New objectives for hydro energy management in power systems with large penetration of Renewables

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***Abstract*—This work focuses on the study of short-term operational planning of hydroelectric power systems, more specifically a hydroelectric system composed of three dams developed from a problem of the discipline of Control and Optimization of Energy Systems.**

This study intends to analyse the implications of the introduction of new objectives in the operational planning of the hydroelectric system in question, to do that the system is modelled and simulated to achieve these objectives. The objectives to be achieved are divided into two parts, a first part where the objective of the systems operation is only the maximization of the total hydro energy value of the system and a second part where the objective of the optimal planning is the maximization of the hydro value, firstly fulfilling the energy requirements imposed on the hydro-power system. Renewables penetration is implemented in this second part by adding renewable power to the system.

The optimization problem is then solved by using linear programming which uses simple algebraic methods and guarantees results assuming that the problem is well formulated, in order to do this it is necessary to model the system through linear approximations.

***Index Terms*—Resource optimization, operational planning, linear programming, hydroelectric power systems, penetration of renewables.**

I. Introduction

A. Scope of the investigation

This dissertation is inserted in the theme of operational planning in hydroelectric systems composed by multiple reservoirs, thematic that has attracted the attention of some researchers in several contexts [1] [2] [3] [4].

This work is based on the analysis of a system composed of several hydro-power plants to which reservoirs are associated, which assume a predetermined disposition.

The objective is to study the operational planning problem of this water system subject to new objectives and restrictions that arise from changes such as the addition of upstream pumping capacity and power constraints corresponding to the energy demand of the system.

As this work intends to analyse the implications of adding new objectives to the initial problem and system, it is chosen to model the system using linear modelling. This modelling uses linear approximations and simpler algebraic methods, logically resulting in less accurate solutions and faster computational times than alternative models. It is considered that for this study the accuracy of the linear modelling is enough to reach meaningful conclusions.

The topic of analysis and comparison of methods is left out of the focus of this study, since it is a topic already addressed by other authors [5] [6] [7].

B. Historical perspective

Traditionally, the organization of the electricity sector relied on two fundamental points: large power stations connected to a grid, away from consumption centres and the existence of a monopoly to manage and operate the system. This sector, like the telecommunications and gas sectors, has undergone deep restructuring in order to create competitive markets [8].

Currently, decentralized production is beginning to become a serious competitor of the traditional system, due to the large investment in renewable energies and the appearance of the possibility of owning small forms of domestic electricity generation (self-consumption). The introduction of new renewable sources outside the major producing centres adds difficulties to the operation of the electrical system, such as the instability in production due to the intermittent nature of the resources themselves.

As soon as renewable production is available this production constitutes the consumption base. This introduces a degree of intermittence in the generating system leading to the oscillation of energy market prices. Hydroelectric power with its almost immediate availability is then used to attenuate these fluctuations and thus regulating the producer system [7].

With the liberalization of the electricity sector, the different producers and operators are competing in a market environment and are subject to price variations, which are also due to the changing nature of demand [7]. In this competitive environment, the subject of operational planning and optimization of available resources is of great interest to competitors in the sense that any improvement affecting the system, even marginal, means an immediate advantage over competition.

C. Motivation and objectives

The problem of operational planning, in this study more focused on the management of reservoirs and the scheduling of hydroelectric power generation, is a subject that is seen by the companies in charge of the operation of the producing systems as being of great importance, as long as this same problem is solved optimally it translates into potentially significant economic benefits.

Currently, the penetration of renewable energies in electrical systems is an issue that is increasingly attracting the attention of both researchers and the general public. This focus is partly due to investments in renewable energy as a strategy for sustainable development and possible efforts to curb greenhouse gas emissions.

Hydroelectric power, besides contributing in energetic terms, also assumes a high importance in the operation of the electric system due to its reliability and availability, where the biggest advantage of these is the flexibility of the operation. The characteristics of these plants allow fine adjustments in production when subjected to variations in the load, the response time is shorter than in thermal power stations and in addition some are equipped with pumping capacity, making them reversible and allowing the balance of load curves, mainly in dry periods [9].

This work is based on the study of this problem of hydroelectric plants as producers and regulators of the electric system, in its operational planning and in its behaviour considering the increasing penetration of renewable energies.

The way in which the present study intends to solve the problem of optimal operational planning, is based on the analysis of the systems behaviour around two fundamental points:

- Situation where the objective of optimal planning is the maximization of the total hydro energy value;
- Situation where the objective of optimal planning is the maximization of the hydro value, firstly fulfilling the energy demand requirements imposed on the water system.

Some case studies are conceived, with varying degrees of penetration of renewable energy in the system to be studied. The focus of this work is not the analysis of optimization methods to solve the operational planning, the method used is considered sufficient enough for the desired effects, the focus is analysis of the solutions obtained by the computational simulation.

