Retail Anatomy

An Analysis on Commercial Activity in Lisbon

1995-2010

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ABSTRACT: The location of commercial activity, and its role within the urban system, has been subject to study since at least the first half of the last century, with prior location models dating even earlier. Those first models are still, however, mandatory references in research, considering the formidable task of understanding the processes that underlie at the foundations of a city. The city is a complex system, where the presence of commercial activity presents itself as a desirable characteristic, whose presence is a both a measure of centrality and liveability: commercial establishments are places of consumption, but also socialization. Economic and social characteristics, accessibility and spatial syntax, along with many other factors, have been used when accounting for their presence, with 200 years of research providing for some relevant insight on the subject. The present dissertation aims to contribute to it, by analysing three commercial censuses carried out in three distinct decades (1995, 2002 and 2010) in the city of Lisbon. Retail location and agglomeration, relocation or extinction, among other phenomena are identified and interpreted, while trying to explain for spatial patterns and establish causal relations between these and other available information (census data from 1991, 2001 and 2011). A better understanding of the processes thus identified can help in developing measures that will contribute for preserving liveability while maintaining relevance, since they can inform urban planners in areas as diverse as urban renewal, mobility issues, or even social intervention.

Keywords: Spatial analysis; location models; urban economics.

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1 INTRODUCTION

On retail location, the cornerstone was set with by von Thünen (1826/1966), in relating rent and location. That relation was established by means of *distance* to the centre. In 1826, though, in an agricultural society, cities were "unimportant in the landscape and viewed as parasitic on the honest toil of agriculture" (Alonso, 1960: 2). The introduction of *central function* and *range of a good*, and a consequent hierarchy of places was vital for location theory (the "isolated state" finds finite application in urban studies, especially in a context in which cities started growing *in continuum*). Differences between Thünen's model, and Central Place Theory developed by Christäller (1933/1966) and further elaborated by Lösch (1940/1954) represent a *shift in society*: from one in which the focus was on production (the city was important because convenient, as a trading centre) and the area around the city, to one in which the focus is on goods and services offered at the city, with its *position within a hierarchy of places* depending on the *order of the goods and services* being offered. Christäller inscribed the honeycomb pattern into planning, with the focus being at the *central place* and not the surrounding plain. Lösch further elaborated that different centres could rely on *different goods* and still maintain its hierarchical position. Higher order goods will be found at higher order centres: their *range* is the sum of the areas that compose the network of influence of all the centres underneath it. Without the constraint of *shape*, Berry and Garrison (1958, 1958a) tested for the inferior limit of that good, introducing the concept of *threshold,* with *range and threshold*, along with *population and distance*, still explaining for the greater part of the location patterns of economic activities. Since the 1960's, and though its many limitations have been recognized, Central Place Theory remains subject of discussion (Fujita & Krugman, 1999: 209): "It is demonstrated that as the economy's population size increases gradually, the urban system self-organizes into a highly regular hierarchical system a la Christäller".

But within the centre, what defines the best use of the land? Alonso (1960; 1964) elaborated on Haig's (1927) work and stipulated that the areas that are best located are occupied by the uses that can *bid for the rent* at that specific location. Land is at its highest and best use when that use maximizes its value: a concept originating from the work of Fisher (1930) and still being used by Fujita (Fujita *et al*, 1988) or, for that matter, to the present day. To the agglomeration of retail activity, specifically, *competitive clustering* (Hotelling, 1927; Eaton & Lipsey 1975) may explain for a good part of it, while *multipurpose shopping* (Lösch, 1940; Eaton & Lipsey 1982; Brown, 1993) and *information spillover* (Caplin & Leahy, 1998; Carter & Vandell, 2005) effects all agree on one point: all theories must incorporate the notion that a consumer will most likely make a trip if he's *informed* about what goods he might expect to find, since that trip will have a cost that ads to the price of the good. *Cost of transportation* ads to the overall cost, so Huff (1963) defined that a larger retail area will attract more customers (for they'll expect a more diverse offer). A *dense and diverse* commercial centre is therefore more likely to attract more trips/costumers, and while most of these seminal theories are many decades or even centuries old, they're still at the basis of current research on the field of location: an implementation of Huff's model was published only last July at Harvard (Sevtsuk & Kalvo, 2017), while diverse spatial data is being used to quantify retail agglomeration at UCL's CASA (Batty *et al*, 2017).

