate results. The inclusion of this forecast model allows forecasting next month’s demand and avoid possible occurrences of stock out.

The stock evolution of this inventory management model and the real stock evolution are represented, in the figure 11.

![Figure 11: Stock Evolution of the different methods](image)

Observing the figure 12, it is possible to verify that the two models have different stock evolutions. The Inventory Management Model based on Forecasting Methods predicted an eventual occurrence of stock out for the period 20, and an order was made in period 19. When the Fixed-Order Quantity Model was applied to the same component for the same period under analysis an order was placed when the final stock reached the reorder point which occurred in period 18. The orders were made in different periods for the different models of inventory management. Considering the Fixed-Order Quantity Model it is possible to verify that the order was made when the quantity in stock was higher when compared with the quantity in stock at the time of order when considered the other Inventory Management Model. Thus, as the order was made when the quantity in stock was higher this model have a higher average stock when compared to the Inventory Management Model based on Forecasting Methods. Since this last Inventory Management Model suggests that the order is made when a stock out occurrence is predicted or when the stock is below the safety stock this model allows to reduce the stock levels. So, it is possible to conclude that
application of a forecast model in inventory management models is an added value since it allows to predict the occurrence of stock out, making an order if this situation is expected reducing at the same time the stock levels.

4.4) Model Comparison for a sample

Considering the Fixed-Order Quantity Model described in the section, this will be applied to a larger sample. The same process will be done for the inventory management model based on forecasting techniques. The sample considered in this study is composed by 360 components. In the figure 13, the results obtained for the average stock and number of stock outs of each model is exposed.

![Figure 12: Average Stock and number of stock outs for each inventory management model](image)

The results obtained show that for the inventory management model based on forecasting techniques the average stock is lower when compared with the Fixed-Order Quantity Model. This fact is associated to the point where the order is placed, because the inventory management model based on forecasting techniques allows to predict the demand for the next period the order it is only placed if a stock out situation can occur or if the stock at the end of the next period is expected to be below the safety stock. As this order point is below the ROP used to control the order point for the usual inventory management model, the inventory management model based on a forecasting method presents a lower stock level. The model based on forecasting techniques do not present stock outs, this can be explained by the service level of the components are very high and the possibility of predicting the occurrence of stock out. For those reasons the inventory management model based on forecasting techniques is better since it allows to avoid those situations.

Evaluating the costs of each inventory management model, the cost of each model will be equal to the sum of the holding cost of all components in the sample. For each stock out occurrence, it will be added a penalty that can lead the aircraft to an AOG situation. The cost assumed for this penalty is considered to be equal to a profit provided for each hour of flight. The results of this cost are presented in table 6.

<table>
<thead>
<tr>
<th></th>
<th>Forecasting M.</th>
<th>FOQM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding Cost</td>
<td>11492</td>
<td>12250</td>
</tr>
<tr>
<td>Penalty</td>
<td>0</td>
<td>35000</td>
</tr>
<tr>
<td>Total Cost</td>
<td>11492</td>
<td>12250</td>
</tr>
</tbody>
</table>

The results on table 6 showed that the inventory management model based on forecasting methods has a lower cost when compared with the Fixed-Order Quantity Model. This inventory management model based on forecasting methods allows reducing the holding cost in 6.12% when compared with the other inventory management model. For the considered sample and considering the Fixed-Order Quantity Model it was verified the occurrence of 7 stock outs, which can lead the aircraft to an AOG situation incurring the company in higher losses. In this case, a penalty was added to this model, this value was assumed to be the profit of each flight hour and since the aircraft, it is in AOG and the company is having losses due to the fact that the plane stopped on the result of the lack of components.

The results were obtained for a representative sample and if this study was extended to a larger number of components more stock outs occurrences could be verified and the penalty associated with that occurrence increases the cost of the model.

5) Conclusions

The objectives defined for this work that aimed at optimizing the company stock through the development of an inventory management system were completed and integrally accomplished. The data collection was successfully carried out allowing the study and its analysis.

The first conclusion of this work is related to the categorization of the demand pattern of consumable components. As it would be expected and according to Ghoobar and Friend [3], most of the inventory components in an airline have a predominantly intermittent demand, representing this 73.9% of components, followed by lumpy, 23%, smooth, 1.8%, and erratic 1.3%. The fact that most of the components present an intermittent demand pattern is related to the great
variability of the consequent demand due to the unplanned maintenance plans.

After the categorization of the demand pattern of the components, the study of the forecasting methods started in order to determine the model that allows obtaining more precise results. In this way, with the purpose to find the best fitted forecasting model in order to improve the forecast of the demand it was selected the forecasting methods that best fit to each of the demand patterns. For an evaluation of the accuracy of the forecasting methods studied, it was necessary to use an error measure, RMSE, and MAE, allowing a comparison between forecasting methods. After the comparison of the different methods, it is concluded that the forecasting method that presents a smaller error for the predictions, revealing to be more accurate was the ARMA model.

The study of the forecasting methods that best adapts to the reality of the company, will allow this model to be implemented in an inventory management model, thus allowing a better control of the stock levels. A stock management model based on a forecasting method suggests that an order is only triggered if it is expected that there will be a possibility of stock out during the forecast period. Once an order is placed only when it is necessary, it will be possible to improve the stock levels as well as avoid the occurrence of stock-out. Thus, it is possible to conclude that inventory management models based on forecasting methods represent an added value for the company. In this particular case of Portugal Airlines, the inclusion of a forecasting method in an inventory management model allows reducing the occurrence of stock out as well as reducing the stock levels.

The size of the sample used in this thesis can be seen as a limitation. The sample considered in this study represents 7% of the 5350 components that can be studied since these components present more than a demand during 52 months allowing their categorization. For the considered sample the forecasting method that provides accurate results was the ARMA model. However, it will be interesting for a further study increase the sample size in order to verify if the ARMA model remains the most accurate method. Thus, the study with a larger sample may result in confirmation of what has been proven in this study or suggest that another forecasting method can be used.

Regarding the inventory management model that considers forecasting methods, it showed to be an added value for the company since the forecasting method allowed to predict the demand for the next month, an order is only made if the occurrence of stock out is expected, allowing to reduce the stock levels. Concerning to the holding costs of the inventory management model based on forecasting methods, this model has a lower holding cost when compared with the Fixed-Order Quantity Model.

6) References