II. System modelling

A. System description

In order to study a system it has to be modelled first. The system to be studied is composed by 3 cascade reservoirs, to which generation turbines are associated, as shown in figure 1. These are run-of-river hydroelectric power plants and therefore have little storage capacity.

The reservoirs R1 and R2 are upstream of R3 and there is a constant inflow of water for each reservoir. Each reservoir is limited by its minimum volume, maximum volume and also by the minimum and maximum turbine flow.

In the figure 1 are also represented the main variables:

- v_i^k - Water volume present in reservoir i, at time k [pu];
- a_i^k - Inflow for reservoir i, at time k [pu];
- t_i^k - Turbine flow through the turbine i, at time k [pu];

Knowing what variables are used to model the system, it is also necessary to explicit the constants that represent the physical limitations of the system:

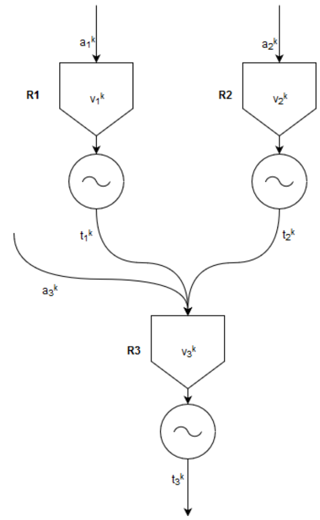

Fig. 1. Initial hydro system layout.

- $\overline{v_i}$ - Upper limit for the volume of water stored in reservoir i [pu];
- $\underline{v_i}$ - Lower limit for the volume of water stored in reservoir i [pu];
- $\overline{t_i}$ - Upper limit for the water flow associated with turbine i [pu];
- $\underline{t_i}$ - Lower limit for the water flow associated with turbine i [pu];

B. Initial problem

The linear modelling method, better known as Linear Programming (LP), is one of most widely used techniques in the context of optimization problems. It is an optimization process that minimizes a linear objective function with variables subject to linear constraints, any non-linearity in either the objective function or the constraints must be approximated by a linear function.

The focus of the problem in question is the control of the flows and stored volumes, the structure of the problem is based on a flow network where the water flows that enter and leave the reservoirs are represented in each hour. The nodes of the network are connected by branches representing the passage of water from one reservoir to another through the associated turbine and also the volume of water stored from one hour to the next.

The network corresponding to the system to be studied is presented in the figure 2, in this first approach a time horizon of 4 hours is considered for illustrative purposes.

The network shown illustrates the linear balance equations of the nodes, where the volume of water in each reservoir is given by:

$$v_i^k = a_i^k + v_i^{k-1} - t_i^k \tag{1}$$

The linear programming problem assumes the form:

$$\min c^T x \quad \text{subject to: } \begin{cases} A \cdot x = b \\ x_{min} \leq x \leq x_{max} \end{cases} \tag{2}$$

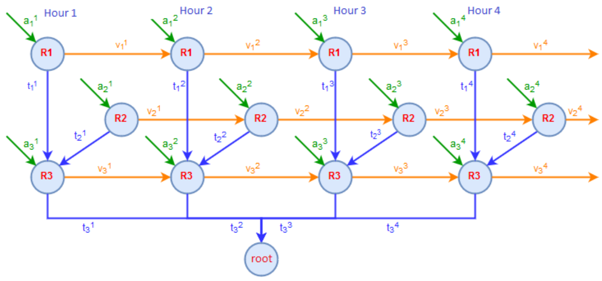

Fig. 2. Linear flow network.

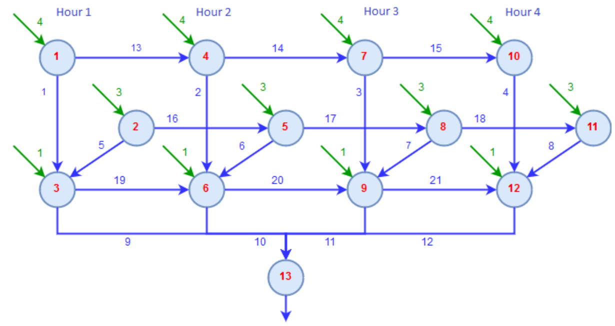

Fig. 3. Linear flow network for the initial problem.

In the case of the problem in question, the vector x corresponds to the decision vector that is constituted by the values of the turbine flows and the volumes of water stored.The matrix A is a nodal incidence matrix, where the branches entering a node assume the value of 1 and those leaving the same node assume the value of -1. The vector b represents the natural inflows of the reservoirs.