On retail location in Lisbon, work by Gaspar (1985) and Barata-Salgueiro (2001) allows for an overview on retail location in this city, but specifically for this article, research by Cachinho and Salgueiro (2016) must be mentioned since the analysis will eventually focus on "*centres of convenience and proximity* (…) "*oriented to the satisfaction of the basic necessities of the population, and therefore with a close connection to the supply of consumer goods of frequent acquisition.*" (Cachinho and Salgueiro (2016: 98).

2 METHODOLOGY

2.1 Case Study

Contributing to research demanded a case study; Lisbon turned out to be a possibility when a database of commercial establishments was made available by the municipality. Even though a great deal is being written about "big data", data collected in a more traditional manner (like door-to-door enquiries) is far from being obsolete. A mix of sources has been used in some very innovative models (Batty *et al*, 2017), but having reliable data at block level is still difficult to obtain, and therefore, relevant for analysis. Data from commercial establishments was available for the 1995-2010 period, as was data about population and housing for the 1991-2011 period, and that amounted to a good starting point. Therefore, population and housing data refers to the censuses of 1991, 2001 and 2011 (INE, 2014), while commercial data pertains to the years of 1995, 2002 and 2010 (CML, 2016).

2.2 Spatial Analysis

A geo-referenced database of establishments containing 14.673 locations in 1995, 16.092 locations in 2002 and 17.035 location in 2010 was used to analyse spatial distribution of establishments. Establishments were compared both globally, disaggregated into "sectors" ("retail" and "restaurant") and into 10 types of activities: Foods (non-prepared: includes groceries and similar establishments, excludes restaurants, cafes, bars and similar establishments); Personal Use Items (mainly clothing and clothing accessories); Household Articles; Health and Hygiene items; Leisure items (sporting goods, bookshops, music stores, etc.); Repairs (all sorts of repairs); Other items (all other items non-included in the remaining types); Restaurants and similar establishments; Cafes and similar establishments; Bars and similar establishments. The first seven types were considered the "retail sector", while the remaining three were considered, globally, the "restaurant sector". This disaggregation was based on available information about economic activities classification (CML/DMEI, 2009), but adapted for the purpose of this article.

2.3 Point Density

Since data with this level of disaggregation has an "intrinsic granularity" with variables presenting "extremely high variances, with a pre-dominance of zero values and hence extremely non-normal frequency distributions" (Thurstain-Goodwin & Unwin, 2000: 309), and considering the final purpose of modelling a regression, using continuous density surfaces presented itself as a way to both go around this problem and

better identify patterns "by smoothing the data to iron out the inevitable discontinuities that take place from data that are originally represented by land parcels and/or the fine scale postal geography" (Batty et al., 2004: 327). Density surfaces were produced for establishments and a set of other variables. In commercial establishments, density represents the number of point features found at a fixed distance (radius) around each cell, per unit area (km^2). For population and housing variables, the value of the variable for each point feature was used for weight.

2.4   Spatial Regression Analysis

Regression analysis was used to test if the location of *one type* of establishment could be related with the presence of other *specific* types. One establishment type was considered the dependent variable, with the remaining used as explanatory variables. Population and Primarily Non-Residential Buildings (with the later posing as a possible source of information for job location) were retrieved from the census databases. These variables account for consumers, and therefore demand – with *demand* being the most important variable explaining the presence of retail (multipurpose shopping, competitive clustering, information spillovers, all rely on *demand externalities*). A classical OLS regression model was built, while testing for spatial dependence, due to the nature of the data. Spatial lag indicates *interdependence between the considered variables*, while error signals for *omitted variables*. Both violate the assumptions of an OLS regression: lag violates the assumption of independent observations, while error violates the assumption of uncorrelated error terms. Diagnostics for spatial dependence and error include using the standard Lagrange Multiplier (LM) and the Robust LM. Since standard LM assumes that the other process (either lag or error) isn't present (which poses an obvious problem) analysis is focused on the Robust LM test.An OLS-type regression is performed, and if lag or error are detected in the diagnostics, a spatial-autoregressive lag model (SAR) or a spatial-autoregressive-error model (SEM) will have to be used.