The function to minimize, $c^T x$ is the cost function, it represents the operating cost of the system and is directly related to the power generated by the turbines. The power generated by each turbine p_i^k is a linear function of the turbine flow, where α_i represents the electromechanical conversion efficiency and also the head contribution, and the power is then given by:

$$p_i^k = \alpha_i \cdot t_i^k \tag{3}$$

It is assumed that there are no costs associated with the storage branches since they do not generate power, only the branches associated with turbines have costs. It is also assumed that these are negative costs, since they represent the sale of energy to the grid and any costs related to the generation are ignored.

The total cost being:

$$c^T \cdot x = -p \cdot \lambda = -\alpha \cdot t \cdot \lambda \tag{4}$$

Where λ is the price of the energy. The vector c only assumes non-zero values for the positions associated with turbine branches, that is, considering that there are only costs for the branches t, one can replace the term x in the expression 4 by the term t, arriving at the expression 5.

$$c^T \cdot t = -\alpha \cdot \lambda \cdot t \Leftrightarrow c^T = -\alpha \cdot \lambda \tag{5}$$

Having presented the general formulation of the linear programming, the linear network of the initial problem is shown in figure 3.

In this problem the initial and final volumes for the reservoirs are zero, there are 13 nodes, where the 13$^{\text{th}}$ node doesn't serve any computational purpose, serves only as balancing node. There are only 21 branches, where 12 represent turbine branches and the other 9 storage branches.

The matrix A first assumes a dimension of 13×21, where later the last line is removed becoming a 12×21 matrix. Similarly the vector b has a dimension of 13, the last element being 0 and the others assuming the values of the inflows, the last position is also removed before solving the optimization problem.

The cost vector has the dimension of the decision vector x, 21, and the first 12 positions represent the costs of the turbine branches and the other 9 are null, representing the storage branches' cost.

It is relevant to mention the Lagrange multipliers γ obtained when solving the problem. The *linprog* function of MATLAB returns these multipliers in a structure, where $\gamma.eqlin$ are the multipliers for the water volume constraints and therefore represent the marginal values of the water, the dimension of this vector is the number of nodes excluding the root node. The $\gamma.lower$ and $\gamma.upper$ are the multipliers for the branches' bounds, they are the marginal values for the decrease of the lower bounds and increase of the upper bounds, respectively, and their dimension is the number of branches.

C. Problem with pumping

To implement the pumping capacity from reservoir R3 to R1 and R2 represented in figure 4, it is necessary to make changes in the system, this is done through the modification of the linear network that represents it. The most obvious change is the addition of pumping branches, however other changes are made. 3 branches representing the volumes of water stored at the last hour of the time horizon are added, so that the initial condition of the final reservoirs' volumes being zero is not necessarily true. The other change is the addition of spillage branches in all reservoirs that do not produce electricity and have a high associated cost that serve mainly for management of the full reservoirs.

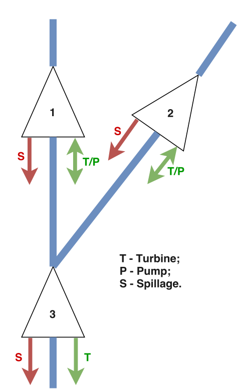

Fig. 4. Layout of the pumping capacity.

The changes to the network are illustrated in figures 5 through 7

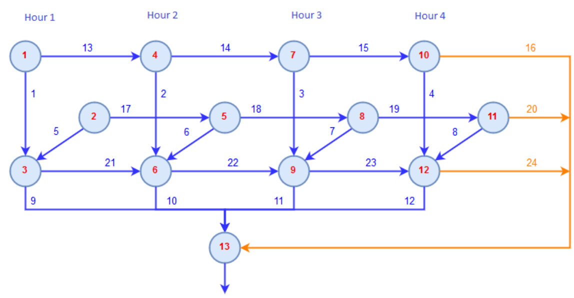

Fig. 5. Linear network with additional storage branches.

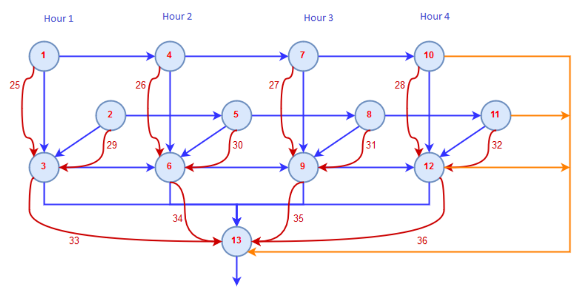

Fig. 6. Linear network with spillage branches.

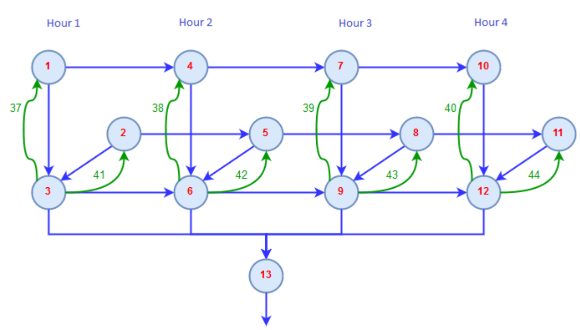

Fig. 7. Linear network with pumping branches.