3   RESULTS

3.1   Spatial Analysis – Point Density

Commercial locations were available as point features. Therefore, point density was used to create *continuous surfaces*, by calculating "a magnitude-per-unit area from point features that falls within a neighbourhood around each cell" (ESRI, 2016). Thus, for commercial establishments, density represents the number of point features found at a fixed distance (radius) around each cell, per unit area (km2). Creating density surfaces also allowed to introduce map algebra into the analysis: by subtracting density raster maps, it is possible to observe where variation occurred, linking data and space.
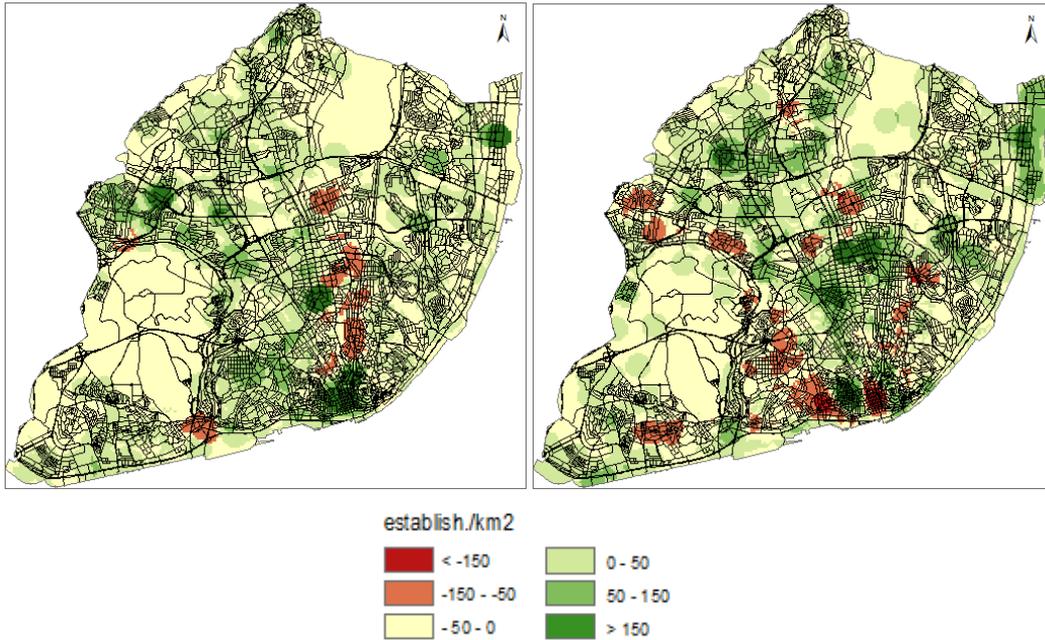
Figure 1 – Total Establishments Density Variation: 1995 to 2002, and 2002 to 2010 (left to right)

Figure 2 presents Total Establishments Density Variation on the analysed periods. The period between 1995 and 2002 can be directly related to a stage in the evolution of shopping malls characterized by the opening of few but (much) larger units when compared with previous decades (Cachinho, 2002). Apart from Bairro Alto-Baixa-Martim Moniz, they amount for all other significant, positive variation that is visible: Colombo can be seen northwest, Vasco da Gama northeast, and close to the centre, in the Saldanha neighbourhood, the opening of Atrium Saldanha, Saldanha Residence and Picoas Plaza is also perceptible. The impact of the shopping malls is evident on Avenida Almirante Reis, Avenida de Roma and Alvalade. This period thus represents (roughly) a decade where commercial activity (both retail and restaurant) was still growing, but malls were presenting themselves as desirable location alternatives.