These changes to the network must be implemented in the mathematical formulation of the optimization problem. The first change affects the matrix A, where 23 new columns are added, 3 of which correspond to the new storage branches, 12 to the spillage and the other 8 correspond to pumping. A has now a dimension of 12×44, after the removal of the last row corresponding to the root node.

The decision vector, the cost vector and the bounds now have a dimension of 44, with the addition of new types of branches new bounds and costs must be defined regarding these. Regarding the spillage branches, the upper bounds assume high values, the costs are very high and also positive, in order to "punish" the choice of these branches. As for the pumping branches, the upper bounds are the same as the bounds of turbine 1, the costs are positive and take into account the pumping efficiency, assuming the form:

$$c = \alpha_0 \cdot \alpha(1) \cdot \lambda \tag{6}$$

Where:

$$\begin{aligned} \alpha_{\text{bomb}} &= \alpha(1) \\ \alpha_0 &= \frac{1}{\eta_{\text{bomb}}} \end{aligned} \tag{7}$$

D. Problem with demand

It is now necessary to implement other constraints in order to make the generated power equal to the demanded power. These new constraints are implemented through new lines in matrix A and in the vector b, the number of lines to add is the same as the time horizon, the number of power constraints is the same as the number of hours in the simulation, for each hour there is a power demand.

In this model there is no pumping capacity, and for a time horizon of 4 hours the matrix A has 36 columns, no pumping branches are implemented.

The generation in each hour is the sum of the power generated by the 3 turbines in that hour.

$$p_{gen}^k = p_1^k + p_2^k + p_3^k \tag{8}$$

Considering that there are no losses in the generation:

$$p_{dem}^k = p_{gen}^k \tag{9}$$

Replacing 3 and 8 in 9, we obtain:

$$p_{dem}^k = \alpha_1 \cdot t_1^k + \alpha_2 \cdot t_2^k + \alpha_3 \cdot t_3^k \tag{10}$$

The introduction of other renewable energy sources in the system is done in a way such that the total power demanded from the system is the sum of the hydro-power and other renewables, fulfilling:

$$p_{dem} = p_{hyd} = p_{tot} - p_{ren} \tag{11}$$

Where p_{hyd} is the power generated by the hydro part of the system, p_{tot} is the total power demanded from the whole system and p_{ren} is the power generated by the alternative renewable source. For a 100% hydro operation p_{ren} is null.

III. Cases Presentation

In the complete study several more cases were structured and simulated, however in this article only the more interesting cases are presented and analysed.

A. Without power demand

In this section two cases are presented to illustrate what happens when the objective of the problem is only the maximization of the total hydro energy value. A time horizon of 24 hours is chosen for both cases and the prices of the energy are real values taken from the iberian electricity market operator (OMIE).

Two cases are compared, with the same hydro conditions expressed in table I, where in the first case there is no pumping capacity and in the second there is.

In the first case the layout of the system is the same as in figure 1 and in the second the layout is the one in figure 4.

Reservoir	1	2	3
Inflow [pu]	2	2	1
Initial Vol. [pu]	4	4	4

TABLE I
HYDRO CONDITIONS FOR THE CASES WITHOUT DEMAND.

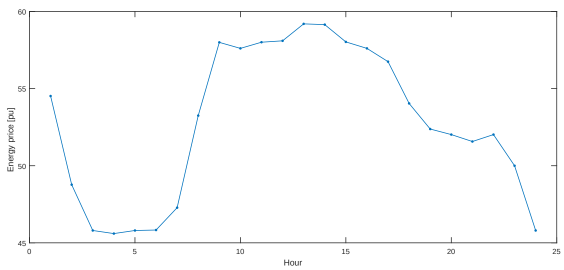

Fig. 8. Energy prices for 26-06-2017 [OMIE].

B. With power demand

In this section are presented the cases where the system is subject to power demands, the objective of the optimization in these cases is the maximization of the value of the hydro energy, after the demand being satisfied.

The two cases presented here both share the 24 hour time horizon, the energy prices and the total load demanded from the system. The first system to be simulated is a system where there is a degree of solar energy penetration, this solar component is simulated as a fixed power to be subtracted from the total demand. Both power curves are adapted to the systems scale from real data of the Portuguese electrical system and form the same day as the prices for the energy.

In the second case, the additional renewable energy source is wind power and its characteristic generation curve is quite different from the solar curve.

For both cases it is necessary to change some of the hydro characteristics because in these cases the hydro power needed to supply the demand will be different, given that the renewable sources chosen present generation curves that are quite different form each other.

The load demanded from the complete system is shown in figure 9.