In the following period, this location effect was extended to other areas traditionally associated with commercial activity, including Baixa, but also Campo de Ourique and Estrada de Benfica. Vasco da Gama became central in a new neighbourhood of Lisbon: Parque das Nações (the former Expo98 grounds). Up north, the residential areas of Telheiras and Lumiar / Alta de Lisboa also revealed some growth dynamics. The city was therefore presented with alternative centres to traditional commercial centres: shopping malls, offering a multitude of shopping opportunities in a controlled environment: an "arranged scenery" destined for consumption (Cachinho, 2006). Vasco da Gama and Colombo, with a vast number of stores, good accessibility and parking possibilities, posed themselves as alternatives to Baixa itself (as stated by Salgueiro (2001)). Focus was then made on Foods, which present a very low percentage of activity inside shopping malls, and might therefore be at the core of *centres of convenience and proximity*. Results are presented on Fig.3.
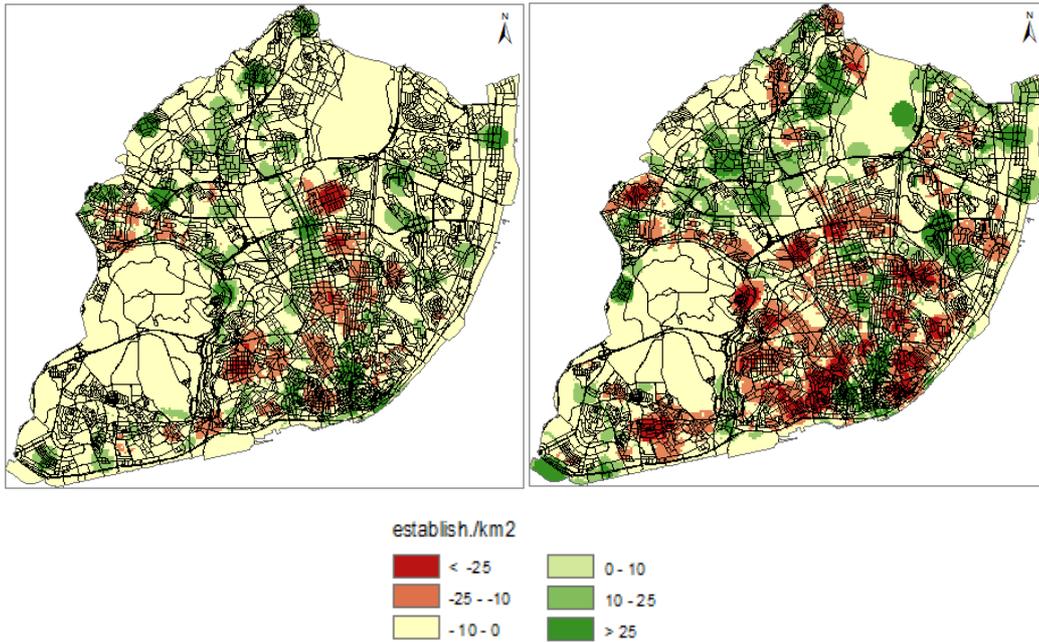
Figure 3 – Foods (as defined) – Density Variation: 1995 to 2002, and 2002 to 2010 (left to right)

Vasco da Gama and Colombo are visible on the first period (the malls in Saldanha don't rely on supermarkets as anchors). Another phenomenon, though, becomes noticeable in the second period: older neighbourhoods of town are losing this type of activity while recent ones are gaining it (with Bairro Alto–Baixa–Martim Moniz appearing as the exceptions, south in green). A preliminary conclusion about Foods can be made: even though, at street level, they suffer from competition from (larger) establishments located in shopping malls, they are needed in residential neighbourhoods, as is noticeable in the ones that are still growing. This would suggest a direct relation between number of establishments and number of residents, and therefore between Foods and Population, dependent of the location of Foods establishments, and not necessarily their sales area. Foods are needed at street level, even if only to a certain number (eventually, the number that is needed to satisfy daily buys, for a certain population located at "reachable distance" – *threshold population*).

3.2 Spatial Regression Analysis

Regression was focused on Foods, considering the analyses that had been made to this point. An OLS-regression model was tested for lag and error, with the results recommending for the use of a spatial error model.

Table 1 – Spatial Error for Foods, 1991-95 and 2010-11

Variable	1991-95			2010-11		
	coefficient	z- statistic	p-value	coefficient	z- statistic	p-value
lambda	0.9802	328.015	0.000	0.9828	353.96	0.000
Constant	-2.3695	-0.533	0.593	-3.9396	-0.948	0.343

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Personal Use Items	-0.0360	-3.336	0.001	-0.0494	-7.173	0.000
Household Articles	0.1687	8.491	0.000	0.0908	5.361	0.000
Health and Hygiene	0.0349	0.834	0.404	-0.0446	-1.419	0.156
Leisure	0.0264	1.197	0.231	0.0846	4.427	0.000
Other Items	0.0313	1.832	0.067	0.1100	9.0842	0.000
Repairs	0.4131	0.034	0.000	0.2355	10.628	0.000
Restaurants (and similar)	0.1637	11.046	0.000	0.0952	9.108	0.000
Cafes (and similar)	0.5585	22.726	0.000	0.3613	19.247	0.000
Bars (and similar)	-0.0423	-0.649	0.516	0.0757	2.475	0.053
Population	0.0013	20.463	0.000	0.0010	15.627	0.000
Primarily Non-Residential Building	-0.0554	-3.969	0.000	-0.0609	-4.030	0.000
% Activity within Malls	-9.8554	-2.257	0.024	-1.8391	-1.449	0.147
Log-likelihood	-10892.379*			-10973.707*		
Akaike info criterion	21810.8*			21973.4*		
Schwarz criterion	21890.5*			22054*		

*The numbers represent values.