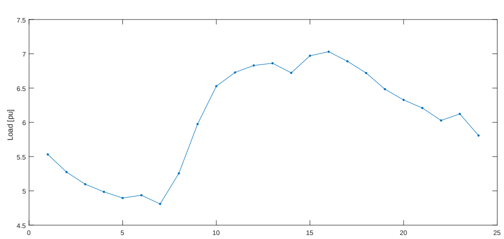

Fig. 9. Total load demanded adapted from 23-06-2017.

As for the changes in the hydro system, they are presented in tables II and III.

The different generation curves for the renewable sources are also adapted from the generation in 23-06-2017, and are illustrated in figure 10 and 11.

Reservoir	1	2	3
Inflow [pu]	4	4	1
Initial Vol. [pu]	10	10	10

TABLE II
HYDRO CONDITIONS FOR THE SOLAR COMPONENT CASE.

Reservoir	1	2	3
Inflow [pu]	3	3	1
Initial Vol. [pu]	10	10	10

TABLE III
HYDRO CONDITIONS FOR THE WIND COMPONENT CASE.

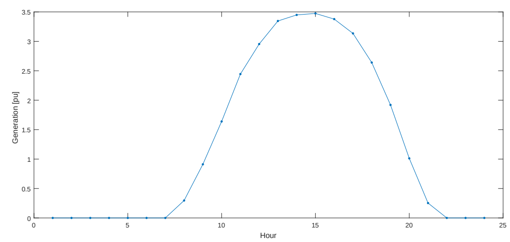

Fig. 10. Solar generation curve adapted from 23-06-2017.

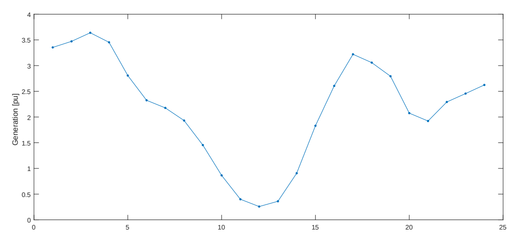

Fig. 11. Wind generation curve adapted from 23-06-2017.

IV. SIMULATION RESULTS

A. Cases without power demand

After simulating the before mentioned cases without power demand, the results are presented as turbine flows, stored volumes, pumped flows, water values (γ) and finally the total benefit of the operation.

1) System without pumping: The total benefit of the operation is for this case 3.938 pu.

In a first analysis of the turbine flows and the stored volumes it is possible to verify that during the first 8 hours, when the price of energy is lower and there is less volume of water in the reservoirs, the system does not generate energy, it simply stores the water in order to turbine it later when the prices are higher.

It is possible to identify at what time the maximum price of energy is verified by analysing when the peak of the generation occurs in figure 12, the turbine flows are all maximized at hours 13 and 14, which are in fact the hours of higher energy prices.

To finish the presentation of the simulation results the values of γ are shown in table IV, these values correspond to $\gamma.eqlin$ mentioned before. Noticeably the values of the water in the upper reservoirs are higher than the values of the lower reservoir, this happens because the upper reservoirs have better water flow to power generated efficiency than the lower one.

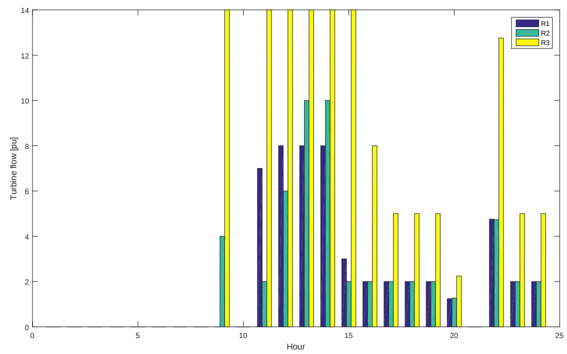

Fig. 12. Turbined flows for the system without pumping or demand.

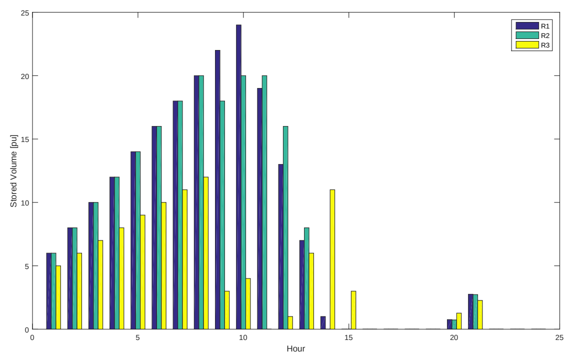

Fig. 13. Stored volumes for the system without pumping or demand.

Hora (k)	γ_1^k	γ_2^k	γ_3^k
1	40.54	30.58	11.53
2	40.54	30.58	11.53
3	40.54	30.58	11.53
4	40.54	30.58	11.53
5	40.54	30.58	11.53
6	40.54	30.58	11.53
7	40.54	30.58	11.53
8	40.54	30.58	11.53
9	40.54	30.87	11.53
10	40.54	30.87	11.53
11	40.54	30.87	11.53
12	40.54	30.89	11.52
13	40.54	30.89	11.52
14	40.54	30.89	11.52
15	40.54	30.87	11.52
16	40.33	30.73	11.52
17	39.73	30.27	11.35
18	37.83	28.82	10.81
19	36.67	27.94	10.48
20	36.41	27.74	10.4
21	36.41	27.74	10.4
22	36.41	27.74	10.4
23	35	26.67	10
24	32.06	24.43	9.16

TABLE IV
WATER VALUES FOR THE SYSTEM WITHOUT PUMPING OR DEMAND.

2) System with pumping: The total benefit of the operation is for this case 3.942 pu, there is an increase of 4 pu from the previous case.

Comparing the turbine flows for this case with the results of the similar case, the main difference is the existence of water flow through R1 at the 9$^{\text{th}}$ hour, where before it did not happen. In addition, the flows through R1 also increase at hours 11 and 15.

In the figure 15 is the answer to the increase of the flow through R1. Pumping from R3 to R1 at hours 4, 5 and 6 is performed, these are hours of lower energy prices. The most significant pumping is done at hour 4, when the price is the lowest.

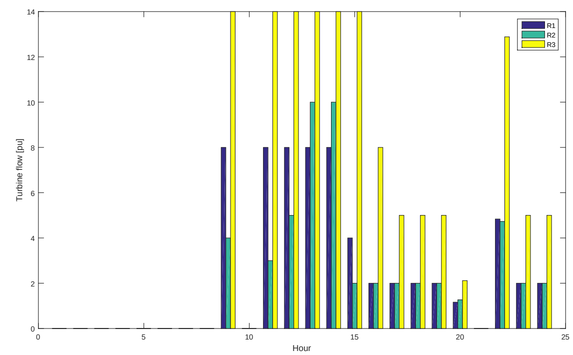

Fig. 14. Turbined flows for the system with pumping and without demand.

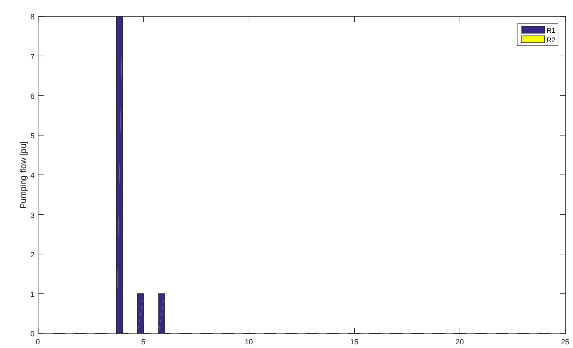

Fig. 15. Pumped flows for the system with pumping and without demand.

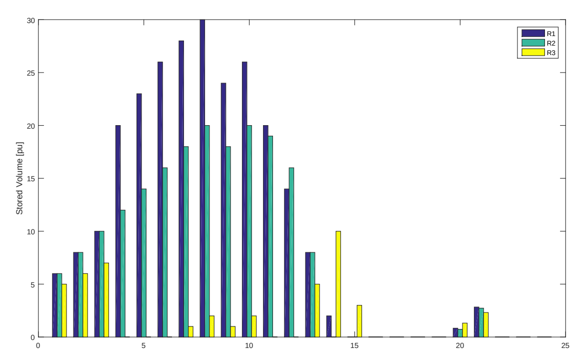

Fig. 16. Stored volumes for the system with pumping and without demand.

The behaviour of the stored volumes is consistent with this operation, after pumping water to R1 this water is stored for later use. In contrast with the previous case, the water that was stored in R3 previously is now pumped to R1 in these lower energy price hours.

Analysing the γ values in table V an increase is noticed in the water value for the R3 reservoir during the hours in which the price of the energy is lower, this means that the water in R3 is more valuable than before as long as the pumping operation is compensatory.

Hora (k)	γ_1^k	γ_2^k	γ_3^k
1	40.54	30.65	12.04
2	40.54	30.65	12.04
3	40.54	30.65	12.04
4	40.54	30.65	12.04
5	40.54	30.65	11.91
6	40.54	30.65	11.89
7	40.54	30.65	11.58
8	40.54	30.65	11.58
9	40.54	30.92	11.58
10	40.54	30.92	11.58
11	40.54	30.92	11.58
12	40.54	30.92	11.55
13	40.54	30.92	11.52
14	40.54	30.92	11.52
15	40.54	30.87	11.52
16	40.33	30.73	11.52
17	39.73	30.27	11.35
18	37.83	28.82	10.81
19	36.67	27.94	10.48
20	36.41	27.74	10.4
21	36.41	27.74	10.4
22	36.41	27.74	10.4
23	35	26.67	10
24	32.06	24.43	9.16

TABLE V
WATER VALUES FOR THE SYSTEM WITH PUMPING AND WITHOUT DEMAND.