Foods maintained a positive and significant relation (p<0.001) with Restaurants (and similar), Cafes (and similar) Repairs and Household Articles when comparing 1991-95 with 2010-11. Leisure also became relevant. As for Personal Use items, one might risk concluding it was a clustered activity at street level in 1995, and has since become clustered at street level and inside shopping malls, maintaining a negative relationship with Foods. Relationship with Other Items is positive and becomes significant: this type was created to include all types that couldn't be included in the other 9 categories. It includes, for example, "Bazares", which in 1995 were just 69 (bout 3% of the total number of establishments) and in 2010 were already 311 (13% of the total number of establishments). "Bazares" are stores selling miscellaneous goods, from clothes to small household articles, usually managed by immigrants, with a large community settled in Intendente-Martim Moniz-Mouraria. It's a street level activity (only 10 out 311 are located inside malls) that almost didn't exist in 1995, but since then, was able to present itself as relevant as street level since. As for relationship with Primarily Non-Residential Buildings, it's significant and negative, in both periods, which implies that Foods are goods whose demand relies on (resident) Population, and not necessarily employment/workers. That is compatible with the nature of the goods being sold. These frequent acquisition goods remained relevant at street level, and not necessarily isolated: while inside higher order clusters they mix with all other types, a small "structure" seems to underlie at lower order clusters: a mix of Foods, Cafes, and Other types, directly related with Population (the "centres of convenience or proximity" (Cachinho & Salgueiro, 2016)).

To validate the results, analysis proceeded with other types that showed significant relationship with Foods. Spatial dependence was tested, and so was model fit, with spatial error models presenting the best results for each individual type.

4   CONCLUSIONS

This article intended to identify retail location patterns in Lisbon in 1995, 2002 and 2010, focusing on cross sectional patterns and their longitudinal changes, and relating them with population and housing data. The analyses did allow for the identification of expected changes (the role attained by shopping malls among higher order clusters), but also for some that weren't quite so obvious (the suggested composition of smaller, "lower order" disperse clusters, and its apparent resilience). A preliminary conclusion is that between 1995 and 2010 there was *a transference of preferred location for retail*, from street to shopping mall, and *a transference of activity at street level,* with the growth in restaurants more than doubling the losses in retail. Retail maintained its relevance, but with a significant part of the commercial activity now occurring at shopping malls. The restaurant sector showed persistent growth, both inside shopping malls and at street level.

The second part was more innovative: an attempt to establish relationships between different types of activities, and between those and demand, via a spatial error regression model, allowed to further elaborate on Lisbon's retail structure composition. Final conclusions were as follows.

1) A higher order cluster of activity is found at Baixa, presenting a *density* of activities that's noticeable when modelling a regression, since Baixa presents itself is a systematic *outlier*. That density would put Colombo and Vasco da Gama at the same level, pertaining commercial centre order, but they don't present the same *diversity* of subtypes of activities. That gives Baixa the highest *range* within centres;
2) Specialized clusters like Almirante Reis and Alvalade create *information spillovers* in which different stores benefit from the presence of a specific type (Household Articles and Personal Use Items); they present themselves immediately located below Baixa in a hierarchy of centres, therefore creating a similar outlier effect when modelling a regression;
3) Smaller clusters, as said, are present in a disperse way and consist of *convenience* goods: Foods, Cafes and Other Items, therefore relying on *proximity* with Population. They are apparently resilient, as demonstrated not only by the regression results, but also by the apparent *population threshold*;
4) Other clusters (e.g. Estrada de Benfica) might rely on a mix of these: some specialization; an interesting externality (e.g. accessibility in Estrada de Benfica); an historic accident (Campo de Ourique). Their composition will probably be site-specific and regression models should take this into consideration.

These considerations are made *for a model that has the restraint of only using demand as a predictor for commercial activity*. A better model can be built considering other variables, especially the ones than can account for commercial clustering: accessibility indicators, space syntax and others, because they *could reduce the effect of the outliers* and clustering in general, thus improving model fit, and therefore, the model (*access to data* will be paramount). At this point, one can risk saying that a *panel data model*, considering *both spatial lag and error*, and *including predictors other than demand*, while considering the conclusions about demand and the relationships established between different commercial types, could produce very satisfactory results. Further developments / implementation of the presented models will be focused on this. The main objective of the article was to gain knowledge about retail activity location within Lisbon. That

objective was, to some point, accomplished. If from a private sector perspective, research on retail location is an investment, from a public perspective, it allows for planning decisions to be taken from a more informed perspective. Retail location can impact on matters as diverse as sustainable mobility, urban regeneration, social interaction or urban logistics. A better understanding on how commercial activity locates can contribute to achieve better planning solutions.

5   POSSIBLE IMPLICATIONS FOR PLANNING PRACTICE

This article created an opportunity for demonstrating the importance of retail related issues when it comes to urban planning practice and urban management. Some of these are presented in this section, and will further be analysed in future studies about retail activity in Lisbon.

1) Baixa possesses a range that explains its resilience, to some extent, considering the diversity of activities present within: only 10 were analysed, but Baixa, especially when considered alongside Chiado, possesses more than 100. There are multipurpose trips that can only be satisfied at Baixa-Chiado – some activity will never be transferred to shopping malls – it's too costly and it possesses a quality of time (in ecology, it'd be "age of the stand") that can't be replicated. On that quality of time and the possibilities it offers for planning: "King of Georgetown" Anthony Lanier seems to be applying that concept to Príncipe Real: by carefully adding high-end retail activities to the ones that already existed and whose pathos can't be replicated inside a shopping mall (antique stores, art galleries, historic bars), Eastbanc is creating a spillover effect that will have housing prices spiralling up, by coordinating an uncoordinated commercial cluster. If the private sector sees this as an investment, the public sector should focus on its importance on the liveability of a city – in this case, it can help in the urban regeneration of several blocks and numerous derelict buildings – the information spillover created by Eastbanc will create a positive effect for other buildings on the area. *Príncipe Real can be a case study for public planning: retail as an anchor for other land-uses. On a downside, this effect might also act as a catalyst, and at the same time a symptom of gentrification; this symptom, though, might also be controlled by public planning if identified.*

2) An interesting attempt has been made at Intendente (City Council services as an anchor), but not for enough time. A "permanent attraction" should be created. At this moment, "A Vida Portuguesa" is the biggest anchor at Intendente. This could be seen as an opportunity for planning: creating partnerships with "anchor stores" or creating attractions that have a permanent spillover effects (if Santos was to be a Design District for years, why wasn't the Museum of Design and Fashion (MUDE) located over there?) or both. It could have a combined effect for producing desirable spillovers. *Combining the potential of retail with that of a museum (like the one Joe Berardo wants to open a stone's throw from Lx Factory) is within the sphere of public planning.*

3) Other pertinent case studies could be found, perhaps, at Almirante Reis and Alvalade: at some point, the joint area of all units located inside those clusters amounted to that of a shopping mall. Considering Huff's model, that would probably explain why Alvalade and Almirante Reis were able to keep part of their trademark activities: they didn't compete store-to-shopping-mall, but cluster-to-shopping mall with Colombo and Vasco da Gama. That could, to some point, explain the resilience of activity at street level, inside specialized clusters, against the effect of shopping malls. And taking into consideration what was

said to this point: if a shopping mall opens with a very disturbing effect on street level commercial activity, why not make them generate a counterpart by creating an underground car park at those areas? It has an impact on traffic congestion: but the effect of people going from city centre to the shopping malls also has an impact. *A balance could be found, within the sphere of public planning.*

4) The fact that the shopping malls in Saldanha have, in recent years, lost most of its activity, reveals the importance of an anchor inside a shopping mall, but also how interesting it'd be to apply a Huff-model to the 3 major shopping mall areas (Colombo, Vasco da Gama and the ones in Saldanha) and analyse if Saldanha ever had any chance at competing with them. Vasco da Gama and Colombo were able to dislocate activity from street level all around Lisbon, but they also depended on the population located north (V. Gama) and west of Lisbon (Colombo). Calibrating a model considering distance/time and population served by those 2 shopping malls (and distance between them), and Saldanha, would provide an idea of the chances of Saldanha in succeeding. If it never had a chance, this should be considered in planning: it could help prevent having semi-empty spaces inside the city. *Competition can't be avoided in a liberal market, but negative effects on the city should be prevented: what will happen if Vasco da Gama (almost 50.000m2 of leasable area) becomes obsolete? This should be taken in consideration by public planners.*

Therefore, further research, and further knowledge about commercial activity location decisions, can be of significant use, and shouldn't be overlooked by the public sector or by academia.

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