B. Cases with power demand

In these cases with fixed power demand the economic benefit of the operation is also fixed because the benefit is directly tied to the power generated by the system. So, these cases will assess how the system behaves when subjected to some penetration of renewable power, given that the economic benefit is expected to be constant.

1) System with solar penetration: The total benefit of this operation is 7.729 pu, this benefit represents the joint operation of both the hydro system and the solar component. The benefit associated with the hydro component is 5.968 pu while the contribution of the solar part assumes a benefit of 1.762 pu, corresponding only to 23% of the total benefit.

Regarding the generated power, by analysing the figure 17 its possible to verify how both components coordinate to fulfil the demand requirements. During the hours of solar availability, when the solar component is not null the hydro subsystem assumes the function of complementing the solar generation, this is to be expected from a real system as well, since the solar component when available integrates the base of generation and hydro acts as a complement to that power.

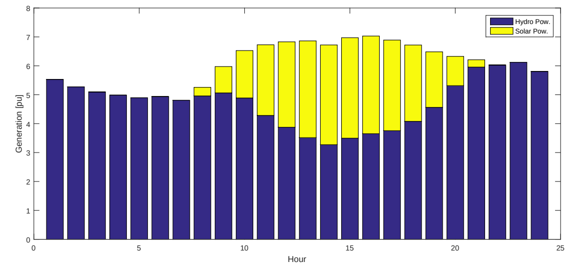

Fig. 17. Power generation with solar component.

In figure 18 the introduction of the solar component is visible, there is a reduction in the turbine flows during the hours where the solar generation peaks, reducing the hydro power generated.

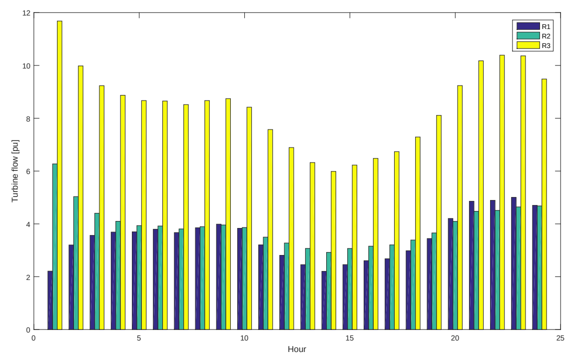

Fig. 18. Turbine flows for the system with solar component.

As for the stored volumes, in figure 19 the results are expected intuitively, when the hydro production is reduced the stored volumes increase. Both in this case and in the next, the water quantities present in the system are not enough to trigger the spillage and thus the benefit remains positive.

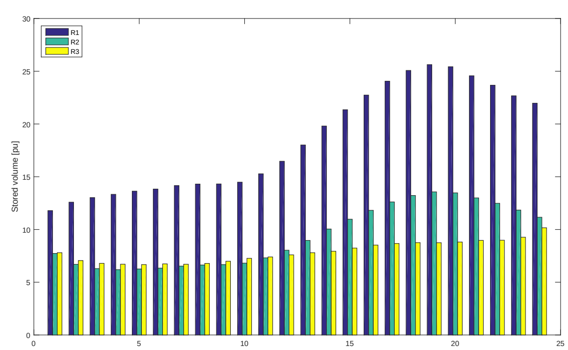

Fig. 19. Stored volumes for the system with solar component.

Regarding the γ values, the water values are null because as the power is imposed to the system as long as there is enough water available to fulfil the demand requisites the water does not have incremental value, the benefit is always the same. The second part of the γ vector corresponds to the power restriction equations, and its values represent the marginal power values, which are also the energy prices in our case, λ.

2) System with wind penetration: Maintaining the power demanded the same as in the previous case, the total benefit of this operation is also 7.729 pu. In this case wind power assumes a higher percentage of the generation, about 35% with a value of 2.684 pu, lowering the hydro component's contribution to 5.045 pu.

The wind power curve has a very distinct form from the solar curve, there is generation during the whole time horizon with bigger intensity during the late afternoon and the night. This is opposite to what happens with the solar.

Like the solar power, the wind power integrates the base of the system's production and the hydro component complements it to fulfil the demand. In the 3rd hour there is a peak of wind power where the wind subsystem even surpasses the hydro production.

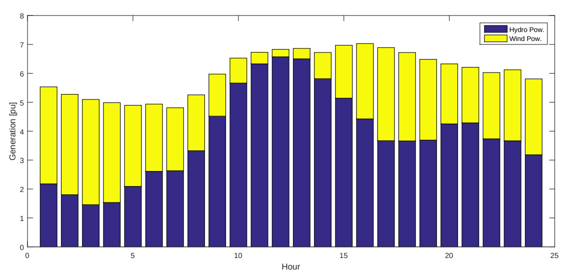

Fig. 20. Power generation with wind component.

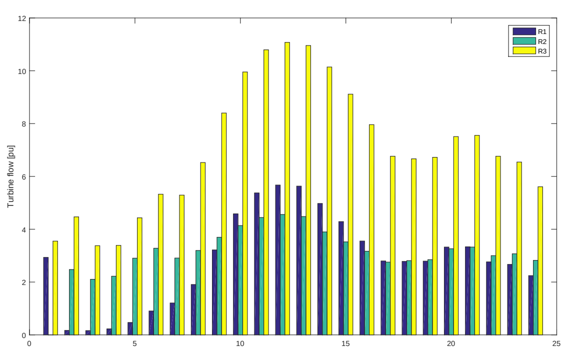

Fig. 21. Turbine flows for the system with wind component.

Both in figure 20 and 21 the influence of the wind power is evident.

Regarding the volumes, during the first hours there is an increase in the stored volumes due to the power demanded from the hydro system being on is minimum, this increase is then reversed when the demand is increased and the volumes then stabilise after the peak of the demand. In this case, like in the previous, the water quantity on the system was designed not to trigger any spillage and thus keeping the benefit positive.

The γ values remain unchanged for this case. In these two cases with the coordinated operation of the different power

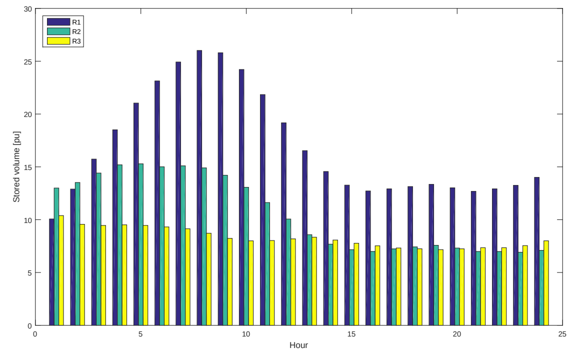

Fig. 22. Stored volumes for the system with wind component.

sources the principal differences in the operation are associated with the proper characteristics of each power source. The power curves for the solar and wind power present very different characteristics, either in availability and in magnitude.

V. Conclusion

In this study a robust linear programming method was structured to solve the problem of operational planning of a hydro power generation system subject to various conditions and limitations. A system of this type is a complex and large system that leads to the application of computational methods to solve it. These methods are of interest to the system operator since in competitive market, such as energy, any solution that results in cost reductions will result in a significant advantage.

The system studied is composed of three dams, where the first two are upstream of the third and the goal is to optimize performance the overall solution for the optimal operation of the system is sought.

The layout of the dams presents a network structure, which is intuitively modulated by a linear network and the consequent resolution of the problem is done using linear programming methods. These methods are valid in the sense that linear programming is more robust and provided that the problem is well formulated it presents rapid convergence. In reservoir dams where variations in volumes do not have a significant impact on fall height, this method may present performance close to that of non-linear methods.

The results obtained for the operation of the system where the objective is the maximization of the value of the water energy, that is, there are no charges to the system, show that in general there is a maximization of the energy production during the hours when the price of the latter is higher. In cases where there is less water availability, the volumes are stored when the energy has the lowest economic value and production takes place only during the hours of higher price. Similarly, with the addition of upstream pumping, in cases where there is less water availability pumping only occurs during hours of lower energy prices, this pumped volume is then used to generate power when the energy prices are higher. The system operation maximizes the economic benefit of the system by maximizing the total value of water.

In the cases where the purpose of the operation is to comply with the power constraints that is, there is a power demand, the results obtained confirm that it is only after the fulfilment of the demand requirements that the management of water volumes occurs. The economic solution of the planning problem is directly connected to the demand that is imposed to the system, provided that the water in the system is sufficient to generate the requested power the benefit is always the same. With the introduction of renewable solar and wind power production it is possible to verify the character of hydroelectric power to compensate for fluctuations in other renewable sources, the latter when available represent the basis of the production and water complements this production in order to meet the system requirements.

The study intends to illustrate the operation of a fictitious system, to be used in a pedagogic environment, it is possible to apply this method to a real system by changing a number of variables and constants, even the network itself. If a more precise solution is desired one can apply a different methodology, for example quadratic programming.

The implementation of the demand constraints as linear equalities limits the programs choices regarding the turbine flows, one way to mitigate this would be the treatment of these constraints as inequalities, allowing for the generated power to be greater or equal to the demanded power.

Another future course for this investigation would be the reformulation of the programming method in order to solve non linearities that appear when there are demand constraints and pumping capacity [10], these non linearities to implement solve the problem of simultaneous operation of a reversible turbine as generator and pump. One possible way to implement this would be recurring to other optimization methods, like mixed integer linear programming for example.

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